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Simple model for predicting the vibration transmission of a squat masonry tower by base forced vibrations

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Abstract

This paper presents the analysis of the dynamic performances of a simple model related to a squat masonry tower situated in the Swabian Castle of Trani (Italy). The main objective of this paper is to introduce a novel strategy based on a simple model validated by experimental data for defining the influence of the excitation frequency on the structural damping dynamic transmission. To this aim, firstly, the accelerations have been acquired simultaneously in 23 points of the tower at different levels, both due to environmental vibrations and to a series of sinusoidal forced vibrations applied at the base by using an electro-hydraulic shaker device specifically designed for the tests. Four different excitation frequencies have been then selected for exciting the structure. An operational modal analysis has been carried out by the environmental recordings and with the different forcing loads obtaining a very good correlation of the identified frequencies in all the cases. Then, a digital filtering process has been applied over all the recorded signals to evaluate the specific contribution for each frequency generated by the shaking device at each level of the tower. Increments of damping ratio have been detected with these forced vibrations at the base. Finally, a simple frame numerical model has been developed to reproduce the dynamic amplification at the most significant locations of the tower. It has been updated not only to have the same main frequencies and modal shapes, but also to get a similar response under forced vibrations at the base. A good correlation has been obtained between the model and the real structure for the base forced vibrations at different excitation frequencies in order to correctly predict the dynamic behaviour of the structure.

Keywords: Damping ratio, masonry tower, squat tower, OMA, dynamic identification.

1. INTRODUCTION

This work presents an extensive experimental analysis aimed at identifying the structure modal parameters; tests have been performed using the data obtained from environmental vibration and forced vibrations. The dynamic behaviour of historical buildings is usually analysed by means of non-destructive tests in order to avoid increasing and/or producing damage in a culturally valuable heritage structure [1], [2]. The aim of the dynamic analysis on historical buildings is to eventually design repair intervention solutions and retrofitting to seismic actions. Retrofitting interventions, in fact, are quite common because old masonry structures are designed and built to resist to vertical loads only. Usually, for this kind of analysis the data are recorded by means of a series of accelerometers installed at specific points of the structure. The recorded data will be then used for Operational Modal Analysis (OMA) [3]–[6], which is utilized to obtain the real values of the modal parameters of the tower. OMA is an efficient method to be utilized in case of historical buildings because it allows us to know the modal parameters of a structure in a non-invasive way. The analysis can be applied not only to historic and cultural buildings, but to modern ones and bridges too [7], [8]. Starting from the acquisition of ambient vibrations in situ, through a simple and accurate process applied to the recorded data it is possible to extract the natural frequencies of vibration, the modal shapes and the modal damping ratios of the considered structure. Thus, the interpretation of these data through OMA allows us to evaluate the characteristics of the materials and the boundary conditions of the structure, in order to establish reliable numerical models by means of procedures of model updating [9], [10].

Slender structures such as towers are particularly suitable for this type of investigation [11]–[13], because if they are subjected to vibrations even of low intensity, they generally produce very clear and sharp signals. On the other hand, in case of squat buildings - like the clock tower considered in this study - it may be necessary to apply a forced excitation in order to obtain a clear signal and then sufficient dynamic information.

Forced vibration tests in situ can be well carried out by means of a mechanical shaking device usually called vibrodine. One important characteristic of the shaking device is the possibility to define and control the signal to be impressed to the structure. By means of a vibrodine it is possible to study the dynamic behaviour of a structure under different forced loads; it can produce different unidirectional harmonic loads with a sweeping frequency depending on the mass of the shaker. Usually a vibrodine consists of moving/rotating masses powered by an electric motor. A method for forced vibration testing of structures is proposed in [14]. The method can identify a linear shaker input motion, which produces a structural response similar to that of the building shaken from the base by an earthquake, and pre-correct the input motion to account for control–structure interaction effects.

Examples of forced excitation by means of a vibrodine are found in [15]–[17]. A shaking device was also utilized to test the civic tower of Rieti (Italy), which was equipped with a Tuned Mass Damper (TMD) system placed on the roof of the building [18].

The use of a vibrodine is especially suggested for the experimental assessment of composite structures [19]–[21]. In the present study, due to the massive structure of the historical building here examined, an appropriately designed and realized electro-hydraulic shaking device has been used to generate forced oscillations on this kind of structures.

In the present paper the equipment and the experimental setup that has been used for in-situ dynamic identification tests on the squat clock tower of Trani's Castle (in Italy) are described and an extensive analysis of the effects of the shaking device on the vibrations of the tower is presented completing the preliminary analysis shown in [22]. The performed analysis may be considered very interesting and innovative in this field of research for the use of a special equipment able to produce forced vibrations on the tower.

Using the vibration acceleration measurements, the modal parameters of the tower are consistently identified through two different output-only procedures: the first, based on the Complex Mode Identification Function, exploits a frequency representation of the response; the second, based on the Stochastic Subspace Identification Method, works in the time domain. All the experimental tests have been analysed and the frequencies of the structure have been identified; moreover, useful indications on the use of the shaking device complete the article. Finally, the analysis is reproduced with the use of a simple Timoshenko frame model calibrated with the main experimental frequencies and mode shapes. This simple model is valid to analyse other possible vibrations transmitted along the tower as low seismic forces.

1.1. Historical considerations

The clock tower here analysed is located on the eastern façade of the Swabian Castle of Trani (South-East of Italy), on its main entrance [23]. The castle is placed just a short distance from the Cathedral of Trani; its location on the edge of the town and the height of the towers allowed to guard the entrance of the port and the access roads to the village. Originally, the castle had a simple and functional quadrangular enhanced layout with four square towers of the same height. In the sixteenth century, with the advent of the first weapons, the castle was adapted to the new defensive techniques, in response to the widespread demand for re-fortification of the Mediterranean coast, under threat from the Turks. These operations involved the rise of the southern front, least favourite course and overlooking towards the open country, and the construction of two bastions at opposite edges to the southwest (a spear) and northeast (square), thus ensuring a complete fire coverage around the perimeter of the fortress. Trani Swabian Castle underwent major works of transformation in 1832 when Ferdinand II Duke of Bourbon converted it in the provincial Central Prison.

In 1844 the need arose to equip the prison with a clock to adjust the internal life of the prison and in 1848 the new clock tower was built to place the clock so that the sound of the latter could reach the whole castle. Following this change of use, the entire castle underwent many changes, the work of the nineteenth century were only superimposed to the old structure. The new clock tower was built on the old walls of the fortress.

The castle was utilized as a prison until 1974 and in 1976 it was delivered to the Superintendence for Architectural, Artistic and Historical Heritage of Apulia. The restoration works started in July 1979 and ended on June 5, 1998. At present the castle can be visited as a historical monument and the clock tower maintains its use as a clock.

1.2. Geometrical description

The clock tower has an overall height equal to 7.20 m from its main door. The tower has a holed square cross section, with the inner side of about 2.05 m, constant long the total height, while the outer side is variable along the height because of the presence of a Trani stone covering. In fact, the outer side is 4.25 m at the base and 3.55 m at the intermediate height of 4.00 m and, from this level up to the top, it remains constant. At the intermediate height of 4.60 m there is also a toroidal stringcourse. Figure 1 shows a schematic view of the general dimensions of this tower.

The tower façades are characterized by openings arranged on two different levels: (i) At the lower level, there is the doorway on the West side, while on the other sides there are archivolted windows with a width of about 1.40 m and an average height of 60 cm. (Figure 2b). (ii) At the upper level there are archivolted windows smaller than the previous ones, with a width of 85 cm and a height of 55 cm, arranged on all sides with the exception of the East one, which is blind and host the large central clock (Figure 2a). On the eastern façade, differently from the other ones, there is a gabled portal with an arched opening in the middle where a bell is located.

1.3. Morphological-structural survey and characterization of the materials

A survey operation was developed for collecting information on the morphology and the dimensions of the tower's structure. The analysis has not been limited to the tower structure only, it has been also extended to the structure of the underlying entrance hall to the Castle at the ground floor.

From the visual inspection of the wall surface and some cross sections in the proximity of the doorway and the windows of the tower, the masonry is composed of a first exterior surface made of squared blocks of Trani stone [24]. The latter has a constant thickness of about 20 cm at the upper level, while at the lower level it has a variable thickness increasing towards the bottom: From the tower doorway

(Figure 3a) the tufa wall can also be noted with a thickness of about 75 cm and above the toroidal stringcourse, the thickness of the tufa wall becomes 55 cm. In order to obtain more information and foresee the composition of the tower's masonry, Ground Penetrating Radar tests have been conducted in different parts of the structure. These tests would confirm the presence of an exterior cover made of Trani stone and an internal tufa wall, which is reasonably homogeneous with only small discontinuities related to the material and its realization (blocks and mortar joints) and not to the hypothesis of a wall with double faces and a central core. The roof structure of the tower consists of a small tufa vault with a thickness of about 20 cm [24].

From a morphological-structural viewpoint, the eastern façade of the tower is in direct continuity with the entrance façade of the Castle, while the other façades rest on the tufa barrel vault of the underlying space, used as an entrance hall to the Castle. More in detail, the base of the tower rests on a thickening of the underlying vault, which forms an arch dropped of 50 cm with respect to the same vault (Figures 3b and 3c).

The space at the ground floor has an almost square cross section with sides of about 6.80 m, its walls are in stone on both sides with squared and irregular shaped blocks having different thicknesses. On the North side, their thickness is 2.05 m, on the South side it is 1.85 m, on the West side it is 0.77 m and on the East side it is 2.20 m. Because of these considerable thicknesses, it can be supposed that the walls are composed by double faces and a central core.

Furthermore, the walls are characterized by the presence of openings with different shapes and dimensions, which allow the entrance from outside (on the East side). The most particular opening is the entrance to the little eastern courtyard of the castle (arched opening on the West wall) and, finally, the access to other spaces at the ground floor (the two openings on the South wall, one of which is now walled up, and only one opening on the North wall).

1.4. Site seismic characterization

This historical tower stands in a seismic Italian region where low and medium accelerations levels could be generated at the base of the tower. To evaluate the possible seismic loads acting on the tower, the seismic classification of the city of Trani is considered according to the Italian seismic classification [25]. Trani is classified between the sites of the third category with a low seismicity. The foundation site for the clock tower has the following coordinates: Lat. 41.28284 - Long. 16.41646. According on the Italian seismic grid and the hypothesis to attribute to the tower a Nominal Life > 100 years and a Class of Use III, the parameters of acceleration to consider in a seismic analysis are shown in Table 1.

To develop the tests on this tower to reproduce forced accelerations at the base sufficiently far from ground accelerations levels imposed for SLO, acceleration levels lower than 0.0005 g have been assumed.

2. ENVIRONMENTAL TESTS

2.1. Main characteristics of the data acquisition system

To evaluate the tower's main frequencies and its experimental mode shapes, the tower has been equipped with 23 high sensitivity seismic accelerometers ICP PCB 393B31 with a sensitivity of about 10 V/g. All the accelerometers were connected with flexible wired cables to a NI data acquisition system, the latter was then connected to a laptop positioned at the base of the tower equipped with a specific acquisition software.

The accelerometers were placed on four different levels at the four lateral sides of the tower: 8 accelerometers at the four corners of the floor over the clock, 6 accelerometers at three corners of the intermediate level, 6 at the three corners of the lower level as part of the clock tower, and 1 accelerometer at the basis as a reference sensor (Figure 4a). Appropriate rectangular blocks where the accelerometers were inserted with screws, were used for ensuring the orthogonality of each couple of accelerometers applied to the same point (Figure 4b).

Finally, two accelerometers were placed on the upper arch for monitoring the oscillation of this part, probably the most significant local modes for a stability analysis. In the following post-processing phase, three accelerometers were discovered not to be properly running during the tests, so the total number of analysed accelerometers is equal to 20.

Figure 4a shows the location on the tower of the position of the 23 accelerometers; the directions of each accelerometer have been indicated by arrows. Points A, B, C, and D have been selected to compare the simple numerical model with the results of the experimental setup.

2.2. OMA analysis and results.

Four consecutive acquisitions of environmental vibrations were carried out on January 23, 2014 by recordings of 15 minutes each with a frequency of 1024 Hz.

ARTeMIS software [26] was used for the extraction of the modal parameters. Two different OMA methods were used for each analysis: Enhanced Frequency Domain Decomposition (EFDD) in the frequency domain and Stochastic Subspace Identification (SSI) using Unweighted Principal Components (UPC) in the time domain. SSI method essentially estimates a state-space model from an observed output correlation sequence using linear algebra techniques [27]. An eigenvalue decomposition of the identified state space model then yields the eigenfrequencies of the analysed structure. The SSI method has the particular advantage of combining high estimation accuracy with high computationally robustness and efficiency becoming a standard for operational modal analysis [28], [29] applied for.

In this case, the SSI method has demonstrated to be able to better estimate the frequencies working at 1024 Hz. Figure 5 shows the SSI diagram of the first environmental test and the frequencies of the tower clearly identified.

The estimated results were consistent for all the environmental tests and a synthesis of the identification results with the SSI method [26] for all the environmental tests is reported in Table 2 (estimated frequencies and corresponding damping values in percentage).

The mode shapes corresponding to the identified frequencies are shown in Figure 6. The first two identified frequencies are related to bending modes (along y –EW and x –NS axis, respectively), the third frequency is related to a torsional mode, the fourth frequency to a second bending mode (y axis), the fifth frequency to a bending-torsional mode, the sixth one to a torsional mode.

3. FORCED BASE VIBRATIONS TESTS

3.1. Description of the shaking device

A special portable device has been designed to introduce forced vibrations at the base of the squat and massive clock tower of the Castle of Trani. To introduce high horizontal forces at low frequencies, the device was built using an electro-hydraulic system, designed by researchers of the Polytechnic University of Bari – Department of Civil Engineering Sciences and Architecture (DICAR) and built by Bosch-Rexhort Company.

Figure 7 shows a scheme of this hydrodynamic double acting symmetrical servo actuator, with two heads mobile masses. This device is situated between two frames to anchor it to the base of the tower by means of four screws. The weight of the shaking device is 1.5 kN with 2 kN of additional steel/plumb masses for the mobile elements, divided into 10 rings of 2 kN/each.

The maximum horizontal displacement of the mobile masses is ±100 mm, measured with an LVDT; the total horizontal force is measured with a load cell with a F.S. of 50 kN. The pressure is measured with a pressure transducer with a F.S. of 400 bar and accuracy of 0.5% F.S.

The device is connected to a diesel motor portable hydraulic pump with a capacity of 40 l/min and a maximum pressure of 120 bar.

With these characteristics the shaking device can produce a sine waveform with a F_{max}= ± 8.4 kN at a frequency of 32 Hz and a stroke of ± 1mm, and a F_{max}=± 3.3 kN at a frequency of 12 Hz and a stroke of ± 2.5 mm. These maximum values can introduce forces at the base of the tower that can simulate small earthquakes and forced accelerations' levels at different positions along the height of the tower.

3.2. Tests description

To develop the forced vibration tests, the electro-hydraulic shaking device was installed on the main entrance of the castle, at the base of the tower (Figure 4a) to act in the E-W direction that is in the direction of the first mode of the tower. The sensors were kept in the same positions of the tests developed under environmental conditions, using the same configuration.

All the forced vibration tests are designed not to produce any damages to the tower or to the castle.

A preliminary test was carried out considering an excitation produced by a 2 kN moving mass having the same amplitude and changing the frequency from 1 Hz to 15 Hz, with a step of 2 Hz modified about every 2 minutes. Figure 8 shows the temporal acceleration values during the tests in points A, B, C and D, respectively. The final 80 seconds of acquisition have been done switching off the diesel motor, only with the pressure stored in the accumulator, in order to evaluate the influence of the motor on the oscillations in the considered positions; it is evident that there is a brusque diminution of the oscillations.

From Figure 8, it is evident that the shaking device amplifies the oscillations in all the positions (with a factor of 5-10 times with respect to the environmental oscillations) nevertheless it is placed at the base entrance without a direct contact with the tower's structure. A relevant amplitude variation according to the shaking device frequency variations was detected; in all the positions the signal increases in the second part when the frequencies of the shaker get to 9, 11, 13 and 15 Hz.

Considering the last 80 seconds of Figure 8 when the pump motor is off, the results clearly show that the dominant effect of the oscillations is due to the pump motor. This preliminary analysis convinced us to use the shaker device with only the accumulator power (motor switched off) in order to excite the structure avoiding the vibrations of the motor.

The preliminary analysis was very useful for arranging further tests without doubts about the possibility of acquiring data only related to the shaker forcing action and not influenced by the pump motor effects.

Short tests were carried out using only the accumulator energy as forcing action. However, the accumulator autonomy was very short and also depending on the frequency; for a frequency of 3 Hz the accumulator had 110 seconds of autonomy, but it decreased to about 50 seconds for 9 Hz, to about 25 seconds for 18 Hz and to only 15 seconds of autonomy for 20 Hz.

Figure 9 shows the acceleration level in positions A, B, C and D for each test in direction E-W; it is evident that the amplitude varies when changing the frequency of the shaking device; in all the positions the maximum amplitude is achieved with a frequency of 18-20 Hz, leading us to consider that this value could be considered close to a frequency of the building. The results here obtained are interesting because they demonstrate the applicability of the shaking device to introduce forced vibrations on the tower when it is not directly in contact with the tower.

3.3. OMA analysis and results with forced vibrations

The estimation methods were also applied to the forced vibration tests; the particularity of the forced tests is that also the forcing vibration of the shaker device is clearly identified in addition to the natural frequencies of the squat tower. The estimated frequencies and their corresponding damping ratios are reported in Table 3 and this table does not present the frequencies directly introduced by the shaking device excitation in each test.

The estimated frequencies under the forced vibrations are consistent with the estimation carried out in environmental conditions, despite the short duration of the forced tests due to the limited power charge of the accumulator [22]. In fact, the length of the considered forced tests is equal to 55 seconds for the test at 3 Hz and about 20 seconds for all the other forced tests. The damping ratios estimation, on the contrary, shows a certain variation evaluated by the same procedure than for environmental tests. Table 4 shows the percentage increment of the damping ratio between environmental and forced vibration tests.

It is worth remarking that the increment of the damping ratio in all situations is due to higher displacements generated by the forced vibrations than the ambient ones. Non-constant and non-linear behaviour of the tower is observed. This behaviour is due to non-linear mechanisms that include rattling, friction, interaction between structural elements, or non-linear stiffening/weakening. Such mechanisms create pseudo-damping, which is not a direct conversion into energy but a systematic behaviour that resembles the effect of a true damping process.

3.4. Filtering process.

We have observed that the forced vibrations generated at the base at a defined frequency produce different levels of acceleration at different quotes of the tower, but each response has several frequency components. To directly analyse the effect of each forced frequency at the base with each response at each position on the tower an innovative strategy is here introduced to use the forced vibration signals in a more specific way.

The approach is based on considering a digital filtering of the acquired signals; in particular a classical windowed linear-phase FIR digital filter has been designed [30]. The filter is normalized so that the magnitude response of the filter at the central frequency of the passband $W_n = [w_1\ w_2]$ is 0 dB. The FIR filter is a non-recursive one because the output signal is a function only of the input signal *u*. The response of such a filter to a generic input signal is a finite sequence of *m+1*samples, where *m* is said the filter order.

In order to generalize the response to any input signal, the system function *H(z)* of the FIR filter has to be evaluated. It is the Z-transform of the impulse response that is the output when the

Kronecker delta function δ is given as an input. The system function $H(z)$ is calculated in (1) where a_0 and b_i (i=0,…,m) are constant coefficients, with a_0, $b_m \neq 0$.

$$H(z) = Z[h(k)] = \frac{1}{a_0} Z\left[\sum_{i=0}^{m} b_i \delta(k-i)\right] = \frac{1}{a_0}\sum_{i=0}^{m} b_i z^{-i} \tag{1}$$

To calculate the frequency response, the variable z must be described as complex: $z = r \cdot e^{jw}$, where r is a magnitude and w is the angle of z. If r =1, then $H(z)$ around the unit circle becomes the frequency response $H(e^{jw})$ that is obtained in (2) substituting e^{jw} for z in (1).

$$H(e^{jw}) = \frac{1}{a_0}\sum_{i=0}^{m} b_i e^{-ijw} \tag{2}$$

The same expression is often given using the Euler's identity $e^{jw} = \cos(w)+j\sin(w)$ as indicated in the rectangular form in (3).

$$H(e^{jw}) = \frac{1}{a_0}\sum_{i=0}^{m} b_i\left[\cos(iw) - j\sin(iw)\right] = \frac{1}{a_0}\sum_{i=0}^{m} b_i \cos(iw) - \frac{j}{a_0}\sum_{i=0}^{m} b_i \sin(iw) \tag{3}$$

The aim of filtering the acquired signal was achieved by the calculation of the coefficients b_i (i=0,…,m) through the *fir1* function in Matlab [31]. A two-element vector [w_l w_h] was passed to this function of the Signal Processing Toolbox in such a way as to specify the band of normalized frequencies for bandpass configuration. This is a Hamming-window based, linear-phase filter with normalized cut-off frequency. The lower and upper bound filters were chosen as in equations (4) and (5), respectively. Different values for the parameter ε were firstly settled for testing the filter capabilities and subsequently the value 0.05 Hz was fixed to obtain the desired bandwidth. By default, the filter is scaled so that the centre of the first passband has a magnitude of exactly 1 after windowing.

$$w_l = 2\left(\frac{f-\varepsilon}{f_s}\right) \tag{4}$$

$$w_h = 2\left(\frac{f+\varepsilon}{f_s}\right) \tag{5}$$

To have an approximately unvaried signal amplitude in the considered range of frequencies, that is to say a desired magnitude $20 \cdot \log_{10}|H(\exp(jw))|$ equal to 0 dB, the high order m = 800 was fixed. Preliminary simulations were carried out in order to test the applicability of the chosen filter. Let us consider the composition x of three sinusoidal signals x_1, x_2, x_3, as expressed in (6), with f_1= 7.5

Hz, f_2=9 Hz, f_3= 10.3 Hz, corresponding, respectively, to the first estimated frequency, to one frequency of the forcing hydraulic shaker and to the second estimated frequency of the tower:

$$\begin{aligned} x_1 &= sen(2\cdot\pi\cdot f_1\cdot t) \\ x_2 &= sen(2\cdot\pi\cdot f_2\cdot t+\frac{\pi}{3}) \\ x_3 &= sen(2\cdot\pi\cdot f_3\cdot t+\frac{\pi}{4}) \\ x &= x_1+x_2+x_3 \end{aligned} \tag{6}$$

Applying to the signal *x* the previously described filter with *m*=800, ε=0.05, and f_2 the central filter frequency, the comparison between the *x* signal and the *x* signal after the filtering is shown in Figure 10 (time domain) and in Figure 11 (frequency domain). It is evident that, after a certain transitory, the filtered signal *x* is exactly equal to the signal x_2 without any phase or amplitude distortion. In Figure 10 (right) it is shown a detail of Figure 10 (left) after a transitory time of 5 seconds where it is evident the perfect overlapping between the filtered signal *x* and the original x_2 signal.

3.5. Application of the filtering process to the forced vibrations data.

The application of digital filters to experimental accelerometers data in environmental conditions have been recently tested by the authors [32], [33]. In [32] the post-processing has permitted us to synchronize different wireless accelerometers by analysing the phase of the signals; in [33] it was demonstrated that, for environmental data, a digital *fir* filter is not able to separate an accurate sinusoidal signal for the effect of close components. In this paper, the digital technique is applied to forced vibrations data, considering the band-pass central frequency coincident with the shaking device excitation frequency. The digital post-processing has been successfully applied to all the acquired accelerometers data. In the following the data referred to four acquisition points, named respectively A, B, C, D as indicated in Figure 4, along the y direction will be analysed.

The filtered signal and its frequency spectrum compared to the original signal and its spectrum is shown for the case of excitation at 9 Hz for points A, B, C, D, in E-W direction and, for point A, also for the case of 16 Hz and 18 Hz excitation in Figures 12-14. It is evident that the digital filter is able to eliminate the external components to the forcing frequency and that the filtered signal is almost sinusoidal with a very low fluctuation regarding its amplitude, considering a transitory time of 1 second.

For all the considered filtered sinusoidal signals it is possible to calculate the Amplitude automatically calculated as the average value of half of the peak-to-peak value all over the full acquisition period, and also an initial Phase angle has been calculated for estimating the delay respect to the initial evaluation time (1 second) of the first maximum.

In order to compare the effect of the digital filter with the environmental situation, in Figure 15 also the case of environmental excitation is shown with reference to the same point A (y direction) and considering the same digital filter centred at the first estimated frequency (7.5 Hz). This demonstrates the importance of the shaking device effect that allows to analyse the data as really forced vibration data, using parameters such as the Amplitude and the Phase Angle also for testing the dynamic behaviour of the structure.

4. SIMPLE NUMERICAL MODEL

A simple numerical model has been developed to analyse the transmission of vibrations at different levels in the tower produced by the vibrations generated at the base. This simple frame model is based on the Timoshenko beam theory and it is composed of 11 nodes with 6 d.o.f. for each node, one linear spring (k1) to simulate the stiffness of the walls and a rotational spring (k2) to simulate the rigid base body placed under the squat tower; at the lower level the structure is considered fully restrained. Figure 16 shows the idealized model and the position of the springs.

The stiffness characteristics in each part of the tower have been calculated according to the proposed section in Figure 16, the density for the material was assumed equal to 1835 kg/m^3, value usually considered for masonry walls [34], and the values for the elastic modulus, k1 and k2, have been determined by an iterative calibration process using the mode shapes and the main frequencies of the tower as objective functions. The calibration process of the stiffnesses k1 and k2 has been carried out by iteratively minimizing the objective function H expressed in Eq. (7) where N is the number of used modes (equal to 5 in this case) and also comparing the experimental and numerical mode shapes with criteria related to their shape:

$$H = \sum_{i=1}^{N} \left(\frac{\omega_i - \bar{\omega}_i}{\bar{\omega}_i}\right)^2 \qquad [7]$$

where ω_i is the FEM frequency and $\bar{\omega}_i$is the experimental frequency.

This model can reproduce the two main frequencies and the two main shapes for bending in E-W and N-S directions. After an iterative process it was obtained that the medium elastic modulus for the tower is equal to 1530 MPa and the spring values k1= 1.0E30 kN/m and k2=1.0 E30 kN/m.

Table 5 shows the comparison between the experimental frequencies and the adjusted frequencies, showing a good approximation with the proposed values for E, k1 and k2.

On the calibrated model some forced vibrations have been introduced at the base of the tower to simulate the real tests developed and to analyse the level of the vibrations at each quote in the tower. A Time History Analysis (THA) was developed by Direct Integration, using sinusoidal functions of 3, 7.5, 9, 18, and 20 Hz with two levels of damping ratio (DR), 3% and 10% (12 analysis). 10% DR is a very high value for structural systems; it has been considered as a higher limit to evaluate the effect of this possible increment when high ground accelerations can be produced due to seismic loads.

THA was developed in two phases for each of the 12 THAs: (i) initially a ramp function simulated the total self-weight of the tower, (ii) in the second phase, on the deformed model, different sinusoidal accelerations have been introduced in different simulations. The damping ratio has been considered proportional to the Mass and the Stiffness matrices for the range frequencies analysed (0.1 – 30 Hz). The obtained values are shown in Table 6.

All the forced ground accelerations have been introduced in E-W direction, the same direction than the shaking device acts in real tests. The maximum values of these forced ground acceleration levels in the simulation (Base point) have been scaled to be equal to those experimentally registered by the acceleration sensor positioned near to the shaker device in each test.

Figure 17 shows the time history accelerations calculated at each position (height) in the tower when the excitation frequency at the base changes between 9 Hz and 18 Hz in E-W direction.

Acceleration levels of Figure 17 are coherent with the results of the experimental results, points located near to the top of the tower are more accelerated than lower points due to the amplification factor. 18 Hz base excitation generates higher amplification than 9 Hz and a diphase has been observed in point D due to the interaction of the excitation ground acceleration with the second mode shape. All accelerations observed for the model with 10% damping ratio are lower than for the model with 3% damping ratio.

An analysis has been developed to evaluate the amplification values of the acceleration at each level with respect to the ground acceleration. Figure 18 shows these results. Maximum amplification is presented when the ground frequency is 9 Hz in all positions, but the highest value is obtained at the top of the tower.

This squat tower has different stiffness at different levels. Figure 19 shows the level of acceleration at each level of the tower, presenting the characteristic low amplified deformation for 3 Hz and 9 Hz. However, at 18 Hz and 20 Hz the main stiffness changes along the height of the tower have been detected in the position of important stiffness changes. With the simulation of a high damping ratio (10%) no special changes have been detected in the behaviour of these accelerations, obviously the maximum values have been reduced.

Figure 19 clearly shows the changes of stiffness of this tower at different levels (6 m, 12.5 m) where the amplification generated by the base excitation is amplified. These situations are singular positions where possible high displacement at the base (seismic loads) can generate initial damages.

5. COMPARISON OF THE MODEL WITH THE EXPERIMENTAL DATA

The comparison of the model with the experimental data may be drawn using the filtering process and comparing the behaviour of the model when it is excited with the same frequency of the forced vibrations generated by the shaker device. In particular, the behaviour of the points previously named A, B, C, D and individuated on the model may be analysed exciting the model with a sinusoidal acceleration with the same procedure of filtering introduced for the experimental forced data. Figure 20 shows the filtering effects on the tower model when an excitation of 9 Hz is given at the basis of the model.

In order to test the accuracy of the model the Amplitude [g] of the filtered signals has been plotted with respect to the position of the four considered points (A, B, C, D) and to the position of the shaking device. The obtained interpolating polynomial function has been compared with the Amplitude experimentally estimated as previously described. The comparison related to the cases of the shaking device acting with frequency 9 Hz, 16 Hz, 18 Hz is shown in Figures 21 and 22.

Moreover, also the phase shift between the phases of points B, C and D and the reference point A has been analysed in the different cases of excitation, comparing the model results with the experimental data (Figures 23, 24).

All the results clearly show that the model is able to estimate with excellent accuracy the dynamic behaviour of the tower for different frequencies of the forcing action. Only in the case of a force acting at 18 Hz the model seems to show lower accuracy than in the other two cases; nevertheless this behaviour may be fully justified considering that around that frequency (18 Hz) there is a torsional natural frequency of the structure that could heavily influence the errors of the simplified model.

Overall, the results are really surprising also considering that the proposed methodology is innovative and it can guarantee the validation of the dynamic behaviour of the structure at different frequencies for forced vibrations.

6. CONCLUSIONS

The analysis of the clock tower of Trani has been performed using the environmental acquisition despite its very squat profile; the modal identification was carried out with two different statistical approaches in different domains: time and frequency.

A specific shaking device (Vibrodine) has been positioned at the base of the structure increasing the amplitude of the oscillations with a controlled excitation frequency. Several frequencies have been applied for analysing the global structural dynamic behaviour. These forced accelerations tests showed a good correlation between the environmental and forced records on the frequency and mode shapes identification. Moreover, the forced tests need low times of forced excitation. For massive structures, it is not possible to register high main frequencies under ambient vibrations; with forced vibrations at the base it has been possible to obtain the main representative frequencies and mode shapes of this stocky and massive tower.

It has been observed that the values of the damping ratio, evaluated by SSI methods, have been increased during the forced vibrations tests from the environmental results. These conclusions have been observed in all the tests and for the sixth main modes analysed. The non-linear behaviour of this masonry structure is detected with a high increment when forced vibrations affect the main frequencies. Tests developed under ambient vibrations usually presents good results to evaluate main frequencies and modal shapes but are not valid for the evaluation of damping ratios under forced vibrations: wind, traffic or seismic loads.

A simple numerical model has been developed and calibrated with the experimental results. The model has been excited at the base with sinusoidal accelerations at frequencies similar to the experimental ones. All the results are coherent with the records at different levels of the tower. To compare the recorded accelerations in the model and the experimental tests a digital filter has been proposed to directly evaluate the amplification at each frequency and at each level without the contributions of other frequencies. The digital filter proposed to individually study the maximum acceleration levels for each frequency demonstrated a good approximation to the maximum values obtained with the original signals and have permitted to compare these results with the simple numerical model presented.

This simple model calibrated with experimental measurements can reproduce the vibration transmission along the height of the tower and can predict the behaviour of the tower under low seismic events. The proposed methodology can be used to analyse acceleration distributions along the height of a squat tower when the ground vibrations occur.

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CAPTIONS OF FIGURES

Figure 1. General geometric description of the squat tower

Figure 2. (a) Main entrance to the Castle and view of the East façade of the clock tower; (b) West and South façades of the clock tower; (c) North façade of the clock tower.

Figure 3. (a) Detail of the masonry thickness from the doorway of the clock tower (b) Entrance hall at the ground floor and view of the arch on which the clock tower rests. (c) View of the clock tower and the entrance hall from the eastern courtyard of the Castle.

Figure 4: a) The scheme of 21 accelerometers' position on the squat tower. b) Detail of accelerometer positioning.

Figure 5: Identification of main frequencies with the SSI method from the environmental vibration test 1.

Figure 6 Environmental test 1: mode shapes corresponding to the identified frequencies.

Figure 7. Shaking device. Left: General scheme. Right: Photo of the real system.

Figure 8: Forced tests with the shaker device and the motor (positions A, B, C, D).

Figure 9: Forced vibration tests with only the shaking device and the motor off (positions A, B, C, D).

Figure 10. Left: Simulated effect of the digital filter. Right: Zoom on the simulated effect of the digital filter

Figure 11. Frequency spectrum on the simulated effect of the digital filter

Figure 12. Filtering effect for point A, y direction, excitation at 9 Hz

Figure 13. Filtering effect for point A, y direction, excitation at 16 Hz

Figure 14. Filtering effect for point A, y direction, excitation at 18 Hz (left) and a zoom of filtered data (right).

Figure 15. Filtering effect for point A, y direction, environmental test 1

Figure 16. Simple model. (a) EW direction view. (b) NW direction view

Figure 17. Accelerations calculated at each level in the tower with the calibrated model. Ground base acceleration for (a) 9 Hz, damping ratio 3%. (b) 18 Hz, damping ratio 10 %. (c) 9 Hz, damping ratio 10 %. (d) 18 Hz, damping ratio 10 %.

Figure 18. Accelerations amplifications at each level (Point positions) with different ground accelerations. Left: with 3% damping ratio. Right: with 10 % damping ratio.

Figure 19. Evolution of accelerations at different levels with different ground acceleration frequencies. Left: with 3% damping ratio. Right: with 10 % damping ratio.

Figure 20. Filtering effect for points A, B, C and D. y direction, excitation at 9 Hz, model data

Figure 21. Comparison of the Amplitude transmitted at different heights with a ground forced vibration. Left: 9 Hz, Right: 16 Hz

Figure 22. Comparison of the Amplitude transmitted at different heights with a ground forced vibration of 18 Hz.

Figure 23. Comparison of the phase shift at different heights with ground forced vibrations. Left: 9 Hz. Right: 16 Hz.

Figure 24. Comparison of the phase shift at different heights with ground forced vibrations of 18 Hz.

Table 1. Ground accelerations of the city of Trani according to the Italian Standards

Limit state	Return period (years)	Ground acceleration (g)
SLO (Operability)	90	0.062
SLD (Damage)	151	0.080
SLV (Protection of life)	1424	0.247
SLC (Collapse)	2475	0.324

Table 2. Frequencies and damping ratios obtained for each test.

Frequency registered	Tests under environmental vibrations							
	Frequency registered (Hz)				Damping ratio (%)			
	1	2	3	4	1	2	3	4
1st	7.52	7.50	7.48	7.53	2.19	2.10	2.25	2.70
2nd	10.32	10.36	10.34	10.31	3.02	3.23	3.35	3.27
3rd	13.28	13.33	13.35	13.51	3.53	3.30	3.52	4.51
4th	15.93	15.98	15.79	16.09	2.99	3.74	3.94	3.27
5th	18.08	18.34	-	-	3.83	3.80	-	-
6th	21.85	21.83	21.83	22.08	2.30	2.57	2.31	2.55

Table 3. Frequencies and damping ratios obtained for forced vibrations tests

Frequency registered	Tests under Forced vibrations							
	Frequency registered (Hz)				Damping ratio (%)			
	3 Hz	9 Hz	16 Hz	18 Hz	3 Hz	9 Hz	16 Hz	18 Hz
1st	7.54	7.549	7.543	7.492	2.85	2.41	2.71	2.48
2nd	10.31	10.32	10.42	10.5	3.54	3.39	2.92	4.33
3rd	13.66	13.58	13.31	13.66	2.90	3.24	2.68	1.92
4th	15.98	16.31	15.99	17.29	3.48	4.18	2.03	2.87
5th	18.16	18.48	18.68	18.61	3.55	3.00	2.51	2.39
6th	22.08	21.9	22.03	21.73	2.69	2.58	2.35	2.54

Table 4: Increment of damping ratio between environmental and forced vibrations tests

Main frequency registered in environmental tests (Hz)		Frequency of the forced vibrations			
		3 Hz	9 Hz	16 Hz	18 Hz
1st	7.53	30.1%	14.8%	20.4%	8.1%
2nd	10.39	17.2%	5.0%	12.8%	32.4%
3rd	13.55	17.8%	1.8%	23.9%	57.4%
4th	16.39	16.4%	11.8%	48.5%	12.2%
5th	18.48	7.3%	21.1%		
6th	21.94	17.0%	0.4%	1.7%	0.4%

Table 5: Main frequencies of the adjusted simple model.

Frequency registered		Tests under environmental vibrations						
		Frequency registered (Hz)					Simple model frequencies	Difference (%)
		Test 1	Test 2	Test 3	Test 4	**Medium Value**		
1st	Bending EW	7.52	7.50	7.48	7.53	7.5	7.52	0.2
2nd	Bending NS	10.32	10.36	10.34	10.31	10.3	10.18	-1.5
3rd	Torsion	13.28	13.33	13.35	13.51	13.4	-	-
4th	Bending EW	15.93	15.98	15.79	16.09	15.9	14.83	-7.5
5th	Bending NS	18.08	18.34	-	-	18.2	18.08	-0.7

Table 6: Mass and Stiffness coefficients to evaluate the damping ratio matrix.

Damping ratio (%)	**Mass Proportional coefficient**	**Stiffness proportional coefficient**
3	0.0376	3.17E-4
10	0.1252	1.058E-3