Estimation of physical parameters of crops by polarimetric SAR interferometry with TanDEM-X sensor data

Master’s Degree in Telecommunication Engineering

Master’s Degree Final Project

Autor:
Noelia Romero Puig

Tutores:
Juan Manuel López Sánchez
Josep David Ballester Berman

July 2018
Acknowledgements

First and foremost, I would like to thank my tutor, Juan Manuel López Sánchez, and my co-tutor, Josep David Ballester Berman, for making me a part of this project and for their expert guidance and encouragement throughout the learning process of this master project. Thank you for the trust and support deposited in me.

A note of thanks for the grant received from the Office of the Vice President of Research and Knowledge Transfer of the University of Alicante for this master studies and initiation into research.

At last, the author would like to thank The German Aerospace Center (DLR), who provided all the TanDEM-X data under the project NTI-POLI6736, and the Federación de Arroceros de Sevilla, for providing the field data.
Abstract

Polarimetric SAR Interferometry (PolInSAR) is a radar remote sensing technique sensitive to different structural variables of scenes with vegetation, such as the height of the plants, the biomass and internal profile, the underlying topography, etc. Historically, it has been used in forests, specially with data measured with airborne sensors, and also, since 2012, with data from the TanDEM-X satellite sensor, formed by two X-band radar satellites with very close orbits. In contrast, the application of PolInSAR to agriculture had not been possible with satellite data due to baseline requirements (separation between satellite positions), since it must be, at least, 10 times greater than in the case of forests.

From April to September 2015 a special campaign was carried out with TanDEM-X during which the satellites were separated at the required distance. The first results obtained when applying these data to the height measurement of rice plants in areas of the provinces of Seville and Valencia, as well as in Turkey, have been recently published. However, there are still many questions to be solved, as well as aspects to improve, to fully validate this technique with satellite data. For instance, the influence of the bistatic configuration on the observations and on the estimations of the variables of the scene must be quantified. Moreover, the estimation of other variables other than height has not been tested yet. Finally, this technique should be tested on other types of crops.

This master’s degree project explores the aforementioned aspects, for which the algorithms of data processing and parameter estimation are implemented, and the different options are evaluated through their validation against field data and from the computational point of view.

The main contribution of this work is the evaluation for the first time of the PolInSAR technique applied to crops with satellite data, as well as the generation of open source (python) for its distribution in repositories.
Resumen

La interferometría polarimétrica SAR (PolInSAR) es una técnica de teledetección con radar sensible a distintas variables estructurales de escenas con vegetación, como la altura de las plantas, su densidad y perfil interno, la topografía subyacente, etc. Históricamente, ha sido empleada en bosques, sobre todo con datos medidos por sensores aerotransportados, y también, desde 2012, con datos del sensor satelital TanDEM-X, formado por dos satélites radar en banda X con órbitas muy próximas. En cambio, la aplicación de PolInSAR a agricultura no había sido posible con datos de satélite debido a los requerimientos de línea de base (separación entre las posiciones de los satélites), ya que ésta debe ser al menos 10 veces mayor que en el caso de bosques.

De abril a septiembre de 2015 se llevó a cabo una campaña especial con TanDEM-X durante la cual se separaron los satélites a la distancia requerida. Los primeros resultados obtenidos al aplicar esos datos a la medida de la altura de plantas de arroz en zonas de las provincias de Sevilla y Valencia, así como en Turquía, han sido publicados recientemente. Sin embargo, todavía quedan muchas cuestiones a resolver, así como aspectos a mejorar, para validar completamente esta técnica con datos satelitales. Por ejemplo, la influencia de la configuración biestática sobre las observaciones y sobre las estimaciones de las variables de la escena debe ser cuantificada. Asimismo, la estimación de otras variables distintas de la altura no se ha probado todavía. Finalmente, debe probarse esta técnica sobre otros tipos de cultivo.

Este trabajo fin de máster explora los aspectos mencionados, para lo cual se implementan los algoritmos de tratamiento de datos y estimación de parámetros, y se evalúan las distintas opciones mediante su validación frente a datos de campo y desde el punto de vista computacional.

La principal aportación de este TFM consiste en la evaluación por vez primera de la técnica PolInSAR aplicada a cultivos con datos de satélite, así como la generación de código abierto (python) para su distribución en repositorios.
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Chapter 1

Introduction

1.1 Context and Motivation

The agricultural sector plays a key role in the social and economic development of a territory, hence providing an improved understanding of the biophysical properties of agricultural crops is of major relevance. In this context, remote sensing presents a valuable tool for monitoring agricultural crop growth and providing additional insights into vegetation state and dynamics. Timely and effective information of these biophysical parameters is fundamental for crop classification and biomass estimation.

Microwave systems, unlike optical ones, have the power of being able to operate at nearly all weather conditions. Moreover, microwaves are sensitive, to a certain extent, to the structural and dielectric properties of the vegetation and the underlying ground, as they have the capability to penetrate the crop canopy.

The potential of active microwave sensors, such as the Synthetic Aperture Radar (SAR), arises from its capability to achieve meter-scale spatial resolution and its sensitivity to crop changes between and within the fields. With the well-known Polarimetric SAR Interferometry (PolInSAR) technique that works combining at least two polarimetric SAR images using interferometry [4], SAR sensors offer a unique opportunity of measuring crop height and other metrics of the vegetation and the underlying soil that can be used as indicators of crop condition.
Despite its potential, particular applications of Pol-InSAR over agriculture are scarce, owing to the absence of suitable data. Back in 2013, PolInSAR data suitable for agricultural applications was available for the first time, thanks to two airborne campaigns: The German Aerospace Center’s CROPEX13 and CROPEX14 [5]. These airborne campaigns yielded the first assessment of the potential of this technique to estimate crop biophysical properties such as vegetation height, water content and biomass. The assessment promised to be a great opportunity to advance the design of future acquisition strategies for both new airborne campaigns and future satellite missions (e.g. TanDEM-L [6]).

The estimation of biophysical variables of scenes with vegetation (forest and agriculture) by means of PolInSAR is based on the inversion of a physical model of the scene that relates the biophysical variables (vegetation height, topography, extinction...) and the observables available in PolInSAR [7]. The most extended approach for this purpose is founded on the Random Volume over Ground (RVoG) model, which was formulated in [8, 9, 10].

The launch of TanDEM-X (see figure 1.1), a satellite formation for high-resolution SAR interferometry in which two identical satellites operate in close formation [11], resulted a turning-point concerning the use of satellite data, since until then all SAR sensors provided revisited times which were too long to avoid an excessive temporal decorrelation [12].

![Figure 1.1: TanDEM-X and TerraSAR-X flying in formation [1].](image-url)
1.2 Objectives

During its first years of operation, the spatial baseline provided by TanDEM-X, 200-300 m, was designed for the generation of a global DEM, also suitable for forest height estimations, but still too short for agriculture [13]. Nevertheless, during the science phase of this mission (April-September 2015), the use of longer baselines, 2-3 km, opened the door to test PolInSAR over agricultural crops with satellite data for the first time.

Before the launch of TanDEM-X, PolInSAR data exploited in studies over vegetation were gathered in repeat-pass mode, where sets of polarimetric images were acquired over the scenes at different times by a radar operating in monostatic mode. However, the standard acquisition mode of TanDEM-X is single-pass as a consequence of its bistatic configuration, which consists in one satellite transmitting and both of them receiving, resulting in one monostatic image and one bistatic image.

In bistatic acquisitions the presence of a double-bounce contribution at the ground affects the interferometric coherence with a decorrelation factor. Although this term appeared in the original formulation of the RVoG model [8] and was analysed later in [3, 2] from a theoretical point of view, until the launch of TanDEM-X, neither this formulation was proven with experimental data nor the RVoG model was inverted with bistatic data.

There are several recent studies that have exploited TanDEM-X data for the retrieval of vegetation height and other parameters of interest (e.g. biomass and water content) of agricultural crops, though in all of them the double-bounce decorrelation term was either ignored [14, 12, 15, 5], or just taken into account in the inversion of the RVoG model [16]. With this project, we aim to elucidate the consequences of its contribution for the first time with real data.

1.2 Objectives

The present project is born with the purpose of assessing the feasibility of the current procedures for estimating the physical parameters of crops by polarimetric SAR interferometry (PolInSAR) with both simulated and experimental data acquired with the TanDEM-X sensor. Moreover, we will extend the study by analysing the influence of the double-bounce decorrelation factor on the inversion of ground topography and vegetation height.
For this purpose, the physical principles of remote sensing with SAR systems, SAR interferometry (InSAR), polarimetric SAR interferometry (PolInSAR), and the basis of the homogeneous volume-over-ground model must be understood. Simulated data with and without the double-bounce decorrelation term are employed as input to inversion procedures which consider or not that aspect.

The difference of the retrieved values with respect to the actual ones, together with the differences between approaches, are evaluated, and the influence of system parameters (baseline and incidence angle) is assessed. Finally, experimental results over rice fields are used to estimate the influence of this aspect on a final application.

### 1.3 Structure of the text

This text provides the steps followed to evaluate the actual estimation procedures to retrieve the biophysical parameters of crops that employ the PolInSAR technique by means of studying the numerical differences among the various inversion models.

In order to understand and interpret the development, Chapter 2 outlines the basic theory of SAR systems, SAR interferometry (InSAR) and polarimetric SAR interferometry (PolInSAR), as well as the basis of the homogeneous volume-over-ground model (VoG).

Chapter 3 provides the methodologies for estimating the PolInSAR model parameters, and the different inversion approaches are presented. Delving deeper, Chapter 4 provides an overview of the datasets used to evaluate the inversion performance of these algorithms, and eventually, the results of these assessments are then detailed and discussed. Last but not least, in Chapter 5 the objectives are evaluated and the final conclusions are drawn.
Chapter 2

Theory

The objective of this chapter is the definition of the basic concepts and formulation that will be used throughout the project. This involves the review of the principles of SAR interferometry (InSAR) and polarimetric SAR interferometry (PolInSAR), and so to provide a general introduction to the basic fundamentals of this technology. For this purpose, understanding the acquisition geometry in SAR systems and the convention used for image processing in a SAR image is necessary.

Once the basic concepts are presented, there is an introduction to the homogeneous volume over ground assumption (VoG). The framework of this model describes, at the same time, several scenarios and their associated models: a Randomly oriented Volume (RV) and an Oriented Volume (OV). Despite we will focus more on the random volume over ground model owing to the fact that it has become the most widely used model for the retrieval of biophysical variables by means of PolInSAR, the principles of both models, RVoG and OVoG, will be presented.

Since the mathematical derivations and the physical background that constitute the analytical expressions in the models are considerable, they entail a thorough definition of many parameters and variables. This is the reason why the reader may be referred to [8, 9, 10, 3, 17] for a broader explanation regarding a complete formulation, parameter definition, criteria and notation aspects.
CHAPTER 2. THEORY

2.1 Synthetic Aperture Radar (SAR)

A Synthetic Aperture Radar (SAR) [18] is a system capable of generating images of the Earth’s surface with meter-scale resolution by means of radar (RAdio Detection And Ranging) technology. The radar sensor lightens a scene and emits electromagnetic pulses to objects in its Line Of Sight (LOS) at a specific frequency and at light speed.

The energy of the radar propagates in all directions, and when the transmitted waves interact with the targets, these last ones scatter back part of the energy received to the antenna, forming echoes. The polarization state of the reflected waves (i.e. the alignment of the electromagnetic field) might be modified due to the physical properties of the targets: shape, orientation and dielectric properties. Thus, the amount of backscattered energy depends on the polarization of the transmitted waves.

SAR systems operate at microwave frequencies, and although this frequency band corresponds to the frequency range between 1 and 100 GHz, SAR applications are found within the range from 1 and 40 GHz. Specifically, the TanDEM-X satellite operates in the X-band, that goes from 8 to 12 GHz and corresponds to a wavelength from 3.8 to 2.4 cm.

These radar systems are able to provide unique images representing the electrical and geometrical properties of a surface in nearly any weather conditions. As active sensors that have its own illumination source, SARs can image in daylight or at night, which makes continuous observation possible.

With respect to data acquisition (see figure 2.1), a radar system consists of a microwave transmitter and a receiver in a moving platform (e.g. an airborne or a satellite system). The direction of the flight path in which the radar moves is known as the azimuth direction, and it coincides with the trajectory and orientation of the velocity vector. On the other hand, the direction perpendicular to the flight path is the range direction, which coincides with the lighting direction pointing at the Earth. The angle of observation, i.e. the incidence angle $\theta$, is defined between the normal to the Earth’s surface and the range direction.

Every radar system is defined by its spatial resolution (i.e. the minimum distance at which two different objects can be detected as separated). In a SAR system, the resolution is
2.1 Synthetic Aperture Radar (SAR)

different for each direction of the image.

Fine resolution in the range direction is achieved by transmitting chirp pulses and, as in a conventional radar, it is inversely proportional to the system’s bandwidth:

\[ \Delta r \sim \frac{c}{2B} \]  

where \( c \) is the light velocity and \( B \) the chirp pulse bandwidth. The factor 2 represents the distance to the target doubled, as the pulse travels through the way forward and the way back.

Resolution in the azimuth, or along-track direction, is increased by synthesizing an artificially large antenna from the echoes received from the sequence of pulses illuminating a target. The pulses in the synthetic aperture contain an unfocussed record of the phase and amplitude history of the target. Therefore, azimuth resolution is limited by the size of the synthetic aperture, directly related with the time a target remains within the radar beam.

After the steps of data acquisition (i.e. the illumination of a scattering object and the collection of the received echoes) and image processing (i.e. focussing the raw data to generate the image of the object) [19], the final Single Look Complex (SLC) images are obtained. Each pixel of these high resolution images contains information regarding amplitude and phase of a resolution cell or location on the Earth.
2.2 SAR Interferometry (InSAR)

SAR Interferometry (InSAR) [19, 20] exploits the phase difference of two or more complex-valued SAR images of the same area to derive more information (compared to using a single image) regarding the imaged objects. In order to provide more information both images have to differ in at least one aspect: the baseline.

The baseline (i.e. angular separation $\Delta \theta$, acquisition time $\Delta t$, frequency $\Delta k$) is precisely the parameter that determines the type of the interferometer (e.g. across-track, along-track, differential...) suitable for a specific application (measurement of DEMs, ocean currents, glacier flows, seismic events, and a great many others). Among all possible InSAR applications, the best known application is the reconstruction of the Earth topography by *across-track interferometry* (see figure 2.2), which is used to explain the InSAR principles.

![InSAR images from sensor ERS-1/2 over the Bachu area, China (100 km × 80 km) (© ESA).](image)

(a) Amplitude  (b) Interferometric phase  (c) DEM

Figure 2.2: InSAR images from sensor ERS-1/2 over the Bachu area, China (100 km × 80 km) (© ESA).

Across-track interferometry is a means to measure the incidence look angle $\theta_0$ as a second coordinate, thus enabling to retrieve the point’s location in space. There are two possible configurations: single-pass and repeat-pass interferometry, both of which use a two antenna arrangement. In single-pass interferometry, the two images are acquired using two antennas simultaneously, normally mounted on the same platform, whereas in repeat-pass interferometry, they are acquired at two different times using one antenna passing over the same area. Increasing the distance (i.e. spatial baseline) between antennas (single-pass) or between tracks (repeat-pass), increases the sensitivity of the interferometric phase to eleva-
Let us approximate the scattering object as a surface describing the Earth’s topography. The distance between both satellite positions is called baseline, and measured perpendicular in its look direction is referred to as effective or spatial baseline. Using images taken at different time intervals (i.e. repeat-pass interferometry), the temporal separation between both acquisitions is known as temporal baseline.

According to figure 2.3, SAR imaging projects the scattering object along circles, where every \((y,z)\)-location of each surface point is reduced to range \(R\) in the SAR image. Given the sensor locations at ranges \(R_1\) and \(R_2\), every point of the Earth’s surface is mapped back into space by triangulation, solving in this way the problem of computing these distance differences.

The two SAR images are formed of a regular grid with complex values (phasors), \(s_1\) and \(s_2\), which can be decomposed in amplitude and phase components,

\[
s_1 = |s_1|e^{j\phi_1},
\]
\[
s_2 = |s_2|e^{j\phi_2},
\]

where \(s_2\) must be aligned and resampled to its corresponding locations in \(s_1\) before obtaining the complex interferogram. The interferogram is generated by the complex multiplication of the conjugate of one grid with the conjugate of the other, yielding the interferometric phase, \(\phi\), to be equal to the phase difference of both SAR images:

\[
v = s_1s_2^* = |s_1||s_2|e^{j\phi}, \quad \text{where} \quad \phi = \phi_1 - \phi_2
\]
The observed phase values $\phi_1$ and $\phi_2$ for the resolution cell are

$$\phi_1 = -\frac{2\pi}{\lambda} \cdot 2R_1 + \phi_{\text{scat},1},$$
$$\phi_2 = -\frac{2\pi}{\lambda} \cdot 2R_2 + \phi_{\text{scat},2},$$

being $\phi_{\text{scat},1}$ and $\phi_{\text{scat},2}$ the contributions of the scattering phases. $R_1$ and $R_2$ are the geometric distances, and the factor 2 refers to the way up and back. Assuming equal scattering characteristics in both images, the interferometric phase is expressed as

$$\Delta \phi = \phi_1 - \phi_2 = -\frac{4\pi}{\lambda} \cdot (R_1 - R_2) = -\frac{4\pi}{\lambda} \cdot \Delta R$$

(2.2.4)

However, in the case of a bistatic system, i.e. TanDEM-X, the factor $4\pi$ turns into $2\pi$ instead.

From figure 2.3 and equation (2.2.4), the phase for each single point in the interferogram is proportional to the path difference between satellite positions and it is not related with the surface’s reflectivity. It is worth noting that the expression above, equation (2.2.4), corresponds to the wrapped interferometric phase, i.e. its values are included within the interval $[-\pi, \pi]$. The unwrapping stage is a crucial and complex step for the topography estimation, i.e. for obtaining DEMs by means of interferometric data [21, 22, 23].

![Figure 2.4: Coordinate InSAR system.](image)

Figure 2.4 represents the defined InSAR coordinates system. An axis system is defined in the master satellite, $S_1$, where $R$ is the range direction and $n$ its perpendicular axis. The projection of the baseline $B$ in the $R$ axis is the parallel baseline, $B_{||}$, and when projected over the $n$ axis the perpendicular one, $B_{\perp}$. The position corresponding to the second
There are several noise and decorrelation sources that affect the SLC images, undermining the quality of the interferometric phase. Hence, it is imperative to define a quality estimator: the interferometric complex coherence, used as a measure for the accuracy of the interferometric phase or as a tool for image classification.

Taking as starting point equation (2.2.2), allow for $s_1$ and $s_2$ to be the backscattered signals from the same resolution cell at two sensor locations separated by a spatial baseline. The interferometric phase $\phi$ is expressed as

$$\phi = \arg (s_1 s_2^*)$$

(2.2.5)

where $*$ denotes the complex conjugation. The interferometric complex coherence $\gamma_{Int}$ is the normalised cross-correlation between these two signals (i.e. SAR images) [19]:

$$\gamma_{Int} = \frac{\langle s_1 s_2^* \rangle}{\sqrt{\langle s_1 s_2^* \rangle \langle s_1^* s_2 \rangle}}$$

(2.2.6)

where the operator $\langle \cdot \rangle$ denotes a spatial averaging between neighbouring pixels (i.e. multi-look) [19]. In the end, from equation (2.2.6) a scalar quantity expressing the degree of correlation between the two SAR images is obtained, as in the following expression

$$\gamma_{Int} = \Gamma \cdot e^{i\phi}$$

(2.2.7)

The interferometric phase $\phi$ is sensitive to height information. The coherence magnitude $\Gamma \in [0, 1]$ is used as an indicator of phase noise, where values of $\Gamma$ close to 1 indicate that the information provided by the phase $\phi$ is precise enough, and thus there exists a high correlation between the images $s_1$ and $s_2$ (i.e. pixels belonging to a well-defined fringe pattern). On the contrary, values close to 0 indicate a high decorrelation degree (i.e. pixels with useless information from an interferometric point of view).

Figure 2.5 summarises the interferometric processing chain, where from the SLC images and orbit information, through the steps of co-registration, orbit generation, filtering and interferogram formation, the final interferogram is obtained, including the phases and
the coherence maps.

Figure 2.5: SAR images from sensor TanDEM-X summarising the interferometric SAR signal processing (© ESA).
2.2 SAR Interferometry (InSAR)

An interferometric coherence map is represented in figure 2.5 (d). The areas where the phase is homogeneous (white areas) are an indicator of high coherence (i.e. high correlation between the two SAR images combined), whereas more noisy areas indicate low coherence values (black areas). In this specific scene, almost all the terrain shows high coherence, and there are just two black areas as a consequence of the low backscattering received from the water.

In the end, after a geo-coding process, the result for the ultimate application is obtained, such as the retrieval of the terrain model (DEM) (see figure 2.5 (e)).

As it has been exposed, coherence is an essential parameter in interferometric applications, since the achievable accuracy of the estimated interferometric phase is degraded by any loss of coherence. As stated in [4, 19, 20], coherence can be expressed as a composition of decorrelation contributions, which are specified following the explanations given in [17, 16] and particularising for TanDEM-X data and the type of scene (agricultural crops):

\[
\gamma_{\text{Int}} = \gamma_{\text{SNR}} \cdot \gamma_{\text{Temp}} \cdot \gamma_{\text{Proc}} \cdot \gamma_{\text{Geom}} \cdot \gamma_{\text{BQ}} \cdot \gamma
\] (2.2.8)

where the total coherence \( \gamma_{\text{Int}} \) and the last term \( \gamma \) are complex numbers and the rest of decorrelation terms are real numbers. The terms are described as follows:

- \( \gamma_{\text{SNR}} \) denotes the decorrelation caused by thermal noise in the sensor, which depends on the signal-to-noise ratio at each pixel. This, together with the vegetation volume, is one of the main decorrelation contributions for TanDEM-X once range filtering has been applied. Under the assumption that signal and noise have the same power in both images (master and slave), the SNR is known to affect the interferometric phase according to the following expression

\[
\gamma_{\text{SNR}} = \frac{\text{SNR}}{1 + \text{SNR}}
\] (2.2.9)

Normally, this decorrelation factor is ignored in interferometric studies, as it only affects to areas with low backscatter. However, it should be taken into account in SAR images acquired in X-band over rice fields, since depending on the season its values can be close to the noise level of TanDEM-X data (see [16]).
• γ_{Temp} is the temporal decorrelation originated by changes in the scene occurred during the time interval between the acquisition of the two images (i.e. repeat-pass interferometry). In a bistatic single-pass interferometer (e.g. TanDEM-X) this term does not affect: γ_{Temp} = 1.

• γ_{Proc} includes any decorrelation caused by the signal processing chain, in which the most important error contribution is usually the one due to the co-registration step. When working with TanDEM-X data, it can be considered negligible (i.e. γ_{Proc} = 1) thanks to the high accuracy of the products provided in CoSSC (Coregistered Single look Slant range Complex) format.

• γ_{Geom}, geometric or baseline decorrelation, is the decorrelation due to the spatial baseline, which causes a wavenumber shift, i.e. a change in the band occupied by the range coordinate spectrum of the two images. This term can be cancelled by performing the spectral shift before the interferogram formation, so that the contributions from both ends of the baseline have the same wavenumber and hence the same coherent phase addition for surface scatterers.

• γ_{BQ} is the loss of the coherence owing to the quatisation of the data with less bits than in the original raw data. It was first defined in [24, 25]. This decorrelation term can be approximately compensated dividing the measured coherences by its value. In particular, when dealing with agricultural crops and either TanDEM-X or TerraSAR-X images, whose products employ a 8:3 block adaptive quantisation, the average value of decorrelation is around 3.5% (i.e. γ_{BQ} = 0.965), as stated in [16].

• γ is the coherence due to the vertical distribution of scatterers of the scene. This is usually named volume decorrelation, γ_{Vol}, as it is present whenever a vegetation or snow/ice volume is in the scene. This is the coherence that will be modelled according to the scene properties. Therefore, is the main term to be estimated from the simulated and measured data. In the following sections, 2.3 and 2.4, it will be explained in detail.
2.3 Polarimetric SAR Interferometry (PolInSAR)

Polarimetric SAR Interferometry (PolInSAR) combines the information provided by polarimetry [26] (i.e. how the polarisation of the radar electromagnetic signal interacts with the scatterer) and interferometry [19, 20] (i.e. height associated with the scatterer).

PolInSAR was first developed back in 1998 and demonstrated using SIRC L-band data [4]. In its original form, it was designed as a more effective mean to separate the different scattering centers present in the vegetation cover. For this purpose, this technique exploits the sensitivity of the interferometric coherence to polarization, thus yielding to further insights into the biophysical properties of a vegetated surface. PolInSAR allows for the quantitative estimation of parameters related to the structure and distribution of scatterers within the canopy (e.g. vegetation height, underlying topography, and plant density), and the relative scattering contributions from the ground and the canopy [15].

The study of the variation of the interferometric coherence with polarisation leads to the development of methods for coherence optimisation [10]. As it will be developed throughout the project, this ultimately yields to the application of optimisation procedures to surface and volume scattering scenarios [9, 7] (e.g. RVoG).

In polarimetric SAR systems, sets of images associated with different polarimetric channels are acquired, that is, multiple images associated with different polarisation states of the transmitted and received signals. Whenever four different types of combinations of polarisation states are acquired ($HH$, $VV$, $HV$, $VH$), the system is known as a full or quad-polarimetric system, whereas if the system simply acquires two combinations, it is called a dual-polarimetric system. In view of the fact that TanDEM-X provides pairs of dual-pol images, in which the user selects the polarisation channels [16], we will focus the formulation taking $HH$ and $VV$, i.e. without $HV$, as the two copolarised channels.

For further explanation of concepts and formulation regarding the electromagnetic wave propagation by means of Maxwell’s equations, vectorial description of the fields or the formulation of the scattering, coherence and covariance matrices, the reader is referred to [26, 17, 27].
Fully polarimetric radar systems (i.e. quad-pol systems) measure $2 \times 2$ complex scattering matrix $[S]$ samples for each resolution cell, containing the full polarimetric information associated with each pixel of the SAR images. The generalisation of the interferometric phase and coherence to the vector case makes use of the coherent scattering vector $\vec{k}$, which is equivalent to a vectorisation of the scattering matrix $[S]$

$$\vec{k} = \frac{1}{2} \text{Trace} ([S] \Psi_P), \quad (2.3.1)$$

where $^T$ denotes the matrix transpose computation, and $S_{ij}$ ($i$, $j$ - H,V) is the complex scattering coefficient for $j$ transmitted and $i$ received polarisation in the HV-polarimetric basis. On the other hand, $\Psi_P$ is the set of $2 \times 2$ orthogonal complex Pauli basis matrices

$$\Psi_P = \left\{ \sqrt{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sqrt{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sqrt{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sqrt{2} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \right\}. \quad (2.3.3)$$

In the case of backscattering in a reciprocal medium, the reciprocity theorem constrains the scattering matrix $[S]$ to be complex symmetric, $S_{HV} = S_{VH}$, therefore, instead of a four-dimensional scattering vector, as in equation (2.3.2), a reduced three-dimensional one can be used:

$$\vec{k} = \frac{1}{\sqrt{2}} [S_{HH} + S_{VV}, \hspace{1mm} S_{VV} - S_{HH}, \hspace{1mm} 2 S_{HV}]^T. \quad (2.3.4)$$

From (2.3.4), according to the dual-pol images provided by TanDEM-X at the two copolarised channels $HH$ and $VV$, each image can be expressed as a two-dimensional scattering vector, $\vec{k}_1$ and $\vec{k}_2$, measured at ends 1 and 2 of the baselines, respectively [16]:

$$\vec{k}_1 = \frac{1}{\sqrt{2}} [S_{HH}^1 + S_{VV}^1, \hspace{1mm} S_{HH}^1 - S_{VV}^1]^T,$$

and

$$\vec{k}_2 = \frac{1}{\sqrt{2}} [S_{HH}^2 + S_{VV}^2, \hspace{1mm} S_{HH}^2 - S_{VV}^2]^T, \quad (2.3.5)$$

where the superscript $n$ from the complex scattering amplitude $S_{ij}^n$ represents the $n$-th image ($n = 1, 2$).
2.3 Polarimetric SAR Interferometry (PolInSAR)

Considering first the general case of a fully polarimetric radar system, i.e. a quad-pol system, by means of the outer product formed from both scattering vectors \( \vec{k}_1 \) and \( \vec{k}_2 \) from equation (2.3.4) for images \( s_1 \) and \( s_2 \), a \( 6 \times 6 \) Hermitian positive semidefinite matrix \([T_6]\) is formed, which is the basic radar observable in PolInSAR, defined as

\[
[T_6] = \left( \begin{array}{c} \vec{k}_1 \\ \vec{k}_2 \end{array} \right) \left( \begin{array}{cc} \vec{k}_1^T & \vec{k}_2^T \end{array} \right) = \begin{bmatrix} [T_{11}] & [\Omega_{12}] \\ [\Omega_{12}]^T & [T_{22}] \end{bmatrix},
\]

where \([T_{11}]\), \([T_{22}]\), and \([\Omega_{12}]\) are \(3 \times 3\) complex matrices given by:

\[
[T_{11}] = \langle \vec{k}_1 \cdot \vec{k}_1^T \rangle \\
[T_{22}] = \langle \vec{k}_2 \cdot \vec{k}_2^T \rangle \\
[\Omega_{12}] = \langle \vec{k}_1 \cdot \vec{k}_2^T \rangle.
\]

\([T_{11}]\) and \([T_{22}]\) are the standard Hermitian coherency matrices containing full polarimetric information of each separate image. \([\Omega_{12}]\) is a new \(3 \times 3\) complex matrix that contains not only polarimetric information, but also the interferometric phase relations of the different polarimetric channels between the images.

Particularising for dual-pol systems, i.e. TanDEM-X, using the scattering vectors \( \vec{k}_1 \) and \( \vec{k}_2 \) from equation (2.3.5) for the outer product, the \([T_6]\) matrix simplifies to a \(4 \times 4\) Hermitian positive semidefinite matrix \([T_4]\), whereas the matrices \([T_{11}]\), \([T_{22}]\) and \([\Omega_{12}]\) simplify to \(2 \times 2\) matrices.

In order to form interferograms, both scattering vectors \( \vec{k}_1 \) and \( \vec{k}_2 \) must be converted into scalars. To this end, unitary complex vectors \( \vec{w} \) are employed to select the desired polarimetric scattering mechanism, yielding to express equation (2.2.1) as

\[
\mu_1 = \vec{w}_1^T \vec{k}_1 \\
\mu_2 = \vec{w}_2^T \vec{k}_2,
\]

which expresses the projection of the scattering vector \( \vec{k}_1 \) and \( \vec{k}_2 \) onto the vectors \( \vec{w}_1 \) and \( \vec{w}_2 \), respectively.
The scalar values $\mu_1$ and $\mu_2$ are linear combinations of the elements of the scattering matrices $[S_1]$ and $[S_2]$. The coefficients of these linear combinations are the entries of vectors $\vec{w}_1$ and $\vec{w}_2$. Combining (2.3.7) with the scalar values in (2.3.8), we can obtain a new expression for the vector interferogram formation as

$$\mu_1 \mu_2^* = (\vec{w}_1^T \vec{k}_1)(\vec{w}_2^T \vec{k}_2)^* = \vec{w}_1^T [\Omega_{12}] \vec{w}_2,$$

from which the interferometric phase is:

$$\phi = \arg(\mu_1 \mu_2^*) = \arg(\vec{w}_1^T [\Omega_{12}] \vec{w}_2).$$

(2.3.10)

From equation (2.2.6), the generalised vector expression for the complex coherence $\gamma$ can be then expressed by

$$\gamma(\vec{w}_1, \vec{w}_2) = \frac{\langle \vec{w}_1^T [\Omega_{12}] \vec{w}_2 \rangle}{\sqrt{\langle \vec{w}_1^T [T_{11}] \vec{w}_1 \rangle \langle \vec{w}_2^T [T_{22}] \vec{w}_2 \rangle}},$$

(2.3.11)

With this last equation we have a more general expression for the interferometric coherence than the conventional scalar expression in (2.2.8). It is important to realise that, if $\vec{w}_1 \neq \vec{w}_2$, the coherence is affected by two different contributions, i.e. the already explained interferometric contribution $\gamma_{Int}$ (2.2.6), and the contribution of the polarimetric correlation $\gamma_{Pol}$ between the two scattering mechanisms corresponding to $\vec{w}_1$ and $\vec{w}_2$:

$$\gamma = \gamma_{Int} \cdot \gamma_{Pol}.$$

(2.3.12)

Only in the particular case of $\vec{w}_1 = \vec{w}_2$, i.e. choosing the same projection vectors at both images, does $\gamma_{Pol}$ become one and $\gamma = \gamma_{Int}$.  

The projection vectors allow for the exploration of the entire polarimetric space, being able to generate resulting interferograms from every possible linear combination of elements of the scattering vectors in the (H,V)-basis. Different combinations show the effect of different sources of decorrelation, provided that some sources of decorrelation affect more some polarisations than others.
2.3.1 Coherence optimisation and the coherence region

The dependence of the interferometric coherence with polarisation leads to the idea of applying polarimetry to optimise the interferogram coherence, presented in (2.3.11). This is carried out by choosing the linear combinations of polarisation channels that yield the highest coherence, i.e. by properly selecting $\vec{w}_1$ and $\vec{w}_2$.

If the interferometric coherence changes slightly with polarisation, then polarimetry plays a weak role. However, if the coherence varies strongly with polarisation, this means that there are relevant changes in the relative positions of scattering mechanisms, which can be exploited for parameter estimation.

This optimisation problem was first solved in [4] by maximising the complex Lagrangian $L$. The starting point is the general expression for the complex coherence, as in equation (2.3.11), from which the optimisation problem is described as

$$\gamma(\vec{w}_1, \vec{w}_2) \Rightarrow \max_{\vec{w}_1, \vec{w}_2} |\gamma|.$$  

(2.3.13)

The general optimisation equation described by the complex Lagrangian function $L$ is the following [17]:

$$L = \vec{w}_1^{*T}[\Omega_{12}]\vec{w}_2 + \lambda_1(\vec{w}_1^{*T}[T_{11}]\vec{w}_1 - 1) + \lambda_2(\vec{w}_2^{*T}[T_{22}]\vec{w}_2 - 1)$$

$$\Rightarrow \begin{cases}
\frac{\partial L}{\partial \vec{w}_1} = [\Omega_{12}]\vec{w}_2 + \lambda_1[T_{11}]\vec{w}_1 = 0 \\
\frac{\partial L^*}{\partial \vec{w}_2} = [\Omega_{12}]^*\vec{w}_1 + \lambda_2^*[T_{22}]\vec{w}_2 = 0
\end{cases},$$  

(2.3.14)

which yields a set of coupled equations for the unknown vectors $\vec{w}_1$ and $\vec{w}_2$ and the Lagrange multipliers $\lambda_1$ and $\lambda_2$.

There are three main approaches to coherence optimisation in polarimetric interferometry. The first one is an unconstrained amplitude optimisation that provides the most general mathematical solution, since it considers full polarimetric diversity (i.e. $\vec{w}_1 \neq \vec{w}_2$), yielding the minimum phase variance interferogram across independent polarimetric variations at either end of the baseline.
The second and third approaches are both constrained ones. They assume that the scattering mechanisms at either end of the baseline are equal (i.e. $\vec{w}_1 = \vec{w}_2$), since for small baselines the optimum scattering mechanisms, in the absence of temporal changes, should be equal.

In particular, the second approach is, as the first one, an amplitude optimisation. As stated in [17], by maximising the real part of the eigenvalue $\lambda(\phi)$ for different values of $\phi$, it maximises the magnitude of the interferometric coherence. This second approach yields to a third important approach to optimisation, based not on coherence amplitude but on phase difference or coherence separation.

From the basis that both SAR images have the same polarisation state (i.e. $\vec{w}_1 = \vec{w}_2$), the third approach seeks the two scattering mechanisms with maximum interferometric separability inside the unit circle, i.e. $\gamma_{\text{max}}(\phi), \gamma_{\text{min}}(\phi)$. Physically, these may represent separated phase centres in a vegetation layer, as we will see in section 2.4.

The drawback of this approach is that it maximises only the real part of the eigenvalue $\lambda$, which means that the maximum is only a local maximum. Hence, to find the true global optima a free phase parameter $e^{i\phi}$ has to be introduced, and the optimisation has to be repeated for different values of $\phi$. This last process of repetition is the general procedure for constrained optimisation, and it is summarised in 2.3.18, see [17] for details.

The coherence region is the geometrical interpretation of the coherence on the complex plane within a unit circle (figure 2.6) [28]. For any given polarimetric interferometry matrix $[T_6]$, there is a sub-region inside the unit circle that encloses all possible values of coherence for all states $\vec{w}$. This is known as the coherence region of the matrix $[T_6]$. In general, the shape and the size of this region are determined by the nature of the scattering processes. The reader is advised to [28, 29] for a detailed explanation.

The boundary of the region can be computed numerically for the constrained case, $\vec{w}_1 = \vec{w}_2$, in which the general optimisation equations of (2.3.14) are simplified to a single eigenvalue equation for $\vec{w}$ as follows
2.3 Polarimetric SAR Interferometry (PolInSAR)

\[
\vec{w}_1 = \vec{w}_2 \quad \begin{cases} 
[\Omega_{12}] \vec{w} + \lambda_1 [T_{11}] \vec{w} = 0 \\
[\Omega_{12}]^T \vec{w} + \lambda_2 [T_{22}] \vec{w} = 0 
\end{cases} \quad (2.3.15)
\]

\[
([T_{11}] + [T_{22}])^{-1} ([\Omega_{12}] + [\Omega_{12}]^T) \vec{w} = -(\lambda_1 + \lambda_2) \vec{w} 
\quad (2.3.16)
\]

\[
[T]^{-1} [\Omega_H] \vec{w} = \lambda(\phi) \vec{w} \quad \begin{cases} 
[\Omega_H] = \frac{1}{2} ([\Omega_{12}] e^{i\phi} + [\Omega_{12}]^T e^{-i\phi}) \\
[T] = \frac{1}{2} ([T_{11}] + [T_{22}]) \end{cases} 
\quad (2.3.17)
\]

\[
\text{max} |\lambda(\phi)| \quad \vec{w}_{\text{opt}} \implies \gamma_{\text{opt}} = \frac{\vec{w}_{\text{opt}}^T [\Omega_H] \vec{w}_{\text{opt}}}{\vec{w}_{\text{opt}}^T [T] \vec{w}_{\text{opt}}} 
\quad (2.3.18)
\]

For each value of \( \phi \), this eigenvalue equation extracts the extreme values (through the maximum and minimum eigenvalues) of the real part of the coherence \([17]\). And for each one of these eigenvalues \( \lambda \), there corresponds an eigenvector that can be used to estimate the corresponding complex coherence:

\[
\frac{\lambda_{\text{max}}, \vec{w}_{\text{max}}}{\lambda_{\text{min}}, \vec{w}_{\text{min}}} \implies \begin{cases} 
\gamma_{\text{max}}(\phi) = \frac{\vec{w}_{\text{max}}^T [\Omega_{12}] \vec{w}_{\text{max}}}{\vec{w}_{\text{max}}^T [T] \vec{w}_{\text{max}}} \\
\gamma_{\text{min}}(\phi) = \frac{\vec{w}_{\text{min}}^T [\Omega_{12}] \vec{w}_{\text{min}}}{\vec{w}_{\text{min}}^T [T] \vec{w}_{\text{min}}} 
\end{cases} \quad (2.3.19)
\]

The boundary of the coherence region can be then reconstructed by estimating for each angle \( \phi \) in the range \( 0 \leq \phi \leq \pi \) the maximum (\( \lambda_{\text{max}} \)) and minimum (\( \lambda_{\text{min}} \)) coherences. This is schematically represented in figure 2.6.
The boundaries of the coherence region define all possible optimum values. Modelling depolarising scatterers at different heights (i.e. volume scattering) results in an elliptical coherence region, which means that both the InSAR coherence and the phase center, as in equation (2.2.7), change with polarisation.

In the next section 2.4, once the fundamentals of the representation of the coherence on the complex plane are understood, we will specify the coherence region obtained with the standard RVoG model.

2.4 The Volume over Ground (VoG) model

The theory presented in this section provides the coherence loci \([10, 7, 3]\) as a limiting form of the coherence region \([28]\) in order to establish strategies for using polarimetric SAR interferometry for physical parameter estimation by means of models of multilayer media (i.e. surface and volume scattering).
Current approaches for retrieving vegetation parameters by using PolInSAR are founded on the inversion of a series of simplified electromagnetic models which treat the vegetation scene as an homogeneous volume over a ground surface [9]. Several works have shown the utility of these models for designing parameter inversion procedures for both forest [10, 7, 30] and agriculture [31, 32, 33].

In general terms, one can conclude that the coherence loci must be somehow related to variations of the vertical structure function $f(z)$ with polarisation. The different models determine this relationship between coherence loci shape and structure function variations for surface and volume scattering scenarios.

The two most important models, widely used in the literature for interpreting the coherence diagram, are: the Random Volume over Ground (RVoG) [8, 9, 10, 7] and the Oriented Volume over Ground (OVOG) [34, 9]. The RVoG model presumes that the wave propagation through the canopy is independent of polarisation due to the random orientation of the targets (e.g. stalks and leaves for crops; branches and trunks for forests) within the vegetation. The OVOG model, on the other hand, acknowledges that different polarisations may propagate differently within the volume layer, which is expected when elements in the canopy show preferred orientations.

In addition, both models are characterised by having a small number of independent physical parameters, usually fewer than observables in the radar data, hence enabling consideration of methods for estimation of such parameters from remotely acquired data. However, in the case of the OVOG model, the polarisation-dependent assumption of the wave propagation through canopies with a preferred orientation (e.g. as wheat in C- and X-bands or maize in L-band) introduces two additional model parameters, i.e. unknowns, accounting for two polarisation states (i.e. perpendicular and parallel to the mean particle orientation), which makes inversion more complicated.

These two models (i.e. RVoG and OVOG) are generated under certain assumptions about the two layer scattering problem (see figure 2.7):

- Assume an exponential structure function $f(z)$ for the direct volume return. This is based on the physical assumption that a layer of uniform density is characterised by a mean wave extinction $\sigma$, though it may nonetheless be a function of polarisation.
• Assume that the vegetation layer is lossy enough and the ground surface rough enough that third- and higher-order interactions can be neglected.

![Figure 2.7: Schematic representation of the geometry of a two-layer scattering problem.](image)

When considering the OV oG model, by allowing polarisation dependence of extinction we are essentially assuming that the volume layer (i.e. layer 1) is an oriented volume, and thus the propagation is not scalar. Nevertheless, by assuming a random volume for layer 1, the propagation factors simplify, as they become independent of polarisation and are a function only of a single mean extinction coefficient $\sigma$. Apart from the RVoG model, there is another approach that also makes use of this assumption, the Interferometric Water Cloud Model (IWCM) [8, 10, 7].

The main difference between these last two models, the RVoG and the IWCM, relies on their assumption regarding the importance of temporal decorrelation. While the RVoG model commonly assumes that $\gamma_{Temp} = \gamma_{SNR} = 1$, which indicates a dominance of volume decorrelation over all other sources, the IWCM normally assumes that $\gamma_{Temp}$ is dominant. From this assumption, together with the fact that the standard acquisition mode of TanDEM-X is single-pass interferometry as a consequence of its bistatic configuration [11], the RVoG model is the one chosen for the retrieval of physical parameters, and hence it is the one in which we will focus hereinafter.
2.4 The Volume over Ground (VoG) model

2.4.1 The Random Volume over Ground (RVoG) model

The Random Volume over Ground (RVoG) model [8, 9, 10, 7] describes the scene as a two-layer medium comprised of an homogeneous volume of scatterers on top of an impenetrable ground layer. In this approach, the structure function $f(z)$ for the two-layer problem has the form shown in figure 2.8.

![Figure 2.8: Schematic representation of the two-layer coherence model for vegetated land surfaces, i.e. RVoG model.](image)

From figure 2.8, equation (2.3.11) and assuming, as usual [4], that $[T_{11}] = [T_{22}]$, the complex interferometric coherence, expressed as a complex scalar magnitude, for a random volume over a ground can be derived as

$$
\gamma = \frac{\bar{w}^*T[\Omega_{12}]\bar{w}}{\bar{w}^*T[T_{11}]\bar{w}} \quad (2.4.1)
$$

From here on, this measurable obtained in (2.4.1) will be treated as the target function to be reproduced with a model that relates the PolInSAR observables to physical parameters of the scattering process.

The matrices in (2.4.1) can be expressed as a function of the physical parameters as follows [7]

$$
[T_{11}] = I_1^V + e^{(-2\sigma h_v)/\cos \theta_0} I_1^G
$$

$$
[\Omega_{12}] = e^{i\phi_2} I_2^V + e^{i\phi_1} e^{(-2\sigma h_v)/\cos \theta_0} I_2^G, \quad (2.4.2)
$$
where
\[
\begin{align*}
I^V_1 &= e^{(-2\sigma h_v)/\cos \theta_0} \int_0^{h_v} e^{(-2\sigma z')/\cos \theta_0} [T_V]dz' \\
I^G_1 &= \int_0^{h_v} \delta(z') e^{(-2\sigma z')/\cos \theta_0} [T_g]dz' = [T_g]
\end{align*}
\]
and
\[
\begin{align*}
I^V_2 &= e^{(-2\sigma h_v)/\cos \theta_0} \int_0^{h_v} e^{(-2\sigma z')/\cos \theta_0} e^{i\kappa z'} [T_V]dz' \\
I^G_2 &= [T_g]
\end{align*}
\]  

From previous equations (2.4.2) and (2.4.3), $\sigma$ is the mean wave extinction in the medium, $\kappa$ the vertical wavenumber of the interferometer (following spectral range filtering) and $\theta_0$ the mean angle of incidence. The angles $\phi_1$ and $\phi_2$ are the phase centers of layers 1 (i.e. within the volume) and 2 (i.e. on top of the layer, at $z_0$), respectively.

Accounting for the contributions of the two layers, $[T_V]$ is the $3 \times 3$ diagonal coherence matrix for the volume scattering, and $[T_g]$ the reflection symmetric ground scattering coherence matrix (i.e. $2 \times 2$ matrices for dual-pol systems), as shown in equation (2.4.4)

\[
[T_V] = m_V \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}, \quad 0 \leq \alpha \leq 0.5
\]

\[
[T_g] = m_g \begin{bmatrix} 1 & t_{12} & 0 \\ t_{12}^* & t_{22} & 0 \\ 0 & 0 & t_{33} \end{bmatrix}
\]

where $m_V$ is the absolute scattering cross-section and $\alpha$ depends on particle shape and varies for single scattering in the range 0 (spheres) to 0.5 (dipole cloud) [10, 17].

From (2.4.4), it follows that in the general case of non-spherical particles, the volume is present in all polarisation channels. Moreover, the wave propagation is scalar, that is, polarisation independent. Therefore, polarisation has no influence on the location of the scattering center and consequently on the interferometric coherence $\gamma_V$, apart from the amount of backscattered intensity.

Similar to the volume, the ground is present in all polarisations and cannot be re-
moved by choosing an appropriate polarisation. However, the main difference is that the backscatter amplitude of the ground varies much more strongly with polarisation than the corresponding volume amplitude. For surfaces, the variation can be up to 25 dB (depending on the surface roughness), while typical variations of the volume scatterer oscillate in the range from 5 to 10 dB [10].

Equation (2.4.2) can be simplified under two assumptions. The first one consists in assuming that the bottom of the canopy corresponds to the ground surface, \( z_0 = 0 \), and the second one that the canopy extends from crown to ground, hence \( \phi_1 = \phi_2 \). Taking both assumptions into account, and combining (2.4.2), (2.4.3) and (2.4.4), we obtain the following expression for the complex coherence [7]

\[
\gamma = \frac{\bar{w}^* T \left( e^{i\phi_0} I_V^T + e^{i\phi_0} e^{(-2\pi h_v)/(\cos \theta_0)} I_G^T \right) \bar{w}}{\bar{w}^* T \left( I_V^T + e^{(-2\pi h_v)/(\cos \theta_0)} I_G^T \right) \bar{w}},
\]  

(2.4.5)

which can be rewritten as the equation of a straight line in the complex plane going through the point \( \gamma_V \) with direction \( (1 - \gamma_V) \) as

\[
\gamma(\bar{w}) = e^{i\phi_0} \gamma_V + \frac{\mu(\bar{w})}{1 + \mu(\bar{w})} \left( \gamma_V + \frac{\mu(\bar{w})}{1 + \mu(\bar{w})} (1 - \gamma_V) \right),
\]

(2.4.6)

where the polarisation dependence is isolated in a single term \( F(\bar{w}) \), which varies within the limits occurring at one end for pure volume scattering (\( \mu = 0 \)), and at the other by pure surface scattering (\( \mu \to \infty \)).

The ground-to-volume scattering ratio \( \mu \) in (2.4.6) includes not only the effects of wave extinction in the medium but, more importantly, the dependence of polarisation:

\[
\mu(\bar{w}) = \frac{2\sigma}{\cos \theta_0 (e^{-2\pi h_v)/(\cos \theta_0}) - 1} \frac{\bar{w}^* T [T_g] \bar{w}}{\bar{w}^* T [T_V] \bar{w}} \geq 0.
\]

(2.4.7)

The polarisations that maximise and minimise the \( \mu \) ratio are of interest in establishing the coherence loci for RVoG. In order to find them, we have to solve the eigenvalue
equation arising from optimisation of the $\mu$ ratio as follows \cite{17}:

$$\max_{\vec{w}} \frac{\vec{w}^T [T_B]\vec{w}}{\vec{w}^T [T_A]\vec{w}} \implies [T_A]^{-1}[T_B]\vec{w}_{\text{opt}} = \lambda\vec{w}_{\text{opt}}. \quad (2.4.8)$$

The solution of (2.4.8) shows that the eigenvalues $\mu_i$ are non-degenerate due to the strong polarisation dependence of ground scattering. Another feature of the solution is that the three eigenvectors obtained are not mutually orthogonal, in contrast to the case of polarimetric interferometry for an oriented volume. And finally, one can also note that for a reflection symmetric ground with azimuthally symmetric vegetation cover (as implied by $[T_g]$), the minimum eigenvalue will be obtained for the HV channel.

Consequently, in (2.4.6) only $\mu$ is a function of polarisation, since the term $\gamma_V$ is defined as a polarisation independent volume integral:

$$\gamma_V = \frac{m_V \int_0^{h_v} e^{(2\sigma\zeta')/\cos \theta_0} e^{ik\zeta'z'} dz'}{m_V \int_0^{h_v} e^{(2\sigma\zeta')/\cos \theta_0} dz'} \quad \text{(2.4.9)}$$

$$= \frac{2\sigma}{\cos \theta_0 (e^{(2\sigma h_v)/\cos \theta_0} - 1)} \int_0^{h_v} e^{ik\zeta'z'} e^{(2\sigma\zeta')/\cos \theta_0} dz'$$

The resulting coherence loci of the RVoG model is a straight line, and its geometrical interpretation is shown in figure 2.9.

Figure 2.9 shows the positions of the coherences measured in a real example of a typical polarimetric interferometric matrix $[T_6]$. The figure represents schematically two coherences that define two points on the boundary of the coherence region, $\gamma(\vec{w}_{\text{max}})$ and $\gamma(\vec{w}_{\text{min}})$. The line containing the maximum and minimum values of the coherence crosses the unit circumference at the topographic phase $\phi_0$. As we vary the polarisation over all possible mechanisms, the interferometric complex coherence will remain contained within the boundary region, i.e. the boundaries of the region define the optimum values. In a real case, despite being contained within the boundaries, some of the coherence candidates will lie along the straight line and some not, as the gray and black crosses in the figure represent. Only those lying on the line defined by the model will be the possible coherence values.
2.4 The Volume over Ground (VoG) model

Figure 2.9: Geometrical interpretation of PolInSAR for the RVoG scattering model.

From figure 2.9, it is important to note that the line that defines the coherence loci is not radial, resulting from the complex nature of the volume coherence, and thus there is a phase as well as an amplitude dependence with polarisation.

According to the idea of a exponential structure function (figure 2.8), contributions from the top of the volume are weighted more strongly in the coherence calculation than those deeper into the volume, as the latter experience a smaller incidence signal owing to wave extinction. The power loss extinction coefficient $\sigma$ (m$^{-1}$ or dB/m) represents the combination of the physical effects of wave attenuation due to absorption of energy by the volume, and scattering loss due to the presence of particles.

Equations (2.4.6) and (2.4.9) demonstrate that the coherence is a function not only of the volume depth $h_v$ but also of the shape of the vertical structure function: $\gamma_{V} = f(h_v, \sigma)$. Figure 2.10 shows an example of how the coherence $\gamma_V$ varies for an exponential profile with varying $\sigma$ and depth $h_v$. 
Figure 2.10: Representation of the RVoG complex volume coherence variation inside the unit circle for varying extinction.

In the figure, the variation of the complex volume coherence inside the unit circle on the complex plane corresponds to four extinction values (0, 0.125, 0.75 and 1.75 dB/m) and layer depths varying from 0 to 1.5 m ($\kappa_Z = -1.31$ rad/m). Here we see that the SINC model spirals quickly from the topographic phase $\phi_0 = 60^\circ$ to the origin (i.e. to zero coherence), while the high-extinction cases show gentler spirals with more rapid phase variation around the unit circle.

So far, we have reviewed the basis of the RVoG model, including an understanding of its coherence loci as a straight line (2.4.6). The formulation of the model also distinguishes between interferometry in single-transmit (single-tx) and alternate-transmit (alternate-tx) [3, 35].

Although both interferometry modes can be used in different configurations, in the case of study, single-transmit is related to single-pass interferometry. The configuration used in this case corresponds to a single-pass bistatic configuration: two different antennas are mounted on the TerraSAR-X and TanDEM-X sensors, respectively, and only one of them is
transmitting, whereas both of them are receiving, resulting in one monostatic image and one bistatic image.

Usually, single-transmit is associated with single-pass interferometry and a bistatic configuration, whereas repeat-pass interferometry is related to a monostatic configuration. In this latter case, the same antenna works as transmitter and receiver passing over the same area at two acquisition times, resulting in two monostatic images.

Moreover, as it was advanced in the introduction, resulting from the bistatic configuration of TanDEM-X, the formulation accounts for two cases of dominant ground return: a direct backscattering from the ground that entails surface effects, and a double-bounce contribution coming from the interactions of the ground with the volume, that entails also volume effects from the interferometric point of view.

In next chapter, we are going to specify the formulation of the RVoG for single-transmit interferometry (i.e. data provided by TanDEM-X) and we are going to distinguish and evaluate the two different cases of dominant ground return.
This chapter details the PolInSAR parameter estimation procedures that have been developed, implemented and evaluated in this project, and that are derived from the theoretical foundations of the previous chapter.

The studies presented to date have demonstrated that vegetation scattering is perhaps the most complex of all scatterers, and simple models for penetration depth are of limited use. The vegetation volume is characterised for being a strong scattering environment, combining multiple wavelengths of penetration with the presence of strong dielectric discontinuities (moist, branches, leaves, twigs, and so on), which are usually larger than the wavelength and act to scatter the waves in a complex manner.

In the previous chapter, we have developed the theory necessary to achieve the understanding of the polarimetric interferometric coherence loci for a two-layer scattering model. We now turn to implement algorithms for the inverse problem, i.e. estimation methods of biophysical parameters of the two-layer model (according to the RVoG approach) from observations of the coherence variation with polarisation [10, 7, 30].

Accounting for the fact that TanDEM-X standard acquisition mode is single-pass as a result of its bistatic configuration [11], in this chapter we are going to particularise the RVoG model considering the presence of a double-bounce contribution at the ground that appears whenever a bistatic configuration is used, and that affects the interferometric coherence with a decorrelation factor.
3.1 Forward model

The following two sections, 3.1 and 3.2, detail the implemented algorithms for estimating physical parameters of crops by means of PolInSAR in order to assess the feasibility of the current RVoG approach taking into account the presence of a decorrelation factor: a double-bounce term present in data obtained with the TanDEM-X sensor, which implies the use of a bistatic configuration resulting from single-pass interferometry.

A robust estimation of RVoG model parameters, such as height, extinction coefficients and ground-to-volume ratios, might help characterise the structural and dielectric features of vegetation, as well as its interaction with polarised waves at different frequencies. Regarding agriculture, structural parameters such as underlying soil moisture, moisture of vegetation layer, height of vegetation layer and soil roughness, among others, may have a direct application in farming management, ecosystem modelling, water cycle management and desertification.

Compared to forest height retrieval, crop monitoring requires more accurate estimates; therefore, larger spatial baselines than those for forest height retrieval need to be used. Moreover, the fast growth cycle of the crops introduces a strong temporal decorrelation, and as a result, only single-pass or short temporal baseline systems are suitable [30].

As we have introduced in sections 2.3 and 2.4, the estimation approaches are characterised by a specific number of independent physical parameters. In particular, the RVoG model (by reference to the schematic diagram in figure 2.8) defines the interferometric coherence at different polarisations as a function of two sensor parameters (i.e. the radar look angle $\theta_0$ and the sensitivity of the interferometric phase to elevation changes $\kappa_Z$) and a series of structural parameters:

- Ground phase $\phi_0$, i.e. the topographic phase at the ground level $\phi(z_0) = \kappa_Z z_0$.
- Height of the volume layer $h_v$, i.e. the vegetation height.
- Extinction coefficient $\sigma$, a quantity accounting for both the wave attenuation through the volume and the single-scattering loss (under the assumption of ignoring multiple scattering effects from the particles within the volume).
3.1 Forward model

- Ground-to-volume ratio $\mu$, the only polarisation-dependent quantity (in RVoG) expressing the ratio of the ground backscattering power attenuated by the volume layer, to the volume backscattering power.

According to the defined two-layer scene (i.e. a vegetation layer and a ground surface), as in figure 2.8, the scattering from the ground is located at a single point in the vertical coordinate $z_0$, whereas the scattering from the volume is distributed according to the scattering function $f_V(z)$. Starting from this assumption, it is possible to express the coherences $\gamma$ that are obtained at different polarimetric channels $\tilde{w}$ as a function of the scene properties and the vertical wavenumber $k_Z$. Taking as starting point equations (2.4.6) and (2.4.9), the most complete expression for a bistatic system, considering that the response from the ground can be composed of two contributions (surface or direct scattering, and double-bounce scattering) is the following [8, 9, 3, 2, 36]:

$$\gamma(k_Z, \tilde{w}) = e^{i\phi_0} \frac{\gamma_V + m_D(\tilde{w}) + \frac{\sin k_z h_v}{k_z h_v} m_{DB}(\tilde{w})}{1 + m_D(\tilde{w}) + m_{DB}(\tilde{w})}$$

(3.1.1)

where the terms $m_D(\tilde{w})$ and $m_{DB}(\tilde{w})$ are the ground-to-volume backscatter ratios corresponding to the direct $D$ and double-bounce $DB$ contributions, respectively. The first term in the numerator, $\gamma_V$, is the coherence that would produce the volume alone (without the presence of the ground), and it is worth to recall from equation (2.4.9) as a function of $f_V(z)$ as:

$$\gamma_V = \frac{\int_0^{h_v} f_V(z) e^{i\nu z} dz'}{\int_0^{h_v} f_V(z) dz'}.$$  

(3.1.2)

The $\sin(x)/x$ term that appears in (3.1.1) before the double-bounce ground-to-volume ratio in the numerator is the decorrelation term present in bistatic configurations. The argument of this term is $k_z h_v$, not $k_Z h_v$. The wavenumber $k_z$ is defined as (see [9, 3, 2]):

$$k_z = k_Z \sin^2 \theta_0.$$  

(3.1.3)

Hereafter we will use $\gamma_{DB}$ to refer to the decorrelation term due to the presence of the double-bounce contribution at the ground:

$$\gamma_{DB} = \frac{\sin k_z h_v}{k_z h_v}$$

(3.1.4)
In many natural scenes, we can expect the ground contribution to be dominated by the direct response of the ground surface, which usually happens due to the small bistatic angle of TanDEM-X combined with the rather low dihedral scattering contributions at X-band (when compared with lower frequencies). In those cases, the coherence expression would be:

\[
\gamma(\kappa_Z, \vec{w}) = e^{i\phi_0} \frac{\gamma_V + m_D(\vec{w})}{1 + m_D(\vec{w})}
\] (3.1.5)

In other scenarios, as in rice fields and mangroves, the flooded ground acts like a mirror. In such a case the double-bounce contribution dominates the ground contribution, and the coherence expression from (3.1.1) results in:

\[
\gamma(\kappa_Z, \vec{w}) = e^{i\phi_0} \frac{\gamma_V + \sin k_z h_v m_{DB}(\vec{w})}{1 + m_{DB}(\vec{w})}
\] (3.1.6)

Whenever there is no clear dominance of one of the two ground contributions, the most general expression for coherence (3.1.1) should be used [2].

The effect of the double-bounce decorrelation term is illustrated in figure 3.1. The true topographic phase \(\phi_0\) is defined by the crossing of the line with the circumference of radius \(\gamma_{DB}\), which is different from the phase \(\phi'_0\) that would have been obtained by the crossing with the unit circumference, as it would be done when the direct ground contribution dominates (3.1.5). Hence, the first effect of the model selection is a bias in the estimation of ground topography. In second place, a wrong topography will influence the estimation of the rest of model parameters. These aspects are discussed in next chapter.

If the radar response of the ground is dominated by the double-bounce [9], the main contributions to the received field are those included in figure 3.2: direct echo from the volume, ground-volume signal, and volume-ground signal. In the figure, \(\vec{R}_1\) and \(\vec{R}_2\) are the two acquisition positions of the SAR sensor at points 1 and 2 of the baseline, respectively, and the blue and red arrows correspond to the path difference for the receiving channels at those points. Moreover, the specular angle \(\theta_{spec}\) is assumed to be the same for both receivers, since in a real case the difference is negligible [9].
3.1 Forward model

Figure 3.1: Unit circle on the complex plane with the representation of the coherences and the line of the RVoG model when the double-bounce ground contribution dominates (3.1.6).

Figure 3.2: Propagation paths for direct (i.e. volume) and double-bounce (i.e. ground-volume and volume-ground) contributions in single-transmit mode. The blue and red arrows correspond to the path difference for the receiving channels at $\vec{R}_1$ and $\vec{R}_2$, respectively [2].

The path followed by the field for each contribution, from the transmitter to the receiver, depends on the interferometric mode. Figure 3.2 illustrates the single-tx mode [2], and table 3.1 summarises the total path length for each contribution in this interferometric mode [3].
Table 3.1: Total propagation path $P(\vec{R})$ for single-transmit interferometry [3].

In order to account for the effect of the double-bounce decorrelation term, the implemented algorithm allows the user to select the corresponding equation for the forward model, (3.1.5) or (3.1.6), to consider a scene in which either the direct or the dihedral contribution is dominant, respectively.

### 3.2 Inversion schemes

There exist different ways to invert vegetation height and the rest of model parameters from PolInSAR data according to the previous expressions [17, 15, 32, 35, 16, 36]. In this work we particularise the inversion procedures for dual-polarimetric TanDEM-X data adapted to the particular properties of rice fields (i.e. dominance of the double-bounce ground contribution), for which the parameterization of the two interferometric coherences in terms of (2.4.6) requires five parameters: crop height $h_v$, extinction $\sigma$, ground topography phase $\phi_0$, and two ground-to-volume amplitude ratios $\mu(\bar{\alpha})$, one for each polarisation.

The present project implements and evaluates two new adapted algorithms: one in which the cost function is the distance of the modelled coherences to the measured coherences, computed for a set of $N_{inv}$ initial guesses, (section 3.2.1) [36, 16], and another one in which the cost function is the topographic phase produced by the modelled coherences (section 3.2.2) [15, 32, 35].

The block diagrams of both proposed algorithms are sketched in figures 3.3 and 3.4, respectively.
3.2 Inversion schemes

Figure 3.3: Block diagram of the first proposed RVoG inversion approach for PolInSAR data and crop scenes. The diagram describes the inversion procedure for a single PolInSAR model scene.
Search space:
Minimisation of the topographic phase pair
produced by the modelled coherences:

Parameter retrieval

Estimates

Figure 3.4: Block diagram of the second proposed RVoG inversion approach for PolInSAR data and crop scenes. The diagram describes the inversion procedure for a single PolInSAR model scene.

The figures demonstrate that both procedures have a common part: taking a set of PolInSAR scenes as input (either from real or simulated data), they first compute the extreme coherences. However, they differ in the inversion approach, i.e. inversion of the model of the scene for the numerical estimation of model parameters (red dashed box). In the next sections 3.2.1 and 3.2.2, each of these inversion procedures is detailed.
The critical part of the inversion approaches is the parameter retrieval stage. It is framed in a dashed box, red in the first approach (figure 3.3), and gray in the second one (figure 3.4). This key part is the new part of each algorithm that has been developed for the first time, regarding the properties of crop scenes and data acquired with the TanDEM-X sensor. Nonetheless, by changing these initial considerations, the developed algorithms can evaluate different scenes (e.g. forest scenes), for different configurations (i.e. either dual- or quad-pol systems), and different inversion approaches (i.e. single-, dual- or multi-baseline PolInSAR inversion approaches).

It is worth to advance that in the first algorithm (figure 3.3), as part of the new strategy developed, we have introduced a set of initial guesses, $N_{inv}$, in order to avoid local minima and ensure that the global solution of the inversion is a global minimum.

When the input PolInSAR data is obtained from real data, they correspond to multi-looked PolInSAR $[T_4]$ matrices, for the specific case of dual-pol TanDEM-X data (i.e. $[T_6]$ matrices for quad-pol interferometry). In such a case, the first step consists of a line fit to the coherences, which can be carried out in several ways (see [17]). In the proposed methodology, this line fit to the coherences consists in generating the border of the coherence region (black ellipse in figure 2.6) and choosing the coherences with maximum and minimum phase to define the line (see [36]). That is, choosing the coherences closest and farthest from the topographic phase $\phi_0$, i.e. coherences with maximum and minimum ground contribution.

On the other hand, if we consider simulated PolInSAR data as input, the extreme coherences are computed from a forward model, either considering a monostatic scene dominated by the direct ground contribution (3.1.5), or a bistatic scene dominated by the double-bounce ground contribution (3.1.6).

Following the assumption that the coherence region of the RVoG model is a straight line along the ground-to-volume amplitude ratio $\mu(\tilde{w})$, the extreme coherences according to the polarisation states $\tilde{w}_{max}$ and $\tilde{w}_{min}$ characterised by the maximum and minimum ground contributions, are denoted as the two extreme interferometric coherences, $\gamma(k_1, \tilde{w}_{max})$ and $\gamma(k_1, \tilde{w}_{min})$, respectively.
Once having the extreme coherences, the next step corresponds to the topography computation using the phases extremes. If we are taking as input $[T_1]$ PolInSAR matrices, this step is included in the line fit to the coherences.

However, if we are going to invert a scene considering a bistatic dihedral configuration, this step is carried out in the numerical minimisation itself. In such case, for each simulation carried out, the inversion of the model computes first the coherence of pure ground contribution as $\gamma_{DB}$, and then the new value of topographic phase $\phi_0$ (see figure 4.8). This is explained in detail in section 3.2.1, with the description of the first implemented inversion approach (corresponding to the block diagram in 3.3).

On the contrary, if the inversion assumes a monostatic direct configuration, the topographic phase $\phi_0$ is then obtained by the intersection of the line defined by $\gamma(\kappa, \bar{w}_{\text{max}})$ and $\gamma(\kappa, \bar{w}_{\text{min}})$ and the unit circle moving from $\gamma(\kappa, \bar{w}_{\text{min}})$ to $\gamma(\kappa, \bar{w}_{\text{max}})$ [8, 17]:

$$\phi_0 = \arg \left\{ \gamma(\kappa, \bar{w}_{\text{max}}) - \gamma(\kappa, \bar{w}_{\text{min}})(1 - F) \right\}, \quad (3.2.1)$$

where, $$F = \left( -B - \sqrt{B^2 - 4AC} \right)/(2A)$$

with coefficients

$$A = |\gamma(\kappa, \bar{w}_{\text{min}})|^2 - 1,$$

$$B = 2\text{Re}(\gamma(\kappa, \bar{w}_{\text{max}}) - \gamma(\kappa, \bar{w}_{\text{min}}))\gamma^*(\kappa, \bar{w}_{\text{min}}),$$

and $$C = |\gamma(\kappa, \bar{w}_{\text{max}}) - \gamma(\kappa, \bar{w}_{\text{min}})|^2.$$

In the case of having as input real data, after the line definition, i.e. (3.2.1) and (3.2.2), the next step is to make an SNR and BQ correction of the two coherences, as defined theoretically in section 2.2, in the expression 2.2.8. This step is schematically represented in figures 3.3 and 3.4 in a black dashed box. See [16] for details.

Once the topographic phase and the extreme interferometric coherences have been estimated accordingly (i.e. crossing the unit circumference or the circumference of radius $\gamma_{DB}$), and compensated with and SNR and BQ correction in the case of real data, they are then compensated by the topographic phase. Thus, the coherence loci as a straight line is defined by this pair of compensated coherences with topographic phase $\phi_0 = 0^\circ$.

Henceforth, in order to distinguish between the initial input coherences to the inversion (either simulated or from real data), and the retrieved coherences that will provide the
parameter set of $h_v, \sigma, \mu_{\text{min}}$ and $\mu_{\text{max}}$ solutions, we are going to refer to the first ones as the “measured coherences”, i.e. $\gamma(\kappa Z, \bar{w}_{\text{max}})$ and $\gamma(\kappa Z, \bar{w}_{\text{min}})$, and to the second ones, as the “modelled coherences”, i.e. $\gamma_V(\kappa Z, h_v, \sigma, \mu_{\text{max}})$ and $\gamma_V(\kappa Z, h_v, \sigma, \mu_{\text{min}})$. This notation is the one followed in the block diagrams of the two inversion approaches, figures 3.3 and 3.4.

Despite the following steps are specific for each inversion approach, there are some a priori information common to both PolInSAR RVoG inversion approaches. Assuming the H-V linear basis, the crop height $h_v$, the extinction coefficient $\sigma$, and the ground-to-volume ratios $\mu_{pp}$ (where $p = H$ or $V$ for dual polarisation), are estimated from a set of initial values that correspond to a specific example. Either to the set of initial guesses (i.e. inversion approach 1) or to the $(h_v, \sigma)$ space (i.e. inversion approach 2), we assign an interval of possible values under certain known criteria.

To ensure physically reasonable solutions for crop scenes (i.e. rice) and enhance inversion speed, both inversion procedures (as schematically represented in 3.3 and 3.4, respectively) are run under the optimisation ranges presented in table 3.2.

<table>
<thead>
<tr>
<th>A priori information</th>
<th>Optimisation ranges</th>
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<tbody>
<tr>
<td>Inversion approach 1</td>
<td>Inversion approach 2</td>
</tr>
<tr>
<td>$h_v \in [0.05, 2.0]$ (m)</td>
<td>$h_v \in [0.05, 2.0]$ (m)</td>
</tr>
<tr>
<td>H-V polarisation basis</td>
<td>$\sigma \in [0.1, 10]$ (dB/m)</td>
</tr>
<tr>
<td></td>
<td>$\sigma \in [0.1, 10]$ (dB/m)</td>
</tr>
<tr>
<td></td>
<td>$\mu \in [-10, 10]$ (dB)</td>
</tr>
<tr>
<td></td>
<td>$\mu \geq 0$</td>
</tr>
</tbody>
</table>

Table 3.2: A priori information and optimisation ranges adapted to crop scenes for both RVoG inversion procedures.

The intervals of the initial parameters presented in table 3.2 are adapted to the specific scene of study (i.e. crop scenes). However, as it has already been mentioned, changing these initial conditions the algorithms can evaluate different scenarios. For instance, changing the vegetation height range $h_v$ and generalising the solutions for the vertical wavenumber $k_v = \frac{\kappa Z h_v}{2}$, we can extrapolate the results for forest.
3.2.1 Minimising the distance of the measured coherences to the modellled coherences

The first implemented method takes as a reference the algorithms proposed by [17, 36, 16], with changes regarding the specific properties of crop scenes (i.e. rice) and the data obtained from the TanDEM-X sensor. The method is described by an iterative optimisation in which the cost function is the distance of the measured coherences to the modellled coherences.

Since we are evaluating the effect of the double-bounce contribution, the algorithm allows the user to choose between equation (3.1.5) or (3.1.6) as the expression to be inverted, as we did for the forward model. When data corresponds to a scene in which the direct contribution is dominant, the model used for the retrieval should consider equation (3.1.5) in accordance. If this is the case, the minimisation of the distance between the measured coherences $\gamma(k_Z, \bar{w})$ and the modellled ones $\gamma_V(k_Z, h_v, \sigma, m_D)$ is the following:

$$\min_{\phi_0, h_v, \sigma, m_D, m_D_{max}} \left\| \gamma(k_Z, \bar{w}_{\text{max}})e^{-i\phi_0} - \gamma_V(k_Z, h_v, \sigma, m_D_{\text{max}}) \right\|, (3.2.3)$$

On the contrary, when the dominant contribution from the ground is the double-bounce, the expression to be inverted should be (3.1.6) instead, yielding to the minimisation

$$\min_{\phi_0, h_v, \sigma, m_{DB}, m_{DB_{max}}} \left\| \gamma(k_Z, \bar{w}_{\text{min}})e^{-i\phi_0} - \gamma_V(k_Z, h_v, \sigma, m_{DB_{\text{max}}}) \right\|. (3.2.4)$$

When the scene evaluated corresponds to a forest, the minimum ground-to-volume contribution can be assumed to be $m_D(\bar{w}_{\text{min}}) = 0$ (see [36]). This assumption is usually satisfied for forest scenes, due to a negligible contribution coming from the ground compared to the contribution of the volume (i.e. tens of meters of vegetation height). However, in view of the fact that we are evaluating crop scenes (i.e. from a few centimetres to a few meters of vegetation height), all polarimetric channels present some contribution from the ground, and hence, neither $m_D(\bar{w}_{\text{min}})$ nor $m_{DB}(\bar{w}_{\text{min}})$ can be assumed to be zero.
In both cases of dominant contribution, there are 5 model parameters (i.e. unknowns) and only 4 real observables (i.e. two complex coherences), so we face an undetermined system to solve.

If the direct ground contribution is dominant, the topographic phase lies on the unit circumference of radius 1 on the complex plane \((\phi'_0\) in figure 3.1). However, if the double-bounce is the dominant ground contribution, the extra decorrelation term \(\gamma_{DB} = \frac{\sin(k_z h_v)}{k_z h_v}\) in the numerator of equation (3.1.6) makes the coherence of pure ground contribution \((m_{DB} \to \infty)\) to not be equal to 1 but equal to \(\gamma_{DB}\). This decrease in the coherence is a result of the differences in the travel paths of the double-bounce contributions when returning to the receiving antennas. The main consequence for the inversion is that the topographic phase is shifted with respect to the point at which the line crosses the unit circumference, as it can be observed in figure 3.1. This is discussed in [3] and [2].

Therefore, depending on the model scene considered, the topographic phase would lie in the circumference of radius either 1 or \(\gamma_{DB}\). If the measured extreme coherences, \(\gamma(k_Z, \bar{w}_{max})\) and \(\gamma(k_Z, \bar{w}_{min})\), correspond to a scene with a monostatic configuration, or when the direct contribution from the ground dominates (3.1.5), the topographic phase can be directly obtained applying the geometrical solution (line fit) described in (3.2.2). Then, in this case, the remaining four parameters can be found numerically by minimising equation (3.2.3), in accordance with the type of scene.

In contrast, this is not fulfilled if the measured coherences are derived from a bistatic configuration, in case the double-bounce ground contribution dominates (3.1.6). Since the double-bounce decorrelation term \(\gamma_{DB}\) depends on the vegetation height, which is one of the unknowns, the topographic phase cannot be estimated directly from the intersection between line and unit circumference.

From the initial guess as input to the numerical optimisation, we take first the value of height vegetation, with which we obtain the corresponding \(\gamma_{DB}\) and find the intersection of the line with the circumference of radius \(\gamma_{DB}\). Hence, the coefficient \(A\) from equation (3.2.2) becomes

\[
A = |\gamma(k_Z, \bar{w}_{min})|^2 - |\gamma_{DB}|^2. \tag{3.2.5}
\]
This provides an initial value for $\phi_0$ (i.e. as in figure 3.1). With that $\phi_0$, we find the set of $h_v$, $\sigma$, $m_{DB_{\text{max}}}$ and $m_{DB_{\text{min}}}$ which provides the minimum distance between the modelled and the measured coherences. Then, the new value of $h_v$ is used to update $\gamma_{DB}$, and the computation of the topographic phase $\phi_0$ and the minimisation of carried out again. The iteration continues until convergence to a solution with minimum distance between model and observations.

In both cases, i.e. monostatic and bistatic configurations, the numerical inversion takes as input the measured coherences with maximum and minimum ground contributions, $\gamma(\kappa_Z, w_{\text{max}})$ and $\gamma(\kappa_Z, w_{\text{min}})$, computed accordingly in the case of simulated data, and a set of initial guesses for inversion $h_v^{(i)}$, $\sigma^{(i)}$, $\mu_{\text{max}}^{(i)}$ and $\mu_{\text{min}}^{(i)}$, $\forall i = 1 \ldots N_{\text{inv}}$. For each scene, and for each initial guess, the minimisation obtains a set of $h_v$, $\sigma$, $\mu_{\text{max}}$ and $\mu_{\text{min}}$ that provides the minimum distance between the modelled and the measured coherences. This procedure is repeated for each set of initial guesses. The set of solutions obtained from each initial guess is stored.

In the end, for each scene, we save every solution from every initial guess, along with the mean, median and standard deviation values of the entire set of initial guesses. This methodology allows us to analyse the results with minimum distance (of all initial guesses), a percentage of them, or their average value.

From this explanation, one can anticipate that if there is a mismatch between the natural scene and the model used for inversion, this would result in an error first in the estimation of the topography and, as a consequence, in the inversion of the rest of model parameters. In next chapter we aim to quantify this error.

3.2.2 Minimising the distance of the topographic phase produced by the measured coherences

The second implemented method takes as a reference the algorithm proposed in [15, 32, 35, 5], again with changes concerning data acquired with the TanDEM-X sensor (i.e. dual-pol images) and the use of the RVoG model (i.e. extinction coefficient is polarisation independent). The method is described by an iterative optimisation in which the cost function
3.2 Inversion schemes

is the topographic phase $\phi_0$ produced by the measured coherences.

The red dashed box in figure 3.4 represents the numerical inversion procedure of this second method. For the sake of interpretation, the inversion scheme is split into three steps described below. Note that the H-V is the known linear polarisation basis, where the notations $HH$ and $VV$ identify the two copolarised channels corresponding to the dual-pol images. The subscript $p$ refers to the receive and transmit polarisation $(p, p) = (H, H), (V, V)$. Moreover, the subscripts of the intermediate estimates are in lowercase, whereas capital letters subscripts are used for the final estimates.

3.2.2.1 Step 1

From the expression of the interferometric complex coherence $\gamma(\tilde{w})$ as a straight line (2.4.6), we can isolate the polarisation dependence as follows:

$$F(\tilde{w}) = \frac{\mu(\tilde{w})}{1 + \mu(\tilde{w})}, \quad 0 \leq F(\tilde{w}) \leq 1.$$  (3.2.6)

The magnitude squared of $\gamma(\tilde{w})$ can be then computed as $|\gamma_{RVoG}|^2$,

$$|\gamma_{RVoG}|^2 = (1 + |\gamma_{RV}|^2 - 2\text{Re}(\gamma_{RV}))F^2 + (2\text{Re}(\gamma_{RV}) - |\gamma_{RV}|^2)F + |\gamma_{RV}|^2.$$  (3.2.7)

For each element of the $(h_v, \sigma)$ space, a quadratic equation in the variable $F_{pp} = F(\tilde{w}_{pp})$ can be derived from (3.2.7), i.e.,

$$f(F_{pp}; h_v, \sigma) = aF_{pp}^2 + bF_{pp} + c = 0,$$  (3.2.8)

where

$$a(\kappa_Z, h_v, \sigma) = 1 + |\gamma_{RV}|^2 - 2\text{Re}(\gamma_{RV})$$

$$b(\kappa_Z, h_v, \sigma) = 2[\text{Re}(\gamma_{RV}) - |\gamma_{RV}|^2]$$

$$c(\kappa_Z, h_v, \sigma) = |\gamma_{RV}|^2 - |\gamma_{RVoG}|^2.$$  (3.2.9)

In the previous expressions (3.2.9) and (3.2.7), the term $\gamma_{RVoG} = \gamma(\kappa_Z, \tilde{w}_{pp})$ refers to the compensated measured coherence, obtained from either real data or computed from
simulated data through the forward model (as in (3.1.5) or (3.1.6)). On the other side, the term $\gamma_{RV} = \gamma_V(\kappa_Z, h_v, \sigma)$ is computed from the volume coherence in (3.1.2) using the $(h_v, \sigma)$ search space and the input vertical wavenumber $\kappa_Z$.

### 3.2.2.2 Step 2

Solving the quadratic equation $F_{pp}$, as in (3.2.8), for each of the two polarisation channels, there is a single ground-to-volume pair $(\mu_{pp}^{(1)}, \mu_{pp}^{(2)})$ related to a single $(h_v, \sigma)$ element of the solution space.

From the ground-to-volume ratio pair, the ground phase solution pair $(\phi_{pp}^{(1)}, \phi_{pp}^{(2)})$ is then computed as

$$\phi_{pp}^{(m)} = \arg \left( \frac{\gamma_{RVoG} \left( 1 + \mu_{pp}^{(m)} \right)}{\gamma_{RV} + \mu_{pp}^{(m)}} \right), \quad \forall m = 1, 2. \quad (3.2.10)$$

These two solution pairs, $(\mu_{pp}^{(1)}, \mu_{pp}^{(2)})$ and $(\phi_{pp}^{(1)}, \phi_{pp}^{(2)})$, correspond to non-negative and real values of $F_{pp}$, discarding all $(h_v, \sigma)$ elements for which solutions to (3.2.8) are imaginary or real negative. That is, for each polarisation channel, $HH$ and $VV$, there is a pair of positive ground-to-volume ratios and topographic phases so that

$$\left( (\mu_{HH}^{(1)}, \mu_{HH}^{(2)}), (\phi_{HH}^{(1)}, \phi_{HH}^{(2)}) \right), \quad \forall (\mu_{HH}^{(1)}, \mu_{HH}^{(2)}) > 0 \quad (3.2.11)$$

and

$$\left( (\mu_{VV}^{(1)}, \mu_{VV}^{(2)}), (\phi_{VV}^{(1)}, \phi_{VV}^{(2)}) \right), \quad \forall (\mu_{VV}^{(1)}, \mu_{VV}^{(2)}) > 0$$

in the $(h_v, \sigma)$ space, respectively.

### 3.2.2.3 Step 3

The set of $(\mu_{pp}^{(1)}, \mu_{pp}^{(2)})$ and $(\phi_{pp}^{(1)}, \phi_{pp}^{(2)})$ duets are used as the possible candidates to find the solution that converges to the minimum topographic phase produced by the modelled coherences. Analogously to the first inversion method, the unknowns are estimated by means of an optimisation problem.
Under the RVoG assumption, i.e. extinction coefficient is independent of polarisation, hence $\sigma_{\tilde{w}_{HH}} = \sigma_{\tilde{w}_{VV}}$, and considering that the topographic phase should be the same for both polarisation channels and equal to zero, $\phi_{\tilde{w}_{HH}} = \phi_{\tilde{w}_{VV}} = 0$, the minimisation problem in the $(h_v, \sigma)$ space results in

$$\min_{h_v, \sigma} \{d_{\min 1}, d_{\min 2}\},$$  \hspace{1cm} (3.2.12)

where,

$$d_{\min 1} = \min \left| \phi_{HH}^{(1)} + \phi_{VV}^{(2)} \right| < d_{\text{threshold}} \hspace{1cm} (3.2.13)$$

From (3.2.12) and (3.2.13), the cost function is the distance, as $d_{\min 1}$ and $d_{\min 2}$, that produces the combination of the contribution of each pair ($m = 1, 2$) of topographic phase candidates for each polarisation state ($HH$ and $VV$).

From the previous step 3.2.2.2, as specified in (3.2.11), we have obtained a pair of topographic phase candidates, associated with a pair of only positive ground-to-volume ratios. However, so as to compute the distance that leads to a minimum topographic phase, the minimisation problem, i.e. equations (3.2.12) and (3.2.13), does not use directly the pair obtained for each polarisation channel, but a combination of the contribution of both $HH$ and $VV$ channels.

The reason of this is that the quadratic equation (3.2.8) results in a solution pair that physically corresponds to the minimum and maximum contributions from the ground, i.e.

$$\begin{align*}
\left( \left( \mu_{HH}^{(1)}, \mu_{HH}^{(2)} \right), \left( \phi_{HH}^{(1)}, \phi_{HH}^{(2)} \right) \right) & \equiv \left( \left( \mu_{HH\min}, \mu_{HH\max} \right), \left( \phi_{HH\min}, \phi_{HH\max} \right) \right), \\
\left( \left( \mu_{VV}^{(1)}, \mu_{VV}^{(2)} \right), \left( \phi_{VV}^{(1)}, \phi_{VV}^{(2)} \right) \right) & \equiv \left( \left( \mu_{VV\min}, \mu_{VV\max} \right), \left( \phi_{VV\min}, \phi_{VV\max} \right) \right). 
\end{align*} \hspace{1cm} (3.2.14)$$

In physical terms, from the two ground-to-volume ratios that we need to retrieve from the fact of using dual-pol interferometry, one needs to correspond to the $HH$ polarimetric channel, and another one to the $VV$ polarimetric channel. Besides, one needs to reflect the maximum contribution from the ground and the other the minimum, respectively. Therefore, any combination of the polarisation channels (i.e. $HH$, $VV$) that results in a pair of ground-
to-volume ratios associated with the maximum and minimum ground contributions \((\mu_{\text{max}}, \mu_{\text{min}})\) is a possible solution.

Moreover, since from the quadratic equation (3.2.8) the imaginary elements of the \((h_v, \sigma)\) space have been discarded, the solution corresponds to only positive values of ground-to-volume ratio pairs, i.e. \(\left(\mu_{\text{HH}}^{(m)} > 0\right)\) and \(\left(\mu_{\text{VV}}^{(m)} > 0\right)\), \(\forall m = 1, 2\), as in (3.2.11), and to pairs for which the topographic phase \(\phi_0\) is zero, due to the fact that it was compensated previously to the numerical inversion. Therefore, the initial threshold distance is set to a value close to zero, so that \(d_{\text{threshold}} \to 0\).

Under the constraint of a positive ground-to-volume pair, the algorithm evaluates if there exists a pair of topographic phase candidates in the \((h_v, \sigma)\) space that satisfies the condition of threshold distance. For that, the possible combinations are evaluated, i.e. the topographic phase candidates obtained from (3.2.11), \(\phi_{\text{HH}}^{(1)} + \phi_{\text{VV}}^{(2)}\) and \(\phi_{\text{HH}}^{(2)} + \phi_{\text{VV}}^{(1)}\). If one of the two combinations is below the threshold, that distance is updated as the new threshold value. If both combinations fulfill the condition, the new threshold distance will be the minimum between the two. In other words, \(d_{\text{threshold}} = \min_{h_v, \sigma} \{d_{\text{min1}}, d_{\text{min2}}\}\), as in (3.2.12).

The first iteration of the minimisation provides an initial set of \(h_v, \sigma, \mu_{\text{max}}, \mu_{\text{min}}\) from the \((h_v, \sigma)\) space, which in turn provides the minimum distance of the two possible combinations, \(d_{\text{min1}}\) and \(d_{\text{min2}}\) in (3.2.13). The resulting minimum distance is taken as input for the following iteration in the \((h_v, \sigma)\) space. Then, the procedure is carried out again for the entire \((h_v, \sigma)\) space until the convergence to a solution of minimum topographic phase produced by the measured coherences.

Eventually, analogously to the first inversion approach, for each scene, we store every solution from every evaluated point in the \((h_v, \sigma)\) space. Moreover, once evaluated the entire \((h_v, \sigma)\) search space, we save the mean, median and standard deviation values of all possible solutions below the condition of minimum distance. Thus, we can later analyse the results taking into account either only the solution of minimum distance of all possible, a percentage of them, or their average value.
Chapter 4

Results

This chapter evaluates and presents the results of the implemented PolInSAR parameter estimation approaches. In particular, the present project focuses, in first place, in the evaluation of the inversion models (i.e. RVoG) for the retrieval of biophysical parameters. In second place, the study is extended to the analysis of the influence of the double-bounce decorrelation term on the inversion of ground topography and vegetation height. The rest of model parameters, including the extinction coefficient and the ground-to-volume ratios are likewise analysed. However, the vegetation height is the parameter best suited to reflect the quality of the inversion algorithms thanks to its high sensitivity to the models of vegetated scenes, as the equations from previous chapters demonstrate.

For this purpose, simulated data with and without the double-bounce decorrelation term are employed as input to inversion approaches which consider or not that aspect. The difference of the retrieved values with respect to the real ones, along with the differences between inversion procedures, are evaluated, and the influence of system parameters (baseline and incidence angle) is assessed. Eventually, experimental results over rice fields are used to estimate the influence of this aspect on a final application.

In this paper we present the results concerning agricultural applications, for which we consider system configuration parameters of TanDEM-X during its science phase, from April to September 2015. The most relevant feature is a large baseline (2-3 km), which is required to provide enough vertical sensitivity to work with this short vegetation, i.e. crops.
Nonetheless, the results are generalised with the use of the parameter $\kappa_v = \frac{\kappa_Z h_v}{2}$ instead of using directly the vegetation height $h_v$. This way, the results obtained can be extended to both agriculture (i.e. short heights) and forest (i.e. much larger heights).

### 4.1 Assessment of the numerical inversion

The possible error sources of the techniques explained in the previous chapter are related mainly with three different aspects [16]. The first question is how well the direct model represents the scene. As figure 3.1 depicts, scenes not properly represented by the RVoG model will provide observations (i.e. coherences) not fitting the expressions examined here, and hence the estimation of model parameters will produce wrong or meaningless values.

Concerning rice scenes, there are several main causes of mismatch between the RVoG model and the scenes. One is the presence of non-exponential scattering profiles, e.g. owing to an heterogeneous vegetation volume along the vertical coordinate caused by different plant elements (stalks, leaves, etc.) at different heights. Another cause is the effect of differential extinction, i.e. vertically polarised waves are expected to be more attenuated than horizontally polarised ones, as a consequence to the dominant orientation of stalks and tillers. The third last cause is related to a non-negligible direct surface component from the ground, i.e. equation (3.1.1) could no be simplified as equation (3.1.6).

The second aspect producing error in the retrieval methodology is associated with all system and data processing aspects, such as the effect of incidence angle and baseline on the sensitivity of the proposed technique to the scene features, but also including the presence of speckle, the multi-looking employed in the calculation of coherences, the compensation of SNR and BQ decorrelations (as in equation (2.2.8)), etc.

At last, the third error source is the numerical inversion itself (i.e. algorithms represented by the red dashed box in figures 3.3 and 3.4). This section provides the numerical assessment of this last aspect.
In our case of study, i.e. single-baseline PolInSAR systems, it is well known that the retrieval of the RVoG parameters does not provide a single solution but a range of possible solutions (see [32, 35, 16]), for which the minimisation in equations (3.2.3), (3.2.4) and (3.2.12) is satisfied. Different combinations of model parameters (i.e. topographic phase $\phi_0$, vegetation height $h_v$, extinction $\sigma$, and ground-to-volume ratios $\mu$) result in very similar model outputs, so the minimisation may fall in local minima or simply provide an arbitrary solution depending on the initial guess of these model parameters, either from the set of initial guesses in the first approach (figure 3.3), or from the $(h_v, \sigma)$ space in the second one (figure 3.4).

As we advanced in the previous chapter, both procedures include a methodology to prevent the minimisation from falling in local minima, or at least, to mitigate its effects. Regarding the first inversion approach (figure 3.3), this is achieved by introducing a set of initial guesses, $N_{inv}$, for each scene. Besides, for each initial guess, the minimisation is carried out under a tolerance constraint, e.g. $10^{-9}$. The minimum distance of every possible solution has to be below this threshold, thus ensuring a good approximation between model and data.

Concerning the second approach (figure 3.4), the methodology to avoid local minima solutions is different. In this second algorithm, the initial threshold distance (i.e. $10^{-3}$) is the tolerance of the minimisation function itself. Depending on the resulting distance produced by the topographic phase candidates (3.2.13), the tolerance (i.e. threshold distance) is adapted as a compromised solution between ensuring global minimum (i.e. more restrictive) and guaranteeing that we are going to find a solution in the $(h_v, \sigma)$ space that provides a distance below the threshold (i.e. less restrictive). Then, once a solution is found, the adaptive threshold distance is updated as a function of the measured coherences.

In order to assess the feasibility of the proposed inversion algorithms, a simulation experiment is carried out. The RVoG procedures presented in figures 3.3 and 3.4 are computed not for a single combination of model parameters, but for a wide set of scene parameters. Each possible combination of scene parameters generates a pair of theoretical coherences, $\gamma(\kappa Z, \bar{w}_{max})$ and $\gamma(\kappa Z, \bar{w}_{min})$ provided by the forward model through either equation (3.1.5) or (3.1.6). With the measured coherences, the numerical inversion (red dashed box in 3.3 and 3.4) is carried out for each case.
Tables 4.1 and 4.2 summarise the set of acquisition parameters and model parameters of the PolInSAR observations simulated with the proposed RVoG models, adapted to the specific type of scene, i.e. rice crops.

<table>
<thead>
<tr>
<th><strong>TanDEM-X system variables</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of incidence $\theta_0$ ($^\circ$) [20, 50]</td>
</tr>
<tr>
<td>Height of ambiguity (m) [3, 5]</td>
</tr>
</tbody>
</table>

Table 4.1: Intervals of input system parameters of the TanDEM-X sensor for crop scenes.

<table>
<thead>
<tr>
<th><strong>Random Volume over Ground model parameters</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vegetation height $h_v$ (m) [0.05, 2]</td>
</tr>
<tr>
<td>Extinction coefficient $\sigma$ (dB/m) [0.1, 10]</td>
</tr>
<tr>
<td>Ground-to-volume ratios $\mu$ (dB) [-10, 10]</td>
</tr>
</tbody>
</table>

Table 4.2: Intervals of model parameters of the PolInSAR observations simulated with the RVoG model for both inversion approaches adapted to crop scenes.

Moreover, for each of the scenes originated from the combinations in tables 4.1 and 4.2, the model is inverted many times. One should note that the intervals that have been employed as input model parameters, from which we obtain the initial theoretical coherences, are in accordance with those employed as optimisation ranges (see table 3.2).

In the first approach (figure 3.3), the scenes are inverted as many times as the number of initial guesses established. In this experiment, we applied the inversion a 100 times for each scene (i.e. $N_{inv} = 100$), and the initial guesses were generated randomly (using a uniform distribution) within the intervals presented in table 3.2 for this first inversion approach. As explained in the previous chapter, from each initial guess we obtain a solution of minimum distance between model and observations. For each scene, we save the solution obtained with every initial guess, together with the mean, median and standard deviation values of the whole set of initial guesses.

In the second algorithm, each scene is inverted for all the parameter sets generated from the $(h_v, \sigma)$ space. In the experiment carried out, the number of parameter combinations for the inversion of a single scene is defined by 351 values of vegetation height, and 301 values of extinction coefficient. For each inversion, the algorithm provides then a solution
corresponding to the minimum distance of the topographic phase produced by the measured coherences (i.e. $\phi_0 = 0$). Analogously to the first approach, we save this solution obtained from each inversion, as well as the mean, median and standard deviation values of all possible solutions computed from the inversion of the entire $(h_v, \sigma)$ space.

Figures 4.1 and 4.2 represent the average and standard deviation of the retrieved heights for all simulated cases (up to a height of 1.5 m) and considering all initial guesses in the first approach (as in 3.3), and the entire $(h_v, \sigma)$ space in the second one (as in 3.4), respectively. The figures show the difference in the vegetation height estimates that exists between taking into account the double-bounce decorrelation term (3.1.6), or not (3.1.5), in the parameter retrieval stage.
Figure 4.2: Numerical assessment of the second inversion algorithm (figure 3.4): vegetation height estimates (mean and standard deviation) for each simulated value. Parameters: \( h_v \) in the range 0.05–1.5 m, \( \sigma = 4.9 \text{ dB/m} \), \( \mu_{\text{min}} = -3 \text{ dB} \), \( \mu_{\text{max}} = 3 \text{ dB} \), \( \text{HoA} = 3 \text{ m} \), \( \text{AoI} = 30^\circ \).

(a) Direct model without DB decorrelation (3.1.5). (b) Direct model with DB decorrelation (3.1.6).

With respect to the average values, figure 4.1 demonstrates that there is not a remarkable bias produced by the inversion approach (figure 3.3) in any of the three methodologies followed. As for the expected standard deviation, the variability of the results is higher in the case 4.1 (b), where the model scene and the expression used for inversion do not match, specially for height values lower than 0.5 m.

Figure 4.2 shows a larger bias produced by the second inversion approach (figure 3.3) for height values larger than 1 m. However, the differences in the retrieved values from both inversion approaches are intrinsic to the methodology followed.

It is important to keep in mind that, so as to evaluate the feasibility of the inversion strategies, we have chosen a scene with typical values of model parameters, i.e. \( \sigma = 4.9 \text{ dB/m} \), \( \mu_{\text{min}} = -3 \text{ dB} \) and \( \mu_{\text{max}} = 3 \text{ dB} \), and system parameters, i.e. \( \text{HoA} = 3 \text{ m} \), \( \text{AoI} = 30^\circ \). Thus, we make sure that we are evaluating the error of the inversion itself, since the RVoG model is expected to work properly for these characteristic parameters. Nonetheless, to fully quantify the error in the parameter retrieval, we have to evaluate scenes with extreme values of these parameters. This is studied in the following sections, together with the differences between approaches and methodologies.
4.1 Assessment of the numerical inversion

As it has been cited in the introduction of this chapter, vegetation height is the most sensitive parameter to the models of vegetated scenes, in particular, to the present homogeneous volume over ground model (i.e. RVoG). Figures 4.1 and 4.2 reflect this. However, figures 4.3 and 4.4 show the average and the standard deviation of the retrieved extinction coefficients, again for all simulated cases and considering all initial guesses in the first approach (as in 3.3), and the entire \((h_v, \sigma)\) space in the second one (as in 3.4), respectively.

The scene chosen corresponds to the same ground-to-volume ratios used in the previous test, i.e. \(\mu_{\text{min}} = -3\) dB and \(\mu_{\text{max}} = 3\) dB, and to an average value of height, i.e. \(h_v = 1\) m. The extinction coefficient ranges within the interval specified in table 4.2, that is, from 0.1 to 10 dB/m.

![Figure 4.3: Numerical assessment of the first inversion algorithm (figure 3.3): extinction estimates (mean and standard deviation) for each simulated value. Parameters: \(\sigma\) in the range 0.1–10 dB/m, \(h_v = 1\) m, \(\mu_{\text{min}} = -3\) dB, \(\mu_{\text{max}} = 3\) dB, HoA = 3 m, AoI = 30°. (a) Direct model and inversion without DB decorrelation (3.1.5). (b) Direct model with DB decorrelation (3.1.5), but inversion without it (3.1.6). (c) direct model and inversion both with DB decorrelation (3.1.6).](image-url)
Figure 4.4: Numerical assessment of the second inversion algorithm (figure 3.4): extinction estimates (mean and standard deviation) for each simulated value. Parameters: $\sigma$ in the range $0.1$–$10$ dB/m, $h_v = 1$ m, $\mu_{\min} = -3$ dB, $\mu_{\max} = 3$ dB, $HoA = 3$ m, $AoI = 30\degree$. (a) Direct model without $DB$ decorrelation (3.1.5). (b) Direct model with $DB$ decorrelation (3.1.6).

The figures demonstrate that the theoretical model of an homogeneous volume over a ground surface is insensitive to the extinction coefficient. This might be the reason why the solutions in figure 4.4 are found in the extreme values of this coefficient. What this tells us is that the evaluated RVoG model, under the present considerations (see tables 4.1 and 4.2), is unable of measuring the amount of extinction suffered by the electromagnetic waves, which accounts for both wave attenuation through the volume and single-scattering loss.

4.1.1 Vegetation height overestimation

In order to evaluate in detail more in detail the error of the inversion itself, figures 4.5, 4.6 and 4.7 present the overestimation in the resulting vegetation height for both implemented inversion approaches, but this time for a wide range of values of height and extinction.

Hereinafter, we are going to refer to the first inversion approach implemented as “inversion approach 1”, the one corresponds to the minimisation of the distance of the measured coherences to the modelled coherences (i.e. schematically represented in figure 3.3 and explained in section 3.2.1). As “inversion approach 2”, we are going to refer to the second inversion approach implemented, the one that minimises the distance of the topographic phase produced by the measured coherences (i.e. represented in figure 3.4 and detailed in section 3.2.2).
Keeping in mind that the numerical inversion itself introduces some error, even in the case in which the model used for inversion goes in accordance with the scene evaluated, this test quantifies how well the direct model represents the scene. For this purpose, the difference in the vegetation height retrieved by employing a direct and an inversion models that do not match, and by employing a direct and an inversion models that do match, is assessed.

Specifically, the overestimation presented here evaluates the difference in the vegetation height estimates when the direct model takes into account the double-bounce contribution (3.1.6) but the inversion ignores it (3.1.5), and when both direct and inverse models correspond to equation (3.1.5).

Figure 4.5: Height overestimation of the inversion approach 1 for four different incidence angles. Parameters: $h_v$ in the range 0.05–2 m, $\sigma$ in the range 0.1–10 dB/m, $\mu_{\text{min}} = -3$ dB, $\mu_{\text{max}} = 3$ dB, HoA = 3 m.
Figure 4.6: Height overestimation of the inversion approach 2 for four different incidence angles. Parameters: \( h_v \) in the range 0.05–2 m, \( \sigma \) in the range 0.1–10 dB/m, \( \mu_{min} = -3 \) dB, \( \mu_{max} = 3 \) dB, HoA = 3 m.

Figures 4.5 and 4.6, represent the height overestimation for vegetation height values ranging from 0.05 to 2 m, and extinction coefficient values ranging from 0.1 to 10 dB/m. On the other hand, so as to be able to see at a glance the difference for the four angles of incidence evaluated (i.e. AoI = 20, 30, 40 and 50°), figure 4.7 represents the height overestimation for the same range of vegetation heights, but for just one average extinction value, \( \sigma = 4.9 \) dB/m. The rest of parameters are the same in all figures, i.e. ground-to-volume ratios -3 dB and 3 dB, respectively, and HoA = 3 m.

The lack of sensitivity with respect to the extinction coefficient is observed in figures 4.5 and 4.6, since basically, the same profile, that is, the same value of overestimation, appears for any value of extinction for all heights. The only changes are observed for AoI = 50°, where there is a slightly greater overestimation for small extinctions and large heights.
Figure 4.7: Height overestimation for four different incidence angles. Parameters: \( h_v \) in the range 0.05–2 m, \( \sigma = 4.9 \text{ dB/m} \), \( \mu_{\text{min}} = -3 \text{ dB} \), \( \mu_{\text{max}} = 3 \text{ dB} \), HoA = 3 m. (a) Inversion approach 1. (b) Inversion approach 2.

For the case AoI = 20\(^\circ\), the height overestimation for all extinctions (figures 4.5 and 4.6) computed from inversion approach 1, is contained within the interval -0.11 and 0.075 m, whereas from inversion approach 2, it ranges between -0.165 and 0.245 m. This is reflected in figure 4.7 for a extinction of 4.9 dB/m. For this specific value of extinction, the minimum and maximum values of the overestimation retrieved from both inversion approaches are -0.08 and 0.03 m, and -0.06 and 0.06 m, respectively.

However, we find these maximum and minimum values of overestimation in the extreme values of the interval of heights evaluated, that is, for very small heights (i.e. below 0.3 m) and for large heights (i.e. above 1 m). Therefore, we know in advance that the RVoG model is not expected to work with the accuracy required for these extreme values of height. For steep incidence angles, e.g. 20 and 30\(^\circ\), and heights between 20 cm and 1 m, we can assume the error of the inversion, i.e. RVoG model, is negligible.

Nonetheless, this error should be taken into consideration for shallower incidence angles. For AoI = 50\(^\circ\), the height overestimation for all extinctions (figures 4.5 and 4.6) retrieved from inversion approach 1, ranges in the interval -0.09–0.168 m, and from inversion approach 2, within -0.14–0.49 m, respectively. This is clearly observed for the selected average extinction, 4.9 dB/m (figure 4.7), where the overestimation is considerably greater, i.e. above 10 cm, than the one obtained for the rest of incidence angles.

In next section, 4.2, the results presented go along with these observations.
4.2 Evaluation with simulated data

4.2.1 Evaluation of the topography estimation

Topography is estimated before the rest of parameters [10], as the block diagram of both inversion approaches shows in figures 3.3 and 3.4, respectively.

As we advanced in the previous chapter, the first test corresponds to the error produced in the inversion of the topographic phase in the case illustrated in figure 3.1, i.e., when data correspond to a scene in which the double-bounce ground contribution is dominant (3.1.6), but the model used for the retrieval considers the ground dominated by the direct contribution (3.1.5). The phase error is defined as

$$
\Delta \phi_0 = \phi_0 - \phi'_0, \quad (4.2.1)
$$

and is translated into topography error as

$$
\Delta z = \Delta \phi_0 / \kappa_Z. \quad (4.2.2)
$$

Since the topographic phase $\phi_0$ is computed before the numerical minimisation, its error is common to both inversion strategies, 3.3 and 3.4.

![Figure 4.8: Topographic error $\Delta z_0$ obtained as function of the normalised vegetation height as $\kappa_v = h_v \kappa_Z / z$ for four different incidence angles. (a) HoA = 3 m ($\kappa_Z = 2.1$ rad/m). (b) HoA = 5 m ($\kappa_Z = 1.25$ rad/m).](image)
Figure 4.8 shows the topographic error $\Delta z_0$ for the range of vegetation heights $h_v$ defined in table 4.2 as a function of the parameter $\kappa_v = \frac{h_v \kappa_Z}{2}$, and for four different incidence angles within the range specified in table 4.1. The height of ambiguity is either HoA = 3 m, which corresponds to a vertical wavenumber $\kappa_Z = 2.1 \text{ rad/m}$, or HoA = 5 m, which corresponds to $\kappa_Z = 1.25 \text{ rad/m}$.

In general, the larger the vegetation height the larger the error in the topography, since $\gamma_{DB}$ decreases with height, and the circumference which fixes the topographic phase shrinks, see figure 3.1. However, for steep incidence angles the error is negligible, e.g. less than 1 cm for 20° and less than 2 cm for 30°. This is a consequence of the conversion factor between $\kappa_Z$ and $k_z$, explained in (3.1.3). When one moves to shallower incidence angles, like 40° or 50°, the error is more noticeable. With HoA = 3 m the error reaches 7 cm and 15 cm, respectively, for vegetation heights around 1.2 m. With HoA = 5 m the maximum errors at 40° and 50° correspond to the maximum height (2 m), with values of 9 and 19 cm, respectively. These values correspond to relative errors around 5% and 12%, respectively, which could be important depending on the final application.

4.2.2 Evaluation of the vegetation height estimation

An error in the topographic phase estimation influences also the estimation of the rest of model parameters. In this section, we show the error produced in the estimation of vegetation height, for which a large number of simulations is carried out with wide ranges of the model parameters, as shown in table 4.2. As mentioned in the introduction of the chapter, the vegetation height is normalised to the vertical wavenumber $\kappa_v = \frac{h_v \kappa_Z}{2} \ (\text{rad})$.

As explained in section 4.1, one must not forget that the inversion of the RVoG model is expected to suffer problems even in the case of ideal data acquired in monostatic mode, due to lack of interferometric sensitivity for very short heights (i.e. below 30 cm, approximately), since a larger baseline is required, and other numerical limitations related to the numerical optimisation. Hence, it is important to separate the errors produced by the presence of the double-bounce term from the intrinsic errors that would appear also in its absence.
4.2.2.1 Vegetation height estimation from inversion approach 1

Figures 4.9 and 4.10 compare the error produced in the estimation of vegetation height using the RVoG model in three cases: when both direct model and inversion correspond to equation (3.1.5), when the direct model takes into account the double-bounce decorrelation term, as in (3.1.6), but the inversion ignores it, i.e. it employs (3.1.5), and when both direct model and inversion correspond to equation (3.1.6).

For the evaluation of this error, we have chosen a height of ambiguity HoA = 3 m, and two angles of incidence (i.e. $\theta_0$), AoI = 30° and 50°. Thus, we can evaluate how the double-bounce decorrelation term affects to the estimation of the physical parameters (i.e. $h_v$) depending on these initial values.

In the first two cases, figure 4.9 (a) and (b), there is a significant overestimation of height when its actual value is below 20 cm (i.e. 0.209 rad), which is a limitation due to the baseline. For short vegetation, the product of height and vertical wavenumber (i.e. $h_v\kappa_Z$) is very small, which means that there is not enough sensitivity to the vertical distribution of scatterers in the scene. As it has been mentioned, this error is actually associated with system and data processing aspects. Nevertheless, it should be noted that this error becomes less noticeable when the inversion model takes into account the double-bounce contribution, as in the third case.

A second behaviour common in the three cases is the error when height is above 1 m (i.e. 1.05 rad) and extinction is either low or high (below 2 dB/m and above 7 dB/m, respectively), so the figures are quite similar in this zone. A more detailed comparison between the figures tells us that the overestimation for low heights is worse in the second case, i.e. when direct and inverse model do not match. However, for these values of height, we know from the assessment of the numerical inversion (figure 4.1) that the estimation of the model starts to be less reliable.

For the rest of vegetation heights, above 20 cm (i.e. 0.209 rad) and below 1 m (i.e. 1.05 rad), there exists a difference around 1–2 cm in the retrieved values, and the error in the estimation of the vegetation height ranges from -5 to 5 cm. Therefore, under the chosen system parameters (i.e. HoA = 3 m and AoI = 30°), for these vegetation heights and for all
4.2 Evaluation with simulated data

Figure 4.9: Error in the vegetation height estimation (as $k_v$) of the inversion approach 1. Parameters: $h_v$ in the range 0.05–1.5 m, $\sigma$ in the range 0.1–10 dB/m, $\mu_{min} = -3$ dB, $\mu_{max} = 3$ dB, HoA = 3 m, AoI = 30°. (a) Direct model and inversion without $DB$ decorrelation (3.1.5). (b) Direct model with $DB$ decorrelation (3.1.5), but inversion without it (3.1.6). (c) direct model and inversion both with $DB$ decorrelation (3.1.6).

extinctions, the height estimation of the evaluated model is quite accurate, and the effect of the double-bounce is not really important.

The results obtained in figure 4.9 are in line with those represented in figure 4.1. When the inversion uses the expression of a monostatic direct configuration (3.1.5), i.e. cases (a) and (b), there is a larger bias for short heights, which disappears otherwise, i.e. case (c). On the other hand, for the highest values, above 1 m, the three methodologies, i.e. (a), (b) and (c), result in greater errors.

Comparing figure 4.9 with 4.10, we can appreciate the difference in the retrieval of height vegetation for both angles of incidence.
The previous test, i.e. figure 4.8 (a), indicates that for AoI = 30°, the error in the topographic phase estimation is lower than 2 cm, so this error will not result in an important bias in the estimation of the vegetation height. On the contrary, for AoI = 50°, the maximum topographic phase error reaches 15 cm, which is no longer negligible. According to these results, figure 4.10 shows that for shallower incidence angles, the error becomes noticeable, and so does the effect of the double-bounce decorrelation term. This is demonstrated in figure 4.10 (b), which compared to 4.9 (b), shows a more pronounced overestimation in the height retrieval.

Figure 4.10: Error in the vegetation height estimation (as \( \Delta \kappa_v \)) of the inversion approach 1. Parameters: \( h_v \) in the range 0.05–1.5 m, \( \sigma \) in the range 0.1–10 dB/m, \( \mu_{\min} = -3 \) dB, \( \mu_{\max} = 3 \) dB, HoA = 3 m, AoI = 50°. (a) Direct model and inversion without \( DB \) decorrelation (3.1.5). (b) Direct model with \( DB \) decorrelation (3.1.5), but inversion without it (3.1.6). (c) direct model and inversion both with \( DB \) decorrelation (3.1.6).
This last figure, 4.10, shows that as the incidence angle increases, the effect of the double-bounce in the estimation of the physical parameters increases too. As we observe in figure 4.10 (c), taking into account the double-bounce contribution in the inversion gets rid of the overestimation for short values of height (below 20 cm), even in the case of an incidence angle of 50°, which is the worst case scenario.

However, similarly to figure 4.9, for values above 1 m, the algorithm starts to accept as valid estimates (i.e. from the initial guesses), solutions that do not have physical sense. This also occurs for all the different incidence angles, from 20° to 50°. The consequence of this is both the overestimation and underestimation in the vegetation height that we observe in figures 4.9 (c) and 4.10 (c). In the following sections, we analyse the distance cost function of the algorithm to check how it is possible that we are considering valid solutions that do not provide the estimates most adapted to the modelled scene.

### 4.2.2.2 Vegetation height estimation from inversion approach 2

Following the methodology of the second inversion approach (section 3.2.2), figures 4.11 and 4.12 compare the error produced in the estimation of vegetation height using the RVoG model in two cases: when the direct model considers a scene dominated by the direct ground contribution (3.1.5), and when the direct model assumes a scene dominated by the double-bounce ground contribution, as in (3.1.6). In both cases, this inversion approach inverts the simulated scene assuming a monostatic configuration, i.e. direct dominant contribution from the ground. The reason why is has not been evaluated the third case, i.e. direct model and inversion with DB decorrelation is due to the computational cost of this model, as following sections demonstrate.

Since we are evaluating again the error of the vegetation height estimation for this second approach (as function of $\kappa_v$), the experiment is carried out under the same system parameters, i.e. a height of ambiguity HoA = 3 m, and two different incidence angles, AoI = 30° and 50°.
Figure 4.11: Error in the vegetation height estimation (as $\kappa_v$) of the inversion approach 2. Parameters: $h_v$ in the range 0.05–1.5 m, $\sigma$ in the range 0.1–10 dB/m, $\mu_{\text{min}} = -3$ dB, $\mu_{\text{max}} = 3$ dB, HoA = 3 m, AoI = 30°. (a) Direct model without $DB$ decorrelation (3.1.5), (b) Direct model with $DB$ decorrelation (3.1.6).

Differently from the retrieved results of height vegetation with the inversion approach 1 in figures 4.9 (a), (b), and 4.10 (a), (b), the overestimation in height for values below 20 cm disappears with this methodology. However, for height values above 1 m, the algorithm starts overestimating, as the inversion approach 1 does.
Another common observation from both inversion approaches is that, for shallower incidence angles, i.e. AoI = 50°, and height values above 1 m, when the dominant contribution from the ground is the double-bounce, but the inversion ignores it (i.e. figures 4.10 (b) and 4.12 (b)), the bias in the estimation of the vegetation height is relevant and cannot be ignored.

4.2.3 Evaluation of the extinction coefficient estimation

This section presents the error in the extinction coefficient that is obtained with the implemented estimation algorithms, i.e. inversion approaches 1 and 2. Following the procedure defined for the evaluation of the vegetation height estimation, the experiment evaluates the same number of simulations with ranges of model parameters specified in table 4.2.

Figure 4.13 compares the error produced in the estimation of the extinction coefficient using the RVoG model for the same three cases contemplated in the evaluation of the height estimation (i.e. section 4.2.2.1). The corresponding system parameters are HoA = 3 m and AoI = 30°, whereas the rest of model parameters, that is, the minimum and maximum ground-to-volume ratios, are -3 dB and 3 dB, respectively.

Analogously, figure 4.14 shows the error in the estimation of the extinction for the two cases contemplated in the inversion approach 2: simulating and inverting a scene dominated by the direct ground contribution (3.1.5), and simulating a scene dominated by the double-bounce contribution (3.1.6), but inverting considering again the direct ground contribution as dominant.

As figures 4.3 and 4.4 demonstrated in the previous section with the numerical assessment of both inversion approaches, the theoretical model of an homogeneous volume over a ground surface is not sensitive to the extinction coefficient under the present considerations (i.e. tables 4.1 and 4.2). Hence, we cannot extract conclusions regarding the estimation of this parameter.
4.2.4 Comparison between inversion procedures

In this section, in order to compare both inversion approaches (figures 3.3 and 3.4, respectively), we are focusing the analysis in a specific scene. Thus, we can evaluate in detail the differences between both minimisation cost functions, i.e. distance between modelled $\gamma_V(k_Z, h_v, \sigma, \bar{w})$ and measured $\gamma(k_Z, \bar{w})$ coherences in inversion approach 1, and topographic phase produced by the modelled coherences in inversion approach 2.

The input system parameters of the TanDEM-X sensor, and the input model parameters of the PolInSAR observations with the RVoG model for the specific scene studied in this test are presented in tables 4.3 and 4.4, respectively.
4.2 Evaluation with simulated data

Figure 4.14: Error in the extinction estimation of the inversion approach 2. Parameters: $h_v$ in the range 0.05–1.5 m, $\sigma$ in the range 0.1–10 dB/m, $\mu_{min} = -3$ dB, $\mu_{max} = 3$ dB, HoA = 3 m, AoI = 30°. (a) Direct model without DB decorrelation (3.1.5). (b) Direct model with DB decorrelation (3.1.6).

<table>
<thead>
<tr>
<th>TanDEM-X system variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of incidence $\theta_0$ (°)</td>
</tr>
<tr>
<td>Height of ambiguity (m)</td>
</tr>
</tbody>
</table>

Table 4.3: Acquisition input system parameters of the TanDEM-X sensor for crop scenes.

<table>
<thead>
<tr>
<th>Random Volume over Ground model parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vegetation height $h_v$ (m)</td>
</tr>
<tr>
<td>Extinction coefficient $\sigma$ (dB/m)</td>
</tr>
<tr>
<td>Ground-to-volume ratio 1 $\mu_{min}$ (dB)</td>
</tr>
<tr>
<td>Ground-to-volume ratio 2 $\mu_{max}$ (dB)</td>
</tr>
</tbody>
</table>

Table 4.4: Model parameters of the PolInSAR observations simulated with the RV oG model for both inversion approaches adapted to crop scenes.

Furthermore, for this test, we have applied a set of $N_{inv} = 100$ initial guesses for each simulated scene in the inversion approach 1, as we did in the previous sections. Regarding the second inversion approach, the $(h_v, \sigma)$ space is likewise defined by 351 values of vegetation height, and 301 values of extinction coefficient.

Tables 4.5 and 4.6 present the parameter set $h_v, \sigma, \mu_{min}, \mu_{max}$ that has been estimated from inversion approaches 1 and 2, for the different methodologies followed in each of them.
In particular, table 4.5 takes as final estimate the solution corresponding to the mean value of all possible solutions. With respect to the inversion approach 1, the mean is computed from all solutions provided by all initial guesses, whereas concerning inversion approach 2, the mean is calculated from all possible solutions in the \((h_v, \sigma)\) search space that satisfy the condition of minimum distance of the topographic phase. On the other hand, table 4.6 considers as final estimate only the solution of minimum distance, considering all initial guesses \(N_{inv}\), and all elements in the \((h_v, \sigma)\) search space, respectively.

The two tables, 4.5 and 4.6, present the parameter solution set for cases (a), (b) and (c), for the inversion approach 1, and cases (a) and (b), for the inversion approach 2. As it has been presented throughout this chapter, case (a) corresponds to a simulation in which both direct model and inversion correspond to equation (3.1.5), case (b), to a simulation in which the direct model takes into account the double-bounce decorrelation term (3.1.6), but the inversion ignores it (3.1.5), and case (c), to a simulation in which both direct model and inversion correspond to equation (3.1.6).

<table>
<thead>
<tr>
<th></th>
<th>RVoG model parameters</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(h_v) (m)</td>
<td>(\sigma) (dB/m)</td>
</tr>
<tr>
<td>Simulated scene</td>
<td>1.00</td>
<td>4.91</td>
</tr>
<tr>
<td>Estimation from inversion approach 1</td>
<td>(a) 0.98</td>
<td>6.88</td>
</tr>
<tr>
<td></td>
<td>(b) 1.01</td>
<td>6.30</td>
</tr>
<tr>
<td></td>
<td>(c) 1.00</td>
<td>5.80</td>
</tr>
<tr>
<td>Estimation from inversion approach 2</td>
<td>(a) 0.95</td>
<td>9.82</td>
</tr>
<tr>
<td></td>
<td>(b) 0.99</td>
<td>9.48</td>
</tr>
</tbody>
</table>

Table 4.5: Studied PolInSAR scene taking as final estimate the mean of all possible solutions.

We can observe in the tables that, regarding inversion approach 1, the more precise case is (c), the one in which both direct and inversion models consider the double-bounce contribution (3.1.6). This is noticeable in both cases, taking as solutions the mean of all the possible (table 4.5), and considering just the one that provides the solution of minimum distance (table 4.6). As expected, the drawback of this methodology is the computational cost, i.e. approximately twice a second more computationally expensive than the other two,
### 4.2 Evaluation with simulated data

#### RVoG model parameters

<table>
<thead>
<tr>
<th></th>
<th>( h_v ) (m)</th>
<th>( \sigma ) (dB/m)</th>
<th>( \mu_{min} ) (dB)</th>
<th>( \mu_{max} ) (dB)</th>
<th>( d_{min} ) (secs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated scene</td>
<td>1.00</td>
<td>4.91</td>
<td>-3.00</td>
<td>3.00</td>
<td>-</td>
</tr>
<tr>
<td>Estimation from inversion approach 1</td>
<td>(a) 0.91 12.01 -1.89 3.60 8.63( \times 10^{-14} ) 0.95</td>
<td>(b) 0.97 8.04 -2.88 2.54 6.65( \times 10^{-14} ) 0.89</td>
<td>(c) 0.99 3.20 -3.41 2.85 5.16( \times 10^{-13} ) 2.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Estimation from inversion approach 2</td>
<td>(a) 0.95 9.34 -2.31 3.36 2.00( \times 10^{-4} ) 14.48</td>
<td>(b) 0.99 7.98 -2.99 2.46 2.01( \times 10^{-4} ) 14.43</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6: Studied PolInSAR scene taking as final estimate only the solution of minimum distance.

(a) and (b).

With respect to the inversion approach 2, both tables 4.5 and 4.6 show similar results. However, according to what we expected from the theoretical point of view, in cases (b), where the direct model considers a scene dominated by the double-bounce ground contribution, but the inversion ignores it, the bias in the estimation of the parameters is greater.

Without a doubt, the main difference between both inversion approaches is that, even following the most expensive methodology in inversion approach 1 (i.e. case (c)), the computational cost is 7 times lower than any of the methodologies followed in inversion approach 2: 2.02 seconds against 14.48 seconds, respectively.

Moreover, it is important to mention that, since we are evaluating a single scene with optimum values of the parameters, the distance of the topographic phase converges quickly to the minimum. However, when evaluating wide ranges of input parameters, this distance becomes adaptive, as explained in section 3.2.2.3, and it is necessary to relax the threshold distance when the algorithm does not find a candidate solution below the threshold. This implies that the computational cost for each scene is even much greater.

For the scene evaluated (see table 4.4), figures 4.15 and 4.16 present all the possible solutions that satisfy the condition of minimum distance in both inversion approaches. For the simulation, we have chosen case (c), in which the direct model and the inversion consider
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the double-bounce term (3.1.6). Whereas for the inversion approach 2, we have selected case (b), where the double-bounce is taken into account, as in 3.1.6.

![Graphs showing the error in the estimation of model parameters](image)

Figure 4.15: Error in the estimation of all model parameters from the inversion approach 1, case (c). Parameters: \( h_v = 1 \) m, \( \sigma = 4.9 \) dB/m, \( \mu_{\text{min}} = -3 \) dB, \( \mu_{\text{max}} = 3 \) dB, HoA = 3 m, AoI = 30°.

The graphs in the figures represent the model values as “simulated” ones in the vertical axis. However, this vertical axis refers to both simulated (i.e. blue) and retrieved (i.e. orange) values, as the legend indicates.

Since we are evaluating a scene with typical values of the parameters, their estimation is very similar for any of the methodologies followed. However, we can see that the variability of all estimates in inversion approach 1 (figure 4.15) is higher than in the inversion approach 2 (figure 4.16). This is due to the number of initial guesses used for inversion (i.e. \( N_{\text{inv}} = 100 \)), together with the fact that the second inversion approach has found only three possible solutions that satisfy the threshold distance (i.e. \( 10^{-3} \)), for this specific scene. The number of possible solutions that this second methods finds depends on the model scene evaluated.
In order to fully complete the study of this model scene, with parameter values $h_v = 1 \text{ m}$, $\sigma = 4.9 \text{ dB/m}$, $\mu_{\text{min}} = -3 \text{ dB}$ and $\mu_{\text{max}} = 3 \text{ dB}$, within the ranges in which the inversion approaches are supposed to estimate properly, we fix one model value and evaluate the estimation for all the combinations. That is, e.g. for a fixed $h_v = 1 \text{ m}$, we represent its estimation for all scenes considering the ranges of $\sigma$, $\mu_{\text{min}}$ and $\mu_{\text{max}}$ presented in table 4.2.

Figures 4.17 and 4.18 show how the estimation of vegetation height evolves for all simulations with this $h_v$ value. The rest of parameters are HoA = 3 m, and AoI = $30^\circ$.

Figure 4.17 shows, at a glimpse, the difference in the three methodologies of the inversion approach 1. While case (c) (i.e. both direct model and inversion use (3.1.6)) shows all estimates ranging from -0.1 to +0.1 m of the simulated value, case (b) (i.e. direct model and inversion do not match), presents the greatest bias in the estimation.
Figure 4.17: Error in the vegetation height estimation of the inversion approach 1. Parameters: $h_{v} = 1$ m, $\sigma$ in the range 0.1–10 dB/m, $\mu_{\min}$ and $\mu_{\max}$ in the range -3–3 dB, HoA = 3 m, AoI = 30°. (a) Direct model and inversion without $DB$ decorrelation (3.1.5). (b) Direct model with $DB$ decorrelation (3.1.5), but inversion without it (3.1.6). (c) direct model and inversion both with $DB$ decorrelation (3.1.6).

Figure 4.18, on the other hand, presents much less variability in the resulting estimates, as we observed in 4.16. Nevertheless, a more detailed examination of this algorithm tells us that, as the inversion approach 1, this one introduces likewise solutions with no physical sense as final estimates.

Both figures 4.17 and 4.18 represent, in orange, the solution corresponding to the global minimum distance from each inversion approach. Analysing these solutions, we can conclude that the solution of minimum distance does not necessarily provide the solution best suited to the simulated (i.e. model) values. In next section, a deeper analysis of this is carried out.
4.2 Evaluation with simulated data

Figure 4.18: Error in the vegetation height estimation of the inversion approach 2. Parameters: $h_v = 1 \text{ m}$, $\sigma$ in the range 0.1–10 dB/m, $\mu_{min}$ and $\mu_{max}$ in the range -3–3 dB, HoA = 3 m, AoI = 30°. (a) Direct model without $DB$ decorrelation (3.1.5). (b) Direct model with $DB$ decorrelation (3.1.5).

4.2.4.1 Distance cost function

In this section we evaluate the minimisation, i.e. the distance cost function, of the inversion approach 1, since on account of the computational cost, this is the approach chosen for the evaluation of the influence of the double-bounce decorrelation term. Thus, the distance cost function is evaluated for the case in which the direct model and inversion use the expression (3.1.6). It is worth remembering that the cost function of this approach is the distance of the measured coherences to the modelled coherences.

The test evaluates four model scenes, i.e. $h_v$ in the range 0.5–2 m, $\sigma = 4.9 \text{ dB/m}$, $\mu_{min} = -3 \text{ dB}$ and $\mu_{max} = 3 \text{ dB}$. For each scene, we evaluate the distance for a set of vegetation heights, ranging from 0.05 to 3 m. Moreover, the estimation depending on four different initial guesses is presented. This is shown in figure 4.19.

As we expected, the minimisation cost function the evaluated inversion approach, figure 4.19, may not provide the solution of minimum distance to the model value, since the solution depends on both the simulated value and the initial guess used for inversion.

Optimum values of vegetation height, i.e. close to 1 m, such as 1.2 m, result in relatively accurate estimates. On the contrary, very low values, e.g. 20 cm, or very high values of vegetation height, e.g. 1.7 m, provide less accurate solutions.
In order to further examine how the cost function depends on the parameters, we analyse for the same range of vegetation heights, i.e. 0.5–2 m, how the distance evolves considering now a high value of extinction, $\sigma = 8.5$ dB/m, and the same values of ground-to-volume ratios, i.e. $\mu_{\text{min}} = -3$ dB and $\mu_{\text{max}} = 3$ dB. This test is represented in figure 4.20.

The main difference of simulating a scene with a high value of extinction, i.e. 8.5 dB/m (figure 4.20), with respect to simulating a scene with an average value, i.e. 4.9 dB/m (figure 4.19), is that the minimum distance to which the cost function converges is higher, specially for higher values of height, e.g. 1.7 and 2 m. The consequence of this for the retrieval of parameters is that the minimisation loses precision, which turns into less accurate estimates.
4.2 Evaluation with simulated data

Figure 4.20: Distance cost function of the inversion approach 1. Parameters: $h_v$ in the range 0.5–2 m, $\sigma = 8.5$ dB/m, $\mu_{\min} = -3$ dB, $\mu_{\max} = 3$ dB, HoA = 3 m, AoI = 30°.

4.2.4.2 Computational cost

Tables 4.5 and 4.6 compare the precision in the estimation of the parameters in both inversion approaches. Besides, they provide as well an idea of the computational cost of each approach. Nonetheless, to be aware of the magnitude of the algorithms, table 4.7 presents the computational cost, measured in days, hours minutes and seconds, of each inversion approach for the configuration (i.e. input initial parameters) that has been used to present the results throughout the entire chapter 4.

The computational cost is presented also for cases (a), (b) and (c) regarding the inversion approach 1, and cases (a) and (b) concerning the inversion approach 2, respectively. The different cases correspond to the different methodologies followed in each inversion approach, as in table 4.5.

As we can observe from the information provided by the table 4.7, the inversion
Inversion approach 1  Inversion approach 2

| TanDEM-X system  | $\theta_0$ ($^\circ$) | 30  | 30 |
| parameters      | HoA (m)              | 3   | 3  |
| RVoG model      | $N_{hv}$             | 40  | 40 |
| parameters      | $N_\sigma$           | 40  | 40 |
|                 | $N_{\mu_{min}}$      | 4   | 4  |
|                 | $N_{\mu_{max}}$      | 4   | 4  |
| Number of initial guesses | $N_{inv}$   | 100 | -  |
| Search space    | $N_{hv_{space}}$     | -   | 351|
| $(h_v, \sigma)$ | $N_{\sigma_{space}}$ | -   | 301|

| Computational | (HH:MM:SS) |
| cost         | (a) 03:39:53 (a) 4 days 11:08:51 |
|             | (b) 03:39:10 (b) 3 days 19:47:46 |
|             | (c) 06:48:13 - |

Table 4.7: Comparison of the computational cost of inversion approaches 1 and 2.

approach 2 provides high precision estimates in exchange of a very high degree of computational cost. Reducing the time the approach takes to find a solution that is below the threshold of minimum distance, may result in solutions that are not physically feasible, even if they are close to the model value.

On the other side, the inversion approach 2 provides precise enough estimates for the optimum range values of the model parameters. Moreover, both inversion approaches have problems for the same range of model parameters, i.e. high values of height vegetation.

### 4.3 Evaluation with real data

In this section we test the influence of the double-bounce decorrelation term on the final height estimates in a real scenario. The test site evaluated consists of an area of $30 \times 30$ km in the mouth of the Guadalquivir river, Sevilla, with center coordinates 37.1 N, 6.15 W, where rice is cultivated annually from May to October, approximately. The specific area is shown in figure 4.21.
In particular, the test takes as input information of three parcels or fields at nine different dates from the 2015 campaign. The parcels respond to the name of Calonge, El Reboso and Minima.

With the purpose of testing the influence of the mentioned double-bounce decorrelation term, the test carries out the vegetation height estimation over the Sevilla test site (see [16]). The estimation is computed following inversion approach 1 (as in block diagram 3.3), for two different methodologies. In first place, using equation (3.1.5) for inversion, i.e. bistatic direct configuration, and secondly, using the expression that takes into account the double-bounce, as in (3.1.6), i.e. bistatic dihedral configuration.

Figure 4.21: Guadalquivir river, Sevilla (Spain). Image from Google Maps.
It is important to clarify that when inverting assuming a bistatic direct configuration, the methodology followed is analogous to the one used when inverting considering a monostatic direct configuration, since in neither case the double-bounce decorrelation term is present. The only case in which is necessary to introduce the double-bounce term in the inversion methodology is in bistatic dihedral configurations. This has to be taken into account whenever we refer to a bistatic direct configuration in the following results.

Figure 4.22 presents the temporal evolution of the height estimates at the Sevilla test site, for the three evaluated parcels, Calonge, El Reboso and Minima, respectively. Moreover, for each field, the figure presents the ground data (i.e. blue), and the PolInSAR estimates, computed from an inversion using a bistatic direct configuration (i.e. black), and a bistatic dihedral one (i.e. green). In the PolInSAR estimates, circles denote average values, and error bars standard deviations, both calculated for all pixels inside a field.

![Figure 4.22](image_url)
We can observe in the figure 4.22 that the three fields show a common behaviour. All parcels present two intervals with different performances. From the third or fourth acquisition onwards, i.e. DoY around 170–180, the retrieved heights and the ground data are close enough, with a difference that ranges around 20 cm. On the contrary, before these dates the estimated heights exhibit a clear overestimation, which is translated in a bias of around 1 m.

As it was demonstrated in the previous chapter with simulated data, this overestimation in the beginning of the season, when the crop height has not reached 30 cm, is a consequence of a lack of sensitivity. In other words, the vegetation crop height must be above 25–30 cm to get accurate results. Below these values, for such short heights, the spatial baselines need to be even longer than the ones provided during the science phase of TanDEM-X (i.e. 2–3 km). Otherwise, as it can be observed in the first two or three acquisitions, owing to a lack of interferometric sensitivity, the retrieved heights are very noisy and strongly overestimated.

With the purpose of evaluating the difference between inverting using a bistatic direct (i.e. monostatic direct), or a bistatic dihedral configuration, and quantifying the error in the vegetation height estimates when ignoring the double-bounce decorrelation term, the following analysis is carried out.

The histograms of the difference between height estimates obtained from an inversion considering a bistatic direct and a bistatic dihedral configuration are shown in figures 4.23, 4.24 and 4.25, for the three parcels and nine different dates, from June to August, respectively. The vertical wavenumber is 2.48 rad/m (HoA = 2.53 m), and the incidence angle is 22.7°.
Figure 4.23: Histogram of the difference between heights retrieved considering or not the double-bounce decorrelation term, over the parcel Calonge.

Figure 4.24: Histogram of the difference between heights retrieved considering or not the double-bounce decorrelation term, over the parcel El Reboso.
4.3 Evaluation with real data

The figures show the vegetation height difference between the two methodologies measured in centimetres. A common observation in the three fields, figures 4.23, 4.24 and 4.25, is that the difference throughout all the acquisitions (i.e. nine dates), oscillates from 2 cm up to 10 cm, approximately.

This deviation is more perceptible in amplitude for those dates in which the average height value is higher, as in the acquisitions of June 4, and of August 9 and 20, where the values are very close to 1 m. Moreover, this higher amplitudes involve a smaller difference (in cm) between methodologies. One can notice this even in the third field, that shows a lower average height value, and therefore, the difference in height estimates is also less pronounced. As expected from the simulated data, for this height values the inversion strategy works properly, introducing a minimum bias, around 1–2 cm.

In general, from these results, except maybe for very specific acquisitions (e.g. height values of 1 m), the error that induces ignoring the double-bounce is significant enough and it should not be ignored.
A last analysis taking as input values solutions obtained from real data is performed to evaluate the difference in the estimates, considering or not the double-bounce term in the inversion, for all parameters of interest, i.e. height vegetation $h_v$, extinction coefficient $\sigma$, and minimum and maximum ground-to-volume ratios, $\mu_{\text{min}}$ and $\mu_{\text{max}}$, respectively.

<table>
<thead>
<tr>
<th>RVoG model parameters</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_v$ (m)</td>
<td>$\sigma$ (dB/m)</td>
</tr>
<tr>
<td>Simulated scene from real data</td>
<td>1.31</td>
</tr>
<tr>
<td>Estimation from bistatic dihedral configuration</td>
<td>1.25</td>
</tr>
<tr>
<td>Estimation from bistatic direct configuration</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Table 4.8: Evaluated PolInSAR scene from parcel Calonge and acquisition 2015/07/29, taking as final estimate the mean of all possible solutions.

The solutions from real data are taken from the first field, i.e. Calonge, and for a date which corresponds to the middle of the season, i.e. July 29. Table 4.8 presents the simulated scene and the inversion results according to the implemented inversion approach 1. The inversion is carried out throughout the expressions (3.1.6) and (3.1.5), respectively.

From this last test, we can evaluate how the bistatic dihedral configuration for inversion turns into more accurate results at the expense of computational cost. The difference between direct and dihedral configurations is even higher for parameters different to height vegetation, i.e. extinction, and ground-to-volume ratios. Regarding these latter parameters, one should notice that the values are expressed in logarithmic scale (i.e. decibels), hence, the difference in linear scale between the estimates is even greater. With respect to the vegetation height, for a simulated value of 1.31 m, the estimates between both configurations (with and without the $DB$ term) differ in just 3 cm.
Chapter 5

Conclusions

Finally, this chapter summarizes the goals achieved in relation to the initial objectives. It includes a proposal of future lines to continue the study of the current procedures for estimating the biophysical parameters of crops by means of PolInSAR.

5.1 Review of the objectives

The aim of the project relies on the development and evaluation of different inversion strategies for the retrieval of vegetation parameters using PolInSAR based techniques. This includes the following specific objectives:

- Understanding the principles of the homogeneous volume-over-ground model used for physical parameter estimation. This itself involves the review and study of the SAR, InSAR and, eventually, PolInSAR principles.

- In second place, the implementation the TanDEM-X PolInSAR data processing algorithms, as well as the techniques of vegetation height estimation, topographic phase, extinction and ground-to-volume ratios, based in TanDEM-X PolInSAR data.

- Evaluation of the estimation algorithms and the direct models (monostatic RVoG and bistatic RVoG) through simulations. Including the evaluation of the quality of the estimations (i.e. precision), and the computational cost.
- Analysis of the influence of the double-bounce decorrelation factor on the inversion of ground topography and vegetation height. This includes the use of experimental data to quantify this influence on a final application.

The implemented inversion approaches for the retrieval of biophysical parameters by means of PolInSAR, have shown precise enough results for a range of specific values of input parameters that define the properties of the model scene.

The inclusion of the double-bounce decorrelation term that appears in bistatic acquisitions, such as in the case of the TanDEM-X sensor, complicates the inversion of the RVoG model when estimating ground topography and vegetation height. As a consequence, the estimation of the rest of model parameters, i.e. extinction and ground-to-volume ratios, gets as well more complicated. That inclusion, despite being more rigorous, is not necessary in many occasions, since the retrieved values, within certain specific ranges, are not very different from the actual ones.

Results have demonstrated that the error becomes more important for incidence angles shallower than $30^\circ$ and large products of vertical wavenumber and vegetation height. Nevertheless, special take should be taken in other applications with different scenarios (e.g. forests), since a priori, the double-bounce decorrelation term cannot be ignored, at least, until the data tested suggests otherwise. This is relevant since, so far, in the retrieval of ground terms, the double-bounce contribution has been mostly ignored, and thus the impact on the parametric inversion must be clarified.

In the end, depending on the final application, a compromise between precision and volume of data to be processed should be taken, since owing to the extra iteration that the double-bounce decorrelation term introduces in the inversion methodology, the computational cost increases.
5.2 Future lines

Finally, a research line is proposed regarding the implemented inversion methodologies. In a common way to the implemented inversion approaches, it is proposed to carry out a more detailed analysis of the minimisation cost function to fully get rid of solutions that do not have physical meaning.

Moreover, the normalised vegetation height as function of the vertical wavenumber can be used in future tests to extrapolate the results for crops (i.e. rice) to more scenarios (e.g. forests). Besides, with the obtained results, further analysis of the overestimation and underestimation in the retrieval of ground parameters can be developed.

With respect to the extinction coefficient, the analysis has demonstrated that, in spite of the fact that it is conveniently introduced in the theoretical RVoG model, at a practical level, we cannot extract conclusions from its behaviour, despite its a priori physical meaning. In this sense, we advise to dig a little deeper into this matter. After all, if we are considering that we have a model to estimate all ground parameters, the definition of that model should be consistent with the observed phenomenon. If this is not the case for any part of the model, it should be made explicit that the role of this or those parameters is not known.

As a complementary way that might be a more compact and perhaps a little more rigorous way of studying the sensitivity of the RVoG model is by the Cramer-Rao limits of the estimates (see [37]). The methodology proposed obtains the minimum limits of variance that could be achieved with the RVoG for the height. This could also be done for other parameters, as well as for the double-bounce model, after obtaining first the corresponding expressions accordingly.

In the end, the exploitation of any future line of research must lead towards the optimisation of the inversion strategies implemented, as well as to the optimisation of the programs and routines themselves.
Bibliography


