INFLUENCES OF RECORDING MATERIALS IN HOE

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ABSTRACT

When recording thick holograms, Bragg's law must be taken into account; this imposes additional limitations on the geometry of the recording sources if the reconstruction geometry is fixed. However, the results we have reported from previous papers do not enable the recording geometry required to give maximum diffraction efficiency of the hologram to be calculated for an arbitrary specified reconstruction geometry when there is a possible change in the thickness and refractive index of the recording medium. Bleached silver halide, dichromated gelatin and silver halide sensitized gelatin are well known photographic techniques for phase holographic formation in commercially available emulsion.

These techniques are applicable to volume phase formation. This occurs when the refractive index and/or thickness of the exposed/non-exposed portion of the emulsion varies relative to the bulk unexposed portion. We present our theoretical expressions of third-order aberrations in off-axis holographic lenses of different r/No, taking into account the index and thickness variations of the recording materials.

1. INTRODUCTION

In recent years much research has been done and many papers have been written on the design and making of Holographic Optical Elements (HOE). As regards the topic we are addressing, two main areas of research focus are of interest. The first area relates to the geometric aspects of the design and making of these elements. Research in this area focuses on the geometry and disposition of beams during the recording and reconstruction of the HOE, and are based to a large extent on image quality studies through reconstituted wave front aberrations. The second research area deals with the analysis of the reconstruction conditions necessary to achieve maximum diffraction efficiency according to the recording material in which HOE related information will be stored. In this second area we are dealing with thick transmission holograms.

This division into two main areas of interest is based on the fact that while the superficial structure of the interference fringes is the element that determines the position and quality of reconstructed wave fronts (first area), it is the internal structure of these fringes (second area) which largely determines diffraction efficiency.

Several papers have been published recently that combine these two aspects. Articles authored by Winick and Assenheimer present the mathematical expressions for recording wave fronts which allow a high degree of efficiency with minimum aberration levels in order to achieve fixed reconstruction geometry in the presence of a construction-reconstruction wavelength shift.

In this paper we will present equations for thick transmission holograms that allow high efficiency not only when the wavelength shifts between the recording and reconstruction stages, but also when the average thickness of the recording material and the average refractive index changes as well. We will also present equations that represent third-order aberrations which appear in a reconstructed wavefront when changes in wavelength, thickness and refractive index are taken into consideration. We want to emphasize that as regards aberrations, variations in thickness and refractive index can be equal in importance or even exceed in importance the utilization of different wavelengths.

We will denote by R, O, C and I the point sources which illuminate the object and reference light beams, and the light beams which reconstruct and form the image, respectively. Their positions may be determined in the XYZ space by finding three parameters: $R_i$, $\alpha_i$ and $\beta_i$, where $i = R, O, C, I$ (Figure 1). $R_i$ is the distance from the source $i$ to the center of the hologram:

$$R_i = \sqrt{x_i^2 + y_i^2 + z_i^2}$$

in this equation $(x_i, y_i, z_i)$ are cartesian coordinates for the point source $i$. The sign of $R_i$ is chosen equal to the sign of the corresponding coordinate $z_i$. The parameters $\alpha_i$ and $\beta_i$ are the angles between beams and their projection over the YZ and XZ planes respectively, thus:

\begin{align}
\sin \xi_i &= \frac{x_i}{R_i} \\
\sin \psi_i &= \frac{y_i}{R_i}
\end{align}

Figure 1: Geometry for recording and readout. An arbitrary point source \( i \) at \( x_i, y_i, z_i \) situated in front of a hologram in the XY plane.

In order to quantify the changes in the refractive index and the thickness of the recording material we will introduce factors that take into consideration the recording medium and possible changes during chemical processing. These parameters are:

- Thickness factor \( T = t_c/t_R \), where \( t_R \) and \( t_c \) represent the thickness of the medium at the recording and reconstructing stages, respectively.

- Index factor \( N = n_c/n_R \), where \( n_R \) and \( n_c \) represent the refractive index of the medium at the recording and reconstructing stages, respectively.

Let us consider a shape factor "q" of HOE that we wish to design. This hologram q factor can be written as:

\[ q = \frac{R_C + R_I}{R_C - R_I} \tag{4} \]

The last factor that we will consider is the wavelength-shift ratio \( \mu_c = \lambda_c/\lambda_R \), where \( \lambda_R \) is the recording wavelength and \( \lambda_c \) is the reconstruction wavelength.

2. HOLOGRAM IMAGERY AND ABERRATIONS

By applying Bragg's law, which ensures a high level of efficiency, and by taking into account the aforementioned factors, we can obtain, for a specified reconstruction geometry, the required expressions for the coordinates of the recording sources which satisfy Bragg's law:

\[ R_R = t \frac{2 \mu \lambda_f}{q \mu_c (T/N) - 1} \tag{5} \]

\[ R_0 = t \frac{2 \mu \lambda_f}{q \mu_c (T/N) + 1} \tag{6} \]
\[
\sin \alpha_R = \pm \left( \frac{1}{2} \mu \right) \left\{ \frac{\mu (T/N) + 1}{\mu (T/N) - 1} \right\} \sin \alpha_C + \frac{\mu (T/N) - 1}{\mu (T/N) + 1} \sin \alpha_T
\]

(7)

\[
\sin \alpha_0 = \pm \left( \frac{1}{2} \mu \right) \left\{ \frac{\mu (T/N) - 1}{\mu (T/N) + 1} \right\} \sin \alpha_C + \frac{\mu (T/N) + 1}{\mu (T/N) - 1} \sin \alpha_T
\]

(8)

\[
\sin \beta_R = \pm \left( \frac{1}{2} \mu \right) \left\{ \frac{\mu (T/N) + 1}{\mu (T/N) - 1} \right\} \sin \beta_C + \frac{\mu (T/N) - 1}{\mu (T/N) + 1} \sin \beta_T
\]

(9)

\[
\sin \beta_0 = \pm \left( \frac{1}{2} \mu \right) \left\{ \frac{\mu (T/N) - 1}{\mu (T/N) + 1} \right\} \sin \beta_C + \frac{\mu (T/N) + 1}{\mu (T/N) - 1} \sin \beta_T
\]

(10)

In these expressions \( f \) is the hologram focal length defined by:

\[
\frac{1}{f} = \frac{1}{R_I} - \frac{1}{R_C} = \pm \mu \left( \frac{1}{R_0} - \frac{1}{R_R} \right)
\]

(11)

As can be seen from the above equations, when the refractive index and thickness change and the recording and readout wavelengths are different, the recording holographic optical element is different from the one we want to reconstruct, thereby assuring the presence of aberrations. If we define the shape factor "p" of the recording element as

\[
p = \frac{R_R + R_0}{R_R - R_0}
\]

(12)

and if we utilize equations (5) and (6), we see that we do not obtain the same result as in equation (4), but rather we find that \( p \neq q \), and only when \( \mu T/N = 1 \) do the two factors coincide.

Note that even when \( \lambda_R = \lambda_C \) (\( \mu = 1 \)), we find that \( p = q \), and only when \( \mu T/N = 1 \) do the two factors coincide.

Aberrations will occur as a result of the change in the geometry of the beams between the recording and reconstruction stages of the HOE. A relation of the equations that correspond to third-order aberrations is as follows. The third-order aberrations, for which \( \Delta_S, \Delta_C \) and \( \Delta_A \) are the spherical, comatic, and astigmatic aberrations, are given by:

\[
\Delta_3 = \Delta_S + \Delta_C + \Delta_A
\]

(13)

Where:

\[
\Delta_S = - \frac{1}{8} \lambda_C (x^2 + y^2)^2 S
\]

(14)

\[
\Delta_C = \frac{1}{2} \lambda_C (x^2 + y^2) (x \xi_x + y \xi_y)
\]

(15)

\[
\Delta_A = - \frac{1}{2} \lambda_C (x^2 A_x + y^2 A_y + xy A_{xy})
\]

(16)

and \( x \) and \( y \) are the hologram coordinates, and \( S, C_x, C_y, A_x, A_y, \) and \( A_{xy} \) are the classical Seidel aberration coefficients. When the reconstruction geometry is given, when a high level of efficiency is desired and when the aforementioned parameters are respected, these coefficients are written as follows:

\[
S = (1/4 \mu^2 T^3) \left\{ 1 - \mu^2 - 3 \mu^2 q^2, [1 - (T/N)^2] \right\}
\]

(17)

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\[
C_x = \frac{(1/4\mu^2)^2}{1} \left[ \mu^2 - 1 \right] (\sin\alpha_C - \sin\alpha_I) + \mu^2 \left[ 1 - \left( T/N \right)^2 \right] \left[ (q - 2)\sin\alpha_C - (q + 2)\sin\alpha_I \right]
\]
\[
C_y = \frac{(1/4\mu^2)^2}{1} \left[ \mu^2 - 1 \right] (\sin\beta_C - \sin\beta_I) + \mu^2 \left[ 1 - \left( T/N \right)^2 \right] \left[ (q - 2)\sin\beta_C - (q + 2)\sin\beta_I \right]
\]
\[
A_x = \frac{(1/4\mu^2)^2}{1} \left[ \mu^2 - 1 \right] (\sin\omega_C - \sin\omega_I) + \mu^2 \left[ 1 - \left( T/N \right)^2 \right] \left[ (\sin^2\alpha_C + \sin^2\alpha_I) - 2q(\sin^2\alpha_C - \sin^2\alpha_I) \right]
\]
\[
A_y = \frac{(1/4\mu^2)^2}{1} \left[ \mu^2 - 1 \right] (\sin\beta_C - \sin\beta_I) + \mu^2 \left[ 1 - \left( T/N \right)^2 \right] \left[ (\sin^2\beta_C + \sin^2\beta_I) - 2q(\sin^2\beta_C - \sin^2\beta_I) \right]
\]
\[
A_{xy} = \frac{(1/4\mu^2)^2}{1} \left[ \mu^2 - 1 \right] (\sin\omega_C - \sin\omega_I) (\sin\beta_C - \sin\beta_I) + \mu^2 \left[ 1 - \left( T/N \right)^2 \right] \left[ (\sin^2\alpha_C + \sin^2\alpha_I) (\sin^2\alpha_C + \sin^2\alpha_I) - 2q(\sin^2\alpha_C - \sin^2\alpha_I) \right]
\]

Knowledge of these aberrations in relation to the parameters \( \mu, N, T \) and \( q \) allows the design of an optimum holographic optical element. By applying this knowledge to the selection and adequate processing of the recording material (\( N \) and \( T \)), as well as to the selection of the wavelengths (\( \mu \)), and by using the appropriate geometric parameters (\( q, f, \alpha_I \) and \( p_I \)), an HOE which performs the desired functions and contains minimal aberrations can be designed and produced.

From this point, we should, in principle, be able to develop two areas of research related to recording materials. The first would entail a complete and detailed analysis of the variations in the index and thickness of those recording materials during different chemical processing. The second would include the study of the recording geometries also in relation to the changes produced in the HOE during the same chemical processing. Later on in this paper we will see that other factors must be considered.

3. EXPERIMENTAL PROCEDURE AND RESULTS

Variations in the average refractive index and thickness of the recording material is present mainly during chemical processing. These changes produce a reordering of the internal structure of the interference fringes. In some instances this reorganization can be so drastic that the end-element and the registered element are completely different. For example, a silver halide sensitized gelatin (SHSG) is first a silver halide emulsion and finally, after adequate chemical processing, becomes dichromated gelatin with an important change in the refractive index. If the HOE is recorded in a dichromated gelatin and an adequate chemical process is applied, the final thickness can be very different from the initial one.

In order to make an experimental determination of the changes in \( n \) and \( t \), we will record transmission holographic gratings with two collimated beams \( R \) and \( O \) propagating in the \( XZ \) plane at a fixed angle of incidence in the air. In this case \( \alpha_R \) and \( \alpha_O \) will be relative to the normal but not equal to each other in absolute value. After processing we will measure the reconstruction angle for maximum efficiency. From equations (7) and (8) we can calculate the parameter \( T/N \) as:

\[
\frac{T}{N} = \frac{\sin\alpha_R + \sin\alpha_O}{2 \cdot \sin\alpha_I} \mu (\sin\alpha_O - \sin\alpha_R)
\]

where, for simplicity's sake, we have considered the usual situation in which \( p_I = 0 \), \( i = R, O, c, I \). If \( \mu = 1 \) and with \( \pm \) sign (reconstruction and recording are similar), we have:

\[
\frac{T}{N} = \frac{\sin\alpha_R + \sin\alpha_O}{2 \cdot \sin\alpha_C} \sin\alpha_O - \sin\alpha_R
\]

By measuring the \( \alpha \) angle which produces maximum efficiency (Bragg's angle) during reconstruction and by knowing the recording angles, \( \alpha_O \) and \( \alpha_R \), we can obtain the corresponding \( T/N \) value for each case.

In general \( T/N \) is a function of the recording medium and the chemical processing used.
as well as of the following factors:

1) Exposure during recording.
2) Beam relationship.
3) Beam polarization.
4) Spatial frequency.
5) The geometric disposition of the recording beams.

In this first stage we are analyzing how points 1 and 4 affect bleached photographic emulsions. We have obtained diffraction gratings at photographic plates Agfa-Gevaert 8656 HD, with 514 nm wavelength from several exposures and spatial frequencies. After exposure, the plates were developed in an ascorbic acid developer. The developed plates were then rinsed briefly and bleached. A rehalogenating R-10 bleach bath was used in these first experiments. Table I shows the results obtained.

Table I: Exposure and corresponding values of T/N for different $\alpha_0$ and $\alpha_R$ angles.

<table>
<thead>
<tr>
<th>Exposure ($\mu J/cm^2$)</th>
<th>$\eta_{\max}$ (%)</th>
<th>T/N</th>
<th>Exposure ($\mu J/cm^2$)</th>
<th>$\eta_{\max}$ (%)</th>
<th>T/N</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>34</td>
<td>0.854</td>
<td>60</td>
<td>30</td>
<td>0.891</td>
</tr>
<tr>
<td>60</td>
<td>27</td>
<td>0.822</td>
<td>125</td>
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<td>23</td>
<td>0.791</td>
<td>165</td>
<td>27</td>
<td>0.785</td>
</tr>
</tbody>
</table>

4. A NUMERICAL EXAMPLE

Let's consider the particular case of a flat divergent symmetric holographic lense ($\alpha_c = -\alpha_R$) where $B_1 = 0$, $i = R$, 0, C, I. Given the current importance of holographic systems in which a small f/No. is required, to obtain maximum efficiency, the holographic lense to be studied will be f/1 (HOE diameter, D, is equal to the focal length of the same, f). The reconstruction shape factor will be q = -1, while the lense to be recorded will be such that p = $-\mu T/N$. Additionally, $\mu = 1$, which will produce $p = -T/N$. As a result of the $f/No.$, $(x = D/2, y = 0)$, the expressions for the spheric, comatic and astigmatic aberrations for the plate point $(x = D/2, y = 0)$ are:

$$\Delta_s = \frac{3D}{512\lambda_c f^3} \left(1 - \frac{T^2}{N^2}\right)$$

$$\Delta_c = \frac{D \cdot \sin\alpha_C}{32\lambda_c f^2} \left(1 - \frac{T^2}{N^2}\right)$$

$$\Delta_A = 0$$

$$\sin\alpha_R = -\sin\alpha_0 = \sin\alpha_C.$$

As numerical values in the f/1 case, we will use $D = 10$ cm, $\alpha_c = 21^\circ$, $\lambda_c = 514$ nm which will produce $\alpha_R = -\alpha_0 = 21^\circ$. In figure 2, the $R_3$ value in relation to T/N for different values if f/No. are given. In figure 3 the third-order aberration for $(x = D/2, y = 0)$ in relation to T/N for different f/No. is given. It is evident that the lower the f/No., the larger the value of $\Delta_3$.

Finally, in Figure 4 the total aberration of the wavefront on the surface of the HOE for the previously described numeric case is shown, with f/1 and the corresponding value (see Table I) of T/N being 0.854.

5. CONCLUSIONS

In these preliminary technical results we have been able to deduce that variations in index and thickness influence the performance of an HOE and that even when the HOE is reconstructed according to Bragg's Law, third-order aberrations are present.
Figure 2: The $R_R$ value in relation to $T/N$ for different values of $f/No$, for the described numeric case.

Figure 3: The third-order aberration for $(x = D/2, y = 0)$ in relation to $T/N$ for different $f/No$. 

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Figure 4: a) Surfaces of equal aberration (in wavelength).
   b) The total aberration of the wavefront on the surface of the HOE for the
   described numeric case.
6. REFERENCES