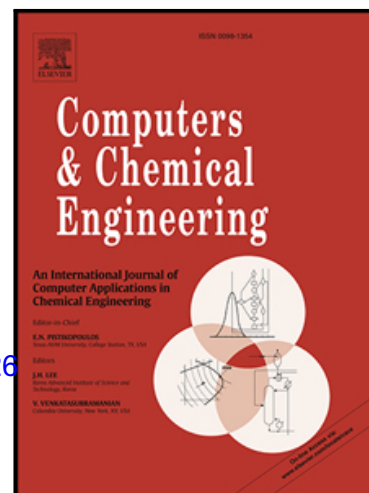


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Highlights

- Novel index based on fuzzy logic to assess the inherent safety of a piece of equipment or plant.
- Disjunctive programming applied to the assessment of the inherent safety of a system.
- Non-linearities can be reduced to the objective function only.
- It can be easily modifiable to the decision maker preferences by weighting the different parameters.

OFISI, a novel optimizable inherent safety index based on fuzzy logic

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Abstract

We develop a novel inherent safety index, OFISI (Optimizable Fuzzy Inherent Safety Index). The index, based on fuzzy logic, assesses the inherent safety of a piece of equipment or process, and ranks different alternatives. Another intention of this index is to provide a systematic methodology to optimize the safety level. The index is divided into three different sub-indices, accounting for different inherent safety design principles, and allows the designer to consider inherent safety at the same level as the other main objectives (economic and environmental) during the early design of a process. The sub-indices are based on logic relationships that can be easily modified. This allows the decision maker to consider various scenarios and assign different levels of importance to the diverse aspects of a plant. Once the index is presented, a case study highlights its usefulness, proving that the safety objective is at the same level as other criteria

Keywords: Fuzzy logic, Inherent Safety, Safety Index, Optimization.

1. Introduction

The consideration of a chemical process from a holistic point of view is one of the major challenges that engineers face during the design stage of a process. There are several details that must be taken into consideration, such as the economic aspect of the plant, the environmental impact, and the safety aspects of both singular pieces of equipment and multiple trains (e.g., distillation trains, reactor trains or heat exchanger network (HEN)). Some works even consider the social repercussion of the proposed plant design. Since there are many ways to weight the relative importance of these criteria, different design solutions are generated.

It is common practice to select the economic profit objective as the unique driving force to guide the optimization process during the conceptual design stage. In some cases, more so over the past few years, the environmental impact has also been added to the set of criteria used to optimize and obtain alternative solutions (Bojarski et al., 2009; Gao & You, 2017; Sabio et al., 2014; Salcedo et al., 2012).

However, neglecting the use of an indicator able to assess the inherent safety level of the plant during the early stages of the process design relegates safety measurements to a later point, once the main variables are fixed by the environmental and economic objectives. Consequently, some possible designs that could have been much safer without incurring a very negative impact on the economic or environmental aspects have already been discarded, as well as the fact that it is important to consider all the aspects in the design early on due to changes being easier and less costly than in later stages. This fact highlights the importance of adding the principles of inherent safety during the early design of a plant / piece of equipment (Ordouei et al., 2014).

The definition of Inherent Safety used in this paper was coined by Kletz (Kletz & Amyotte, 2010), when he stated the four principles of an inherently safer design. These are:

- **Intensification or Minimization**, based on the idea of reducing the inventory of a plant. This is considered one of the most important steps to enhance the safety of process units and chemical plants.
- **Substitution**, based on the idea of exchanging a material with a less hazardous one, whenever this exchange is possible. Studying different possible reaction paths to obtain the same chemical compound is an example of applying this principle.
- **Attenuation or Moderation**, based on the idea of softening the operating conditions of the process. If the use of hazardous materials is unavoidable, we must ensure that their operating conditions are not unnecessarily dangerous.
- **Limitation** of effects, based on the idea of what would happen in the worst case scenario; a process safety incident. We must ensure that if an accident were to happen, it results in the least possible damage. Examples of the execution of this principle include the location of the different pieces of equipment in the plant, the safety distances among different tanks, columns, etc.

There are other principles, such as **simplicity**, which usually are obtained as a byproduct of following these main four. This concept is based on the idea that simple plants have fewer opportunities to fail than complex plants, due to having less equipment, or just simpler equipment. It is considered that in this work, this principle is followed by intensification and substitution, since applying these two usually result in a simpler plant (Kletz & Amyotte, 2010).

The main idea behind these principles is to design the piece of equipment, process or system without considering the different additional layers of protection that will be placed upon it. If the process unit is already safer than another alternative without the need of complex control loops, it is expected to be even safer when those are added. Having this idea in mind, we consider safety a core principle of the design of the plant, weighting it at the same level as the other key design principles.

Normally, safety indices such as Dow's Fire & Explosion Index (AIChE, 1994) or Safety Weighted Hazard Index (F. I. Khan et al., 2001) are evaluated once the early design is set. As said above, this early design

normally takes as main objective the economic one and, in some cases, it considers the environmental impact as well. Therefore, the main variables, such as operating temperature and pressure, reaction route and process units to include, among others, are already decided before the safety analysis has started.

Safety analyses are commonly performed with techniques such as the Hazard and Operability (HAZOP) method, where a committee of experts, with the aid of Safety Indices (SI), usually Dow's Fire & Explosion Index (Dow F&EI), discuss what and where the weak points of a plant are. Plenty of keywords are thrown in these meetings, such as "moderate hazard" and "small hazard", which cannot be immediately quantified. Therefore, the use of fuzzy logic to assemble spoken language with mathematical rules is an attractive idea. Previous works have already focused on these subjects (Gentile et al., 2003; Howard & Seraji, 2002; Olinderson et al., 2017), proving that with the use of colloquial language we can establish the inherent safety of a piece of equipment.

Fuzzy logic is set in the theory of the possibility (Zadeh, 1999). This approach differs with the probabilistic approach, more used in statistics, in that we are not that concerned about the measure of the information, but its meaning. Language elements, for example the term "high hazard", may not mean the same to different people, but everyone agrees that it is something that the design must avoid. This does not mean that the probabilistic approach fails when assessing the safety of equipment. It has been thoroughly proved that it is a powerful tool to perform a more accurate safety analysis of an existent plant by considering all the possible risks that are present in a design. However, when dealing with early design, the information is not completely available nor measurable, which makes obtaining a probabilistic statement of the safety of the plant very difficult.

The field of fuzzy logic is based upon the concept of membership to a set. In Boolean logic, there are only two possible outcomes. Let $x \in X, y \in Y$, where X, Y are two sets whose $X \cup Y \neq \emptyset$. If we define a Boolean variable that relates the membership of each value to a set (Z), since partial membership is not allowed, it will only provide outputs in the form of $\{0,1\}$. Therefore, x may be a member of the set $X \cup Y$, but we do not know in which grade, only if it belongs ($Z = 1$) or if it does not belong ($Z = 0$). The same thing happens with y . Fuzzy logic allows the concept of partial memberships, which defines if a value belongs to only one set (membership to a set equals 1 and all the others are 0), partially belongs to one or more sets (membership between 0 and 1) or if it does not belong to a set at all (membership equals 0). A typical example is that of the environmental temperature. Let μ_A be the membership function value to a fuzzy set A . We can define three intuitive fuzzy sets for the environment temperature. If we take a crisp input, which is a value that is not fuzzy, for example, a temperature of 28 °C, we can calculate its membership to each fuzzy set, as shown in Figure 1.

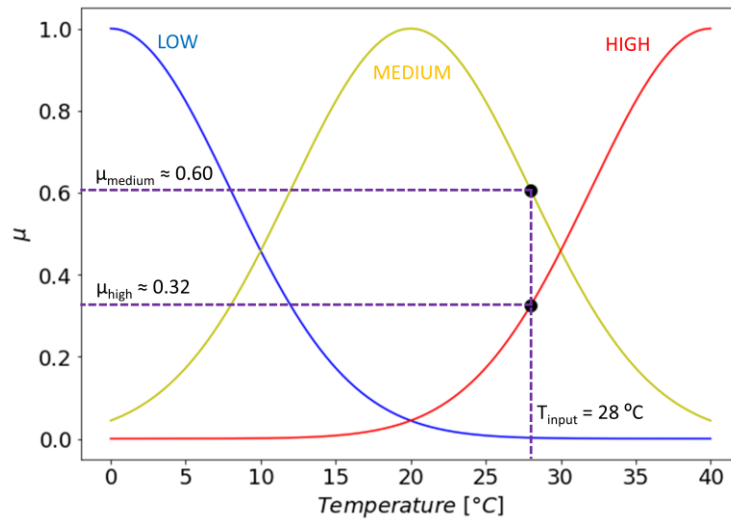


Figure 1. Environmental temperature ranges represented as fuzzy sets

In the figure, it can be seen that there are two membership functions, each one referring to different fuzzy sets. The input temperature belongs both to the “medium temperature” fuzzy set and to the “high temperature” fuzzy set with a different membership value, rather than 1 for both, which is what we would have obtained using Boolean logic.

The basic operations of a fuzzy set are extensions of those in normal sets (Zadeh, 1965), as shown in Figure 2. We can intersect fuzzy sets and obtain the union of two sets in the same manner, as well as having sets included in others, or sets that are the complement of others.

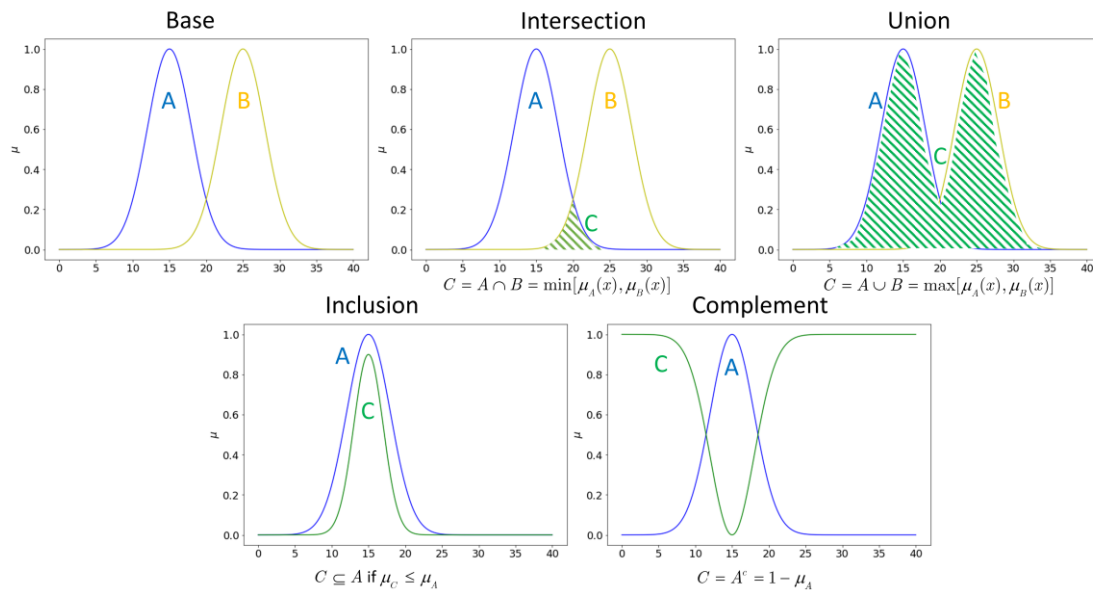


Figure 2. Basic operations of fuzzy sets

The usefulness of fuzzy logic is that, by inference, it allows us to define cases and relations between process variables and a dimensionless index without the need of using non-linear or non-convex expressions (or at least minimizing its use). This allows the engineer to optimize different designs systematically with the aid of Mixed-Integer Linear Programming (MILP) and Mixed-Integer Non-Linear Programming (MINLP) formulations. This results in a combination of Boolean logic and Fuzzy logic able to model a continuous and optimizable system.

For the inference, we rely on Mamdani & Assilian's inference method (Mamdani & Assilian, 1975), which comprises the following steps:

- Input fuzzification. The inputs are taken as crisp values and they are fuzzified with the corresponding membership functions.
- Establishing the relationships among the different inputs and outputs and implication. This is achieved by using the same operators as in Boolean logic, being these the operator "OR (\vee)" and the operator "AND (\wedge)". The operator "OR", in the scope of this work, can be seen as a maximum, while the operator "AND" can be seen as a minimum.
- Aggregating the conclusions. Once the relationships are established, we obtain a number of outputs that must be aggregated. There exist different fuzzy aggregation operators, such as the maximum, the sum and the probabilistic sum. In this article, we show the methodology using the maximum operator. This is known as max-min composition.
- Defuzzification. Once we have an aggregated fuzzy set of outputs, we want to obtain a crisp value that states the safety of the system. In the same manner as the aggregation step, there are different methods to perform the fuzzification. In this work, we use the centroid of the resultant polygon as the defuzzification tool.

This is not the only method used for fuzzy reasoning, however, it is the most common one, since it is based on a simple structure of max and min operations (Gentile et al., 2003), which are appropriate to convert into a mixed integer programming model.

A problem present in the measure of the inherent safety level of a design is that an important amount of safety indices, even those based on fuzzy logic, are calculated with non-linear or non-convex correlations. This is essential when we work with very different factors, namely temperature, molar or mass flows, pressure, etc. and the objective is to obtain a single number that characterizes the safety level. However, there are multiple works in the area despite this shortcoming. Jafari et al (2018) performed a review of the different approaches and indicators that are being used to determine the inherent safety level, where it can be seen that there are multitude of possible approaches, such as relative rankings statistical methods, numerical descriptive method and many others. Another important review of safety indices was published by Roy et al. (2016). While this one is not focused directly on inherent safety, some indices can be used in order to assess the inherent safety level at an early stage in development (Vázquez, Ruiz-Femenia, et al., 2018). Some examples include the Hazard Identification and Ranking (HIRA) index, by Khan and Abbassi (Faisal I. Khan & Abbasi, 1998), which then influenced

other indices, such as the I2SI (Faisal I. Khan & Amyotte, 2004, 2005), the Quantitative Index of Inherently Safer Design (QI2SD) (Rusli et al., 2013) and the Inherent Safety and Key Performance Indicators (IS-KPIs) (Tugnoli et al., 2009) among others. The first index for inherent safety, ISI (Heikkilä, 2000), which was partially based on the PIIS index (Edwards & Lawrence, 1993), aimed to quantify the factors that define the four principles of inherent safety. This index was further developed into the Comprehensive Inherent Safety Index (Gangadharan et al., 2013).

Indices based on fuzzy logic, such as the work from Gentile et al. (2003), included the possibility of converting linguistic variables into fuzzy rankings. A safety index based on fuzzy logic was developed by Olindersson et al. (2017) to enhance the safety of ship traffic. Other index based on fuzzy logic was used to assess the safety of the landing site risk (Howard & Seraji, 2002).

All these indices succeed in assessing both the safety and the inherent safety of a series of process units, with results backed by its use in actual chemical plants. However, these indices are developed mainly from the point of view of safety, not from the point of view of mathematical optimization. Numerous correlations are non-convex, which in turn complicates the optimization. As stated above, the main source of these non-convex equations is the desire of grouping different variables with different units in a single output. This works admirably well when studying an already built unit, or a group of different, albeit small amount of cases. However, when trying to systematically consider the safety as a part of an optimization problem, all these nonconvexities and nonlinearities make it hard to obtain a solution.

In this work, we propose a different approach to this lumping, considering each of the first three main principles of inherent safety as separated mostly linear inherent safety objectives inside the main index. By doing this, the results show a dimensionless intensification index, a dimensionless substitution index and a dimensionless attenuation index. The reason for not considering a dimensionless limitation index is that the variable that would be the most adequate to be fuzzified as input is the distance among process units. The indices are considered to be calculated at such an early level in design that we do not have the number of process units nor if all the units will be in the final design or not, so trying to model the distance among possibilities adds significant complexity.

All the indices shown in this article are designed with the sole purpose of obtaining comparative results. While they could be modified in order to provide a global index, the basic idea is to obtain different indices with systematic models, which have the minimum possible amount of non-linearities and are systematically optimizable. In order to deduct these indices, we studied the most used and novel safety indices, such as Dow's F&EI and Safety Weighted Hazard Index (SWeHI), among the others mentioned above, in order to collect the most repeated variables used to assess the safety of a process unit. The main idea was to replicate what a safety meeting is about, key-wording different variables of the process and assigning a safety measure, and provide a more mathematical approach based on fuzzy logic for those main variables, while keeping it the most simple and early-design friendly possible, as well as optimizable.

Accordingly, the major contributions of this paper are:

- We acknowledge the need of considering safety as one of the main driving forces in the design of a piece of equipment / plant, rather than just an additional constraint that must be satisfied once the main design is decided by other criteria, such as economic or environmental.
- We propose an index, OFISI (Optimizable Fuzzy Inherent Safety Index), formed by three different dimensionless sub-indices, which presents a high degree of freedom in its definition. This allows the user to tackle very different systems with only a small change in the parameters. These indices are based on the most basic variables that can be taken into account at a very early design of the equipment or plant.
- We avoid most of the non-linearities that other indices incur due to the need of using different variables, by grouping these variables into different indices and inferencing these with fuzzy logic relationships. This allows the indices to be included in an optimization model which is then solved to obtain the optimal design choices.

The remainder of this article is organized as follows. First, the intensification index is introduced, as well as a step-by-step guide of the Mamdani & Assilian's method. In the next section, the substitution index is presented, and a small example is given. In the following part, the attenuation index is developed and three examples showing its adequacy are presented. The next section introduces the model for the optimization of the intensification index, which shares a very similar structure with the rest of the optimization models. Finally, we draw some conclusions.

2. Intensification index

As stated in the introduction section, intensification is the act of reducing the inventory of a plant or piece of equipment. The inventory can be strictly defined as the amount of chemical that the process unit / plant handles over a defined period of time. It is obvious that the higher the amount of chemical the piece of equipment has, the higher the consequence of a loss of containment event. However, in this work, we consider that this measurement alone is not enough to provide an interesting and reliable intensification index.

We define another variable alongside the inventory, which takes into account the energy holdup contained in the chemicals. This allows the user to introduce different relationships between these two inputs, which results in greater differentiation among alternatives of the process. Since the relationships are established with logic operators, the model does not need any non-linear transformation.

The intensification index is broken down into two sub-indices, which are shown in Eq.(1).

$$II = (II_{inventory} \wedge II_{energy}) \quad (1)$$

Where:

- II stands for Intensification index.

- $II_{inventory}$ stands for the sub-index that quantifies the inventory to treat in the equipment.
- II_{energy} stands for the energy sub-index, which quantifies the amount of energy in the equipment, both from the chemicals inside it and from the possible reactions that may occur inside the process unit. As it is expected, endothermic reactions are considered safer than exothermic reactions.

In fuzzy logic, we must relate keywords to intervals in order to fuzzify the input value. The output of the Intensification Index, from Mamdani & Assilian's method, is provided in a fuzzified form, which then has to be defuzzified. Therefore, the fuzzy sets of the output must be defined. Their structure is shown in Table 1, where \sim means "approximated", since it is a Gaussian function.

Table 1. Fuzzy distribution of the output intensification index

Level of risk	Value of the index	Fuzzification
Minimal Risk	0 – ~ 10	Gaussian
Low Risk	4 – 15	Triangular
Medium Risk	15-35	Triangular
Moderate Risk	35-45	Triangular
High Risk	~ 40 -50	Gaussian

These values are plotted in Figure 3 in order to provide a visual representation of the fuzzy sets.

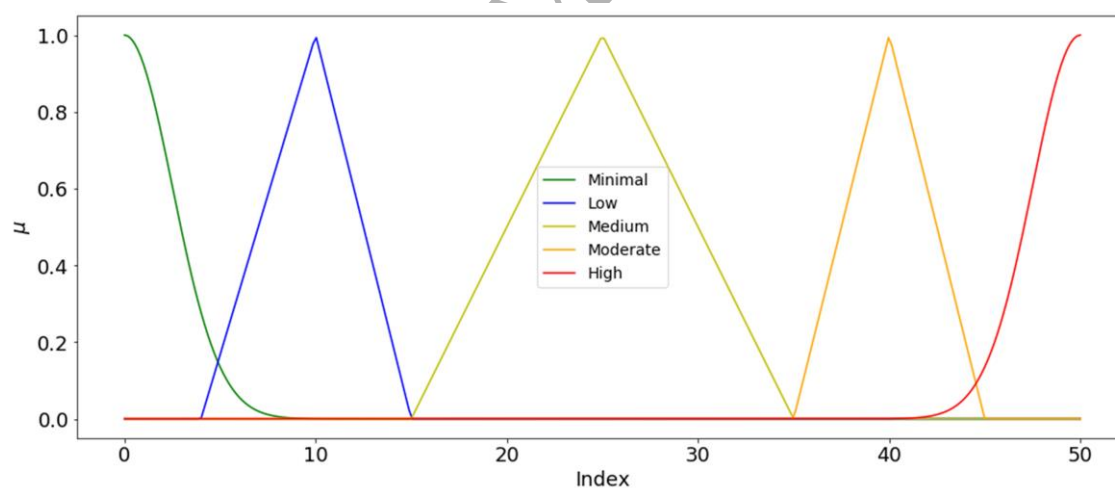


Figure 3. Fuzzified intensification index

The values in the extremes of intervals, such as 15 and 35, can be taken as any of the two sets, since this is the output and not the input. The input must be clearly defined and not have coincident intervals. In order to fuzzify the input values, we must establish some bounds for the main variables. We define three levels of input for both the main variables $II_{inventory}$ and II_{energy} . These levels, as well as their bounds and types of fuzzification functions are shown in Table 2.

Table 2. Fuzzification of the inputs for the intensification index

Semantic pointer	Range	Fuzzification
Low (A)	0 – 60 % of the interval	Gaussian
Medium (B)	5 – 95 % of the interval	Triangular
High (C)	40 – 100 % of the interval	Gaussian

As an important note, these values and ranges are not established as the norm by the authors. It is the duty of the current safety expert of the plant to define these intervals correctly, as well as the type of fuzzification distribution. In this paper, in order to illustrate the methodology, we choose these intervals.

The two main variables of the index are calculated as shown in Eq.(2) and Eq.(3).

$$II_{inventory} = FE \times V \times \rho + \dot{m}_{t=10min} \quad (2)$$

$$II_{energy} = \sum_{i=1}^C \Delta Hc_i x_i - \Delta Hr \quad (3)$$

Where:

- ΔHc_i is the combustion enthalpy of the chemical i of C components in the equipment, in kJ/kg.
- ΔHr is the reaction enthalpy in the equipment, in the case that there exists any reaction taking place, in kJ/kg. Endothermic reactions are considered positive, so they diminish the value of the index. Exothermic reactions are more hazardous, so they increment the value of the index. If there are multiple reactions, the summation of their enthalpies is considered as the reaction enthalpy in the equipment.
- FE is the equipment factor. In the case of reactors and tanks, it can be taken as 0.8. In the case of columns, 0.3 is a possible conservative value (Vázquez, Ruiz-Femenia, et al., 2018). Of course, these two values are examples, since it is normal for the process units to not be 100 % full. If you are using the reactant volume directly, omit this factor.
- V is the volume of the process unit, in m^3 .
- ρ is the density of the chemical inside the process unit, in kg/m^3 .
- x_i is the mass fraction of the chemical $i \in C$ inside the process unit. If these data are not available, the user can assume the mass fraction of the exit stream as this parameter.
- $\dot{m}_{t=10min}$ is the mass net outflow of the process unit in an interval of ten minutes, in kg. The interval of 10 minutes is taken due to the fact that indices, such as Dow's F&EI, consider that as the studied interval in case of a leak or spill.

As such, the main parameters are the different enthalpies, $\Delta H_c, \Delta H_r$, and the amount of chemical stored in the piece of equipment. The bounds of this fuzzification must be carefully chosen attending to the system's characteristics. If the interval is too large, all the results provided by the index will be practically equal. If the chosen interval is too small, the process variables might get out of bounds. The fuzzified sets defined in Table 2 are plotted in Figure 4 and Figure 5.

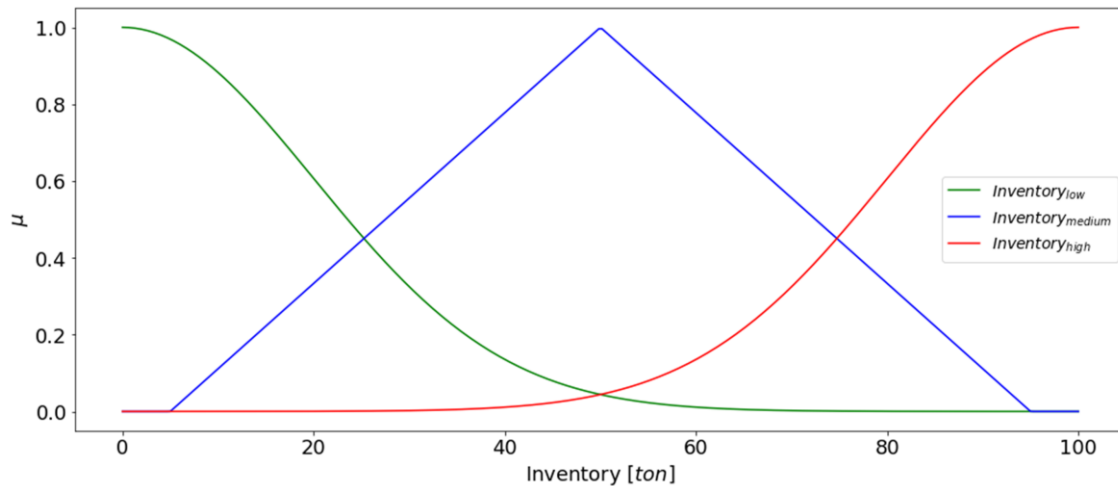


Figure 4. Fuzzified inventory sets

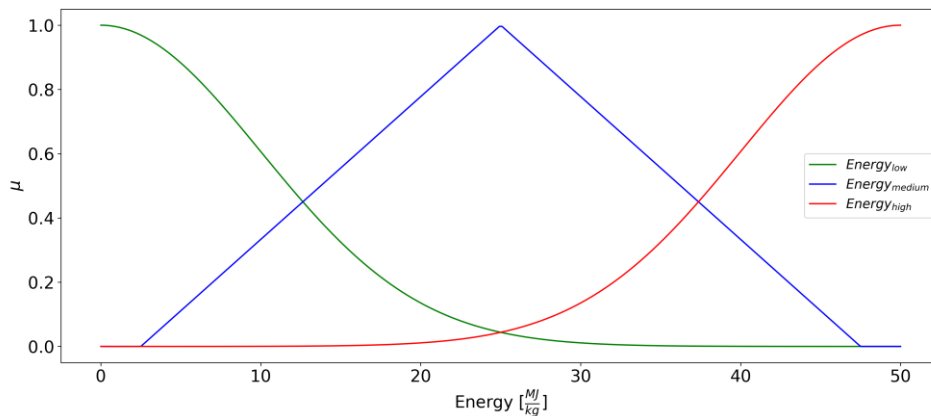


Figure 5. Fuzzified energy sets

At this point, we have both inputs and outputs fuzzified. For the next step we introduce the logical relationships that act as a logical nexus among them. One of the major advantages of this index is the possibility of modifying the logical relationships depending on the studied plant. This allows us to introduce versatility, allowing the user to define, for example, which of both sub-indices is more critical to the safety of the plant and how they interact with each other. In this paper, as an example, we consider the relationships shown in Table 3. For this case, we give the same importance to the inventory

and energy sub-indices. Therefore, a relationship of $(A \wedge B)$ must give the same output as $(B \wedge A)$. Again, it is the duty of the safety expert to design these relationships. The only direction that can be given, is that they must be consequent. If $(A \wedge A)$ gives a minimal risk, then $(B \wedge B)$ must give a higher risk, since B is a riskier state than A , due to the fact that there is more energy and inventory.

Table 3. Relationships among inputs and output for the intensification index

Level of risk	Implications (Inventory – Energy)
Minimal	$(A \wedge A)$
Low	$(A \wedge B) \vee (B \wedge A)$
Medium	$(B \wedge B) \vee (A \wedge C) \vee (C \wedge A)$
Moderate	$(C \wedge B) \vee (B \wedge C)$
High	$(C \wedge C)$

We now illustrate Mamdani & Assilian's method step by step with an example. We consider a piece of equipment with certain characteristics, which holds a certain chemical. There is neither a chemical reaction occurring in the interior of the unit nor any mixture of different chemicals. The unit has a void fraction of 20 % of its volume, which means that the equipment factor can be taken as 0.8. The chemical inside is considered as moderately dangerous, since the amount of energy holdup that it stores is quite important. All the needed properties of this equipment and chemical are shown in Table 4. Notice that this table includes what we consider as the upper bounds for both sub-indices. The lower bound is considered as 0.

Table 4. Properties of the equipment and the chemical inside

Property	Value
Enthalpy of combustion $\left[\frac{kJ}{kg} \right]$	30000
Enthalpy of reaction $\left[\frac{kJ}{kg} \right]$	0
Mass fraction of chemical	1
Equipment factor $[FE]$	0.8
Volume of the equipment $[m^3]$	35
Density of the chemical $\left[\frac{kg}{m^3} \right]$	1000
Outlet mass flow $\left[\frac{kg}{h} \right]$	200
Upper bound for the inventory sub-index $[kg]$	100000

$$\text{Upper bound for the energy sub-index } \left[\frac{kJ}{kg} \right] \quad 50000$$

2.1 Minimal risk relationships

Looking at Table 3, we see that the only relationship that produces a minimal level of risk is that of both the inventory and energy having a membership function with their respective low fuzzy sets membership value different to 0. Mamdani & Assilian's method relates the operator implication by clipping the minimum value among the inputs. In this case, this operation is shown in Figure 6.

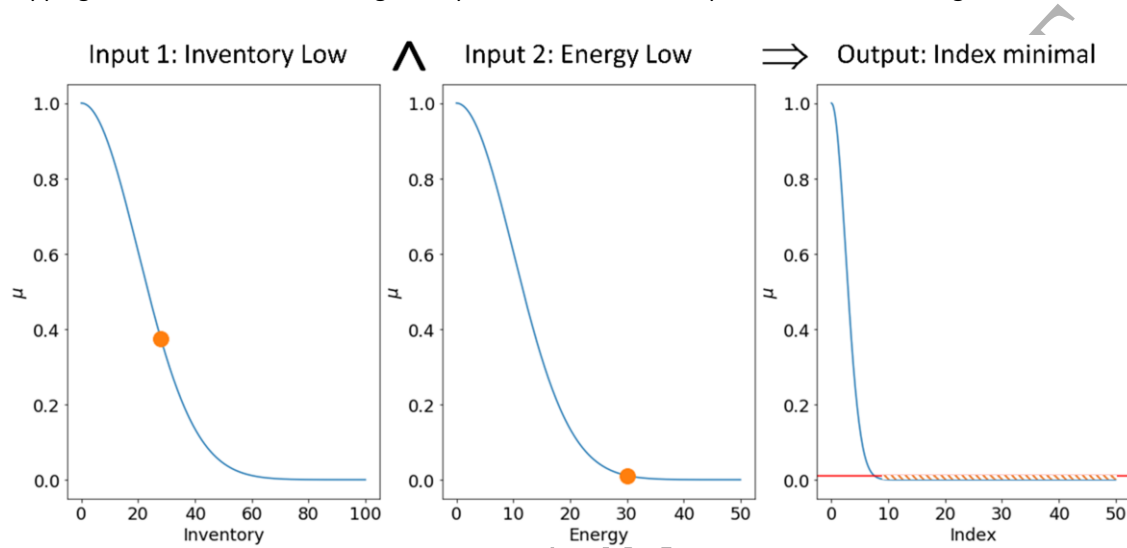


Figure 6. Minimal relationship: inventory low, energy low

As a result, we obtain a value of the output membership function in the minimal risk fuzzy set of the index, around 0.003.

2.2 Low risk relationships

We have two relationships that result in a low risk. These are: 1) if the inventory is high and the energy is low, and 2) if the energy is high and the inventory is low. As in the previous case, the implication operator can be seen as a clipping of the value obtained by the logical "AND" over the output fuzzified set. In order to see this, we plot the relationships and their implications in Figure 7.

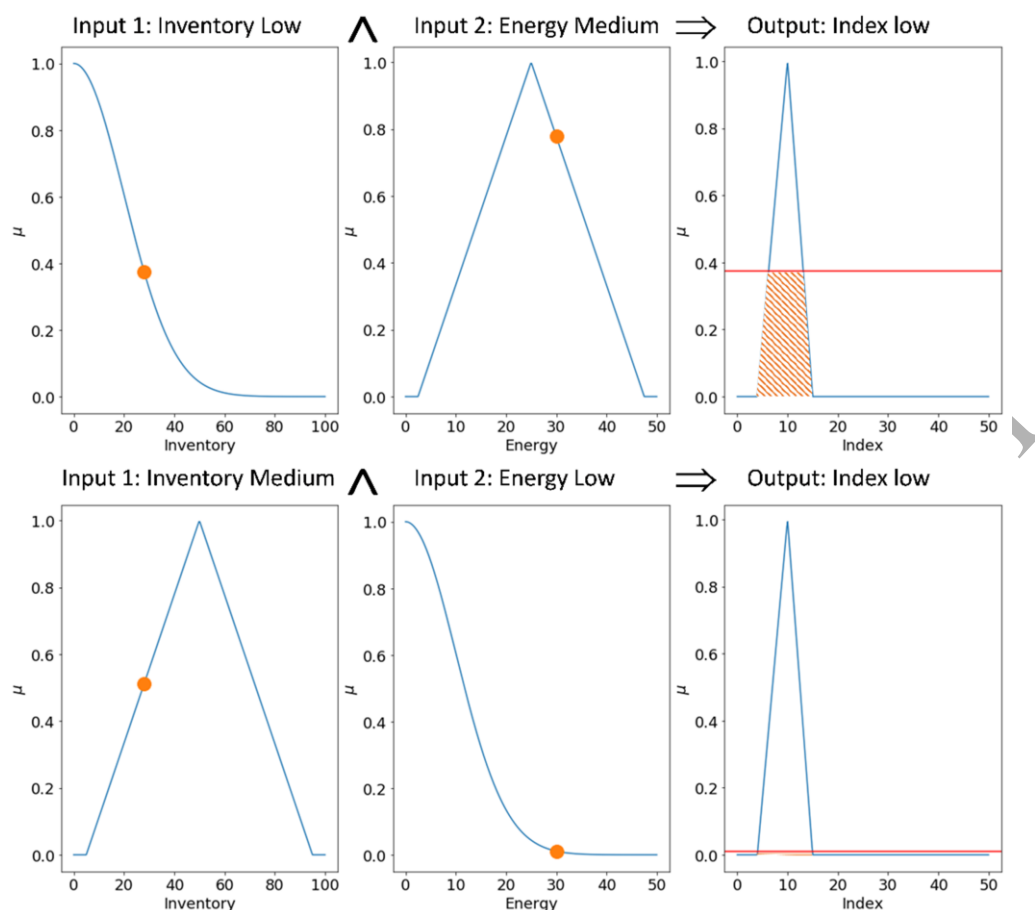


Figure 7. Low relationships: one high and the other low

2.3 Medium risk relationships

Here we have the highest amount of relationships, with three possible combinations that result in an output of medium risk. These being: 1) inventory and energy medium, 2) inventory high and energy low, and 3) inventory low and energy high. These are plotted in Figure 8.

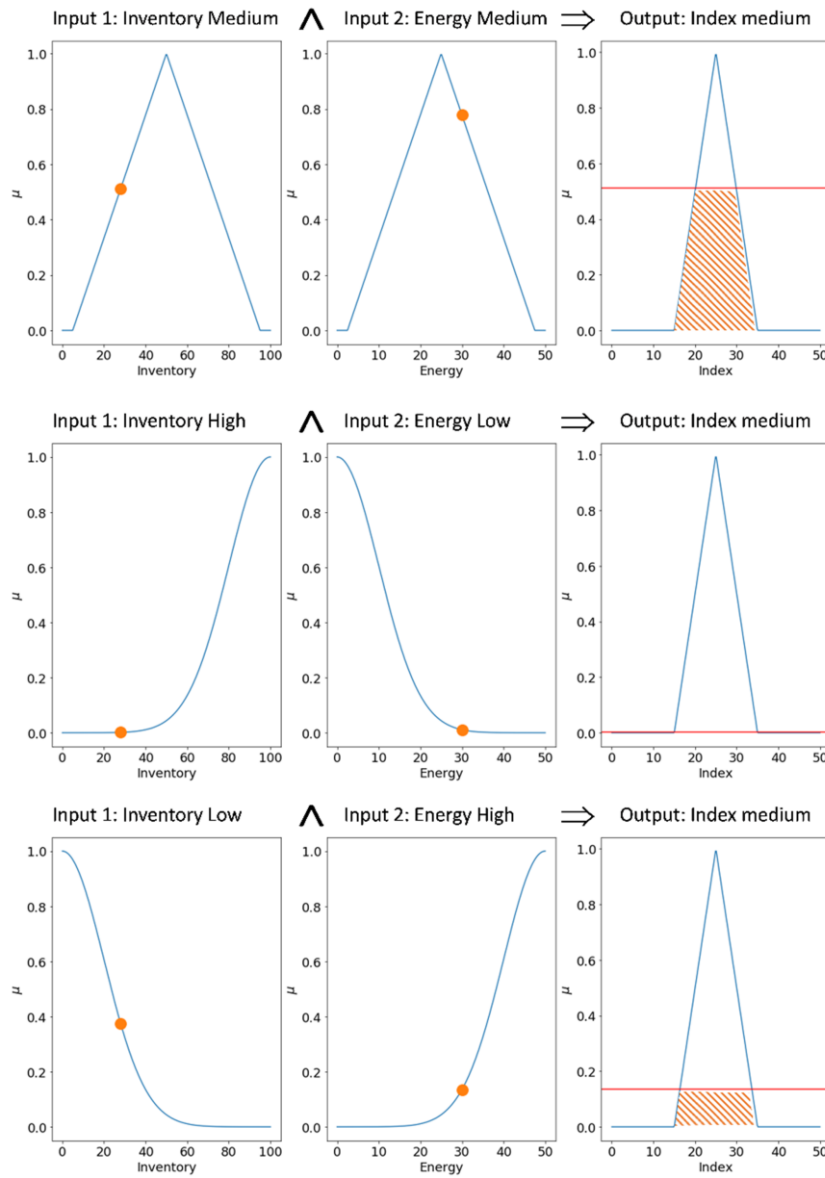


Figure 8. Medium relationships: two medium, one high and one low

2.4 Moderate risk relationships

Only two relationships result in a moderate risk index. These are: 1) inventory high and energy medium, and 2) inventory medium and energy high. These are plotted in Figure 9.

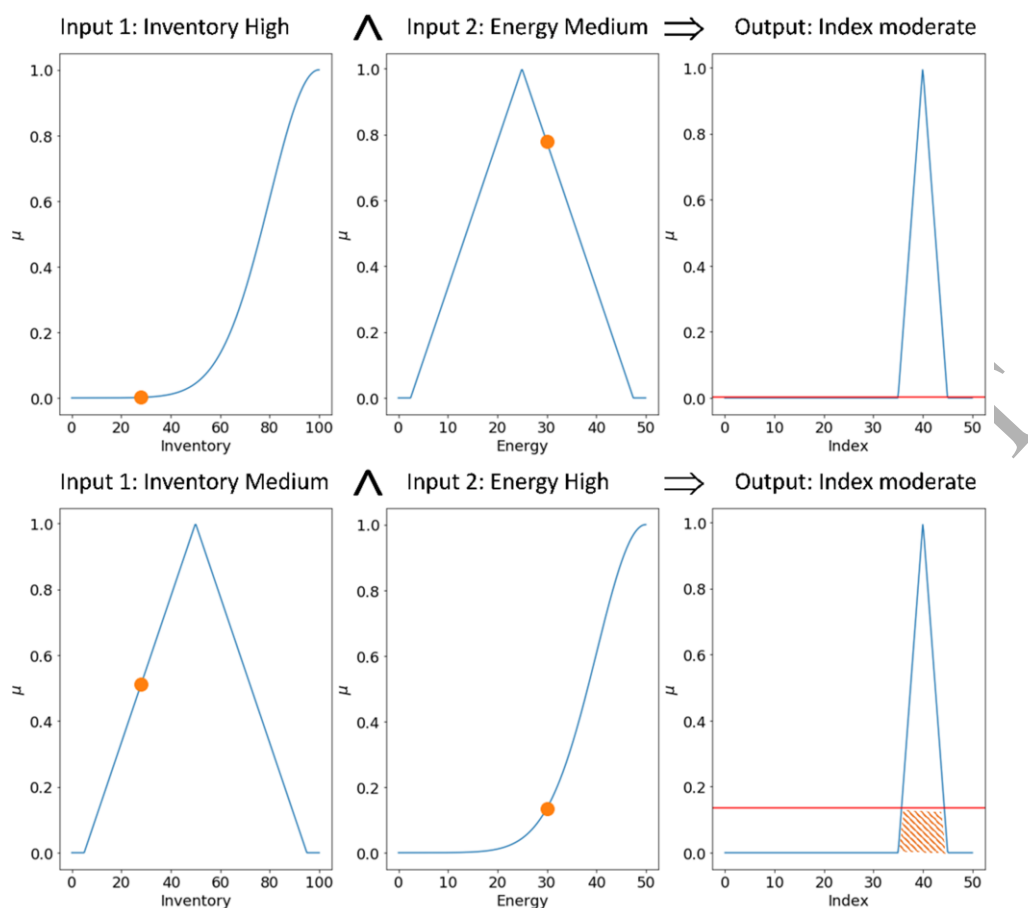


Figure 9. Moderate relationships: one high and the other one medium

2.5 High risk relationships

Only one relationship is cataloged as high risk, this being, if both the inventory and energy levels are high. This is plotted in Figure 10.

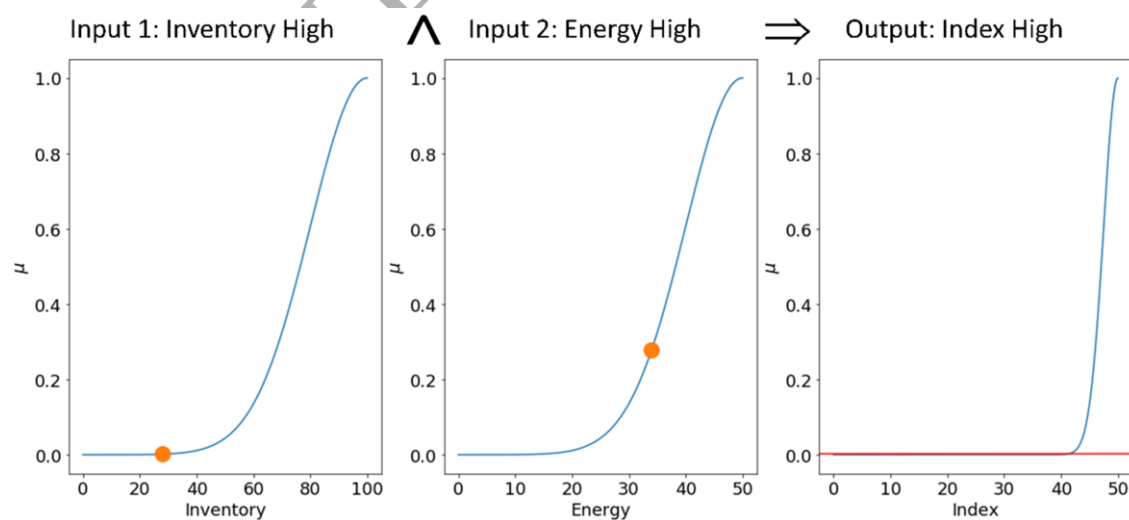


Figure 10. High relationship: both high

2.6 Aggregation and defuzzification

Following Mamdani & Assilian's method, the aggregation of all the outputs obtained from the different relationships is performed by using a maximum operator. The aggregated output is shown in Figure 11.

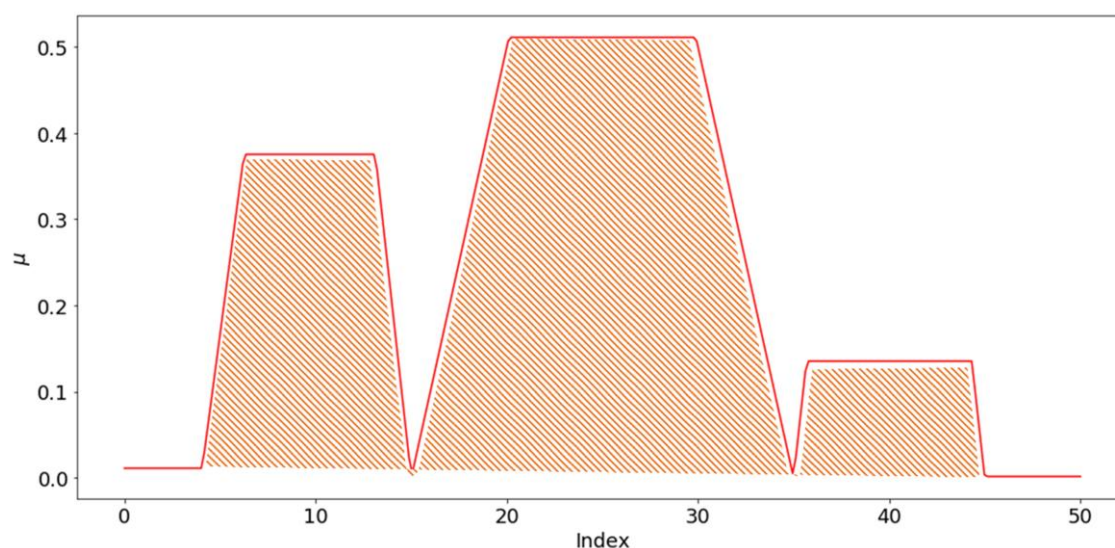


Figure 11. Aggregated Intensification index

Defuzzification can be achieved by different methods. The most direct one is using the concept of the centroid of the remaining polygon. This can be calculated as shown in Eq.(4).

$$\text{Defuzzified Index} = \frac{\int \mu \cdot \text{Index} \cdot d\text{Index}}{\int \mu \cdot d\text{Index}} \quad (4)$$

Using the centroid, the index has a value of 22.26. It is worth remarking that this is not a global index, but one to compare alternatives. As such, the chosen limits for the inventory and energy are very important. Choosing a wide range of values will result in small to no differences among alternatives, while choosing a too small range will result in some alternatives staying out of bounds, and thus not calculating its safety index correctly.

3. Substitution index

The idea behind the substitution principle is to substitute the chemicals that are hazardous, either due to their toxicity and risk to the human health or due to their explosion and flammability characteristics (or even both). These two factors of the chemicals are taken into account considering the numbers assigned to them by the National Fire Protection Association (NFPA) (2012), as shown in Eq.(5).

$$SI = (SI_{NH} \wedge SI_{NF}) \quad (5)$$

Where SI_{NH} represents the part of the index that depends on the health number from the NFPA-704 norm and SI_{NF} represents the part of the index that depends on the flammability number from the NFPA-704 norm.

Contrary to the intensification index, the limits here are clearly established. These numbers rank from 0 to 4, being 0 the most innocuous one and 4 the most dangerous. These will be modeled as in the previous case and fuzzified with a Gaussian distribution on the extremes and triangular on the middle.

For the cases where the equipment has a mixture of compounds, a simple arrangement is to weight each contribution according to the mass fraction of each compound, as shown in Eq.(6)

$$\begin{aligned} SI_{NH} &= \sum_{i=1}^C x_i NH_i \\ SI_{NF} &= \sum_{i=1}^C x_i NF_i \end{aligned} \quad (6)$$

Where x_i, NH_i, NF_i stand for the mass fraction and respective numbers of the chemical $i \in C$, where C is the set of components in the unit.

The fuzzification of both the NH and NF is performed in the same manner as it was in the intensification index. It is shown in Figure 12.

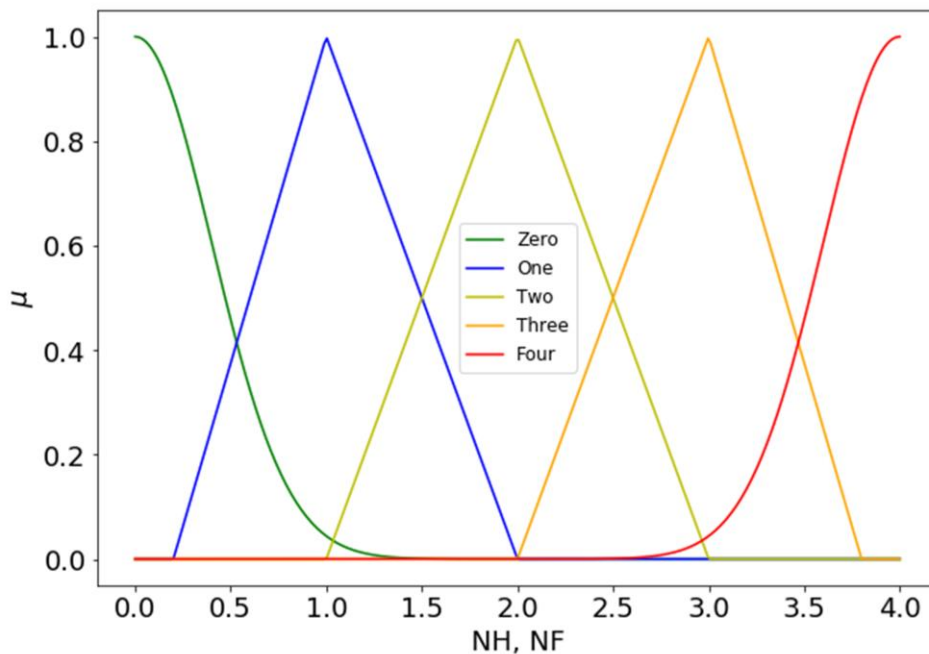


Figure 12. Fuzzy sets for NH, NF

In this example, we work under the consideration that both the risk for the human health and the risk due to the flammability are equally important. In order to diminish the number of fuzzy relationships, we first classify these two parameters between a maximum (N^{\max}) and a minimum (N^{\min}). Again, this

methodology of giving one more importance than the other is not the only option. Every situation may require a different approach to the indices. Depending on the kind of equipment or plant, it may be recommended to consider more dangerous the flammability number than the health number, or vice versa. Here, we only show one of these possibilities.

Therefore, the number of logical relationships is equal to the number of combinations with repetition of a multiset of size 2 from the set $S := \{0, 1, 2, 3, 4\}$, i.e., a sequence of 2 not necessarily distinct elements of the set S , where order is not taken into account. The number of such 2-combination with repetition is denoted by $\left(\binom{5}{2}\right)$, and can be computed in terms of binomial coefficients using the following expression:

$$\left(\binom{n}{k}\right) = \binom{n+k-1}{k} \rightarrow \left(\binom{5}{2}\right) = \binom{5+2-1}{2} = \frac{6!}{2!4!} = 15$$

This result can be verified by listing all the 2-combination with repetition of the set S , as shown in Table 5.

Table 5. Relationships among the inputs and output for the substitution index

Level of risk	Implications ($N^{\max} - N^{\min}$)
Minimal	$(1 \wedge 0) \vee (0 \wedge 0)$
Low	$(1 \wedge 1) \vee (4 \wedge 0) \vee (3 \wedge 0) \vee (2 \wedge 1) \vee (2 \wedge 0)$
Medium	$(2 \wedge 2) \vee (4 \wedge 1) \vee (3 \wedge 2) \vee (3 \wedge 1)$
Moderate	$(3 \wedge 3) \vee (4 \wedge 2)$
High	$(4 \wedge 4) \vee (4 \wedge 3)$

Since the development of the index is identical to the development of the intensification index, it is not worth repeating it. Mamdani & Assilian's method methodology does not change either, and therefore we only provide the result for the following example case.

We consider a unit with three different chemicals inside. The properties needed to calculate this index are shown in Table 6.

Table 6. Properties of the equipment and chemical inside

Property	Value
$ C $	3
x_i	[0.4, 0.4, 0.2]
NH_i	[4, 4, 0]
NF_i	[3, 3, 0]

Where C is again the set of components and $|C|$ is the cardinality of the set, this being, the number of elements in it. After following the methodology, we obtain the aggregated output index shown in Figure 13.

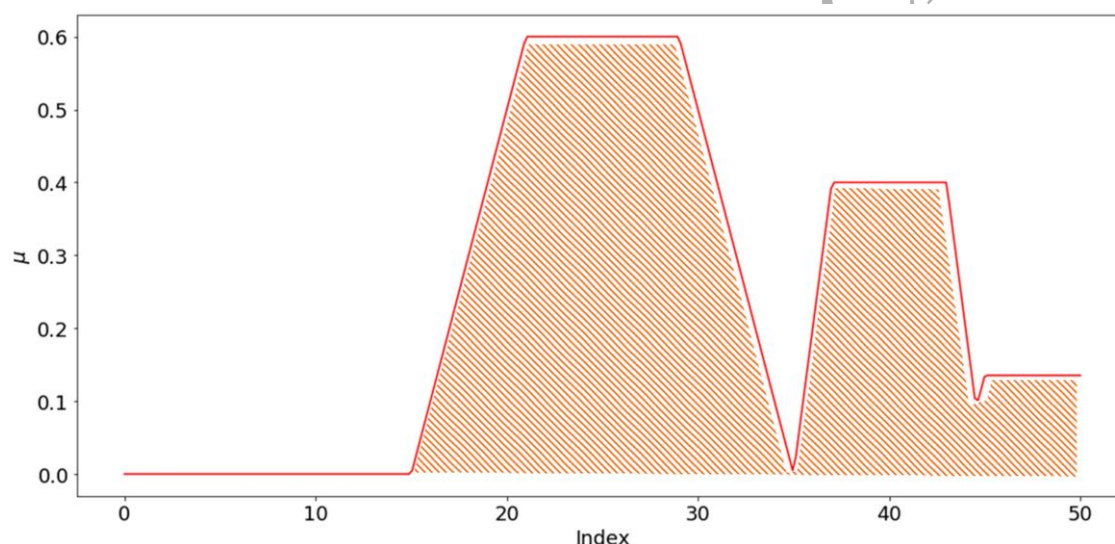


Figure 13. Aggregated Substitution index

After defuzzifying it using the centroid of the polygon, we obtain a value for the substitution index of 30.19.

It is worth mentioning that in this case, the obtained values of the index are in a much wider range than in the previous case. This means that an index with a resultant value rounding 25 can be due to the fact that one of its NFPA numbers is around four and the other one is zero when the number of compounds is two ($|C| = 2$). This result does not imply that there is no risk. Actually, it is a very risky material in regards to its flammability or toxicity. But it is not the “worst” material the index can rank, and it needs to be able to rank any material. Hence, we once again remark that the numbers given by this index cannot be considered as universal safety indices, but only as comparative ones for the different possibilities of a piece of equipment or a plant.

If there are multiple alternatives where different chemicals with very similar NFPA numbers are used, it may be desirable to exchange the minimum input with the NR number from the materials, in order to conserve the sensitivity of the index. This NR number refers to the number for reactivity or instability.

4. Attenuation index

In this section we consider the two parameters that most affect the safety of a process unit; temperature and pressure of the unit. The simplest approach would be to only consider the temperature and pressure at which the unit is operating and setting a maximum and a minimum value for the fuzzy functions like in previous indices. However, this option does not provide a result that compares alternatives in a truly attenuating way. For example, a subcooled liquid working at 200°C is, normally, safer than an overheated steam working at 180°C. As such, some penalizations and rewards must be included in order to consider the difference among the different thermal states of the chemicals.

The index is composed of two sub-indices. These are shown in Eq.(7).

$$AI = (AI_T \wedge AI_P) \quad (7)$$

The formulae of those two sub-indices are shown in Eq.(8).

$$\begin{aligned} AI_T &= T_{op} + Pen_T \cdot \max(0, T_{op} - T_b) + R_{w_T} \cdot \max(0, T_b - T_{op}) \\ AI_P &= P_{op} + Pen_P \cdot \max(0, P_{vap} - P_{op}) \end{aligned} \quad (8)$$

Where:

- T_{op}, T_b : These parameters stand for the operating temperature and the boiling temperature at the operating pressure in K.
- Pen_T, R_{w_T} : These parameters stand for a penalization in the temperature due to being higher than the boiling temperature and a reward to the index due to working with subcooled liquid. Example values are 0.5 and 0.1 respectively, if we want to penalize harsher than we want to reward. These parameters must be chosen by safety experts.
- P_{op}, P_{vap} : These parameters stand for the operating pressure and the vapor pressure at the operating temperature in bar.
- Pen_P : This parameter stands for a penalization in the pressure due to working at volatile conditions. In this example, we consider a value of 0.5.

As was the case with the intensification index, the bounds that we establish for our fuzzy sets must be previously set. These are shown in Table 7.

Table 7. Fuzzification of the inputs for the attenuation index

	Range in temperature (°C)	Range in pressure (bar)
Atmospheric	~30	~1
Low	30 – 200	1 – 19
Medium	140 – 350	11 - 35
High	340 - > 400	28 - > 40

We can see by looking at these intervals that we work with a very wide range of values. If we are considering as low index temperatures up to 200 °C and pressures up to 19 bar it means that the worst possible studied case may reach very high values. If we are comparing processes that operate in a more restricted interval, these bounds must be revisited.

Using a Gaussian membership function at both ends (Atmospheric and High) and triangular membership functions for the middle values, the resultant plot of these fuzzy sets is shown in Figure 14 and Figure 15.

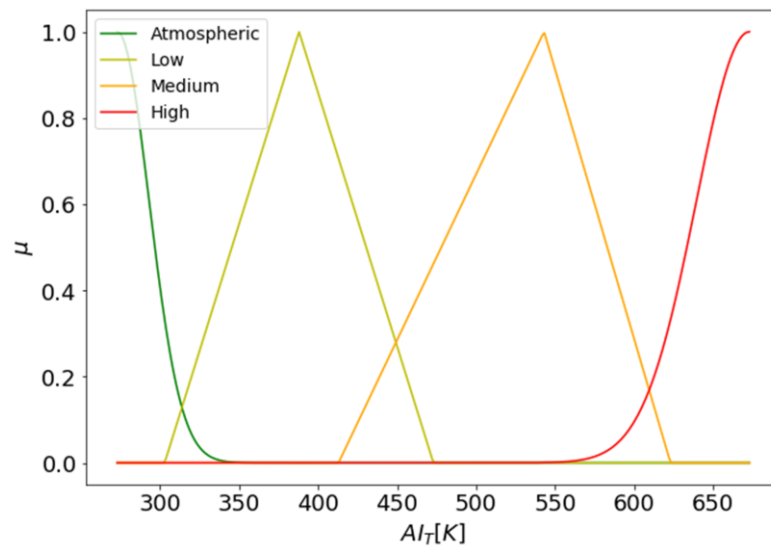


Figure 14. Fuzzy sets of the temperature sub-index

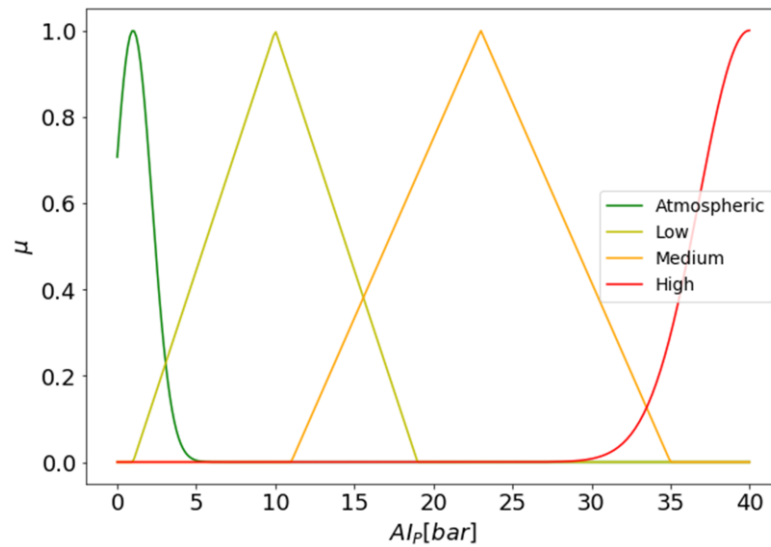


Figure 15. Fuzzy sets of the pressure sub-index

Now, we establish the relationships among the inputs that result in the different outputs. In order to show the versatility gained thanks to the use of fuzzy logic, let the process be one, such that its risk is more sensitive to the increase in temperature than to the increase in pressure. If we map letters to the different fuzzy sets, such that; A = Atmospheric, B = Low, C = Medium and D = High, the resultant relationships are shown in Table 8.

Table 8. Relationships among the inputs and the outputs of the attenuation index

Level of risk	Implications (T – P)
Minimal	$(A \wedge A) \vee (A \wedge B)$
Low	$(B \wedge B) \vee (B \wedge A) \vee (A \wedge C) \vee (B \wedge C)$
Medium	$(C \wedge C) \vee (A \wedge D) \vee (C \wedge B) \vee (B \wedge D) \vee (C \wedge A)$
Moderate	$(D \wedge A) \vee (D \wedge B) \vee (C \wedge D)$
High	$(D \wedge D) \vee (D \wedge C)$

As a first example, we consider that inside the equipment there is a saturated liquid, with an operating temperature of 160 °C and an operating pressure of 16 bar. After doing the appropriate inferences, we obtain the aggregated fuzzified index shown in Figure 16.

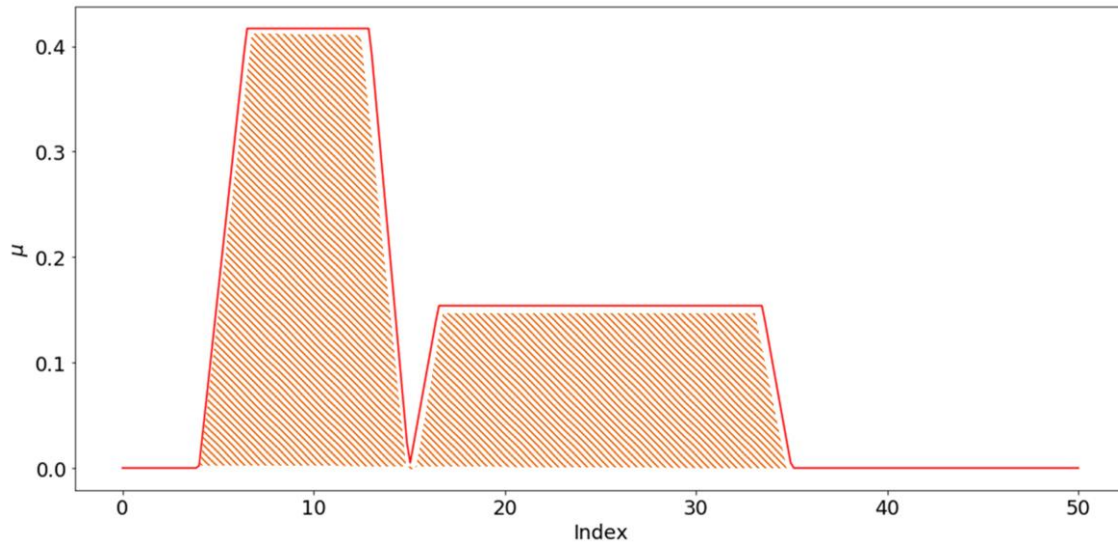


Figure 16. Aggregated attenuation index of a saturated liquid stream

Defuzzifying using the centroid we obtain an index of 16.36.

Now we consider that the operating temperature and operating pressure are the same as in the previous case, but the stream is an overheated vapor. Thus, let its boiling temperature be 140 °C and its vapor pressure 18 bar. Its aggregated attenuation index is shown in Figure 17.

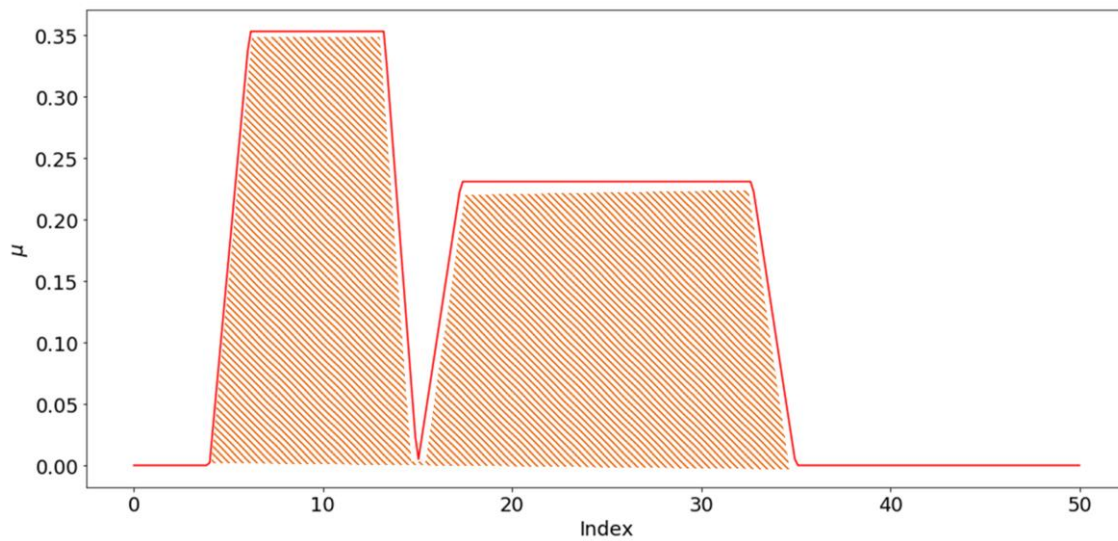


Figure 17. Aggregated attenuation index of overheated vapor

As it is expected, thanks to the penalties in the functions, the index takes into account the state of the stream and penalizes the overheated stream with a higher index of 18.23, obtained after defuzzifying using the centroid.

If we consider a subcooled liquid in the same operating conditions, with a boiling temperature of 180 °C and a vapor pressure of 14 bar, we obtain the aggregated index shown in Figure 18.

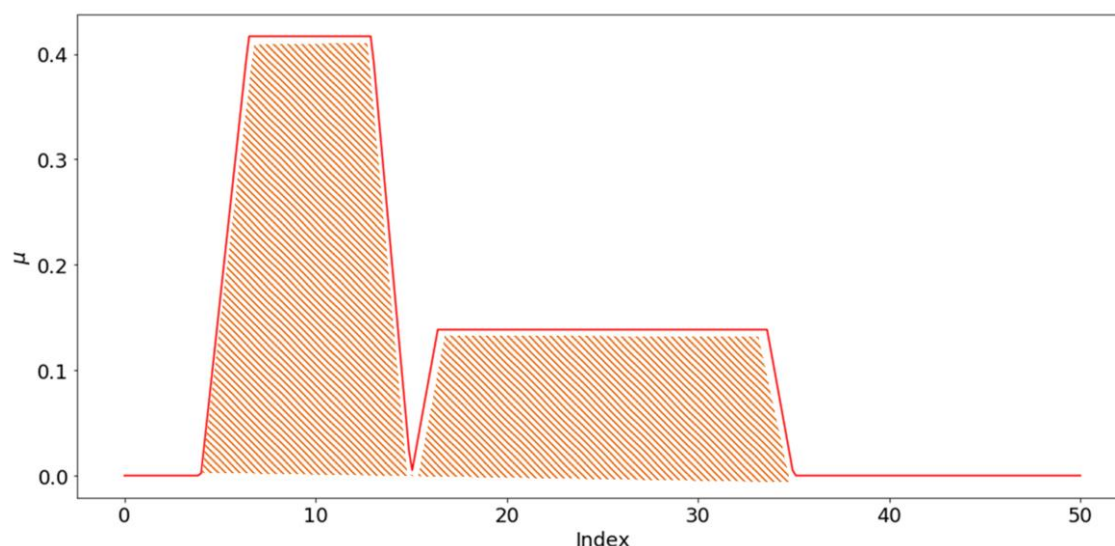


Figure 18. Aggregated attenuation index of subcooled liquid

Now the defuzzification using the centroid results in an index of 16.00. As it is expected, the subcooled liquid is considered safer than the saturated liquid, and of course, than the overheated vapor.

The index penalizes and rewards correctly the differences between the saturation properties and the operating properties. The penalization and reward factor are chosen by the decision maker, and they depend on the type of process that the piece of equipment is going to perform, since it is not considered equally dangerous having vapor in a distillation column, where it is supposed to be, as having it in a pressurized reactor where the reaction is expected to be carried out in liquid phase.

5. Recommended intervals for the fuzzy sets

As stated above, the intervals considered in this paper for the fuzzy sets that result from them are examples. It is expected that experts in safety still assess what they consider dangerous for a given system. Fuzzy logic only allows the user to transform spoken concepts to mathematical models, but still has to be delimited by the logic of an expert.

That being said, some recommendations can be given when talking about safety intervals if no expert input is available. These are based on the normal expected operation range of process units, or even can be partially based on other safety indices which have been proven valuable to assess the safety level. Since these interval decisions are stated before the optimization model starts running, they can include non-convex equations.

For the intensification index, the inventory sub-index, we can start by looking at references such as Turton's (Turton et al., 2012). There, we find appendices with expected ranges of capacity for several units. It is true that some are not in standard volume measures, such as the heat exchangers, but for those the vessel size and the amount of chemical that can have at the same time are the ones which have to be taken care of. Therefore, from that reference, some examples in ranges would be as shown in Table 9. As it is expected, this sub-index is vastly influenced by the density.

Table 9. Estimation of the inventory range for some units in volume

Unit	Interval	Low	Medium	High	Units
Vessels	0.1 – 628	0.1 – 377	31 – 597	252 – 628	m^3
Tanks	90 – 40000	90 – 24036	2085.5 – 38000	16000 – 40000	m^3
Towers	0.3 – 520	0.3 – 312	27 – 494	208 – 520	m^3
Reactors	0.04 – 60	0.04 – 36	3 – 57	24 – 60	m^3
Crystallizers	1.5 – 30	1.5 – 18.6	2.92 – 28.58	12.9 – 30	m^3

The energy sub-index is a bit more complex, especially when there are mixtures present in the process unit. There are a huge amount of different chemical compounds with different characteristics. While it is recommended to tailor this by the needs of the studied process unit and plant, it can be argued that at least the minimum heat of combustion, unless in a specific process that involves dangerous chemicals, would be 0. The highest, if there is no other input, can be taken as the energy of combustion of the hydrogen. The intervals are shown in Table 10.

Table 10. Estimation of the energy range

Interval	Low	Medium	High	Units
0 – 142	0 – 85.2	7 – 134.9	56 – 142	MJ / kg

Looking at these intervals they may seem overly wide. However it is important to point out that intervals do not define the fuzzy set per se. The fuzzy function (Triangular, Trapezoidal, Gaussian) is needed, and there are infinite of these functions that may fit an interval. An example appropriate for these uses would be the ones shown in Figure 4 and Figure 5. The logic dictates that the maximum point of belonging to the Low fuzzy set should be at the start of the interval, and so for the High fuzzy set and the end of the interval.

For the substitution index, there is no possible doubt. There are only five groups strongly defined by the NFPA, so those must be always the possibilities. The maximum belonging value should always be at the number they represent, as they are shown in Figure 12.

For the attenuation index, it is recommended if no further information to use the intervals presented in Table 7.

The interval selection is one of the most critical parts that must be considered in order to obtain coherent results. However, it is out of the scope of this paper to claim some formula to define the perfect interval and fuzzy function. Some recommendations include using techniques such as Analytic Hierarchy Process (Saaty, 2008) (AHP) or other decision making tools. The work here presented is

centered on presenting a systematically optimizable index, or at least a methodology, to obtain an inherent safety index that can be studied alongside other process variables.

6. Optimization models

This novel index, OFISI, presents two key advantages. First, its indices (intensification, substitution and attenuation) are easily modifiable just by redefining the membership functions of each fuzzy input set and the relationships that joint them to form an output. Secondly, these sub-indices, and by extension the main index, can be optimized using a mostly linear model. The non-linearities are reduced to the objective function calculation phase and to the possible Gaussian membership functions that the user may include. The former is where the majority of the problems during optimization lie.

It is true that non-differentiable minimizers (or maximizers) are produced due to the use of fuzzy inference, since the maximum and minimum operators are non-smooth. Nevertheless, using disjunctive programming (Grossmann & Trespalcios, 2013; Raman & Grossmann, 1991) we can model these discontinuities with the aid of Boolean variables. In order to illustrate this concept, in the following paragraphs we develop the optimization model of the intensification index step by step. For the other sub-indices the steps are analogous, albeit some may require more disjunctive programming extensions during the modeling of the input membership functions, such as the substitution index if we considered the explained approach of dividing the relationships depending on the maximum and minimum number.

6.1 Input Membership functions model

We assume that we have a series of technologies with different amounts of inventory and energy. First, we model the equations to determine the membership function.

Let I_t be the inventory sub-index of a technology t in mass units, E_t the energy sub-index of a technology t in energy units. These variables are the same as the previously defined $II_{inventory}$, II_{energy} for a defined technology t , but we change the nomenclature in this section in order to increase the clarity of the equations. I_{max} , E_{max} are both predefined parameters that bound the membership functions with mass and energy units respectively. The Gaussian representation of the membership function of the inventory takes the form shown in Eq.(9):

$$\mu_{low}^{Inventory} = e^{-\frac{(I_t - m_{low})^2}{2k_{low}^2}}$$

$$\mu_{high}^{Inventory} = e^{-\frac{(I_t - m_{high})^2}{2k_{high}^2}}$$
(9)

Where m_{low} , m_{high} , k_{low} , k_{high} are parameters of the Gaussian distribution. In the case of this paper, $m_{low} = 0$, $m_{high} = I_{max}$, $k_{low} = k_{high} = 0.2I_{max}$. The case of the energy is identical, as shown in Eq.(10).

$$\mu_{low}^{Energy} = e^{-\frac{(E_t - m_{low})^2}{2k_{low}^2}} \quad (10)$$

$$\mu_{high}^{Energy} = e^{-\frac{(E_t - m_{high})^2}{2k_{high}^2}}$$

The case of the triangular membership function requires a different approach. Since we cannot define non-differentiable functions, we must use disjunctive programming. Therefore, a disjunction is formulated, as shown in Eq. (11)

$$\left[\begin{array}{l} IT_1, ET_1 \\ \mu_{medium}^{Inventory} = 0 \\ \mu_{medium}^{Energy} = 0 \\ I_t \leq LB \cdot I_{max} \\ E_t \leq LB \cdot E_{max} \end{array} \right] \vee \left[\begin{array}{l} IT_2, ET_2 \\ \mu_{medium}^{Inventory} = SLOI_1(I_{max} - LB \cdot I_{max}) \\ \mu_{medium}^{Energy} = SLOE_1(E_{max} - LB \cdot E_{max}) \\ LB \cdot I_{max} \leq I_t \leq MP \cdot I_{max} \\ LB \cdot E_{max} \leq E_t \leq MP \cdot E_{max} \end{array} \right] \vee \left[\begin{array}{l} IT_3, ET_3 \\ \mu_{medium}^{Inventory} = SLOI_2(I_{max} - MP \cdot I_{max}) \\ \mu_{medium}^{Energy} = SLOE_2(E_{max} - MP \cdot E_{max}) \\ MP \cdot I_{max} \leq I_t \leq UB \cdot I_{max} \\ MP \cdot E_{max} \leq E_t \leq UB \cdot E_{max} \end{array} \right] \vee \left[\begin{array}{l} IT_4, ET_4 \\ \mu_{medium}^{Inventory} = 0 \\ \mu_{medium}^{Energy} = 0 \\ I_t \geq UB \cdot I_{max} \\ E_t \geq UB \cdot E_{max} \end{array} \right] \quad (11)$$

$$\frac{1}{MP \cdot I_{max} - LB \cdot I_{max}} = SLOI_1$$

$$-\frac{1}{UB \cdot I_{max} - MP \cdot I_{max}} = SLOI_2$$

$$\frac{1}{MP \cdot E_{max} - LB \cdot E_{max}} = SLOE_1$$

$$-\frac{1}{UB \cdot E_{max} - MP \cdot E_{max}} = SLOE_2$$

In this disjunction, IT, ET are Boolean variables that refer to each of the inventory and energy possibilities. A crisp value can be located before the triangular membership function of that fuzzy set, and thus the membership function of that fuzzy set equals 0. This behavior is shown in IT_1, ET_1 . An input can also be located either in the ascending part of the triangular membership function (IT_2, ET_2) or in the descending part (IT_3, ET_3) of the triangular membership function and the value of the membership function of the studied fuzzy set is obtained as shown in the equations inside these blocks. If the crisp input value is in the vertex of the triangle, the “exclusive or” Boolean symbol \vee allows the model to choose only one. Finally, the value of the input can be located after the triangular membership function, and thus the value of the membership function of that fuzzy set is equal to 0.

Obviously, each Boolean variable is associated with the equations in which that Boolean variable will have an impact. Hence, IT refers to the first and third equation in each disjunction while ET refers to the second and fourth equations. UB, MP, LB refer to the values that delimit the triangular membership function. Previously in section 2, where the intensification index was defined, we chose

$UB = 0.95, LB = 0.05, MP = 0.50$ for the only triangular membership function, i.e., the one that models an input of “Medium”.

In order to transform the disjunction into a set of algebraic equations with the aid of binary variables, we use the convex hull reformulation. In the case of the inventory for a single technology t , we obtain the equations shown in Eq.(12).

$$\begin{aligned}
 \mu_{1medium}^{Inventory} &= 0 \cdot y1_t \\
 \mu_{1medium}^{Inventory} &\leq y1_t \\
 I1_t - (LB \cdot I_{max}) y1_t &\leq 0 \\
 \\
 \mu_{2medium}^{Inventory} &= SLOI_1(I2_t - LB \cdot I_{max} y2_t) \\
 \mu_{2medium}^{Inventory} &\leq y2_t \\
 I2_t - (LB \cdot I_{max}) y2_t &\geq 0 \\
 I2_t - (MP \cdot I_{max}) y2_t &\leq 0 \\
 \\
 \mu_{3medium}^{Inventory} &= SLOI_2(I3_t - MP \cdot I_{max} y3_t) \\
 \mu_{3medium}^{Inventory} &\leq y3_t \\
 I3_t - (MP \cdot I_{max}) y3_t &\geq 0 \\
 I3_t - (UB \cdot I_{max}) y3_t &\leq 0 \\
 \\
 \mu_{4medium}^{Inventory} &= 0 \cdot y4_t \\
 \mu_{4medium}^{Inventory} &\leq y4_t \\
 I4_t - (UB \cdot I_{max}) y4_t &\leq 0 \\
 \\
 \mu_{1medium}^{Inventory} + \mu_{2medium}^{Inventory} + \mu_{3medium}^{Inventory} + \mu_{4medium}^{Inventory} &= \mu_{medium}^{Inventory} \\
 y1_t + y2_t + y3_t + y4_t &= 1 \\
 I1_t + I2_t + I3_t + I4_t &= I_t
 \end{aligned} \tag{12}$$

As for the energy part of the index, the methodology is exactly analogous, as shown in Eq.(13). It is worth remarking that each of these equations applies to each of the available technologies, so the inclusion of a new index that takes a set of technologies into account will be necessary when optimizing various technologies. As they are shown here, they refer to a unique technology t .

As such, by using these equations, and generalizing for a set t of T technologies, we calculate the membership function to each input fuzzy set for each technology, denoted as

$$\mu_{low}^{Inventory}, \mu_{medium}^{Inventory}, \mu_{high}^{Inventory}, \mu_{low}^{Energy}, \mu_{medium}^{Energy}, \mu_{high}^{Energy} \quad \forall t \in T.$$

6.2 Relationships among fuzzy sets

Once we have the input membership functions modeled, we require to model the relationships among the sets following Mamdani & Assilian's method. We define the following subsets;

$r_{min}, r_{low}, r_{medium}, r_{mod}, r_{high} \subset R$, where R is the set of all the existent relationships defined by the user.

$$\begin{aligned}
 \mu_{1medium}^{Energy} &= 0 \cdot w1_t \\
 \mu_{1medium}^{Energy} &\leq w1_t \\
 E1_t - (LB \cdot E_{max})w1_t &\leq 0 \\
 \\
 \mu_{2medium}^{Energy} &= SLOE_1(E2_t - LB \cdot E_{max}w2_t) \\
 \mu_{2medium}^{Energy} &\leq w2_t \\
 E2_t - (LB \cdot E_{max})w2_t &\geq 0 \\
 E2_t - (MP \cdot E_{max})w2_t &\leq 0 \\
 \\
 \mu_{3medium}^{Energy} &= SLOE_2(E3_t - MP \cdot E_{max}w3_t) \\
 \mu_{3medium}^{Energy} &\leq w3_t \\
 E3_t - (MP \cdot E_{max})w3_t &\geq 0 \\
 E3_t - (UB \cdot E_{max})w3_t &\leq 0 \\
 \\
 \mu_{4medium}^{Energy} &= 0 \cdot w4_t \\
 \mu_{4medium}^{Energy} &\leq w4_t \\
 E4_t - (UB \cdot E_{max})w4_t &\leq 0 \\
 \\
 \mu_{1medium}^{Energy} + \mu_{2medium}^{Energy} + \mu_{3medium}^{Energy} + \mu_{4medium}^{Energy} &= \mu_{medium}^{Energy} \\
 w1_t + w2_t + w3_t + w4_t &= 1 \\
 E1_t + E2_t + E3_t + E4_t &= E_t
 \end{aligned} \tag{13}$$

As an example, we start with the relationships that produce a minimal risk level. As commented in previous sections, only one of the pairs among II_{energy} and $II_{inventory}$ results in a minimal risk. This being, when both the inventory and energy level are low.

We define the model as if there were more than one relationship here, in order to give it a general sense. Therefore, we need to model all the relationships. From Mamdani & Assilian's method, we need a minimum operator. As a result, for the relation between low inventory and low mass, the resultant value is $\min(\mu_{low,t}^{Energy}, \mu_{low,t}^{Inventory})$. We cannot use a minimum operator in an optimization model, since those operators are discontinuous. Therefore, for a single technology and a single relation, the disjunction that is equivalent to this minimum operator is shown in Eq.(14).

$$\left[\begin{array}{c} Ymin_1 \\ \mu_{Minimum}^{Relation} = \mu_{low}^{Energy} \\ \mu_{low}^{Inventory} \geq \mu_{low}^{Energy} \end{array} \right] \vee \left[\begin{array}{c} Ymin_2 \\ \mu_{Minimum}^{Relation} = \mu_{low}^{Inventory} \\ \mu_{low}^{Inventory} \leq \mu_{low}^{Energy} \end{array} \right] \quad (14)$$

In order to perform the convex hull reformulation, we first need to disaggregate the variables. The algebraic disaggregated expression, considering multiple possible relationships and multiple technologies, is shown in Eq.(15).

$$\left. \begin{array}{l} \mu_{Minimum,r,t}^{1Relation} = \mu_{r,t}^{1Energy} \\ \mu_{r,t}^{1Inventory} \geq \mu_{r,t}^{1Energy} \\ \mu_{r,t}^{1Inventory} \leq ymin1_{r,t} \\ \mu_{r,t}^{1Energy} \leq ymin1_{r,t} \\ \mu_{Minimum,r,t}^{1Relation} \leq ymin1_{r,t} \end{array} \right\} \forall r \in r_{min}, \forall t \in T \quad (15)$$

$$\left. \begin{array}{l} \mu_{Minimum,r,t}^{2Relation} = \mu_{r,t}^{2Inventory} \\ \mu_{r,t}^{2Inventory} \leq \mu_{r,t}^{2Energy} \\ \mu_{r,t}^{2Inventory} \leq ymin2_{r,t} \\ \mu_{r,t}^{2Energy} \leq ymin2_{r,t} \\ \mu_{Minimum,r,t}^{2Relation} \leq ymin2_{r,t} \end{array} \right\}$$

Then, we must aggregate the disaggregated variables. In order to do this, we know that the only relationship is low energy – low inventory, which is denoted as AA. As such, the aggregation equations are shown in Eq.(16).

$$\left. \begin{array}{l} \mu_{AA,t}^{1Inventory} + \mu_{AA,t}^{2Inventory} = \mu_{low,t}^{Inventory} \\ \mu_{AA,t}^{1Energy} + \mu_{AA,t}^{2Energy} = \mu_{low,t}^{Energy} \end{array} \right\} \forall t \in T \quad (16)$$

$$\left. \begin{array}{l} \mu_{Minimum,r,t}^{1Relation} + \mu_{Minimum,r,t}^{2Relation} = \mu_{Minimum,r,t}^{Relation} \\ ymin1_{r,t} + ymin2_{r,t} = 1 \end{array} \right\} \forall r \in r_{min}, \forall t \in T$$

Since there is only one relationship, we obtain the membership to the minimal risk group by

$$\mu_{Minimum,t} = \mu_{Minimum,AA,t}^{Relation} \quad \forall t \in T \quad (17)$$

This is the simplest case. All the other cases that have only one relation are analogous to this one, changing only the denomination of the variables and Boolean variables and the values introduced in the first two equations at the right hand of Eq.(16). Now we develop both the most difficult case and the general case, where there are multiple relationships for the same level. In our example, the case with the highest number of relationships is the one of the medium risk level.

Mamdani & Assilian's method of composition recalls that, whenever there are multiple relationships, a maximum operator must be used. The first block of equations is analogous to Eq.(15). It is shown in Eq.(18).

$$\left. \begin{aligned} \mu_{Medium\ r,t}^{Relation} &= \mu_{r,t}^{Energy} \\ \mu_{r,t}^{Inventory} &\geq \mu_{r,t}^{Energy} \\ \mu_{r,t}^{Inventory} &\leq y_{med1_{r,t}} \\ \mu_{r,t}^{Energy} &\leq y_{med1_{r,t}} \\ \mu_{Medium\ r,t}^{Relation} &\leq y_{med1_{r,t}} \end{aligned} \right\} \forall r \in r_{medium}, \forall t \in T \quad (18)$$

$$\left. \begin{aligned} \mu_{Medium\ r,t}^{Relation} &= \mu_{r,t}^{Inventory} \\ \mu_{r,t}^{Inventory} &\leq \mu_{r,t}^{Energy} \\ \mu_{r,t}^{Inventory} &\leq y_{med2_{r,t}} \\ \mu_{r,t}^{Energy} &\leq y_{med2_{r,t}} \\ \mu_{Medium\ r,t}^{Relation} &\leq y_{med2_{r,t}} \end{aligned} \right\}$$

The aggregation equations, while more complex than in the previous case, are still analogous, as shown in Eq.(19).

$$\left. \begin{aligned} \mu_{BB,t}^{Inventory} + \mu_{BB,t}^{Inventory} &= \mu_{medium,t}^{Inventory} \\ \mu_{BB,t}^{Energy} + \mu_{BB,t}^{Energy} &= \mu_{medium,t}^{Energy} \end{aligned} \right\} \{BB\}subset$$

$$\left. \begin{aligned} \mu_{AC,t}^{Inventory} + \mu_{AC,t}^{Inventory} &= \mu_{low,t}^{Inventory} \\ \mu_{AC,t}^{Energy} + \mu_{AC,t}^{Energy} &= \mu_{high,t}^{Energy} \end{aligned} \right\} \{AC\}subset \quad \forall t \in T$$

$$\left. \begin{aligned} \mu_{CA,t}^{Inventory} + \mu_{CA,t}^{Inventory} &= \mu_{high,t}^{Inventory} \\ \mu_{CA,t}^{Energy} + \mu_{CA,t}^{Energy} &= \mu_{low,t}^{Energy} \end{aligned} \right\} \{CA\}subset$$

$$\left. \begin{aligned} \mu_{Medium\ r,t}^{Relation} + \mu_{Medium\ r,t}^{Relation} &= \mu_{Medium\ r,t}^{Relation} \\ y_{med1_{r,t}} + y_{med2_{r,t}} &= 1 \end{aligned} \right\} \forall r \in r_{medium}, \forall t \in T \quad (19)$$

Now it is time to tackle the difference. In the previous case, minimal risk, we had only one single relationship, AA, which allowed us to establish the membership function of that subset as the one and only for the minimal risk. Now, we have three different relationships, BB, AC and CA, and Mamdani & Assilian's method requires us to consider the maximum of those three as the membership function for the medium risk level. It can be done using disjunctions, as well as the minimums were done, and it is fairly easy considering that we only aim to obtain the maximum, without ordering the values, which would need more nested relations. This method, since it takes the maximum of the previous composed minimums, is known as the *max-min composition*.

For the medium relations BB, AC and CA the disjunction is shown in Eq.(20), where Y^{max} is a binary variable.

$$\left[\begin{array}{c} Y_{Medium, BB, t}^{max} \\ \mu_{Medium, t}^{BB} \geq \mu_{Medium, t}^{AC} \\ \mu_{Medium, t}^{BB} \geq \mu_{Medium, t}^{CA} \\ \mu_{Medium, t} = \mu_{Medium, t}^{BB} \end{array} \right] \vee \left[\begin{array}{c} Y_{Medium, AC, t}^{max} \\ \mu_{Medium, t}^{AC} \geq \mu_{Medium, t}^{BB} \\ \mu_{Medium, t}^{AC} \geq \mu_{Medium, t}^{CA} \\ \mu_{Medium, t} = \mu_{Medium, t}^{AC} \end{array} \right] \vee \left[\begin{array}{c} Y_{Medium, CA, t}^{max} \\ \mu_{Medium, t}^{CA} \geq \mu_{Medium, t}^{AC} \\ \mu_{Medium, t}^{CA} \geq \mu_{Medium, t}^{BB} \\ \mu_{Medium, t} = \mu_{Medium, t}^{CA} \end{array} \right] \quad (20)$$

This can be reformulated in a general form for all the groups of output membership with a convex hull. Using this case as an example, consider that there are the following groups of output membership $Minimum, Low, Medium, Moderate, High \in G$ and we have the relationships $r_{min}, r_{low}, r_{medium}, r_{moderate}, r_{high} \subset R$. If we consider a set that correlates each group of membership with its correspondent relations, being for this example one relation in the Minimal output membership group, two in the Low output membership group, three in the Medium group, two in the Moderate group and one in the High group, we end up with the relations show in (21).

$$\begin{aligned} REL_{G,R} &= \{REL1_{G,R}, REL2_{G,R}, REL3_{G,R}, REL4_{G,R}, REL5_{G,R},\} \\ REL1_{G,R} &= \{(Minimum, r_{min,1})\} \\ REL2_{G,R} &= \{(Low, r_{low,1}), (Low, r_{low,2})\} \\ REL3_{G,R} &= \{(Medium, r_{medium,1}), (Medium, r_{medium,2}), (Medium, r_{medium,3})\} \\ REL4_{G,R} &= \{(Moderate, r_{moderate,1}), (Moderate, r_{moderate,2})\} \\ REL5_{G,R} &= \{(High, r_{high,1}), (High, r_{high,2})\} \end{aligned} \quad (21)$$

The convex hull reformulation can be stated as shown in Eq.(22), where $y_{g,r,t}^{max}$ refers to the binary variable that is activated, this being, has value 1, in the output membership group g , for the relation r within the technology t .

$$\begin{aligned} \mu_{g,r,t}^{Relation} &= \sum_{r' \in REL'} \mu_{g,r',t}^{RelationDisaggregated} & \forall t, REL_{g,r} \\ \mu_{g,r,t}^{RelationDisaggregated} &\geq \mu_{g,r',t}^{RelationDisaggregated} & \forall t, (g, r, r') \in REL_{g,r} \wedge REL'_{g,r'} \\ \mu_{g,t} &= \sum_{REL'_{g,r'}} \mu_{g,r',t}^{Disaggregated} & \forall t, g \\ \mu_{g,r,t}^{Disaggregated} &= \mu_{g,r',t}^{RelationDisaggregated} & \forall t, REL_{g,r} \\ \sum_{r \in REL_{g,r}} y_{g,r,t}^{max} &= 1 & \forall t, g \\ \mu_{g,r,t}^{Disaggregated} &\leq y_{g,r,t}^{max} & \forall t, REL_{g,r} \end{aligned} \quad (22)$$

There are also other methods to perform the aggregation. A different approach is to consider the sum of the membership functions instead of the maximum, using then a *sum-min composition*. The equation to calculate this sum as an example for the *Medium* group of membership is shown in Eq.(23).

$$\mu_{Medium,t} = \sum_{r \in r_{medium}} \mu_{Medium\ r,t}^{Relation} \quad t \in T \quad (23)$$

It is easier to perform the *sum-min composition* than the *max-min composition*. If the model presents convergence problems, it is recommended to switch to a *sum-min composition*.

6.3 Technology selection

All the convex hull reformulations performed previously considered that we have a number of technologies t with the corresponding parameters to calculate the index. Keeping that in mind, we now need to choose among the different technologies the one that provides the minimum intensification index. Therefore, we need to introduce a new binary variable Z_t , whose value is 1 when technology t is chosen and 0 otherwise. We introduce as well a new variable called μ_{Index}^{Chosen} that will act as the membership function of the chosen technology. As such, for the minimal index, we obtain the disjunctive model shown in Eq.(24).

$$\left[\begin{array}{c} Z_{tec1} \\ \mu_{Minimal}^{Chosen} = \mu_{Minimal,tec1} \\ \mu_{Low}^{Chosen} = \mu_{Low,tec1} \\ \mu_{Medium}^{Chosen} = \mu_{Medium,tec1} \\ \mu_{Moderate}^{Chosen} = \mu_{Moderate,tec1} \\ \mu_{High}^{Chosen} = \mu_{High,tec1} \end{array} \right] \vee \dots \vee \left[\begin{array}{c} Z_{tecT} \\ \mu_{Minimal}^{Chosen} = \mu_{Minimal,tecT} \\ \mu_{Low}^{Chosen} = \mu_{Low,tecT} \\ \mu_{Medium}^{Chosen} = \mu_{Medium,tecT} \\ \mu_{Moderate}^{Chosen} = \mu_{Moderate,tecT} \\ \mu_{High}^{Chosen} = \mu_{High,tecT} \end{array} \right] \quad (24)$$

After applying a convex hull reformulation for T technologies, disaggregating the variables into $\mu\mu_{g,t}^{Chosen}$, $\mu\mu_{g,t}$ the disjunction results in Eq.(25).

$$\left. \begin{array}{l} \mu\mu_{g,t}^{Chosen} = \mu\mu_{g,t} \\ \mu\mu_{g,t}^{Chosen} \leq n_g^{rel} Z_t \end{array} \right\} \forall g \in G, t \in T$$

$$\mu\mu_{g,t',t} \leq n_g^{rel} Z_t \quad \forall g \in G, t' \in T$$

$$\sum_t \mu\mu_{g,t',t} = \mu_{g,t'} \quad \forall g \in G, t' \in T$$

$$\sum_t \mu\mu_{g,t}^{Chosen} = \mu_g^{Chosen} \quad \forall g \in G \quad (25)$$

A new parameter, n^{rel} , appears in these equations. This parameter acts as an upper bound for the disaggregated membership function variables in the disjunctions. At first, it would seem intuitive that this parameter always has the value of 1, and this is true when the user applies the *max-min composition*, but it is not necessarily correct when applying the *sum-min composition*. Therefore, the recommendation is to match this parameter to the number of relations for each index. For example, we

may have two relations for the *Minimal* index. This would result in $n_{Minimal}^{rel} = 2$. By doing this, we ensure that we will always maintain the feasibility of the problem, considering that while the sum may be bigger than 1, the individual membership function for each relation in the sum will be never bigger than 1.

We must also add the restriction that only one technology is chosen. This restriction is shown in Eq.(26)

$$\sum_t Z_t = 1 \quad (26)$$

6.4 Objective function

In the examples from previous sections, we explained how we truncate the membership function of each risk and aggregate them into a single figure. Then we calculate the centroid and that value is taken as the index.

When optimizing, this is extremely difficult to write and can incur errors easily. We are not obtaining the graph as such from this model. Therefore, the centroid must be simplified. As such, it is better to simplify how the aggregated output is represented and how the centroid is calculated.

First, we want to simplify the aggregated output to a bi-dimensional parallelogram with a known centroid. The simplest approach is to use rectangles instead of the resulting trapezoids and triangles. A representation of this simplification can be seen in Figure 19.

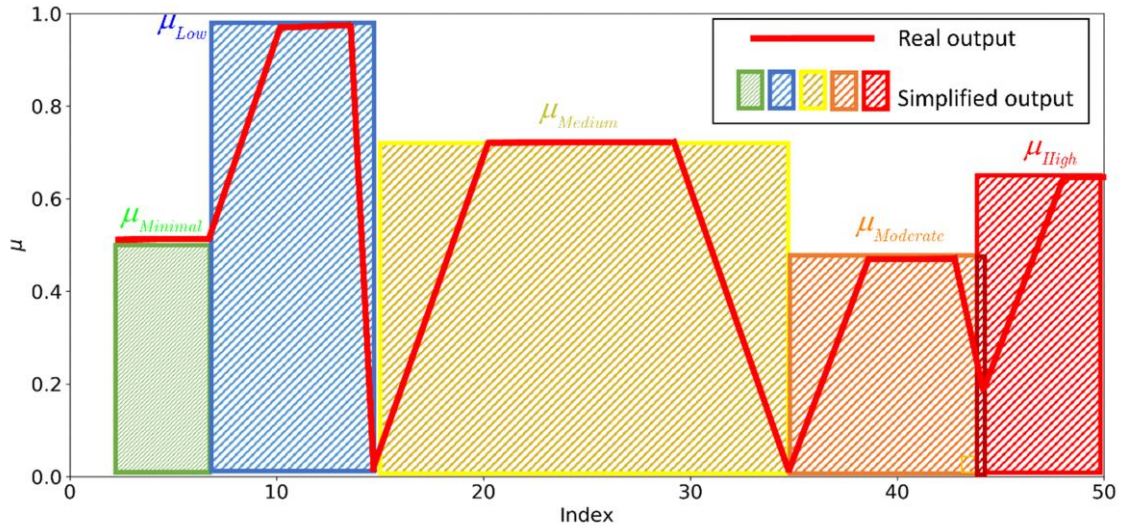


Figure 19. Simplification of the output

Since in the fuzzification of the output (Figure 3) the groups are already separated, we can easily define the centroid of each subgroup (Minimal, Low, Medium, Moderated, High). It is true that there are small overestimations and clipping if we maintain the Gaussian function for the minimal and high risk values of the index, but since the numbers are only for comparative purposes among alternatives, this small error is not significant.

For a surface S formed by i sub-surfaces $S_i : \sum_i S_i = S$, the centroid can be obtained as

$C_S = \sum C_{S_i} \frac{S_i}{S}$. Therefore, we obtain the centroid with the following expressions, shown in Eq.(27).

$$\begin{aligned} z = & C_{Minimal} \frac{Base_{Minimal} \mu_{Minimal}^{Chosen}}{S_{Total}} + C_{Low} \frac{Base_{Low} \mu_{Low}^{Chosen}}{S_{Total}} \\ & + C_{Medium} \frac{Base_{Medium} \mu_{Medium}^{Chosen}}{S_{Total}} + C_{Moderate} \frac{Base_{Moderate} \mu_{Moderate}^{Chosen}}{S_{Total}} \\ & + C_{High} \frac{Base_{High} \mu_{High}^{Chosen}}{S_{Total}} \end{aligned} \quad (27)$$

$$S_{Total} = \sum_{g \in G} Base_g \mu_g^{Chosen}$$

Thanks to the simplification performed, we already know a considerable amount of these parameters. In this example:

- $C_{Minimal} = 2.5$, $Base_{Minimal} = 5$
- $C_{Low} = 10$, $Base_{Low} = 10$
- $C_{Medium} = 25$, $Base_{Medium} = 20$
- $C_{Moderate} = 40$, $Base_{Moderate} = 10$
- $C_{High} = 50$, $Base_{High} = 5$

The centroid for the case of *High* is moved to 50. If it were kept with a value of 45, the index would never reach 50 even when the system is in the extreme point of the boundary. The centroid in the Minimal case is moved to 2.5 so we do not have problems with zeros in the resultant index.

Since these equations are non-linear, the initialization of the variables is very important. So is choosing appropriate boundaries. In our experience, solvers like Antigone (Misener & Floudas, 2014) and Baron (Tawarmalani & Sahinidis, 2005) (global solvers) have no problem finding the global optimum and are very fast. However, solvers like standard Branch & Bounds (SBB) (Drud, 2001; Land & Doig, 1960) and Dicopt (Duran & Grossmann, 1986) require the addition of a constraint in order to not incur numerical errors. This added constraint is shown in Eq.(28). Thanks to this addition, we give a minimum value to the area so it will not result in errors of division by zero. If in the optimal solution this constraint is active, the value must be diminished. The initialization is also very important in this case.

$$S_{Total} \geq 0.001 \quad (28)$$

7. Case study and results

We develop a simple case study to show a possible application of the index and how its results are interpreted. Instead of building a case study from scratch, we use the superstructure defined by Ruiz-

Femenia et al. (2013) as a base. The original problem is a scheduling problem related to the election of which technologies should be chosen and where to locate them in order to satisfy the demand. In our case, we want to study the inherent safety level of the different technologies. We show the results of the intensification and substitution index, since the data required in order to calculate the attenuation index is not readily available. It is considered that those sub-indices are enough to clearly distinguish the alternatives in technologies.

The idea of the indices is to apply each one to a piece of equipment, namely the one that differentiates the technology. As the superstructure presented in Figure 20 shows, we only have general data of inputs and outputs of the process. Therefore, some considerations must be assumed in order to calculate the inherent safety value of each technology. We consider a case where the plant must withhold enough material to produce a determined amount of product solely by its reserves. Thus for each technology, we have normalized inventory, energy amount and safety numbers from the NFPA-704. We consider that the determined amount of product that each technology must produce is 20 tons. As such, the amounts of the byproducts and the required quantities of raw materials are determined by the numbers shown in the superstructure.

As it was mentioned in the definition of the intensification index, we must consider limits for the inventory and energy. These will be set considering the numbers shown in Table 11. We pick a range in inventory from 0 to 80 tons, and a range in energy from 30 to 50 MJ/kg. We solve the model shown in section 5, both for the intensification and substitution indices. We include a binary cut strategy (Dakin, 1965) in order to sort out all the solutions, instead of obtaining only the optimal one. The results of this case study are shown in Table 12.

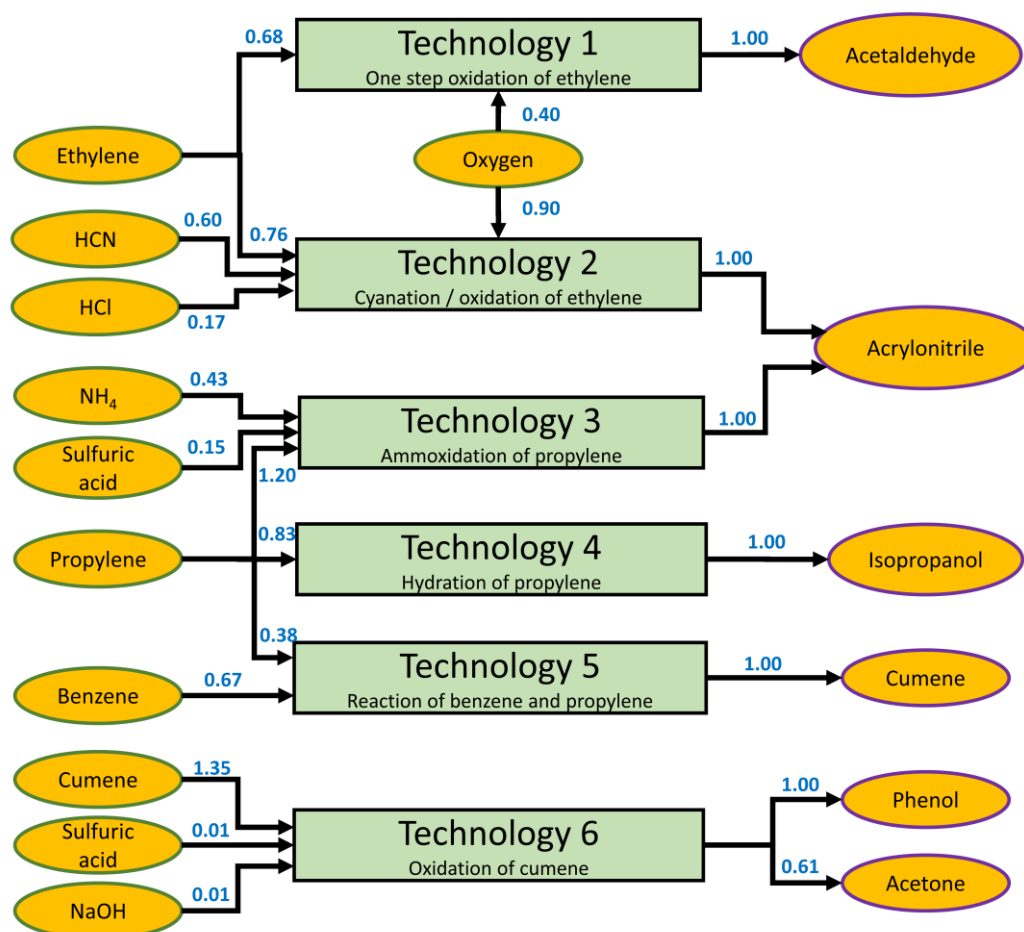


Figure 20. Superstructure of technologies

Table 11. Parameters of the technologies

Technology	Intensification index		Substitution index	
	Inventory (ton)	Energy (MJ/kg)	NF	NH
1. One step oxidation of ethylene	41.60	36.32	3.23	1.86
2. Cyanation / oxidation of ethylene	68.60	36.80	2.46	3.02
3. Ammoxidation of propylene	55.60	36.31	2.96	2.50
4. Hydration of propylene	36.60	40.23	3.45	1.00
5. Reaction of benzene and propylene	41.00	44.10	3.18	1.81
6. Oxidation of cumene	59.60	36.91	2.64	2.14

The strategy to aggregate the two sub-indices may differ from user to user. A simple equally weighted addition in order to obtain a general index can be enough to classify the different technologies. If both indices have the same tendency, one can be removed without affecting the dominance structure (Guillén-Gosálbez, 2011; Vázquez, Fernández-Torres, et al., 2018), which reduces the number of objectives if the MOO is performed alongside economic and environmental objectives.

Table 12. Results of the optimization

Technology	Intensification Index	Substitution Index
1	22.35	25.99
2	27.58	28.34
3	24.32	28.58
4	24.64	24.28
5	27.30	25.73
6	25.70	26.01

In this case study, it can be seen that all the results orbit the middle of the index. This is due to the chosen bounds of the input fuzzy sets. The important conclusion is that the index has enough sensitivity to correctly rank the alternatives.

The user may require to give more importance to the intensification index than to the substitution index, so a non-equal weighted sum would be a preferred option. Another option is the worst case scenario, where we consider the highest index of each category. The result of these different possibilities is shown in Table 13.

Table 13. Results of the optimization with different considerations in the calculus of OFISI

Best to worst position	OFISI (50/50 Weight)	OFISI (70/30 Weight)	OFISI (Worst case)
# 1	Technology 1	Technology 1	Technology 4
# 2	Technology 4	Technology 4	Technology 1
# 3	Technology 6	Technology 3	Technology 6
# 4	Technology 3	Technology 6	Technology 5
# 5	Technology 5	Technology 5	Technology 2
# 6	Technology 2	Technology 2	Technology 3

As a thought experiment, we see what happens when the intervals are tighter. Now the inventory is set from 30 to 70 ton, while the maximum and minimum energy remain in the range of 30 – 50 MJ. The bounds of the substitution index remain the same, so we only change the indices for the intensification index, which are shown in Table 14.

Table 14. Results of the optimization with changed boundaries

Technology	Intensification Index
1	19.61
2	30.81
3	23.58

4	16.80
5	23.57
6	25.61

We see that this may even change the best technology result. Now it is technology 4, while with the previous boundaries, it was technology 1. This is why the election of the boundaries is so important, and so is the fuzzification of the inputs. The second boundaries of 30 – 70 ton for the inventory establish that this is a case where up until 30 ton, there is no real problem of safety for the inventory. If we revisit section 6, where we introduced some recommended intervals, looking at Table 9, and assuming density of a standard liquid, 800 – 1200 kg/m³, these would be even low inventories for tanks. However they would be very high for other units, such as crystallizers. Since we do not have more information for this case, and we are assuming the amount of raw material needed to produce 20 tons, we can expect that the results shown in Table 12 represent better the safety of the process.

In order to obtain some comparison results, we suppose a process unit fully formed where the reactions take place and calculate Dow's F&EI for each of these technologies. Of course, this is different than the proper way of calculating the complete index for a known and existent process unit, but in order to obtain comparable results at an early stage is considered enough. The results are shown in Table 15.

Table 15. Simplified Dow's F&E index of the case study

Technology	F1	F2	F3	Dow's Index
# 1	2.50	4.67	11.68	280.23
# 2	2.85	4.73	13.48	323.47
# 3	2.50	4.99	12.47	299.19
# 4	2.50	4.71	11.78	247.34
# 5	2.50	5.23	13.08	274.67
# 6	2.50	5.34	13.36	320.58

While the results are very high for a Dow Index, it must be remarked that the control credits were not considered. From this index, the order of the technologies from best to worst is: #4, #5, #1, #3, #6, #2. If we compare this to the results shown in Table 13 and Table 14, we see a similar behavior. Both #4 and #1 tend to be among the best solutions, and #2 is practically always the worst. However, there is a huge difference between OFISI and the simplified Dow's Index when it comes to #5, which ranks up to second place using Dow's and falls when using OFISI. This is due to the fact that, when using Dow's Index, the enthalpy of combustion is a continuous variable that does not really change the index greatly when the possibilities move in the same neighborhood. In this case, from 36.32 MJ/kg to 44.10 MJ/kg. However, when using fuzzy logic, it is possible to move completely to another fuzzy set by a small change, and this incurs a greater impact than a completely continuous variable in the end result. It must be noted that some considerations in regards of the draining, spilling, accessibility, etc. were supposed equal for all

the technologies, due to the lack of data in the early design. The results are not exactly parallel, since both indices give different importance to the conforming variables, but OFISI provides, at least, similar results to the ones expected by a more complex index, even if it is not developed to the fullest of its extent. The advantage is that OFISI is easier to include in a optimization model than Dow, due to the nonlinearities being reduced to the objective function and possible Gaussian fuzzy sets, in contrast to the F2 term, which includes nonlinearities both in the pressure and energy calculations.

8. Conclusions

In this paper, we propose and develop OFISI, a novel index based on the principles of inherent safety that allows the user to perform a comparative optimization among different alternatives or technologies for a plant or a process unit. It is based upon the most basic variables related to inherent safety, and it displays great versatility while maintaining a mostly linear structure. This advantage allows us to formulate a systematic optimization model. Due to this versatility and the number of parameters that are chosen by the user in the optimization model, it cannot be expected to perform as a global safety index in the same extent as much more well-grounded indices, such as Dow's, which require more knowledge about the process and its variables once it is already designed. Therefore, the main purpose of this index is to classify the inherent safety level of different alternatives at an early design stage. By doing this, inherent safety objectives can be included in a MOO model at the same level as economic and environmental indicators.

OFISI is divided into three different sub-indices, which account for different principles of inherent safety. The union nexus among them is the use of fuzzy logic. Thanks to this approach, we can establish simple and intuitive relationships among the inputs and the output of the indices, which can also be easily altered in order to account for differences in the particularities of each process.

How to add these three sub-indices in order to obtain a value of the OFISI index is another choice of the decision maker. There are multiple strategies for the resolution of MOO problems, for example, the use of the ϵ -constraint method (Haimes et al., 1971) and augmented ϵ -constraint method (Mavrotas, 2009; Mavrotas & Florios, 2013), aggregation by means of a weighted sum or, worst case and best case scenarios comparison. Another option is to check if the reduction of the dimensionality of the problem is feasible, with the aim of considering only the most significant of the three sub-indices whenever possible. There are different methods to achieve this objective reduction, for example Deb's algorithm (Deb & Saxena Kumar, 2005), which is based on the Principal Component Analysis (PCA), or methods based on the dominance structure (Brockhoff & Zitzler, 2006), which are based on the calculus and minimization of the δ -error (Guillén-Gosálbez, 2011; Vázquez, Fernández-Torres, et al., 2018).

From the results displayed in the case study section, we conclude that it is possible to use OFISI to classify different technologies regarding its inherent safety levels without the need of complex indices. The index proves to be directly optimizable, having only difficulties with the objective function being

non-linear even when considering the simplification of the defuzzification using the centroid. Despite this difficulty, we were able to correctly optimize all the models.

Notation of the optimization model

SETS

T	$\{t : \text{Set of technologies}\}$
G	$\{g : \text{Set of groups of the output fuzzy set}\}$
R	$\{r : \text{Set of relationships among fuzzy sets}\}$
$REL_{G,R}$	$\{g, r : \text{Set that correlates each group of the output fuzzy set with its relations among fuzzy sets}\}$

PARAMETERS

UB, LB, MP	Parameters that describe the triangular fuzzification of the inputs. They stand for upper bound, lower bound and middle point respectively
m, k	Parameters that describe the Gaussian fuzzification of the inputs
n_g^{rel}	Number of relationships in the output group of membership g
$C_g, Base_g$	Centroid and Base of the simplified rectangles in the output group g

VARIABLES

IT_t, ET_t	Entrance inventory and energy of each technology t . These are analogue to $II_{inventory}, II_{energy}$
$\mu_{InputGroup,t}^{Inventory}, \mu_{InputGroup,t}^{Energy}$	Membership functions of each group of the input fuzzification for each technology t , both for the inventory and energy characterizations
$\mu_{g,t,r}^{Relation}$	Output membership function to each relation r in each group g within each technology t
$\mu_{g,t}$	Resultant output membership function to the group g after composition, for each technology t
μ_g^{Chosen}	Output membership function to the group g for the chosen technology

$y1_i, y2_i, y3_i, y4_i$	Binary variables that help to determine in which part of the triangular fuzzification is the inventory input located, for a set technology and input fuzzification
$w1_i, w2_i, w3_i, w4_i$	Binary variables that help to determine in which part of the triangular fuzzification is the energy input located, for a set technology and input fuzzification

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