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EXPERIMENTAL STUDY OF ENTROPY FOR HOLOGRAPHIC OPTICAL ELEMENTS

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ABSTRACT

When aberrations are present the principal maximum of the axial irradiance does not lie at the Gaussian image point but at an axial point closer to it. In general there may be more than one diffraction focus and a criteria is needed for the election of the best image point in the sense of minimal aberrations, we have proposed an entropy function as merit function for study the best image plane. In this paper we present an experimental procedure for the measure of the entropy of an different imaging planes for holographic optical elements. The point spread function of the holographic lens is recorded with a CCD camera for different imaging planes, taking as origin the Gaussian image plane. The CCD camera is controlled with a computer, and axial displacements of 1 μm are possible. With a computer program, the Point Spread Functions obtained are normalized to the total intensity of the plane, and then we have a probability distribution associated to every imaging plane. Using the definition of entropy, we numerically calculate the value of this magnitude. We represent the entropy as a function of distance from the Gaussian image plane and compare the results with teorethical predictions. A good agreement between theory and experience is found, so the concept of entropy could be used for find the best image plane in holographic optical elements.

Keywords: holographic optical elements, aberrations, diffractinal analysis, entropy function.

I. INTRODUCTION

An evaluation of the classical or holographic optical systems imaging quality is possible by using the diffraction theory of aberrations [1-5]. The diffraction image given by an optical system from a point is known as the point spread function (PSF) and this optical response function can be calculated from the design data [6]. This is still of great use to study the performance of an optical system with aberrations. By using this theory it is possible to directly calculate the light intensity distribution on the image plane. In previous papers we used the diffraction theory to address a series of questions including intensity distribution on an image plane and its characterization through a series of statistical moments [7] and the axial irradiance [8] of a HOE. Recently, the concept of entropy has been used to locate the position of the best image plane for a HOE in the presence not only of small but also large aberrations [9]. In this paper we are going to use an experimental optical set-up to find out how all these theoretical results can be experimentally applied. We will begin with a detailed description of the experimental set-up and then we will present the most relevant results we obtained. Let us outline the propierties of hologram aberration derived by Champagne [10]. We assume that the HOE is located on the the XY plane and that the origin $Q(x_q, y_q, z_q)$, -with $q = r$ (reference), o (object), c (reconstruction) and i (image) - of a spherical wavefront is defined in terms of the parameters R_q , α_q , and β_q , as can be seen in Figure 1. Recording and reconstruction wavelengths are λ_r and λ_c , respectively. The pupil function $U(x, y)$ having the phase aberration Δ at a point (x, y) in the hologram plane, $\Delta(x, y)$, is represented as:

$$U(x, y) = \exp [i \Delta(x, y)] \quad (1)$$

The phase aberration Δ is related to the wavefront aberration W according to $\Delta = (2\pi/\lambda_c)W$, where $W(x, y)$ can be written as:

$$W = r_c - r_i \pm \mu(r_o - r_r) - [R_c - R_i \pm \mu(R_o - R_r)] \quad (2)$$

where $\mu = \lambda_c/\lambda_r$. The geometry for calculating the diffraction patterns is shown in Figure 2. We introduce a local coordinate frame $X'Y'Z'$ fixed to the Gaussian image point G as the origin. The Z' axis is defined by the principal ray which runs from the center of the hologram to the Gaussian image point and we choose the $X'Y'$ plane as the image plane, so that (x', y') are coordinates of a image point H in this plane while the coordinates of this image point in the

XYZ coordinate system are (x_i, y_i, z_i) . The intensity of the image in a plane normal to the chief ray (Z' axis) at a distance z' from the center of the HOE may be written as [7]:

$$I(x', y', z') = \frac{1}{|B|^2} \left| \iint_{S'} \exp[i \Delta(x, y; x', y', z')] dx dy \right|^2 \quad (3)$$

where S' represents the area of the exit pupil where the integration is done and B is the amplitude at the Gaussian image point ($x' = y' = 0$) in the absence of aberrations. Numerical calculations are necessary for obtaining equation (3). Intensity can be interpreted as a probability density function, provided that it is adequately normalized as:

$$P(x'_j, y'_k; z') = \frac{I(x'_j, y'_k; z')}{\sum_{p=1}^P \sum_{q=1}^Q I(x'_p, y'_q; z')} \quad (4)$$

For an image plane situated at a z' position, we can define the entropy of the image formed on this plane as:

$$S(z) = - \sum_{p=1}^P \sum_{q=1}^Q P(x'_p, y'_q; z') \ln [P(x'_p, y'_q; z')] \quad (5)$$

Entropy should be interpreted as a property of the way energy is distributed on an image plane intensity, and, in this sense, The lowest entropy plane will be the best image plane in the sense of minimal aberrations.

Figure 1: Geometry used for the HOE analysis. Figure 2: Geometry used for calculating the diffraction patterns.

II. EXPERIMENTAL

A holographic lens was recorded with monochromatic light from a 40 mW He-Ne laser which had a recording wavelength of $\lambda_r = 633$ nm, a beam ratio of 1:1 in the central zone of the lens and an exposure of 40 mW/cm². The diameter of the lens was $D = 10.45$ mm and the recording material that we used was Agfa 8E75 HD photographic emulsion processed with a PAACM developer and a rehalogenating bleach. Details of the processing schedule as well

as the developer and bleach formulas are given in Tables I and II.

During recording, a collimated beam was made to cross a divergent beam in a totally asymmetrical set-up with the distance from the source point of the plate to the center of the plate being 40 cm. Once processed, the plate was reconstructed with light from the same laser that was used in the recording stage ($\lambda_c = \lambda_r = 633$ nm). As was stated previously, this light was repositioned so that a convergent wave was reconstructed which was made from the divergent recording wave with maximum diffraction efficiency.

Table I: Processing schedule

S T E P	
1.	Develop with PAACM developer for 4 minutes at 20 °C
2.	Rinse in running water for 1 minute
3.	Bleach with R-10 for 1 minute at 20 °C
4.	Wash in running water
5.	Dry in air

Table II: Developer and bleach formulas

<i>Developer PAACM</i>	
Ascorbic Acid	18 g
Sodium carbonate	120 g
Phenidone	0.5 g
Methol	2 g
Destilled water to make	1 l
<i>Bleach R-10</i>	
Sulphuric Acid	10 cc
Potassium Dichromate	2g
Potassium Bromide	35 g
Destilled water to make	1 l

The holographic lens is reconstructed with a - 413 cm divergent beam. During reconstruction the lens is rotated 180° to obtain a convergent image wave. The parameters of the lens shown in Table III are used. Using the recording geometry shown in Table III, we can deduce that the dominant aberration in the recorded holographic lens is astigmatism.

Table III: Recording and reconstruction geometry parameters of the lens

$R_r = \infty$	$\alpha_r = 30^\circ$	$\beta_r = 0^\circ$
$R_o = 40$ cm	$\alpha_o = 0^\circ$	$\beta_o = 0^\circ$
$R_c = -413$ cm	$\alpha_c = 0^\circ$	$\beta_c = 0^\circ$
$\lambda_r = \lambda_c = 633$ nm	$D = 10.45$ mm	

In order to calculate entropy experimental diffraction figures must first be obtained. Figure 3 shows the experimental set-up that was used to obtain entropy. In alignment with the optical axis of the lens we find a CCD camera which is also mounted on a micropositioning device which allows the distribution of intensities to be captured in any image plane perpendicular to the optical axis. The CCD camera is connected to a PC with a digitalized card. The image is initially digitalized with the software package. In order to calculate entropy a total of 22 images were captured. The position of the CCD camera was moved 1 mm between images, covering a total distance along the optical axis of 22 mm. In order to calculate entropy it is necessary to work with the discrete distribution of probability associated with each image plane, $P_{jk} = P(x'_j, y'_k; z')$ and the entropy associated with the plane, S , can be numerically calculated by using equations (4) and (5).

Figure 3: Optical set-up used to obtain entropy.

Figure 4 shows the entropy obtained experimentally when the intensity was corrected for background noise in each plane. As can be seen, the curves coincide both qualitatively and quantitatively. As was indicated previously, when this recording and reconstruction geometry is used, the lens is essentially astigmatic. Therefore the lens has two Sturm focal points (Sturm's focals) and the circle of minimum astigmatic confusion is found exactly half way between these two focal points.

III. FINAL REMARKS AND CONCLUSIONS

We performed an experimental study of a transmission HOE using Agfa 8E75 HD photographic emulsion and an R-10 rehalogenating bleach without a fixation step. The diffractive analysis of these elements was investigated, and the entropy calculated from the light intensity distribution on an image plane measurements are provided. Using the values of the entropy we have obtained the best imaging plane for an holographic lens whose dominant aberration is astigmatism. The analysis shown in this paper give us important evidence about the possibilities of the entropy for the study of imaging quality in holographic systems. Finally, the experimental study presented in this paper can also be extended to conventional optics.

Figure 4: Comparison between the theoretical calculated entropy and the experimental entropy.

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