

# The power law model applied to the marathon world record

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## ABSTRACT

In September 2013 the world record in the marathon men's race was broken. The aim of this study is to apply to the 2013 Berlin Marathon a mathematical model based on the power law that analyses the marks distribution and checks its connection. The results show that the correlations obtained in all the different categories have been very significant, with a result of ( $r \geq 0.978$ ;  $p < 0.000$ ) and a linear determination coefficient of ( $R^2 \geq 0.969$ ). As a conclusion it could be said that the power law application to the 2013 Berlin Marathon Men's race has been an useful and feasible study, and the connection between the data and the mathematical model has been so accurate. **Keywords:** Ranking, Power law, Marathon, World record, Space-time distribution.

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## INTRODUCTION

The aim of this study is the analysis of the 2013 Berlin Marathon Men's race in which the world record was broken and it was close to the invincible barrier of 2 hours. This fact motivates us to know the characteristics of this great achievement. This study is based on the time and space marks obtained at the end of the race. That is the reason why we thought on applying a mathematical model based on the power law. This method has been already applied to the 2010 London Marathon Women's race (Fernández-Revelles, 2013).

When the power law is applied to real data, there is a tendency of applying it only to central values since it could appear some errors in the values that are placed at the rank extremes, the ones which are related to the distribution line (Amaral, Scala, Barthelemy, y Stanley, 2000; Newman, 2005). For that reason, a great number of investigations related to the distribution lines or extreme values have been appeared (Hong, Ha, y Park, 2007; Laherrere y Sornette, 1998; Naumis y Cocho, 2007, 2008).

The power law is mostly used in the scientific field of the bibliometric studies. This type of studies are investigations in which the obtained data is organised in a ranking hierarchically (Campanario, 2010a, 2010c, 2010d, 2011a, 2011b; Coile, 1977; Edwards y Collins, 2011; Egghe, 2010b, 2011b, 2012; Garfield, 1980).

Sports and other areas related to the topic have not stayed on the sidelines on the power law usage. That is why it exists a large quantity of studies dealing with sports world records and its analysis. For that reason, it is necessary to look for mathematical methods that can be adapted to the current marks and it gives us the chance of studying its possible evolution and even the possibility of prognosticating future records (Carbone y Savaglio, 2001; Fernández-Revelles, 2013; Katz y Katz, 1999; Savaglio y Carbone, 2000). There are other studies which have studied the prediction of future records, as well, but without using the power law (Joyner, Ruiz, y Lucia, 2011).

Not always it is used the same power law. Each case uses the most appropriate one in order to explain each investigation item. In our case, we want to analyse the time distribution marks of the marathon race in which the world record was broken. In this case, we have used a mathematical method based on the power law that was used in the recent article about the 2010 London Marathon (Fernández-Revelles, 2013). The power law used here is the 'Beta rank', a law which improves the Zipf law (del Rio, Cocho, y Naumis, 2008). This law is also called 'U-like function' (Naumis y Cocho, 2007, 2008) because it has two parameters; 'S-shape' (Egghe, 2009a, 2011a, 2013) because of its shape; or 'semi-log rank-order distribution' (Campanario, 2010d; Mansilla, Koppen, Cocho, y Miramontes, 2007) because it is a law that is represented by a scatter plot in which the x axis has a linear scale and the y axis has a logarithmic scale.

### **Objectives**

The objectives of this study are:

- Objective 1: To apply to the 2013 Berlin Marathon Men's race in which the world record was broken, a mathematical method based on the power law for the distribution marks.
- Objective 2: To check the connection between the mathematical method and the marathon marks.

## METHODOLOGY

### **Sample**

The sample chosen in our study is the 2013 Berlin Marathon Men's race that was held on the 29<sup>th</sup> September, 2013 and the data was downloaded the 30<sup>th</sup> September 2013 from (SCC-Events, 2013). This marathon has a great importance because the men's world record was broken.

The age categories in which the Berlin Marathon Men's race is divided are from 18 to 39 years-old, from 40 to 44 years-old, from 45 to 49 years-old, from 50 to 59 years-old, from 60 to 64 years-old, from 65 to 69 years-old, and 70 or more years-old.

In our study, it has been used the data from all the participants – from all ages – who finished the race. We obtained 9,135 different time marks from 27,549 participants who finished the race.

In the global data analysis, all the age categories are included but in the detailed analysis the categories from 60 years-old on are excluded because the number of participants was limited.

### **Procedure**

The data processing started by organising the downloaded data in a Microsoft Excel 2007 document. We only focussed on the seconds marks and the km/h marks of each participant recorded at the end of the race. The marks were hierarchically organised in a dynamic table in which the best mark was at the top of the table and the worst mark was at the end of it. This is a methodology widely used in the rankings distribution studies (Campanario, 2010c; Fernández-Revelles, 2013).

We used the statistical programme SPSS (Version 18.0, Chicago, IL, USA) to organise the data in variables. These variables were the order variable, that is to say, the ranking which established the hierarchical mark order; and the variables for time marks calculated in seconds and the speed variable calculated in km/h. We repeated this process for the global study and for each of the studied categories.

In this study it was used the power law equation (see Equation 1), which is the one that best adapts to this type of curves (del Rio, et al., 2008; Fernández-Revelles, 2013; Mansilla, et al., 2007) given that the parameters  $a$  and  $b$  are associated to the attachment lines of the left-hand side and the right-hand side respectively:

$$f(r) = K \frac{(N + 1 - r)^b}{r^a}$$

Equation 1. Power law for curves with increasing distributions

The previous equation is an improvement of the power law used in the magazines impact factor analysis (Lavalette, 1996; Popescu, 2003). Furthermore, this equation has been widely used for curves with decreasing values (Alvarez-Martinez, Martinez-Mekler, y Cocho, 2011; Campanario, 2010a, 2010b, 2010c, 2010d; del Rio, et al., 2008; Egghe, 2009a, 2009b, 2011a; Fernández-Revelles, 2013; Mansilla, et al., 2007; Naumis y Cocho, 2007, 2008; Waltman y van Eck, 2009).

We do not have almost any evidence in any publication of a case in which appears a curve with increasing data. It only appears in the 2010 London Marathon Women's analysis (Fernández-Revelles, 2013). The 2010 London Marathon Women's analysis transformed the equation and we followed the same methodology in order to respond to our needs (Equation 2):

$$f(r) = K \frac{r^a}{(N + 1 - r)^b}$$

Equation 2. Power law for curves with decreasing distributions

As it can be observed, the elements and parameters of both power law equations that form this linear function are the same but with a different order:

*f*: function

*r*: ranking or position in the ranking

*f*(*r*): function depending on the ranking or position in the ranking

*K*: constant

*N*: sample data number

*a* and *b*: linear function parameters

Then, we used the marathon race data to calculate the curve model. In order to accomplish this objective, we followed the method described by Juan Miguel Campanario (2010a, 2010c, 2010d), and the one used in the 2010 London Marathon Women's race analysis (Fernández-Revelles, 2013). Following these methods, we did some calculations that consisted on transcribing the power law into equations in a Microsoft Excel 2007 document. When these calculations were done, we calculated the constant *K* and the parameters *a* and *b* in a free web service of massive calculation called 'zunzun' (Phillips, 2010). This service provided us the parameters previously mentioned and the coefficient of linear determination  $R^2$ .

When we obtained the value of these parameters, we applied the power law to each ranking datum in order to obtain the function value in each point and in that way we obtained the values of our mathematical model. This process was done in a Microsoft Excel 2007. After that, we calculated the Pearson correlation between the marathon data and the calculated model in order to obtain the data that established the relationship level between the data obtained directly from the marathon and the data of our model.

Finally, a scatter plot was drawn using the original data and the calculated model in order to compare both of them. In the *x* axis appeared the ranking and in the *y* axis appeared the speed or the time mark, either from the marathon data or from the model data. With the purpose of creating a chart with a more extended *y* axis, we needed to apply a logarithmic scale as it is mentioned by many authors (Alvarez-Martinez, et al., 2011; Campanario, 2010a, 2010b, 2010c, 2010d; del Rio, et al., 2008; Egghe, 2009a, 2009b, 2011a; Fernández-Revelles, 2013; Mansilla, et al., 2007; Naumis y Cocho, 2007, 2008; Waltman y van Eck, 2009).

This process was repeated in the global study and in the detailed one – time and speed average marks –. We used the most appropriate power law in each case.

## RESULTS

*Objective 1: To apply to the 2013 Berlin Marathon Men's race in which the world record was broken, a mathematical method based on the power law for the distribution marks.*

In our study we needed to apply two different mathematical models as the characteristics of the marks distribution were different. One mathematical model was used for the rankings whose values were decreasing like in the case of the speed average marks, and the other one for the rankings whose values were increasing like in the case of the time marks.

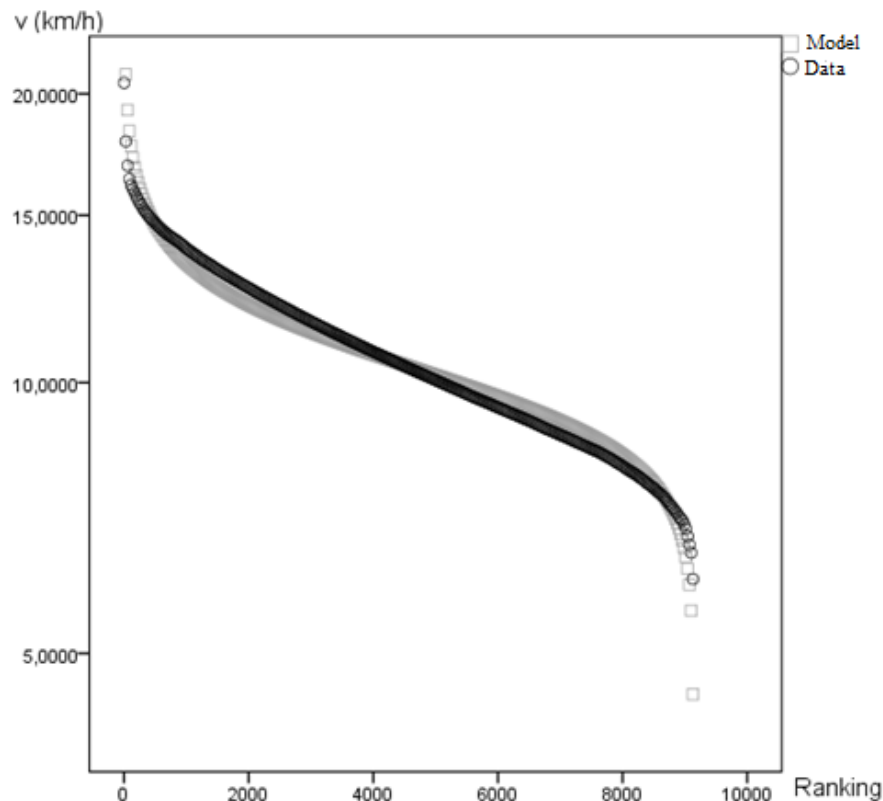


Figure 1. Distribution of the final speed average in all the categories

When we applied the mathematical model for decreasing values to the speed average data (see Equation 1), we obtained (see Table 1 and Figure 1) negative values in the parameter  $a$  of the equation and positive values in the parameter  $b$ . We could see that the absolute values of both parameters were very similar, almost without any difference. We could also see that the absolute value of the parameters was  $a > b$  in the group called 'All of them' in which all the categories were included, and in the group from 18 to 39 years-old. In the other categories, the result was just the opposite, the absolute value of the parameters was  $a < b$ .

When we applied the mathematical model for increasing values to the time marks (see Equation 2), we obtained (see Table 2 and Figure 2) positive values in the parameter  $a$  and negative values in the parameter  $b$ . In this case we could see that the absolute value in the parameters was  $a < b$  in the group called 'All of them' and in the group from 18 to 39 years-old. In the other categories the result was just the opposite, the absolute value of the parameters was  $b > a$ .

Table 1. Parameters obtained with the mathematical model for decreasing values, in this case for the speed average

Categories	Magnitude	<i>N</i>	<i>K</i>	<i>b</i>	<i>a</i>	<i>R</i> <sup>2</sup>
All of them	v(km/h)	9,132	11.8737	0.1077	-0.1234	0.9693
18-39 yr	v(km/h)	5,741	11.5231	0.1010	-0.1078	0.9842
40-44 yr	v(km/h)	3,774	9.7063	0.1024	-0.0856	0.9860
45-49 yr	v(km/h)	3,801	9.4273	0.1002	-0.0825	0.9851
50-54 yr	v(km/h)	3,007	9.0508	0.0978	-0.0776	0.9848
55-59 yr	v(km/h)	1,703	8.9384	0.0935	-0.0759	0.9858

Table 2. Parameters obtained with the mathematical model for increasing values, in this case for time marks

Categories	Magnitude	<i>N</i>	<i>K</i>	<i>b</i>	<i>a</i>	<i>R</i> <sup>2</sup>
All of them	t(s)	9,132	12,793.1764	-0.1077	0.1234	0.9693
18-39 yr	t(s)	5,741	13,182.4185	-0.1010	0.1078	0.9842
40-44 yr	t(s)	3,774	15,649.8508	-0.1024	0.0856	0.9860
45-49 yr	t(s)	3,801	16,113.0670	-0.1002	0.0825	0.9851
50-54 yr	t(s)	3,007	16,783.2586	-0.0978	0.0776	0.9848
55-59 yr	t(s)	1,703	16,994.2779	-0.0935	0.0759	0.9858

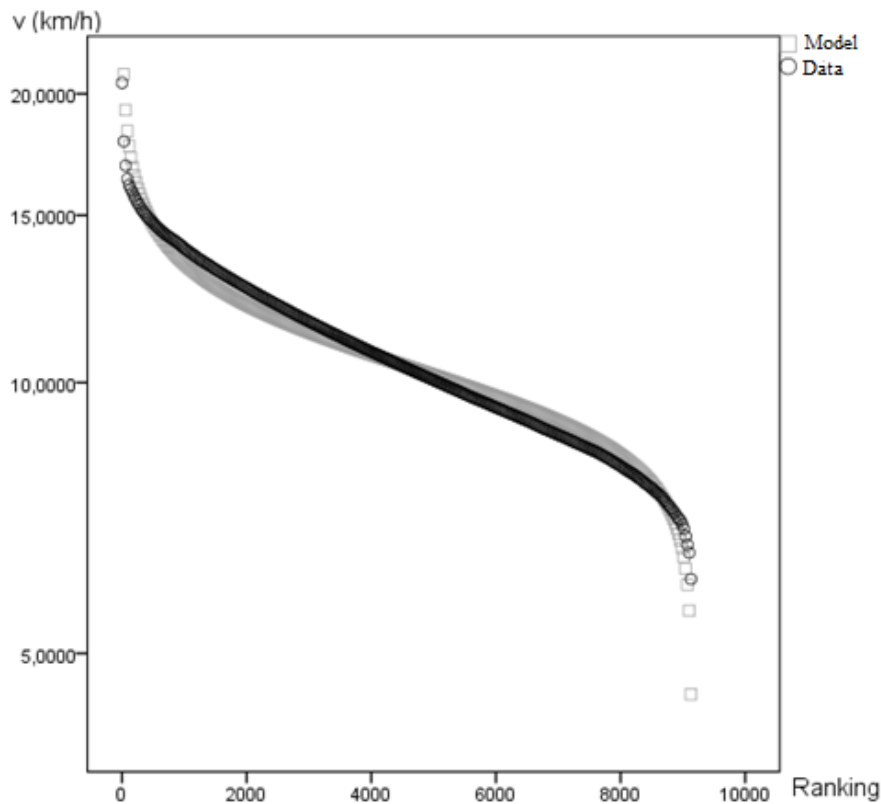


Figure 2. Time marks distribution in all the categories

*Objective 2: To check the connection between the mathematical method and the marathon marks.*

The correlations obtained in all the categories, either in the speed average to which we have applied the mathematical model for decreasing values, and in the time marks to which we have applied the mathematical model for increasing values, were very significant with a result of ( $r \geq 0.978$ ;  $p < 0.000$ ), and a linear determination coefficient of ( $R^2 \geq 0.9693$ ) (See Table 3 and Table 4).

Table 3. Correlations between the speed average data and the mathematical model

Categories	N	r	p	R <sup>2</sup>
All of them	9,132	0.978*	0.000	0.9693
18-39 yr	5,741	0.990*	0.000	0.9842
40-44 yr	3,774	0.990*	0.000	0.9860
45-49 yr	3,801	0.991*	0.000	0.9851
50-54 yr	3,007	0.981*	0.000	0.9848
55-59 yr	1,703	0.992*	0.000	0.9858

\* Significant correlation at level  $p < 0.001$

Table 4. Correlations between the time marks and the mathematical model

Categories	N	r	p	R <sup>2</sup>
All of them	9,132	0.978*	0.000	0.9693
18-39 yr	5,741	0.990*	0.000	0.9842
40-44 yr	3,774	0.990*	0.000	0.9860
45-49 yr	3,801	0.991*	0.000	0.9851
50-54 yr	3,007	0.981*	0.000	0.9848
55-59 yr	1,703	0.992*	0.000	0.9858

\* Significant correlation at level  $p < 0.001$

## DISCUSSION

We can apply the two different mathematical models of the power law to the speed distribution ranking and to the time distribution ranking data obtained in the Marathon Men's race because its connection (in either of the categories  $R^2 \geq 0.9693$ ) is similar to previous investigations related with other investigation topics.

It can be found similar results in studies related to the rank distributions universality with a linear determination coefficient ( $R^2 \geq 0.978$ ) (Naumis y Cocho, 2007); in studies related to the language ( $R^2 \geq 0.964$ ) (del Rio, et al., 2008); and in bibliometric studies related to the change in the magazines impact factor ( $R^2 \geq 0.9460$ ) (Campanario, 2010a). Although in our case the highest linear determination coefficient found has been in the category from 40 to 44 years-old ( $R^2 = 0.9860$ ) – in the speed average as well as in the time marks –, this result has not reached other results that appear in other studies ( $R^2 \geq 0.99$ ) (Campanario, 2010c).

The equation parameters analysis, its interpretation and its usage is going to help us to exploit the mathematical model in the marathon race study and its relationship with the world record. This mathematical model represented in a graphic form shows us a central zone and a curve at the beginning and another at

the end of the representation. The left line represents the parameter  $a$  and the right line represents the parameter  $b$  (Mansilla, et al., 2007). The similarity of the absolute value in the parameters  $a$  and  $b$  provides lines with a similar angle (Campanario, 2010c). In our case this means that there is a mark accumulation very similar in both lines since the difference in the absolute value between both parameters are always interior to 0.021 (see Table 1 and Table 2).

If the absolute value of the parameter  $a$  is inferior to the one of the parameter  $b$ , the left line will be smaller than the one from the right side as it happens in the Beethoven's Quartet Opus 131 analysis ( $a=0.20$  y  $b=1.81$ ) (del Rio, et al., 2008), or vice versa as it happens in the Spanish townships population analysis ( $a=0.86$  y  $b=0.40$ ) (Alvarez-Martinez, et al., 2011). If in our study the parameter  $a$  were higher than the parameter  $b$ , we would have higher marks than the ones we found at the end of the race.

It is very difficult to interpret the data by only visualising the parameters values of the equation. In order to improve the interpretation, it is necessary to use some graphs focussed on the lines, on the mark in which the cut-off point is produced, and on the place where the curve begins (Egghe, 2009a, 2010a, 2013). However, it arises some doubts about the curvilinear line origin and how to calculate it.

When we carry out a deeper analysis of the power law parameters related to the speed average and we compare it to the 2010 London Marathon Women's race analysis (Fernández-Revelles, 2013), we can see that the parameter  $a$  in both cases is negative and the parameter  $b$  is positive. In the same way, the absolute value is very similar in both cases, which means that there is a certain degree of symmetry in both lines.

In the 2010 London Marathon Women's race analysis, the parameter  $a$  is the one that is closer to the  $y$  axis (Fernández-Revelles, 2013). In this analysis, the parameter  $a$  values fluctuate from  $a=-0.1136$  – in all the categories – to  $a=-0.0962$  – in the categories from 45 to 49 years-old and from 50 to 54 years-old which had the same value –. However, in our study the parameter  $a$  values have a larger fluctuation, in this case from  $a=-0.1234$  – in the category 'All of them' – to  $a=-0.0759$  – in the category from 55 to 59 years-old. The largest values of the parameter  $a$  in the absolute value show a larger difference between the marks – a difference on the arrival times or a bigger space gap between the runners at the end of the race.

The quick rhythm established by the head of the race is related to the highest value that appears in the absolute value of the parameter  $a$ . It also gives a higher time allowed to the better marks. In that way, the higher values – in the absolute value – in the category that goes from 18 to 39 years-old with a result of  $a=-0.1078$  and in the category called 'All of them' with a result of  $a=-0.1234$ . This means that there were good marks not only in the category that goes from 18 to 39 years-old but also in other categories.

However, we find that in the 2010 London Marathon Women's race study (Fernández-Revelles, 2013) the parameter  $a$  value in the absolute value of the category that goes from 18 to 39 years-old is  $a=-0.1167$ . This result is bigger than the one we find in our study which has a result of  $a=-0.1078$ . In the case of the 2010 London Marathon Women's race, the world record was not broken and the race rhythm was lower, so they obtained this result because there was a bigger space gap between the runners in this category.

The analysis of the parameter  $b$ , the one which is the most distant from the  $y$  axis, is important to analyse because it is related to the worst marks of the race and its study could be interesting for the marathon race organization.



The constant  $K$  is related to the speed average. In our study the values fluctuate from the categories with a higher  $K$  value that are the 'All of them' category with a result of  $K=11.8737$  and the category that goes from 18 to 39 years-old with a result of  $K=11.5231$ , to the category with a lower  $K$  value that is the category that goes from 55 to 59 years-old with a result of  $K= 8.9384$ . Nevertheless, in the 2010 London Marathon Women's race study (Fernández-Revelles, 2013) we find that the values fluctuate from  $K=8.236$  in the 'All of them' category and  $K=8.9056$  in the category that goes from 18 to 39 years-old to  $K= 8.0730$  in the category that goes from 50 to 54 years-old. We could see that the constant  $K$  is higher in the 2013 Berlin Marathon Men's race than in the 2010 London Marathon Women's race because the speed average – 10.7 km/h – is higher in the marathon men's race than in the women's one – 8.88 km/h. It has been demonstrated that there is a relationship between the constant  $K$  and the speed average of the race. In our study, we can see that the speed average is not always higher when the constant  $K$  is higher as well. For example, in the case of the 'All of them' category the constant  $K$  is  $K=11.8737$  and the speed average is 10.70 km/h but in the category that goes from 18 to 39 years-old the constant  $K$  is  $K=11.5231$  and the speed average is 11.14 km/h.

Something similar happens in the time distribution marks but in this case the sign of the parameters  $a$  and  $b$  is negative although the values are the same.

Comparing the correlations between our study and the 2010 London Marathon Women's race one (Fernández-Revelles, 2013), we can see that the 'All of them' category results in our investigation are  $r = 0.978$  and  $p = 0.000$  and in the 2010 London Marathon Women's race are  $r = 0.985$ ; and  $p = 0.000$ . In both cases the correlations between the data and the proposed model are very significant.

After the analysis of the obtained data and the curves visualisation, we think that future studies should focus on the study of the ranks and marks of the left-hand side. This line is the one that is closer to the  $y$  axis and it depends on the parameter  $a$  of the power law equation. They should also focus on the study of the constant  $K$  influence and in the analysis of other variables that could affect it.

If this methodology is applied to future studies related with the analysis of marathon races, it will be opened a new line of work that could reveal some secrets that lead to break records or even to predict them.

## LIMITATIONS

This study has some limitations. In order to apply the power law to the marathon race, it will be needed more studies related to this topic. The limited number of previous studies that had used the same power law do not enrich the discussion. Due to these limitations we have the need of applying this method to future marathon races in order to obtain more results and to have more relevant conclusions.

## CONCLUSIONS

The power law application to the 2013 Berlin Marathon Men's race has been an useful and feasible study, and the connection between the data and the mathematical model has been so accurate.

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