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The Role of Topology in Organic Molecules: Origin and Comparison of the Radical Character in Linear and Cyclic Oligoacenes and Related Oligomers

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Abstract

We discuss the nature of electron-correlation effects in carbon nanorings and nanobelts by the analysis tool known as fractional occupation number weighted electron density ($\rho^{\text{FOD}}$) and the RAS-SF method, revealing for the first time significant differences in static correlation effects depending on how the rings (i.e. chemical units) are fused and/or connected until closing the loop. We choose to study in detail linear and cyclic oligoacene molecules of increasing size, and relate the emerging differences with the difficulties for the synthesis of the latter due to their radicaloid character. We finally explore how minor structural modifications of the cyclic forms can alter these results, showing the potential use of these systems as molecular templates for the growth of well-shaped carbon nanotubes as well as the usefulness of theoretical tools for molecular design.

Key words: Oligoacenes, Cyclacenes, bi- or poly-radicals, fractional orbital occupation, FT-DFT, RAS-SF.
1 Introduction

Nowadays, there is a growing and worldwide interest for the study of cyclic organic molecules of limited size, constituted by fused or bound benzene rings,1–8 as molecular templates (i.e. chemical precursors) for the synthesis and growth of size-specific Single-Walled Carbon Nano Tubes (SWCNT), which could be afforded under certain experimental conditions still under investigation.9 Probably the best known examples are the CycloParaPhenylene (CPP) compounds (see Figure 1) leading to SWCNT with armchair edges in this case. Other interesting family is formed by CyclaCene (CC) molecules of increasing size, see also Figure 1, which would ideally give rise to zig-zag SWCNT of controlled diameter and shape. It should be indicated that [n]CPPs are experimentally produced at the gram-scale and can be synthesized following several routes. On the other hand, many groups have historically attempted to synthesize [n]CC compounds without success,10 whereas pristine linear acenes have been synthesized up to hexacene11 and very recently heptacene,12,13 and up to nonacene using stabilization strategies based on chemical substitution.14 Furthermore, the molecule [3]cyclobenzo[a]anthracene, an isomer of [12]cyclophenacene, has been successfully synthesized,15 recrystallized and its structure has been confirmed by X-ray data. This achievement paves the way for the production of belt-shaped compounds, constituting thus a major breakthrough possibly inspiring other discoveries soon.

The strong tendency of some of these organic compounds to react with the environment has been associated with their diradical (or polyradical) character, which for instance would explain why the stability of linear acenes decreases as the number of fused rings increases. Therefore, in order to ra-
tionally design stable nanostructures to be used as molecular building blocks for the synthesis of nanotubes, it is necessary to comprehend their electronic properties in general, and the physical origin of their diradicaloid nature in particular. From the theoretical and computational point of view, elucidating the bi- or polyradical character of organic molecules is a longstanding research effort from decades ago, whose origin can be traced back to difficulties of theoretical methods to include Strong Correlation Effects (SCE), also called static, left-right, nondynamical, or multicenter, in a cost-effective and systematic manner. These effects often manifest into a small or even vanishing one-electron energy gap, i.e. the energy difference between frontier molecular orbitals, posing consequently a great challenge to Density Functional Theory (DFT) methods within the Kohn-Sham (KS) formulation due to its intrinsic one-determinantal nature and/or the lack of a known exact density functional. Knowing a priori if a system is affected or not by SCE is not an easy task. Some global diagnostics exist in the literature like $T_1/D_1$ or $A_\lambda$, but they are based on costly calculations which might hamper the judicious choice of single- or multi-reference electronic structure methods for large systems or for a fast screening of compounds. Note also that only very recently local descriptors of electron correlation have appeared too.

Previous considerable and remarkable efforts extended the standard KS formalism to cope with these SCE, making use of the on-top pair density, the ensemble of pure determinantal states, the adequate and balanced coupling between some ab initio method with a density functional (e.g. DFT-MRCI, CAS-DFT, CASCI-DFT, or spin-projected UHF-DFT), the spin-flip approach, the Becke’05 correlation model, the spin densities
Based on natural orbitals, the pair natural orbital functional theory, the fractional-spin occupations, or the recently developed Multi-configuration Pair-Density Functional Theory (MC-PDFT) to explicitly mention just a few of them.

Interestingly, another route recently pursued circumvents these difficulties and aims at simulating SCE by fractional orbital occupations, as it may be naturally arising in the systems affected by it, thus avoiding issues as the double-counting of correlation contributions (i.e. static and dynamic) and benefiting from a more practical implementation. This kind of methods provide fractional orbital occupations by a Fermi-Dirac distribution under the effect of a fictitious temperature (several thousands of K) to force that occupation, with these techniques generally referred as Finite-Temperature (FT-) DFT or Fermi-Smearing and Thermally-Assisted-Occupation (TAO) DFT. However, it was not until recently that this theory has been proposed to serve as an interpretative tool to predict, analyze, and thus rationalize, SCE in any chemical system, which we would like to further exploit here for disclosing structure-property relationships of organic molecules of cyclic topology.

In the following, we will introduce the theoretical framework employed before presenting results for \([n]\)CPP and \([n]\)CC molecules with \(n = 5 - 12\) units, the latter extensively compared with the parent linear compounds. We try next to disclose the role played by cyclic topology, and conclude presenting results for other nanobelts recently synthesized efficiently. We will compare systematically results achieved from the FT-DFT method with those from the Restricted-Active-Space (RAS) Spin-Flip (SF) method, which will
allow to merge the best of both worlds, DFT and wave function based models, while achieving results of high accuracy. These two methods bring two rather different perspectives for the description of electronic systems were strong correlation might be important, and their combined use might result in an clear description of molecular systems. Moreover, the FT-DFT analysis provides a general framework for the quantification of the (poly)radicaloid character for electronic densities obtained within different methods. Overall, these results should help to find out why some nanorings are synthesized and others not, as well as displaying the usefulness of these theoretical tools for this exciting field of knowledge.

2 Theoretical framework

2.1 The FT-DFT method and the FOD analysis

In standard KS-DFT calculations, the ground-state electronic density is built upon the lowest $N$ occupied spin-orbitals $\varphi_i$ as:

$$\rho_{\text{KS-DFT}}(\mathbf{r}) = \sum_{i}^{N} |\varphi_i(\mathbf{r})|^2,$$

with the corresponding energy levels given by $\varepsilon_i$. Since degeneracies do not often arise in common systems, the orbital occupation numbers are 0 or 1. Deviations from this situation are typically associated with the presence of strong electron correlations and can be rectified by the introduction of fractional occupation numbers:

$$\rho(\mathbf{r}) = \sum_{i}^{\infty} f_i |\phi_i(\mathbf{r})|^2,$$

employing hereby a finite-temperature DFT approach with $f_i$ the fractional occupation numbers ($0 \leq f_i \leq 1$) of a new set of orbitals $\phi_i$. These orbitals
are self-consistently obtained by minimizing the Gibbs electronic free energy 
\( G_{el} = E_{el} - T_{el}S_{el} \) of the system at a fictitious pseudo-temperature (i.e. 
electronic) called \( T_{el} \). The fractional occupation numbers \( f_i \) are determined 
from the Fermi-Dirac distribution:

\[
    f_i = \frac{1}{1 + e^{(\epsilon_i - E_F)/\theta}},
\]

(3)
depending critically on the \( \epsilon_i - E_F \) difference and on \( \theta = k_B T_{el} \), which actually 
depends on the exchange-correlation functional used for these calculations.

Under this approach, it is possible to define a Fractional Orbital Density 
\( (\text{FOD}) \) as an analysis tool of static correlation: 63,64

\[
    \rho^{\text{FOD}}(r) = \sum_i (\delta_1 - \delta_2 f_i) |\phi_i(r)|^2,
\]

(4)
where ideally \( M \to \infty \) and is the dimension of the finite basis set in practice, \( \delta_1 \) and \( \delta_2 \) are chosen to become \((1,1)\) if the single-particle energy level 
\( (\epsilon_i) \) is lower than the energy of the Fermi level, \( E_F \), or \((0, -1)\) otherwise.

Representation of natural orbitals with fractional occupations and/or the as-
associated electron density, i.e. \( \rho^{\text{FOD}} \), has repeatedly shown to be a very useful 
and intuitive computational tool to understand the distributions of unpaired elenctrons in molecular systems. 66–68 Note finally that the integral of \( \rho^{\text{FOD}}(r) \) 
over all the spatial space yields the highly useful \( N_{\text{FOD}} \) value, that is, an es-
timation of the number of strongly correlated electrons. The \( S_{el} \) entropy, 
connected with the symmetrized von Neumann entropy, is related with the 
entanglement of electronic states and may be also obtained from the corre-
sponding \( f_i \) values, as explained and shown in the Supporting Information.
2.2 The RAS-SF method

Within the realm of wave function approaches able to deal with degeneracies or near-degeneracies, Spin-Flip (SF) methods have shown to be a reliable approach in many cases. In particular, the definition of a Restricted Active Space (RAS) in conjunction with a SF excitation operator (promotion of $\alpha$ electrons into empty $\beta$ orbitals), i.e. the RAS-SF method, represents a balanced and computationally affordable approach to the study of molecular systems with radical or polyradical character. One of the typical metrics employed to evaluate the amount of polyradicalism of RAS-SF electronic states is through the expression introduced by Head-Gordon, to quantify the number of unpaired electrons:

$$N_U = \sum_i (1 - \text{abs}(1 - n_i)),$$

where $\{n_i\}$ are the electron occupancies of the RAS-SF natural orbitals ($0 \leq n_i \leq 2$). Close inspection of Eqs. (4) and (5) reveals the equivalence of $N_U$ and the integration of $\rho_{\text{FOD}} (N_{\text{FOD}})$. Therefore, in the following we will exclusively use the $N_{\text{FOD}}$ label to quantify the number of unpaired electrons of electronic states computed with both approaches, i.e. FT-DFT and RAS-SF. Figure S1 shows the correlation between both approaches applied to the case of oligoacenes.

3 Computational details

The geometries of the compounds were optimized at the (restricted) M06-2X/6-31+G* level, using the Gaussian09 package and the ultrafine grid, and their nature confirmed as global minima by the corresponding all-real vibrational frequencies. The use of a broken-symmetry guess to optimize the
ground-state geometry of the molecules did not appreciably alter any of the results exposed here. The choice of the M06-2X functional \(^{73}\) is motivated by a previous study showing that this method provides sufficiently accurate structures for cyclic nanorings compared with the long-range dispersion-corrected M06-2X-D3(BJ) extension. \(^{74}\) Then, the FT-DFT calculations were done at the (unrestricted) TPSS/def2-TZVP level using the ORCA 4.0.0.2 package \(^{75}\) with the default temperature \(T_{el} = 5000\text{K}\) as originally recommended for this functional. The isocontour values for the FOD plots were set to 0.005 \(\text{e/bohr}^3\), except for borderline cases for which a value of 0.002 \(\text{e/bohr}^3\) is instead fixed to better appreciate the corresponding isosurfaces. The FOD plots were generated with UCSF Chimera \(^{76}\) (version 1.12).

The RAS-SF calculations were done with a development version of the Q-Chem program \(^{77}\) and with the 6-31G(d) basis set. RAS-SF calculations used the lowest ROHF (Restricted Open-Shell) triplet state as a reference configuration in all the systems but the cyclic acenes with an odd number of rings, for which the lowest ROHF quintet orbitals were employed instead. The restricted orbital space considered 8 electrons in 8 \(\pi\)-orbitals for the RAS2 subspace, as recently recommended for CASSCF, and all virtuals and doubly occupied \(\pi\)-orbitals for the RAS1 and RAS3 subspaces, respectively. Excitations from core electrons (1s C orbitals) were disregarded. Fragment decomposition of the RAS-SF ground state function of even \([n]CC\) molecules have been done considering a RAS2 subspace with 2 electrons in 2 \(\pi\)-orbitals. Molecular orbital energies for the Hückel model were obtained with \(\alpha < 0\), \(\beta = \alpha/10\) and \(\gamma = \beta/5\) for the Coulomb and resonance integrals.
4 Results and discussion

4.1 Cycloparaphenylenes

A relatively large number of cycloparaphenylenes have been synthesized efficiently and following different routes since 2008, as said before, with multiple possibilities for cutting-edge applications such as: (i) self-aggregation to form stable molecular crystals\textsuperscript{78} with specific porosity arising from well-defined microscopic orientations,\textsuperscript{79} (ii) nanocontainers for encapsulating pristine or fine-tuned fullerenes,\textsuperscript{80–82} and (iii) creation of (sub)-nanochannels or tweezers of controlled length and/or chirality\textsuperscript{83} after dimerization. These applications thus showing their versatile chemical character and some of their nanofunctions. The FOD plots (not shown here) for the set of the \([n]CPPs\) studied, with \(n = 5\) – 12, become visible only for a cut-off of 0.002 e/bohr\textsuperscript{3}, indicating thus a minor polyradicaloid nature as it was expected from their allowed synthesis and applications, which may be also seen as a proof of concept. The corresponding \(N_{FOD}\) values (Table S1) for all the studied compounds are comprised between 0.467 and 0.636 and thus admittedly low, although slightly increasing with the system size. Actually, from these results and its comparison with standard molecules,\textsuperscript{63} it can be inferred a negligible presence of SCE in agreement with their afforded synthesis by many groups and in different conditions.

4.2 Linear and cyclic oligoacenes

4.2.1 Generalities

We analyze and compare next linear and cyclic oligoacenes of increasing size, noticing that some of the FT-TPSS results for the odd-numbered linear systems (i.e. anthracene, pentacene, and heptacene) were also included.
in previous publications\textsuperscript{63,64} although not compared with the parent cyclic forms neither with RAS-SF results. According to the Clar’s sextet rule long acenes tend to be rather reactive since only one aromatic sextet ring can be drawn for resonance structures with no radical centers (see Figure S2 in the Supporting Information). This simple rule-of-thumb suggests that the cyclic oligoacenes will be even more reactive than their linear counterparts, since it is not possible to form a Clar’s sextet without unpaired electrons for the \([n]\)CC family.

\subsection{Geometrical and electronic features}

To better understand the electronic structure of oligoacenes, we first explore their geometrical characteristics. The different contour conditions between the two families are reflected on the distribution of C-C bond lengths (Figure 2), with optimized geometries here in good agreement with previous DFT calculations.\textsuperscript{84} In the cyclic molecules all edge and bridging bond distances are equal between themselves, the former being close to the typical distance in benzene (e.g. 1.41 Å for [11]CC) suggesting linear chain conjugation at the edges, an aspect with key implications that will be further analyzed in more detail along the study. The interchain bond (bridge) is much longer, e.g. 1.47 Å for [11]CC, corresponding to the bond distance of a single bond between two sp\(^2\) C atoms. On the other hand, the linear acenes exhibit alternating edge distances with the two bonds at the molecular ends close to a double bond situation (Figure 2a). The bridging distances (Figure 2b) follow a continuous profile with shorter bonds at the ends and larger distances (closer to the cyclic case) at the center of the molecule.
This different structural topology between linear and cyclic acenes, namely the border vs. cyclic conditions, results in non-equivalence between central and edge C atoms in the linear family, and complete equivalence between all outer and inner (bridging) carbons for the cyclic acenes. This also has, in addition to the geometrical results described above, a strong impact on the electronic properties. In particular such differences results in qualitatively different molecular orbital diagrams, as recovered by the simple Hückel model as a proof of concept (Figure 3). The linear arrangement of benzene rings results in a set of non-degenerate orbitals around the Highest Occupied and Lowest Unoccupied Molecular Orbitals (HOMO and LUMO, respectively) with decreasing HOMO-LUMO gaps as the conjugation size of the system increases. On the other hand, the higher symmetry in the cyclic family results in sets of degenerate frontier orbitals. Moreover, the electronic distribution around the Fermi energy level is qualitatively different for molecules with an odd or even number of six-member rings. Cyclic acenes with an even number of rings present two electrons in two degenerate orbitals, predicting a strong diradical character, while \([n]CC\) molecules with odd number of rings exhibit a pair of degenerate HOMOs and LUMOs with smaller energy gaps for larger systems. Hence, the Hückel model suggests that the electronic structure of cyclic oligoacenes hold sensible differences with respect to the linear counterparts, and that the odd and even members of the series follow rather different electronic structure patterns.

4.2.3 Evidences about the polyradical character

The spatial distribution of the unpaired electrons for linear oligoacenes represented by (FT-TPSS) FOD plots (Figure 4) and RAS-SF natural or-
bitals (Figure 5) suggests a growing polyradicaloid character with the number of fused rings, in qualitative agreement with the HOMO-LUMO gaps from the Hückel model, the symmetrized von Neumann entropy (see Figure S3) and the Hückel-based bond orders (see Figures S4-S5). Actually, Figure S6 also shows the exponential decay of $N_{FOD}$ for linear oligoacenes as a function of the HOMO-LUMO energy difference. The unpaired density mostly lies on the non-bridging C atoms situated at both sides (up and down) of the molecule and mainly localizes on the central rings with virtually no contribution from the atoms at the molecular ends (left and right). This picture is obtained by both DFT (FT-TPSS) and wave function (RAS-SF) calculations, although the former produces larger delocalized FODs, which can be possibly associated with the lack of exact-exchange. The computed $N_{FOD}$ values by both methods (FT-DFT results in Table S2 and RAS-SF results in Table S5) confirm the increasing diradicaloid nature (and SCE) along the linear oligoacene series, with values close to two unpaired electrons for the longest members of the series. The shortest oligoacenes (i.e. from naphthalene to tetracene) exhibit much weaker SCE, with $0.08 < N_{FOD} < 0.49$.

The unpaired electron density for the cyclic forms is delocalized along the entire nanoring and for the most part equally shared amongst the non-bridging C atoms (Figures 6 and 7). In general, the computed orbital occupancies and energies follow the scheme predicted by the Hückel model for the odd-numbered cyclic acenes, i.e. doubly degenerate HOMOs and LUMOs, while the frontier orbitals of $[n]CC$ with $n = 6, 8, 10$ and 12 are no longer degenerate (as in Hückel) and present natural orbital occupancies and $N_{FOD}$ values approximately corresponding to the case of two unpaired electrons (Tables S3 and S6), hence they can be labelled as diradicals (or
diradicaloids). On the other hand, \([n]\)CC with \(n = 5, 7, 9\) and 11 hold four natural orbitals with occupancies with sizable departure from the closed-shell configuration. These results suggest that odd cyclic oligoacenes contain four strongly correlated electrons and they should be treated as tetraradicaloid systems rather than diradicaloids.

Figure 8 shows the size-evolution of \(N_{FOD}\) along the two series, linear and cyclic acenes, obtained with the FT-TPSS method. The zig-zag profile drawn by the \([n]\)CC family exposes the electronic differences between molecules with even and odd number of rings, with much larger values for odd \(n\) indicating close to four unpaired electrons in \([11]\)CC. For linear oligoacenes there is a kind of cumulative effect, i.e. each C atoms contribute more and more to the radicaloid nature of the whole molecule and its number increases as the system grows. For cyclic acenes, the increase in the \(N_{FOD}\) values with the number of rings is mostly due to an increase in the total number of unpaired electrons, as will be discussed in the next section, despite each C atom having a lower radicaloid character on average (see also Figures S3 and S5). Note finally that the highest value of \(N_{FOD}\) here obtained (4.259 for the \([11]\)CC case) is significantly larger than any of the \(N_{FOD}\) values reported before for prototype systems displaying a truly multireference character, such as \(p\)-benzyne or the transition state of the Be–\(H_2\) complex, as well as for studied biological systems such as the methylcobanimide cation.\(^{63}\)

### 4.2.4 Interaction between radical rings

It is worth noting that in cyclic acenes with an even number of fused rings, the HONO (LUNO) presents "bonding" ("anti-bonding") interactions
between up and down edge (i.e. non-bridging) C atoms (Figure 7 (left)),
while in linear and odd cyclic molecules the HONOs (LUNOs) correspond
to the out-of-phase (in-phase) arrangement between up and down π-orbitals.
This observation suggests that interaction between unpaired π-electrons at
the edge carbons in even \([n]CC\) lifts the degeneracy obtained with the Hückel
model (Figure 3). Actually, if we add an additional (weak) interaction be-
tween each of these atom pairs in the Hückel Hamiltonian, we obtain a sit-
uation closer to the electronic structure calculations (Figure 9). The same
interaction for odd \(n\) results in a decrease of the HOMO-LUMO gap.

To further explore the idea of interacting radical centers at the molecular
dges we analyze the ground state RAS-SF wave functions of \([n]CC\) with
\(n = 6, 8, 10\) and 12 by means of fragment decomposition.\(^{85,86}\) In the fragment
basis of up and down chains, the total wave function can be expressed in
terms of covalent and ionic contributions as:

\[
\Psi = c_{\text{cov}} |\phi_u^{(1)} \phi_d^{(1)}\rangle + c_{\text{ion}} (|\phi_u^{(2)} \phi_d^{(0)}\rangle + |\phi_u^{(0)} \phi_d^{(2)}\rangle),
\]

where \(\phi_u\) and \(\phi_d\) indicate up and down radical rings, the superindexes \((m)\)
indicate the number of electrons in each ring, and \(c_{\text{cov}}\) and \(c_{\text{ion}}\) are the coef-
ficients for covalent and ionic contributions. The presence of ionic contribu-
tions is related to the ability of the two radical rings to share their electron
density, thus it can be used as a measure of the strength of the inter-radical
interaction. Stronger interaction between radical centers will result in larger
orbital gaps and weaker diradical character. The correlation between ionic
contributions and singlet-triplet gaps in Figure 10 rationalizes how the inter-
radical ring interactions, equivalent to \(\gamma\) in the Hückel model, control and
tune the strong diradical character in even cyclic acenes, with stronger in-
teractions resulting in lower diradical character (larger HOMO-LUMO gaps).
4.2.5 The singlet-triplet energy gap

The diradical (or diradicaloid) character of ground state singlet wave functions has been systematically associated to small energy separation between the ground state singlet ($S_0$) and the lowest triplet state ($T_1$). Here we calculate the (vertical) singlet-triplet energy gap $\Delta E_{ST} = E(T_1) - E(S_0)$ in linear and cyclic oligoacenes by means of the FT-TPSS and RAS-SF models. Previous calculations for the longstanding quest of the ground-state nature of $[n]$CCs, and related isomers,\textsuperscript{87,88} have shown the following: (i) a general failure of DFT calculations for predicting correctly the nature of their ground-state\textsuperscript{89,90} unless for some last-generation orbital-optimized DFT-based methods\textsuperscript{91} and/or double-hybrid functionals;\textsuperscript{92} (ii) a significant multireference character of the prototype $[6]$CC molecule, with a $\Delta E_{ST}$ value of less than 0.04 eV at the MRMP2/6-31G*/CASSCF(8,8)/6-31G* level;\textsuperscript{93} (iii) the existing difficulties for selecting systematically and routinely the (often large) CAS active spaces needed for the systematic study of $[n]$CCs of increasing size;\textsuperscript{84,94} and (iv) the adequate alternative provided by TAO-DFT based methods\textsuperscript{95} at a substantially lower computational cost.

Figure 11 displays the $\Delta E_{ST}$ results of the FT-TPSS calculations performed in this study, from which the odd-even pattern is also seen for $[n]$CC systems as well as the rapid decay for the linear systems in close agreement with RAS-SF and recent TAO-LDA results\textsuperscript{95} (Table 1). It is worth noting the rather constant (and small) energy gap obtained for all the even cyclic acenes, which supports the idea of weak interactions between radical rings as the origin for singlet state stabilization. We also compare in Table 1 the
results obtained here with the best available, up to the best of our know-
edge, multireference ab initio calculations,\textsuperscript{84,94} to find a reasonable (semi-
quantitative) agreement between this and the other approaches with some
due caution with respect to the evolution with system size of the CASPT2
results. Overall, the cost-efficient FT-TPSS method exhibits very good re-
results in the computation of singlet-triplet energy gaps at an impressively low
computational effort.

4.2.6 \textbf{Tetraradical character of odd cyclic acenes}

As discussed above, the electronic structure analysis of the odd cyclic
acenes suggests that they can be considered tetraradicaloids. The tetraradi-
cal character can be associated with small singlet-quintet energy difference,
like singlet-triplet gaps relate to diradical character. Hence, we compute the
energy gap to the lowest quintet state (Q\textsubscript{1}) for [n]CC with n = 5, 7, 9 and
11 (see Table 2) to find that: (i) the vertical energy to Q\textsubscript{1} decrease with the
molecular size, in good agreement with an increase of the tetraradical char-
acter with n; and (ii) the energy gaps to the lowest quintet are considerably
low and similar to energy differences to T\textsubscript{1} for the largest cyclic acenes, i.e.
n = 9 and 11.

4.3 \textbf{Relating }N_{\text{FOD}}\textbf{ to the bi- or polyradical character}

The open-shell singlet biradical character of organic molecules has been
recently related to the \(N_{\text{FOD}}\) values, thanks to the development of an experi-
mental route in last years to gauge its character:\textsuperscript{96}

\[ y = 1 - \sqrt{1 - \left( \frac{E_{S_{1u}S_{1g}} - E_{T_{1u}S_{1g}}}{E_{S_{2g}S_{1g}}} \right)^2} , \]  

(7)
where all the excitation energies of the second right-hand side can be obtained from two-photon absorption, ESR, and phosphorescence experiments, and $0 \leq y \leq 1$ as limits for pure closed-shell or open-shell (biradical) nature.\textsuperscript{97,98} According to the data reported before\textsuperscript{64} for a set of some polycyclic aromatic molecules (including naphthalene, anthracene, fluorene, TIPS-pentacene, and chrysene) and a substituted hexaphyrin molecule, there is a linear relationship between $y$ and the $N_{\text{FOD}}$ values (see Figure S7 in the Supporting Information). Following that trend, we propose to distinguish between highly (slightly) pronounced polyradical ground-state character for organic molecules displaying $y \geq 0.5$ ($y < 0.5$) which is, in turn, roughly related to values of $N_{\text{FOD}} \geq 1.5$ ($N_{\text{FOD}} < 1.5$).

Following this approximate rule-of-thumb, we can label all $[n]CC$ ($[n]CPP$) systems as having a pronounced (negligible) polyradical character, whereas for the case of linear oligoacenes there is a distinction between short (from naphthalene to hexacene) and long (from octacene to largest systems) molecules, with heptacene situated at the borderline ($N_{\text{FOD}} \approx 1.5$) as also displayed in Figure 8. Actually, whereas systems as tetracene and pentacene are easy to functionalize and are being routinely exploited in many fields (e.g. Organic Electronics\textsuperscript{99,100}), the situation becomes more involved for larger acenes. As a matter of example, heptacene has long been pursued and it was not until very recently that it could be isolated\textsuperscript{12} and reported in bulk and at room-temperature.\textsuperscript{13} Photogeneration of octacene and nonacene has made these systems accessible experimentally under the high-vacuum conditions required for matrix isolation,\textsuperscript{101} with these and longer acenes constituting still a challenging synthetic effort.\textsuperscript{102}
4.4 Molecular engineering of nanobelts

Herein we apply the FT-TPSS and RAS-SF approaches combined with the FOD analysis to the study of the electronic properties of other molecular nanobelts with distinct structural topologies, and we try to establish connections between their polyradicaloid character and structural motifs. First, we consider the distortion of the perfect loop in \([n]CC\) to achieve the molecules named as cyclo[a]decacene and cyclo[a]undecacene, see their chemical structure in Figure 12. The rather small computed singlet-triplet energy gaps and considerable amount of unpaired electrons obtained by FOD analysis (Table 3) suggest important polyradical character pointing towards potential weak molecular stability that might limit their successful synthesis.

The different chemical topology at the kink breaks the perfect equivalence between benzene rings, which can be seen as the result of boundary conditions much like the molecular ends in linear acenes. Such contour conditions break the degeneracies between frontier orbitals (HOMOs and LUMOs) obtained in \([n]CC\) molecules. As a result, cyclo[a]decacene and cyclo[a]undecacene present strong polyradical character with unpaired electrons localized mainly on the up and down (non-bridging) C atoms further from the kink. It is important to notice that their HONO and LUNO look like the ones obtained for linear acenes for both odd and even number of rings, that is a HONO and LUNO with out- and in-phase interaction between radical rings (Figure 13). These results clearly indicate that their electronic structure is closer to the linear acenes than to the \([n]CC\) series.

Actually, we may conceive some other nanobelts with additional kinks, as those presented in Figure 12 too and dubbed as \([3]\)cyclobenzo[a]anthracene
and [3]cyclochrysene. Note how the FOD plots, which needed to be done with a cut-off of $\sigma = 0.002$ e/bohr$^3$ to become visible, are delocalized but still absent from some C atoms. The $N_{FOD}$ values are significantly lower than in any of the other studied nanobelts here, and closer to the $[n]$CPP series, indicating a rather weak diradicaloid character. Our results confirm the close relationships between SCE and structural motifs, hopefully paving also the way towards the molecular engineering of this kind of compounds.

5 Conclusions

We have studied a set of cyclic molecules of ongoing interest, named as $[n]$CPP or $[n]$CC, envisioned to act as molecular precursors for the fine-tuned synthesis of nanotubes of uniform diameter and controlled edges. The accurate modelling of these molecules might need the use of methods incorporating strong correlation effects, and we have thus systematically employed the tool known as FOD and the RAS-SF method to reveal this nature. While the former tool provides not only a real-space representation of these electron-correlation effects, but some global indicators (e.g. the integrated number of electrons arising from the fractional occupation of orbitals, $N_{FOD}$) allowing to distinguish and then classify the molecules as bi- or poly-radicals, the latter provides a multi-configurational wave function description able to accommodate and describe to a great detail a several strongly correlated electrons.

Our calculations suggest a strong relationship between polyradical character and the structural motifs forming the nanoring, with a particular distinction between cyclic phenylenes or cyclic acenes. The latter are known to be a longstanding synthetic target not still achieved, closely related with
the longest parent linear compounds as the calculations unveiled. Detailed analysis of the ground and low-lying states of these molecules suggests that odd and even \([n]\)CC oligomers should be treated as separate families with diradical and tetraradical character, respectively. They exhibit qualitative differences in their electronic structure around the frontier orbitals, resulting in a small and rather constant singlet-triplet energy gaps for the even series and with small and decreasing singlet-quintet gaps for the odd cyclacenes.

We have also tried to relate the cyclic molecular shape and the envisioned synthesis of nanobelts, starting from the \([n]\)CC forms, with some kinks or defects being prone to break the strong correlation effects found in pristine \([n]\)CC compounds.

## Acknowledgements


## Associated content

The Supporting Information contains in this order: (i) Figure S1 with the correlation between \(N_{FOD}\) values obtained by FT-DFT and RAS-SF methods; (ii) Figure S2 illustrating the resonance structures found for linear and cyclic acenes, with Clar’s sextet ring indicated in blue; (iii) a discussion about the symmetrized von Neumann entropy, and its evolution (Figure S3).
as a function of the number of C atoms for linear and cyclic oligoacenes; (iv) Figures S4-S5 showing the evolution of the radicaloid character for linear and cyclic oligoacenes, as deduced from bond orders obtained at the Hückel level; (v) Figure S6 displaying the relationship between $N_{FOD}$ and the HOMO-LUMO energy difference for linear oligoacenes; (vi) Figure S7 with the relationship between $y$ (biradical character) and $N_{FOD}$ values; (vii) Tables S1-S4 with the energies and fractional occupation numbers of the (H-4)OMO to (L+4)UMO window of orbitals for each of the $[n]$CPP, linear and cyclic compounds, and the other engineered nanobelts; (viii) Tables S5-S6 with the natural occupation numbers of the (H-3)OMO to the (L+3)UNO window of orbitals for each of the linear and cyclic compounds; and (ix) Table S7 with the optimized Cartesian coordinates of all relevant compounds.

**Author contributions**

A.P.G. and M.E.S.S. contributed equally to this work. The manuscript was written through contributions of all authors. All authors have given approval to the final version of the manuscript.

**References**


[25] Lee, T. J.; Taylor, P. R. A Diagnostic for Determining the Quality of


[40] Kazaryan, A.; Heuver, J.; Filatov, M. Excitation Energies from Spin-Restricted Ensemble-Referenced Kohn–Sham Method: A State-


Table 1: Comparison between calculated $\Delta E_{ST}$ values (in eV) for $[n]$CCs of increasing size ($n = 5 - 12$).

<table>
<thead>
<tr>
<th>System</th>
<th>RAS-SF</th>
<th>FT-TPSS</th>
<th>CASPT2/6-31G*&lt;sup&gt;a&lt;/sup&gt;</th>
<th>NEVPT2/ANO-DZP&lt;sup&gt;b&lt;/sup&gt;</th>
<th>TAO-LDA&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 5$</td>
<td>0.854</td>
<td>0.302</td>
<td>–</td>
<td>–</td>
<td>0.26</td>
</tr>
<tr>
<td>$n = 6$</td>
<td>0.264</td>
<td>0.419</td>
<td>0.52</td>
<td>0.36</td>
<td>0.43</td>
</tr>
<tr>
<td>$n = 7$</td>
<td>0.356</td>
<td>0.294</td>
<td>0.52</td>
<td>–</td>
<td>0.26</td>
</tr>
<tr>
<td>$n = 8$</td>
<td>0.255</td>
<td>0.404</td>
<td>0.63</td>
<td>0.41</td>
<td>0.48</td>
</tr>
<tr>
<td>$n = 9$</td>
<td>0.164</td>
<td>0.255</td>
<td>0.35</td>
<td>–</td>
<td>0.12</td>
</tr>
<tr>
<td>$n = 10$</td>
<td>0.258</td>
<td>0.304</td>
<td>0.49</td>
<td>0.44</td>
<td>0.39</td>
</tr>
<tr>
<td>$n = 11$</td>
<td>0.112</td>
<td>0.232</td>
<td>0.32</td>
<td>–</td>
<td>0.11</td>
</tr>
<tr>
<td>$n = 12$</td>
<td>0.228</td>
<td>0.231</td>
<td>0.60</td>
<td>0.33</td>
<td>0.28</td>
</tr>
</tbody>
</table>

<sup>a</sup> Values taken from Ref. 84

<sup>b</sup> Values taken from Ref. 94

<sup>c</sup> Values taken from Ref. 95
Table 2: RAS-SF calculated energy gaps (eV) to the second triplet ($T_2$) and lowest quintet states ($Q_1$) for odd $[n]$CCs.

<table>
<thead>
<tr>
<th>System</th>
<th>$\Delta E_{ST_2}$</th>
<th>$\Delta E_{SQ_1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n = 5$</td>
<td>1.147</td>
<td>2.068</td>
</tr>
<tr>
<td>$n = 7$</td>
<td>0.812</td>
<td>0.842</td>
</tr>
<tr>
<td>$n = 9$</td>
<td>0.568</td>
<td>0.399</td>
</tr>
<tr>
<td>$n = 11$</td>
<td>0.421</td>
<td>0.284</td>
</tr>
</tbody>
</table>
Table 3: Comparison between RAS-SF and FT-TPSS calculated $\Delta E_{ST}$ values (in eV) and $N_{FOD}$ for cyclo[a]decacene and cyclo[a]undecacene molecules.

<table>
<thead>
<tr>
<th>System</th>
<th>RAS-SF</th>
<th>FT-TPSS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta E_{ST}$</td>
<td>$N_{FOD}$</td>
</tr>
<tr>
<td>cyclo[a]decacene</td>
<td>0.138</td>
<td>1.82</td>
</tr>
<tr>
<td>cyclo[a]undecacene</td>
<td>0.151</td>
<td>1.99</td>
</tr>
</tbody>
</table>
Figure 1: Chemical structures (from top to bottom) of $[n]$CPP, linear oligoacene, and $[n]$CC systems, with H atoms omitted for clarity.
Figure 2: Consecutive C-C edge (top) and bridge (bottom) bond lengths for the linear (undecacene, red) and cyclic ([11]CC, black) oligoacenes.
Figure 3: Hückel molecular orbital energy diagram around the Fermi energy level ($E = \alpha$) for linear (left) and cyclic (right) acenes with an odd ($n = 5$) and even ($n = 6$) fused rings.
Figure 4: Plots of the FOD density ($\sigma = 0.005 \text{ e/bohr}^3$) for oligoacenes ranging from pentacene (top) to undecacene (bottom), as obtained from the FT-DFT method.
Figure 5: Plots of RAS-SF frontier natural orbitals for hexacene (top) and heptacene (bottom).
Figure 6: Plots of the FOD density ($\sigma = 0.005 \text{ e}/\text{bohr}^3$) for $[n]$CCs of increasing size ($n = 5 - 12$), as obtained from the FT-DFT method.
Figure 7: Plots of RAS-SF frontier natural orbitals for $[n]$CC with $n = 8$ (left) and $n = 9$ (right).
Figure 8: Evolution of the $N_{FOD}$ values, as obtained from the FT-DFT method, as a function of the oligomer size for both linear and cyclic acenes. The dashed red line corresponds to $N_{FOD} = 1.5$. 
Figure 9: Effect of the radical center interaction ($\gamma$) in the Hückel frontier orbitals for $[n]$CC with $n$-odd (left) and $n$-even (right). Introduction of $\gamma \neq 0$ in linear acenes systematically reduces the HOMO-LUMO gaps.
Figure 10: Correlation between ionic contributions ($\omega_{\text{ion}} = 100 \times |c_{\text{ion}}|^2$) and singlet-triplet energy gaps for the RAS-SF ground state wave functions of [n]CC molecules with n-even as indicated in Eq. (6). Note that values in the graph have to be considered only qualitatively since fragment decomposition has been done with a small active space (see Computational details). Inset shows fragment orbitals for the case with $n = 8$. Dashed line corresponds to the regression line only plotted for guidance.
Figure 11: Evolution of the $\Delta E_{ST}$ values, as obtained from the FT-DFT method, as a function of the oligomer size for both linear and cyclic acenes.
Figure 12: Chemical structures and plots of the FOD density, as obtained from the FT-DFT method, for: (a) cyclo[a]decacene ($\sigma = 0.005 \text{ e/bohr}^3$), (b) cyclo[a]undecacene ($\sigma = 0.005 \text{ e/bohr}^3$), (c) [3]cyclobenzo[a]anthracene ($\sigma = 0.002 \text{ e/bohr}^3$), and (d) [3]cyclochrysene ($\sigma = 0.002 \text{ e/bohr}^3$).
Figure 13: Plots of RAS-SF frontier natural orbitals for cyclo[a]decacene (left) and cyclo[a]undecacene (right).