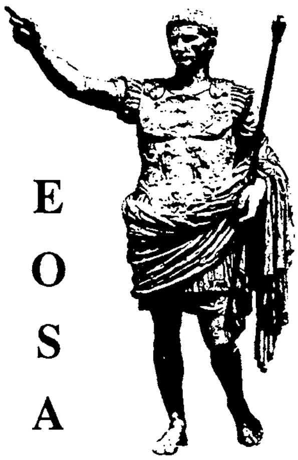
EUROPEAN OPTICAL SOCIETY

ANNUAL MEETINGS DIGEST SERIES: VOLUME 3 ISSN: 1022-0151

EOS

EOS ANNUAL MEETING

1993



E
O
S
A
M MCMXCIII

Zaragoza, 6 - 9 July 1993

Organised by: Universidad de Zaragoza

With the collaboration of: Sociedad Española de Optica

Sponsored by: Comisión Interministerial de Ciencia y Tecnología
Ministerio de Educación y Ciencia
Diputación General de Aragón
Ayuntamiento de Zaragoza
Universidad de Zaragoza
Academia de Ciencias de Zaragoza
iberCaja

EOS Annual Meetings Digest, Vol.3. Zaragoza 1993.

DIFFRACTIONAL ANALYSIS OF HOLOGRAPHIC OPTICAL ELEMENTS RECORDED ON CYLINDRICAL SUBSTRATES

L. Carretero, A. Beléndez (†), R. Fuentes and A. Fimia

Laboratorio de Optica. Departamento Interuniversitario de Optica
Universidad de Alicante. Apdo. 99. Alicante E 03080. SPAIN

(†) Departamento de Ingenieria de Sistemas y Comunicaciones
Universidad de Alicante. Apdo. 99. Alicante E 03080. SPAIN

Abstract

The image quality of holographic optical elements recorded with spherical wavefronts on cylindrical substrates is analyzed using diffraction theory. The computation of point spread functions for aberration-free and angular misadjustment cases is done by using a computer program.

1.- Introduction

The aberrations in holographic optical elements (HOE's) on planar substrates and recorded with point sources have been analyzed in a number of papers [1, 2]. More recently, aberrations for HOE's on substrates of any shape have been investigated [3, 4]. These studies consider both the obtention of the aberration's coefficients and the calculation of ray tracing spot diagrams. However, these methods do not allow for the direct calculation of the intensity distribution on the image plane. For planar HOE's light intensity distribution on the aberration spot has been analyzed in some papers [5].

In this paper, an analysis of the influence of aberrations on the imaging quality of HOE's recorded on cylindrical substrates is carried out by calculating the aberrational diffraction pattern on the gaussian image plane using a computer program.

2.- Theoretical analysis

The analysis of the characteristics of HOE's recorded on cylindrical substrates is somewhat more complex that the planar substrate case. This is due to the fact that we must specify the phase functions of the recording and reconstruction wavefronts on the cylindrical surface of the HOE. The relationship between the ϕ_c phase of the replay wavefront of wavelength λ_c at the HOE and the ϕ_i phase of the image wavefront at the HOE is given by:

$$\phi_i = \phi_c + (\phi_o - \phi_r)$$

where ϕ_o and ϕ_r are the phases of the object and reference waves of wavelength λ_r, respectively. When the desired image phase ϕ_d differs from the actual image phase, ϕ_i, we encounter aberrations, and therefore the wavefront aberration Δ can be calculated as $\Delta = \phi_i - \phi_d$. We will assume that all waves are spherical (including the plane wave, which is considered a spherical wave with infinite curvature radius). Let the phase of a wavefront throughout a certain volume be $\phi_q^*(\xi, \eta, \zeta)$ (the subscript q refers to the particular wave: r, c, i, or d). The phase wavefront ϕ_q evaluated at the surface $\zeta = \zeta(\xi, \eta)$ of the cylindrical substrate is given by $\phi_q(\xi, \eta) = \phi_q^*[\xi, \eta, \zeta = \zeta(\xi, \eta)]$, which we refer to as the surface phase function. Then, the wavefront aberration on the surface of the hologram is:

$$\Delta(\xi, \eta) = \Delta^*[\xi, \eta, \zeta = \zeta(\xi, \eta)]$$

The intensity, I, at any given point (x, y) on the image plane XY is calculated using the formula:

$$I(x, y) = \frac{1}{N}\left| \iint_{\Sigma} f(\xi, \eta) \exp[i\Delta(\xi, \eta; x, y)]\, d\xi d\eta \right|^2$$

where N is a normalized factor that makes I equal to 1 in the centre of the aberration-free image. Σ denotes the surface of the cylindrical hologram, $f(\xi, \eta)$ takes into account any variation in the amplitude on the surface of the hologram, and $\Delta(\xi, \eta; x, y)$ is the wavefront aberration and is a function of points (x, y) on the image plane. In order to evaluate this integral, we consider the numerical method proposed by Chung and Hopkins [6] for conventional optical elements.

3.- Numerical example

As a numerical example we will assume that the cylindrical hologram acts as a lens that transforms an input plane wave (that forms a 30° angle with the ζ axis) into an output convergent wave (with the point source separated 40 cm from the hologram centre and situated on the ζ axis). The recording and the reconstruction wavelength is the same and equal to 633 nm. The radius of the cylindrical substrate is 20 cm and the pupil has dimensions of 4 x 4 cm. We can also assumed that $f(\xi, \eta) = 1$.

Figure 1 shows the aberration-free irradiance distribution on the XY gaussian image plane. In this situation all recording and readout parameters of the HOE are the same. As we can see, this figure is similar to the diffraction pattern of a rectangular aperture. In Figure 2 we have plotted the aberrational diffraction pattern obtained when there is an angular misadjustment of the reconstruction wave of 1° (it forms a 31° angle with the ζ axis). This irradiance distribution is evaluated in the shifted gaussian image plane.

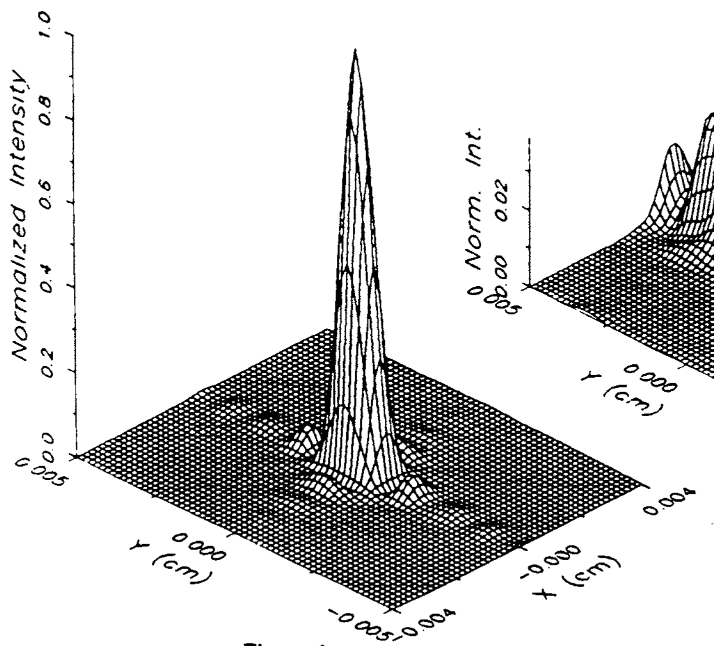


Figure 1

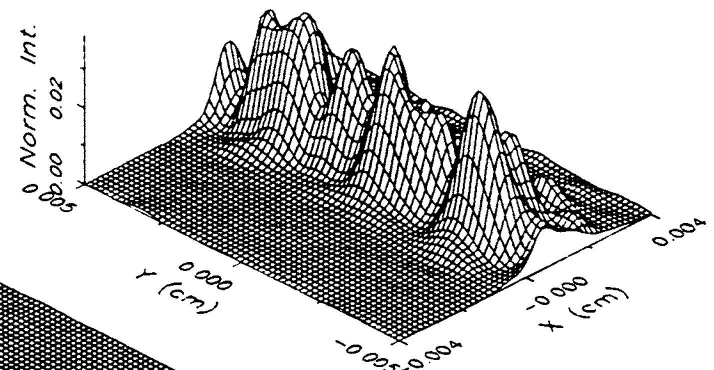


Figure 2

4.- Conclusions

The analysis presented in this paper is a useful method for estimating the imaging quality of HOE's recorded on cylindrical substrates. This evaluation can be completed considering another aspect such as the line of sight of the HOE, the second-, third- and four-order moments of light intensity distribution on the image plane, the fractional encircled energy, the Strehl ratio, the non-uniform amplitudes across the pupil of the HOE or a wavelength shift between recording and readout. These questions will be the subject of subsequent research.

References

[1] E. B. Champagne, *J. Opt. Soc. Am.*, **57**, 51 (1967).
[2] J. N. Latta, *Appl. Opt.*, **10**, 666 (1971).
[3] P. E. Verboven and P. E. Lagasse, *Appl. Opt.*, **25**, 4150 (1986).
[4] J. Masajada and J. Nowak, *Appl. Opt.*, **30**, 1791 (1991).
[5] J. Nowak and M. Zajac, *Optica Acta*, **30**, 1749 (1983).
[6] C. S. Chung and H. H. Hopkins, *J. Modern Opt.*, **35**, 1485 (1988).