A new theoretical matrix formula for IOL calculation using the optimal bending factor

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S u m m a r y

By applying the Gaussian optics the authors propose a new matrix formula model for IOL calculation which allows us not only to obtain the optical power of the IOL for a particular eye, but different optical designs taking into account the bending factor of a lens. This option improves the selection of the IOL selecting these lens as a function of the best possible optical image, a function which is dependent on the bending factor of the IOL.

This formula constitutes a new approach to IOL calculation and may be particularly useful for the design of IOLs for abnormally short or long eyes. IOL powers might not be the only factor to be considered in IOL calculation, but rather a combination of optimal power together with the optimal design. Such new approach may improve the optical as well as the clinical results of IOL implantation in the aphakic patient.

KEY WORDS: *IOL, Aphakia, Bending factor, Optical powers, Retinal image, Optical aberrations, Matrix calculation.*

INTRODUCTION

– Intraocular lens (IOL) implantation now brings to the ophthalmic surgeon the unique opportunity to achieve two relevant goals in the aphakic eye. One is the restoration of the patient's vision. The second is the correction of the preexisting spherical ametropies by accurately choosing the optical power of the IOL for each eye.

The dioptric power of an IOL is now calculated using one of several formulas. The most popular are formulas such as SRK and others (1) based on statistical regression calculations. Mathematical matrix calculation when applied to the calculation of the power and design of optical systems offers an adequate approach to the optical study of the aphakic eye (2).

In this paper the eye is assimilated to a centered system having several surfaces separating areas of different indexes of refraction (3). Applied to this centered system is the paraxial approximation, in which the rays of light stand at a small angle to the axis, and the angles of incidence on the surfaces are small (4). In a recent publication, Colliac et al have reported on an optimized matrix formula for IOL calculation (5). However, their method is based solely on the optical power calculation for convex-plain IOL s and hence the applications of such a formula are limited to one type of IOL design.

We report herein a new matrix formula which can be applied to the calculation of the optical power of any type of IOL shape or design. With this formula we can obtain different types of IOLs for a given optical power and this allows us to choose the IOL with the best optical properties

and which most improves the optical image for a given eye thereby minimizing the optical aberrations of the lens taking into account the eye in which the lens is implanted.

MATERIAL AND METHODS

Matrix calculation

The eye is assimilated to a centered system having several surfaces separating areas of different indexes of refraction. Optically, there is a homographic relation between the object and the image (6); an object point and its image form a conjugate couple and under the inverse return principle the relationship remains the same if the object becomes image and the image object. The matrix notation determines the IOL that should be implanted in a given aphakic eye.

A matrix is a set of numbers ordered in rows and columns. A two dimensional matrix has two rows and two columns. Such matrixes may be associated with optical surfaces which separate two media with different refraction indexes. The numbers that constitute a matrix contain all the information about geometrical characteristics of surfaces (7).

Let us now considerate the "S" surface represented in figure 1. This surface separates two media whose refraction indexes are n and n' . The matrix which can be associated with this type of surfaces is the refraction matrix $\mathcal{R}$;

$$\mathcal{R} = \begin{bmatrix} a_0 & b_0 \\ c_0 & d_0 \end{bmatrix} \text{ where } a_0 = 1; \quad b_0 = 0; \quad c_0 = \frac{P}{n'}; \quad d_0 = \frac{n}{n'} \quad [\text{I}]$$

$P = (n' - n)/r$ is the optical power of this type of surface and r is the radius of curvature, hence, the final matrix would be:

$$\mathcal{R} = \begin{bmatrix} 1 & 0 \\ \frac{P}{n'} & \frac{n}{n'} \end{bmatrix} \quad [\text{II}]$$

If a beam of light moves from one of the surfaces to the other or even from one point to another, as is shown in figure 2, the matrix which would be associated with this movement of the light beam would be the translation matrix $\mathcal{T}$:

$$\mathcal{T} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \quad \text{where } \begin{matrix} a = 1, & b = -t \\ c = 0, & d = 1 \end{matrix} \quad [\text{III}]$$

and t is the distance between two surfaces and $\delta = t/n$ is the reduced thickness. The resulting matrix would be:

$$\mathcal{T} = \begin{bmatrix} 1 & -t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\delta n \\ 0 & 1 \end{bmatrix} \quad [\text{IV}]$$

If we have a centered optical system which is formed by several surfaces that separate different refractive media, the matrix that

corresponds to the set is a two dimensional matrix which is obtained by multiplying the matrixes that correspond to the different surfaces and movements made by the beam of light. It is important to notice that this product is noncommutative but it is associative and, thus, the multiplications should be made in the order that has been indicated as it is the one that corresponds to the path of the light beam.

Calculation of the ocular matrix

Now let us consider the optical system of the aphakic eye with a posterior chamber IOL. In figure 3 we have represented an aphakic eye with such an IOL and we have divided it into a set of optical surfaces that separate transparent media with different refraction indexes.

The cornea has been represented by two dioptrios, as has the IOL to be calculated. Following the definition of the surfaces and distances named in figure 3, will calculated the following matrixes:

Cornea associated matrix:

The matrix consists of the product of three matrixes: first, the refraction matrix at the first surface $\mathcal{R}_1$, second, the translation matrix of the first to the second surface, $\mathcal{T}_2$ and third, the refraction matrix of the second surface, $\mathcal{R}_3$. Mathematically we shall express the final matrix as:

$$C = \mathcal{R}_3 \mathcal{T}_2 \mathcal{R}_1 = \begin{bmatrix} 1 & 0 \\ \frac{P_3}{n_4} & \frac{n_2}{n_4} \end{bmatrix} \begin{bmatrix} 1 & -t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{P_1}{n_2} & \frac{n_0}{n_2} \end{bmatrix} \quad [\text{V}]$$

The final result is the matrix:

$$C = \begin{bmatrix} 1 - \delta_2 P_1 & -\delta_2 n_0 \\ \frac{1}{n_4} \left[P_1 + P_3 - \delta_2 P_1 P_3 \right] & \frac{n_0}{n_4} \left[1 - \delta_2 P_3 \right] \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} \quad [\text{VI}]$$

The matrix associated with the translation from the second surface of the cornea to the first of the IOL:

$$\mathcal{T}_4 = \begin{bmatrix} 1 & -t_4 \\ 0 & 1 \end{bmatrix} \quad [\text{VII}]$$

An IOL associated matrix:

As when calculating the cornea-associated matrix, here too we shall multiply two refraction matrixes which correspond to the IOL surfaces r_5 and r_7 , and a translation matrix which falls between the first and the second surfaces of the IOL, $\mathcal{T}_6$:

$$L = \mathcal{R}_7 \mathcal{T}_6 \mathcal{R}_5 = \begin{bmatrix} 1 & 0 \\ \frac{P_7}{n_8} & \frac{n_6}{n_8} \end{bmatrix} \begin{bmatrix} 1 & -t_6 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{P_5}{n_6} & \frac{n_4}{n_6} \end{bmatrix} \quad [\text{VIII}]$$

$$L = \begin{bmatrix} 1 - \delta_6 P_5 & -\delta_6 n_4 \\ \frac{1}{n_8} \left[P_5 + P_7 - \delta_6 P_5 P_7 \right] & \frac{n_4}{n_8} \left[1 - \delta_6 P_7 \right] \end{bmatrix} = \begin{bmatrix} l_1 & l_2 \\ l_3 & l_4 \end{bmatrix} \quad [IX]$$

The final result is obtained by multiplying the matrix of cornea, C,
the matrix of translation $\mathcal{T}_4$ and the matrix of IOL, L:

$$M = L \mathcal{T}_4 C = \mathcal{R}_7 \mathcal{T}_6 \mathcal{R}_5 \mathcal{T}_4 \mathcal{R}_3 \mathcal{T}_2 \mathcal{R}_1 \quad [X]$$

$$M = \begin{bmatrix} l_1 & l_2 \\ l_3 & l_4 \end{bmatrix} \begin{bmatrix} 1 & -t_4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 & c_2 \\ c_3 & c_4 \end{bmatrix} = \begin{bmatrix} l_1 & l_2 \\ l_3 & l_4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad [XI]$$

RESULTS

This matrix formula allows us to determine the power of the emmetropizing implant. If we consider that the light beam that enters this optical system is collimated and that the eye is corrected for vision (emmetropia) and if we know the distance from the last IOL surface to the retina l' , we can then obtain the optical power P_7 of the second surface of the IOL to be selected for implantation:

$$P_7 = \frac{n_8 \left[1 - \delta_6 P_5 \right] a - n_8 n_4 \delta_6 c - l' \left[a P_5 + n_4 c \right]}{l' \left[a - \delta_6 P_5 a - n_4 c \delta_6 \right]} \quad [\text{XII}]$$

where

n_8 = refractive index of the vitreous humor

δ_6 = reduced thickness of the IOL

P_5 = optical power of the first IOL surface

n_4 = refractive index of the aqueous humor

a and c = the result of multiplying the $\mathcal{T}_4$ matrix by C

$$a = c_1 - c_3 t_4 \quad \text{in equation [XI]}$$

$$c = c_3 \quad \text{in equation [XI]}$$

Once P_7 has been calculated it is possible to find P , the optical power of the IOL, which should be selected in order to correct the aphakic eye using the equation:

$$P = P_5 + P_7 - \delta_6 P_5 P_7 \quad [\text{XIII}]$$

We have developed a computer program to perform all the above mathematical calculations so that IOL can be implanted in several clinical types of aphakic eyes. The refraction index and the thickness of the different transparent media of the eye were taken as follows (8)(9):

Refractive index:

$$n_0 = 1 = \text{refractive index of the air}$$

$$n_2 = 1.3771 = \text{refractive index of the cornea}$$

$$n_4 = 1.3374 = \text{refractive index of the aqueous humor}$$

$$n_6 = 1.49 = \text{refractive index of the implant}$$

$$n_8 = 1.336 = \text{refractive index of the vitreous humor}$$

Thickness:

$$t_2 = 0.552 \text{ mm} = \text{thickness of the cornea}$$

$$t_6 = 0.7 \text{ mm} = \text{thickness of the implant}$$

All the other data as axial length of the eye, anterior chamber depth and optical power of the cornea are particular data of each eye and should be determined by means of an ultrasonic biometer and a keratometer.

By using the proposed equation [XII] we can obtain the values of P_7 taking into account P_5 ; likewise, the r_7 values regarding from r_5 .

If we already know the values of r_7 and r_5 we can obtain the value of P which corresponds to each coupled curvature radii by using equation [XIII]. In this manner we can obtain a family group of lenses, all of them having the same optical power but with different curvature radii. This family group of curvature radii will allow us to choose those coupled radii

which will result in the best image quality at the retina for each optical power. In other words, it will be possible to decide which IOL design best improves the quality of vision. Because this design does influence the quality of the optical image at the retina, we propose an additional parameter. This parameter is known as the bending factor:

$$q = \frac{r_7 + r_5}{r_7 - r_5} \quad [XIV]$$

Data consisted of four types of eyes with respective axial length ranges of 20.95 - 30.58 mm, and with respective average keratometry ranges of 47.01 - 40.75 D. The results of these eyes are shown in table I. As it can be observed different shaped eyes are emetropized by different designs of IOL. If we wish at the same time an optimal quality of the retinal image, we should use the defined bending factor. Currently most, if not all, IOL are made in one of three forms: convex-flat (curved surface toward the cornea), flat-convex (curved surface toward retina), and equi-convex. These forms have probably not been based on optical design decisions. A previous study (10) has shown that the optimum IOL shape for an on-axis object is close to being convex-flat, with the more curved surface facing the cornea, if we used an eye similar to the Gullstrand number 1 schematic eye (11). The bending factor is $q = 1.1$. However an abnormally short eye (hyperopic) obtain an optimal optical behaviour for an biconvex design of the IOL, while a previously abnormally long eye (highly myopic) obtain such optimal optical behaviour for a convex-concave design.

DISCUSSION

The matrix formula that we have reported herein, we call the Alicante formula, is more broadly applied than others previously reported from the geometrical optic point of view, as it considers all the alternative optical surfaces that can be obtained for a given clinical type of eye. However in agreement with other IOL calculation formulas, the one that we propose includes some theoretical simplifications, such as making the calculations in the paraxial zone and considering the eye as a centered optical system. These considerations allow more simplified calculations and accurate enough results for clinical application purposes. We can assert that the matrix formula proposed here is an exact model for the study of the aphakic eye from the Gaussian Optics point of view.

If we compare our matrix formula with previous reports (5) we obtain a generalization of its results, such as when we make $P_7 = 0$ in equation [XII] and obtain an expression for P_5 which coincides with the results of Colliac for the optical power of a convex-flat IOL. However other previous studies have related the quality of the optical image at the retina to the type of IOL used for the correction of surgical aphakia. In fact, spherical and coma aberration depend on the curvature radius of the IOL. Some years ago convex-flat IOLs were employed but in others studies it was shown that for standard sized eyes, biconvex IOLs allow for an improvement in optical results (12). In fact, the optimal quality of the retinal image depends on the shape and design of the IOL and it would be more accurate in this regard to be able to choose for non-standard sized

eyes the optimal IOL for a particular eye from alternative shapes for the same optical power that we can derive from the matrix formula that we propose here. Each lens is labeled by a number q . This is the bending factor. This is an important parameter because spherical and coma aberration are a function of the bending factor q . For minimum aberration and non-standar sized eyes the bending factor is $q \neq 1.1$. This can be consider a new approach in IOL calculation, considering not only the most adequate IOL in terms of optical power calculation but also in terms of IOL shape and design.

We have proved in this paper that a convex-flat design is the best IOL to be inserted in normally lengthed eyes, but that such a design is not optimal from the optical point of view for abnormally shortened highly elongated eyes. To the best of our knowledge, such finding had not been previously reported and indicates that in the selection of the best IOL for a given eye, not only the optical power of the IOL to be considered but IOL design may improve the optical as well as the clinical results of IOL implantation in the aphakic patient.

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Figure captions:

Figure 1.- "S" surface separating areas of different indexes of refraction n and n' .

Figure 2.- Movement of the light beam from one surface S to another S' separate a distance t . n is the refractive index between these surfaces.

Figure 3.- Diopters of the pseudophakic eye. Refractive indexes, radii of curvature, thickness of the media.

TABLE I: Clinical data for four types of eyes and calculations of IOL power for emmetropia

Eye	AL	K	r ₁	ACD	r ₅	r ₇	P	q	
Previously hiperopic eye	20.95	47.01	7.18	3.41	5.00	29	25.5	1.42
					5.30	44		1.27	
					5.60	84		1.14	
					5.90	456		1.03
					6.20	-152		0.92	
					6.50	-69		0.83	
					6.80	-46		0.74
Previously emmeropic eye	23.65	43.81	7.70	4.0	7.00	82	19.7	1.19	
					7.30	124		1.13	
					7.60	429		1.04
					7.90	-337		0.95	
					8.20	-127		0.88	
					8.50	-80		0.81	
					8.80	-60		0.74
Previously myopic eye	26.97	42.5	7.94	3.96	11.7	71	11.0	1.40
					12.0	86		1.27	
					12.6	133		1.21	
					12.9	179		1.16	
					13.2	268		1.10	
					13.5	510		1.05
					13.8	3687		1.01	
Previously highly myopic eye	30.58	40.75	8.28	3.0	17.0	34	4.6	3.00
					18.0	38		2.76	
					19.0	44		2.55	
					20.0	49		2.36	
					21.0	56		2.19	
					24.0	86		1.77
					28.0	187		1.35	
					31.0	567		1.12	
					32.0	1397		1.05	
					33.0	-3727		0.98

AL = Axial length in millimeters
K = Preoperative mean keratometry in diopters
r₁ = Preoperative mean anterior radius of curvature of the cornea in millimeters
ACD = Anterior chamber depth from corneal vertex to implant vertex in millimeters
P = IOL power in diopters
q = Bending factor

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