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EFFECTIVE HOLOGRAPHIC GRATING MODEL TO ANALYZE THICK HOLOGRAMS

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ABSTRACT

In this paper, we present a model in which we associate a recorded and processed volume holographic grating and an "effective diffraction grating". The "effective grating" is determined by the grating vector after processing and has an "effective thickness". This effective thickness can then be used to express the relation that exists between the components of the grating vector before and after processing. This thickness determines the direction of the reconstruction beam when Bragg's Law is complied with. Experimental results with holographic gratings are presented.

1. INTRODUCTION

There are two important questions associated with the making of volume holographic optical elements (HOEs). One question is the design and analysis of the HOE¹. In this area we can consider the recording and reconstruction conditions (geometry and/or wavelength) of the element, its function in an optical system, the aberrations associated with HOEs and the necessity of fulfilling Bragg's Law during the readout in order to achieve high diffraction efficiency. The second question is the recording material used to record the HOE. In order to analyze a recording material for the fabrication of a HOE, we must consider several holographic parameters such as diffraction efficiency, angular and spectral selectivities, signal-to-noise ratio, environmental stability and refractive index and thickness change during the fabrication process. Some papers^{2,3} have been published that combine these two aspects in the fabrication of a HOE, but all of them are theoretical studies and they do not present experimental results.

In this paper we consider an important aspect of the second question: Variations in the average refractive index and thickness of the holographic recording material introduced by the photochemical process in a HOE. These changes produce a reordering of the internal structure of the interference fringes so we can see that the processed and recording elements are different. As a result of the variations in thickness and refraction index the reconstruction geometry corresponding to maximum diffraction efficiency has to be changed with respect to the obtention geometry, which evidently gives rise to the appearance of aberrations in the HOE during the reconstruction stage⁴.

In order to study the influence of the process in a volume holographic grating, we have developed a geometrical model⁵ in which we associate a recorded and processed grating and an "effective diffraction grating". The "effective grating" is determined by the grating vector after processing and has an "effective thickness". This thickness determines the direction of the reconstruction beam when Bragg's Law is complied with. The model has been experimentally analyzed by means of diffraction gratings made with different

recording geometries and exposures, and with processings typical of bleached emulsions, both in solvent and rehalogenation processings, and by studying the variations in the angle of reconstruction corresponding to maximum diffraction efficiency. These angle variations have been quantitatively related to the "effective thicknesses" of the model.

In the following we shall briefly present the "Effective Holographic Grating Model" (E.H.G.M.) applied to the case of holographic gratings. We shall also present some experimental results regarding the meaning of the "effective thickness" in transmission gratings formed in bleached photographic emulsions.

2. THEORETICAL DEVELOPMENT OF THE MODEL

2.1.- Description of the processing effect in recording material

The HOEs are usually fabricated in volume phase materials such as photographic emulsions and dichromated gelatin because only volume phase holograms can produce diffraction efficiencies of nearly 100%⁶. In the photochemical processing of these recording materials we use liquid solutions. There are three important consequences of this processing on the recorded element and on the material: (1) A change in the average refractive index; (2) a deformation of the recording material, and (3) a non-uniformity of the interference fringes through the material thickness. In our model we don't include the last aspect, which is more important in absorption holograms.

The deformation phenomena is characteristic of gelatin-based materials with liquid solution processing. The gelatin is elastic in behavior, and stresses on the volume of the material appear during processing. If we record a diffraction grating, the periodical structure of interference fringes will modify the elastic behavior of the medium. Equally, changes in the gelatin will modify the recorded interference fringes' structure. Due to the compensation of elastic forces during processing, there will be a deformation of the volume material and a change in the interference fringes' orientation. As a results of these variations, the reconstruction geometry for maximum diffraction efficiency will be different from the recording geometry.

2.2.- Theory

In the first place we consider the interference fringes (named Bragg planes) in a holographic grating produced by two plane waves, as shown in Fig. 1. We place the X axis in the plane of incidence and parallel to the surface of the hologram in contact with the glass backing, the Z axis perpendicular to the surface of the hologram, and the Y axis perpendicular to the paper. The initial thickness of the medium is t_R and the final thickness, before processing, is t_C . In this figure $\mathbf{K}$ is orientated perpendicular to the Bragg planes and is the grating vector of the recording grating, d_x and d_z are the distances between the Bragg planes in the X and Z directions, Λ is the grating spacing and ϕ is the angle of Bragg plane inclination with respect to the Z axis. The expression of the grating vector is⁶:

$$\mathbf{K} = \frac{2\pi n_R}{\lambda_R} \begin{bmatrix} \sin \alpha'_R - \sin \alpha'_O \\ 0 \\ \cos \alpha'_R - \cos \alpha'_O \end{bmatrix} \quad (1)$$

where α'_R and α'_O are the angles of incidence of the reference and object waves in the medium, respectively, with respect to the Z axis, and λ_R is the wavelength in free space in the recording process.

In order to quantify the deformation of the holographic grating due to the processing, we introduce a model based on three hypotheses. These hypotheses are:

- 1.- A change in the average refraction index of the material due to processing. n_R and n_C will be the average refractive index of the medium before and after processing, respectively.
- 2.- A homogeneous deformation of the recording material: The "Bragg planes" before and after processing are "planes".
- 3.- Due to the processing the position of the points in the medium in contact with the glass backing do not change. If these positions change, the grating will be destroyed.

Now, we want to write the analytic expression that relates the (x, y, z) coordinates of a point of the medium before processing with (x^* , y^* , z^*) coordinates of the same point after processing, when we apply hypotheses 2 and 3 of the model.

Fig. 2 shows the holographic grating after processing. Now, K^* , ϕ^* , d_x^* , d_z^* , Λ^* , t_C and n_C are the new parameters of the grating.

Fig. 1.- Recording geometry of holographic grating (before processing).

Fig.2.- Holographic grating after processing.

The relationship that exists between the coordinates of the points on the grating before and after processing can be expressed in the following manner⁵:

$$x^* = x + z \cdot \tan \delta \quad (2)$$

$$z^* = T \cdot z \quad (3)$$

Similarly, the relationship between the grating vectors $\mathbf{K}$ y $\mathbf{K}^*$ are expressed as:

$$K_x^* = K_x \quad (4)$$

$$K_z^* = (-\tan \delta \cdot K_x + K_z) / T \quad (5)$$

where $T = t_C / t_R$ y δ is the "cut" or "shear" angle, similar to those described in the theory of elasticity as can be seen in Fig. 3.

Fig. 3.- Holographic grating (a) before and (b) after processing.

From a physical point of view it is clear that the shear effect is due to the inclination itself of the Bragg planes, and therefore angle δ must be a function of angle ϕ . This effect can be explained by considering the turning of the Bragg planes, or, in other words, the tendency of the medium to move laterally due to the planes themselves. Moreover, this δ angle is small (only a few grades) due to the elasticity of the recording material (in the emulsions, especially in gelatin bases).

2.3.- The introduction of effective thickness

In the previous section we found the relationship that exists between the points on a holographic grating before and after processing by using T and δ . While the value of T can be determined through experimentation by measuring the "physical" thicknesses of the medium before and after processing, the direct measurement of δ is obviously quite complicated. In order to relate this "shear angle" with the parameters characteristics of the grating, we can refer to Fig. 4 shows the change in an Bragg plane OP due to processing when affected by the lineal transformation given in equations (2)-(3).

Fig. 4.- Relation between the grating parameters before and after processing.

The following relationship can be obtained from Fig. 4:

$$\operatorname{tg} \delta = T \cdot \operatorname{tg} \phi^* - \operatorname{tg} \phi \quad (6)$$

This expression relates the angle of Bragg plane inclination before and after processing and the real quotient of the thicknesses, T , with the δ angle. By introducing equation (6) into equations (4)-(5), the components of the grating vector after processing are expressed as follows:

$$K_x^* = K_x \quad (7)$$

$$K_z^* = [\operatorname{tg} \phi - T \cdot \operatorname{tg} \phi^*] \cdot K_x + K_z \quad (8)$$

Keeping in mind that $\operatorname{tg} \phi = -K_z / K_x$ and $\operatorname{tg} \phi^* = -K_z^* / K_x^*$, it is possible to uncouple equation (8) so that:

$$K_z^* = \frac{\operatorname{tg} \phi^*}{\operatorname{tg} \phi} K_z \quad (9)$$

and from Fig. 4 it is easy to see that the quotient of the tangents can be expressed as:

$$\frac{\operatorname{tg} \phi^*}{\operatorname{tg} \phi} = \frac{t_R}{t_e} \quad (10)$$

where $t_e = ON$ gives the "effective thickness" of the holographic grating. If we call $T_e = t_e / t_R$, the equation (9) becomes:

$$K_z^* = \frac{1}{T_e} K_z \quad (11)$$

The effective thickness plays a fundamental role in volume holographic gratings because it is directly related to the reconstruction angle which complies Bragg's Law. If this angle is α_C , the following equation can be obtained⁵ for transmission gratings by developing equation(11):

$$\text{sen } \alpha_C = \frac{1}{2} \left[\frac{N}{T_e} + \mu \right] \text{sen } \alpha_R + \frac{1}{2} \left[\frac{N}{T_e} - \mu \right] \text{sen } \alpha_O \quad (12)$$

and it is possible to determine the T_e/N quotient through experimentation by using:

$$\frac{T_e}{N} = \frac{\text{sen } \alpha_R + \text{sen } \alpha_O}{2 \text{sen } \alpha_C + \mu (\text{sen } \alpha_O - \text{sen } \alpha_R)} \quad (13)$$

where $N = n_C / n_R$ and $\mu = \lambda_C / \lambda_R$. In other words, the measurements of the recording angles and the reconstruction angle in compliance with Bragg's Law does not give the actual thickness but rather the effective thickness.

For a reflection grating, the following is obtained by applying equation (11):

$$T_e = \frac{\mu}{N} \quad (14)$$

Nevertheless, in cases of reflection, good experimental results for T can be obtained with the $T = \mu/N$ equation.

Additionally, by using expressions (6) and (10), we can conclude:

$$T_e = \frac{\text{tg } \phi}{\text{tg } \delta + \text{tg } \phi} T \quad (15)$$

Equation (15) produces the following conclusions:

- The greater the difference between angles δ and ϕ , the more similar the values of T and T_e .
- When ϕ is small, the difference between T and T_e is greater.
- When ϕ is around 90° (reflection grating), the T and T_e parameters coincide, so that for reflection gratings $T_e \approx T$.
- When $\phi \approx \delta$, then $T_e \approx 0.5 T$.
- In symmetric gratings ($\phi = 0^\circ$), δ will equal 0° .

Fig. 5 shows some of the theoretical conclusions of the model. The effective thickness has been shown as a function of the Bragg plane inclination angle using equation (15) at a fixed cut angle $\delta = 1.6^\circ$ (Fig. 5.a) and at a fixed real thickness, $T = 0.85$ (Fig. 5.b).

Fig. 5.- T_e as a function of the angle ϕ for (a) a fixed angle δ , and different values for parameter T , and (b) a fixed T and different values for angle δ .

These figures show that "cut-type" deformations which have not been previously dealt with in the literature and which in spite of being negligible in terms of relative displacement, can indeed be significant in terms of angles, as long as the Bragg plane inclination angle is not very large (as is the case in transmission holograms).

3. EXPERIMENTAL RESULTS

The model has been experimentally analyzed by means of diffraction gratings in photographic plates Agfa-Gevaert 8E75 HD, with 633 nm wavelength, made with different recording geometries and exposures, and with processings typical of bleached emulsions, both in solvent and rehalogenation processings, studying the variations in the angle of reconstruction corresponding to maximum diffraction efficiency. These angle variations have been quantitatively related to the "effective thickness" using equation (13) with $\mu = 1$.

Fig. 6 shows the experimental results obtained for the effective thickness (T_e) and the physical thickness (T) as a function of exposure for a transmission grating with recording angles of $\alpha_R = 5^\circ$ and $\alpha_0 = -35^\circ$, which produces a Bragg plane inclination angle of $\phi = 8.8^\circ$. AAC developer and rehalogenation bleaching have been used in the processing (Fig. 6a) as have AAC developer and solvent bleaching (Fig. 6.b). Fig. 7.a and 7.b show the values of the δ angle as a function of exposure which correspond to Fig. 6a and 6.b, respectively.

(a)

(b)

Fig. 6.- Effective thickness (T_e) and real thickness (T) as a function of exposure for (a) rehalogenation bleaching and (b) solvent bleaching.

(a)

(b)

Fig. 7.- Cut angle, δ , as a function of exposure for (a) rehalogenation bleaching and (b) solvent bleaching.

4. CONCLUSIONS

In this paper we have introduced a new model for holographic gratings (the "effective Holographic Grating Model" E.H.G.M.) which is more complete due to the fact that the model takes into account deformations in the recording material which have not been previously contemplated in the literature. These deformations can play a fundamental role in the reconstruction geometry of a holographic grating for maximum diffraction efficiency. The introduction of a new parameter, the effective thickness of the holographic grating, which characterizes recording material deformations due to processing, allows us to compare the components of the grating vector before and after processing. Since reconstruction geometry for maximum diffraction efficiency depends upon the grating vector, this effective thickness conditions the reconstruction stage of the holographic grating. Moreover, this model justifies the experimental results that have been obtained in the analysis of more than 240 transmission holographic gratings submitted to both rehalogenation and solvent bleaching, some of which have been cited as examples in this paper.

It is possible to apply E.H.G.M. to Holographic Lenses⁵ by using an approximation of the local grating which produces a set of relationships between the coordinates of the recording and reconstruction sources when spherical wave fronts are used and Bragg's Law is complied with in the reconstruction stage. In these expressions T_e , N and μ parameters all play a similar role. In such a case it is possible to find geometries in which the combinations of these three parameters gives way to situations in which the aberrations due to changes in geometry and wavelength are minimized, and, at the same time, high diffraction efficiencies are achieved. This opens a new door in the field of HOE design in which the recording material plays an essential role.

5. REFERENCES

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