Manufacturers Mergers and Product Variety in Vertically Related Markets

Chrysovalantou Milliou and Joel Sandonis*

January 2017

Abstract

We study final product manufacturers’ incentives to introduce new products into the market and how they are affected by a merger among them. We show that when manufacturers distribute their products through multi-product retailers, a manufacturers merger, although it leads to an increase in the wholesale prices, it can enhance product variety. The merger generated product variety efficiencies though arise only when vertical relations are present: when manufacturers sell directly their products to consumers, a merger never results into more product variety. Still, both with or without vertical relations, a manufacturers merger is harmful to consumers and welfare.

Keywords: horizontal mergers; product variety; vertical relations

JEL classification: L11; L13; L41; L42

---

*Milliou: Department of International and European Economic Studies, Athens University of Economics and Business, Athens 10434, Greece, e-mail: cmilliou@aueb.gr; Sandonis: Department of Economics, University of Alicante, 03080 Alicante, Spain, e-mail: sandonis@ua.es. We would like to thank Ramon Fauli-Oller for his useful comments. The first author gratefully acknowledges financial support from the Research Support Program - Action 1 of the Athens University of Economics and Business. The second author gratefully acknowledges financial support from Generalitat Valenciana under grant PROMETEO/2013/037 and from Spanish Ministries of Education and Science and Economics and Competitiveness under project ECO2012-34928.
1 Introduction

Mergers among final product manufacturers constitute a common business strategy. They take place in almost every market. For instance, in the mobile phone manufacturing market, Sony and Ericsson merged with each other, and so did Colgate-Palmolive and Sanex in the market of personal care goods, Oracle and PeopleSoft in the software programs market, and Panasonic and Sanyo in the electronics manufacturing market.¹

One of the recent concerns of the U.S. antitrust authorities regarding manufacturer mergers is whether and how they alter the merged firms’ decisions regarding their product lines, and in turn, how they affect product variety in the market.² This concern has been explicitly expressed in the most recent U.S. Horizontal Merger Guidelines (2010), which state that the authorities should focus not only on the impact of horizontal mergers on cost-related efficiencies, but also on their potential impact on product variety. Furthermore, in the assessment of a number of merger cases, the U.S. antitrust authorities, have incorporated the merger’s impact on product variety. For example, in 2003, the Federal Trade Commission (FTC) challenged the merger of Nestlé Holdings, Inc. and Dreyer’s Grand Ice Cream, Inc., in the super-premium ice cream market, citing that “the market for super-premium ice cream is already highly concentrated, and this deal will reduce the number of significant competitors from three to two” and it would “lead to anticompetitive effects... including less product variety and higher prices.”³

In this paper, we study the relationship between manufacturers mergers and product variety. A key novelty of our approach is that we consider a setting where manufacturers operate in a vertically related market, i.e., in a market where the production and the distribution of products are undertaken by different firms that operate at distinct market levels. In such a setting, a manufacturers merger can affect not only the number of products offered in the market but also the terms of vertical trading, which, in turn, can determine the efficiency of the distributors - retailers and the final prices. Our purpose is to address a number of fundamental questions of both theoretical and practical importance, such as: How the investments in new product introduction are affected by the intensity of market competition? What is the relationship between product variety and the terms of vertical trading? Whether and how a merger among manufacturers alters the investments in product introduction? Does a manufacturers merger harm consumers and total welfare?

To address the above, we construct a framework in which two upstream manufacturers initially produce two horizontally differentiated goods, which they distribute to consumers

¹For additional examples, see e.g., Froeh et al. (2007).
²Note that many final product manufacturers produce a number of different versions of the same final product. Kellogg’s, for instance, offers more than 20 different varieties of cereals, Ben & Jerry’s sells 39 varieties of ice-cream, and Colgate and Crest each delivers more than 35 types of toothpaste. Product proliferation though is not restricted to supermarket goods. It is observed in several product categories, such as sunglasses, apparel, watches, and consumer electronics. In many of these product categories, the manufacturer’s products differ, not in quality, but in other features, such as scent, color, or design.
³Similar arguments were also used in the decisions regarding the merger among Whole Foods Market and Wild Oats Market in the supermarket industry, the merger among Oracle and PeopleSoft in the software industry, and the merger of Kimberly-Clark Corp and H.J. Heinz Co. in the food industry.
through two competing multiproduct retailers. Manufacturers decide, first, whether they will merge, and second, whether they will introduce additional product varieties into the market after incurring the respective fixed costs. Next, manufacturer(s) set the wholesale prices of their products and, in turn, the retailers choose their quantities. In order to examine the role of vertical relations and the potential importance of accounting for them and for vertical trading in the analysis of the merger implications, we also analyze the benchmark case in which manufacturers operate in a one-tier market, i.e., they sell directly their products to consumers.

The introduction of a new product into the market gives rise to two opposite effects, regardless of whether manufacturers have merged or not and of whether they operate in a vertically related market or in a one-tier market. First, by increasing product variety, product introduction expands the market size. We refer to this positive effect as expansion effect. Second, product introduction causes a negative cannibalization effect: the new product that a manufacturer introduces steals away demand from its already existing product(s). Besides these effects, product introduction gives rise also to a competition effect when the manufacturers remain separated. This effect corresponds to the intensification of competition among the manufacturers that results from the fact that the new product competes against the product(s) of the rival manufacturer. Therefore, product introduction can bring about, on the one hand, the gains of increased demand, and on the other hand, the losses of increased competition (either intrabrand alone or both intrabrand and interbrand).

We demonstrate that when the manufacturers do not sell directly their products to consumers, product introduction also causes a decrease in the wholesale prices. This effect - the wholesale pricing effect, however, is present only when the manufacturers remain separated. When they are merged, there is an "indifference result": the number of products has no impact on the wholesale prices. Intuitively, in the non-merger case, product introduction, by causing the competition effect, reinforces the incentives of each manufacturer to behave more aggressively and lower the wholesale prices of its product(s). This does not occur in the merger case, since the merged manufacturers fully internalize the competition effect. In both the merger and the non-merger case though, the wholesale prices exceed the manufacturer's marginal cost of production; hence, neither the industry profits are maximized nor the manufacturers manage to fully appropriate them.

Not surprisingly, the wholesale prices can be affected not only by product variety, but also by the upstream market structure. In particular, we demonstrate that the wholesale prices are higher when the upstream merger materializes. In other words, double marginalization is more severe then. This result is a straightforward implication of the fact that, as mentioned above, the merged manufacturers internalize the externality that they would otherwise impose on each other by offering lower wholesale prices in order to promote their

---

4In many product categories, competing manufacturers sell the same versions of their products through the same competing retailers. For instance, both Kellogg's and Nestlé sell various versions of cereals through the same supermarket chains. Similarly, tv models by both Samsung and Sony can be found in both WalMart and Best Buy.
products against the products of their rival.

Importantly, our analysis reveals that a manufacturers merger can affect product variety. In fact, we show that product variety increases when manufacturers merge as long as products are sufficiently close substitutes and the investments in product introduction are not too costly. Why is that? The merger allows the manufacturers to extract a higher share of the industry profits, through the higher wholesale prices, as well as to internalize the competition effect which is strong when products are not too differentiated. When, instead, products are sufficiently differentiated, product variety can be higher in the non-merger case. This occurs not only because the negative competition effect is weak then, but also because the manufactures overinvest in product introduction, being trapped in a prisoners’ dilemma situation.

The presence of vertical relations is crucial for the emergence of the product variety related merger induced efficiencies: in markets in which the manufacturers sell directly their products to consumers, product variety is never higher when the manufacturers are merged. This is so because in such markets, in contrast to what happens in vertically related markets, there are no wholesale prices and the manufacturers fully capture the profits that arise from their production activities. As a consequence, in one-tier markets, the wholesale pricing effect is absent and the manufacturers do not capture a larger share of the industry profits by merging. It follows from this that the exclusion of vertical relations in the analysis of manufacturers mergers could result in different conclusions regarding their implications.

A manufacturers merger, although it can enhance product variety in a vertically related market and increase the manufacturers profits, it hurts the retailers and the consumers, and reduces total welfare. In other words, the increase in the severeness of the double marginalization problem always dominates the merger’s potential efficiency gains. Clearly, this suggests that in cases in which the merger enhances significantly the market power, the merger induced product variety synergies cannot be used as an argument in favor of the merger in its evaluation by the antitrust authorities.

Our prediction that a manufacturer merger can affect the number of products in the market seems to accord well with a number of empirical studies. In particular, Alexander (1997) and Watson (2009), studying respectively the music distribution industry and the eyewear retailing market, find a non-monotonic relationship between concentration and product variety. George (2007), instead, in her study of the U.S. daily newspapers market, concludes that more concentrated markets tend to have more variety both in terms of the number and of the variety of topics covered. Focusing on the issue of product variety in response to a merger, Berry and Waldfogel (2001) find that mergers in the U.S. radio broadcasting market prompted an increase in both variety per station and overall variety. In contrast, according to Gortz and Gugler (2006) and Fan (2003) mergers in the Austrian gasoline market and in the U.S. daily newspapers market caused a decrease in their product variety.

Although the empirical evidence indicates that mergers can affect product variety, the existing theoretical work on horizontal mergers has little to say about this. The standard
merger theory focuses almost exclusively on horizontal pricing effects (e.g., Reynolds et al., 1983, Davidson and Deneckere, 1985) and/or on cost related efficiencies (e.g., Farrell and Shapiro, 1988, Perry and Porter, 1985). A notable exception is the paper of Lommerud and Sorgard (1997) which studies how a merger affects firms’ decisions regarding the expansion of their product lines. Specifically, Lommerud and Sorgard consider a market with three firms that produce differentiated goods. They assume that each firm initially offers one product and that there is a fixed non-sunk cost of marketing a brand. They find that whenever a merger among two firms is profitable, it either has a negative impact or no impact at all on product variety and that it is often detrimental to welfare. When instead the merger triggers the introduction of a new product by the outsider and, thus, causes an increase in the product range, the merger is unprofitable.

Another exception is a recent paper by Chen and Schwartz (2013), who consider mergers to monopoly and assume that initially only a single product is offered by all the market participants. They find that when product introduction is sufficiently non-drastic, the incentives to innovate are stronger when the merger takes place, and subsequently, that consumer welfare and overall welfare can be higher under monopoly than under more rivalrous regimes. Importantly, all the above mentioned papers consider only one-tier markets. Therefore, in contrast to us, they do not take into account the fact that product manufacturers often do not sell directly their products to consumers. By not doing so, clearly, they do not explore the role of vertical relations and trading for the merger implications on product variety.

Our paper contributes to the recently growing literature on horizontal mergers in vertically related industries. Horn and Wolinsky (1988), Ziss (1995), O’Brien and Shaffer (2003), Inderst and Wey (2003), Froeb et al. (2007), Milliou and Petrakis (2007), and Milliou and Pavlou (2013), similarly to us, study mergers in the upstream market. Within this literature, only Milliou and Pavlou (2013), in line with us, analyze the potential efficiency gains of upstream mergers. However, Milliou and Pavlou (2013) consider a market with exclusive relations pre-merger and cost-related efficiency gains. We complement their work by using a less restrictive market structure, and importantly, by analyzing product variety related

---

1 We should note that there exists a vast theoretical literature on the product line decisions of competing firms (e.g., Brande and Eaton, 1984, Raubitschek, 1987, Martinez-Giralt and Neven, 1988, Gilbert and Matutes, 1993, De Fraja, 1993, Dobson and Watson, 1996, Ottaviano and Thissen, 1999, Ju, 2003, Avenel and Caprice, 2006, Anderson and De Palma, 2006, Gandhi et al. 2008, Cuninal and Granerod, 2012). This literature, however, does not examine the impact of market structure changes on these decisions. Moreover, there is an extensive theoretical literature on product variety driven not by product proliferation but by firms’ market entry decisions (e.g., Salop, 1979, Schmalensee, 1978, Lancaster, 1979, Eaton and Lipsey, 1979, and Innes, 2008).

2 Greenes et al. (1998) analyze a similar topic in a vertical product differentiation framework.

3 Note that there is a number of papers (e.g., Ganchi et al., 2008, Mazzetti et al., 2012) which examine how a horizontal merger can affect product variety not through altering the number of products but through altering the product characteristics. In other words, they examine whether post-merger, the merged firm has incentives to reposition its products in terms of their degree of product differentiation.

efficiency gains of upstream mergers. The only papers that, to the best of our knowledge, consider product variety issues within the literature on horizontal mergers in vertically related industries are the papers by Inderst and Shaffer (2007) and Fauli-Oller (2008), which focus though on downstream mergers and not on upstream mergers. Both of these papers demonstrate that a merger among retailers allows them to commit to not selling one of the goods supplied by manufacturers, and thus, that such a merger can result in a welfare-detrimental decrease in product variety. These papers differ from ours not only because of their different focus, but also because in their setting there is no product introduction; the number of products manufactured by the upstream firms is exogenous and the downstream firm(s) choose how many of them they will distribute.\footnote{In a related paper, Milliou et al. (2010), similarly to us, endogenize upstream product innovation. However, they focus on the role played by economies of scope and upstream entry and not by upstream mergers.}

The rest of the paper has the following structure. In Section 2, we describe our model. In Section 3, we perform the equilibrium analysis. In Section 4, we examine the merger implications on the wholesale prices and on product variety. In Section 5, we explore the merger incentives and impact on retailers, consumers and welfare. In Section 6, we discuss the role of vertical relations. Finally, in Section 7, we conclude.

2 The Model

We consider a market consisting of two upstream product manufacturers, $M_1$ and $M_2$, and two downstream retailers, $R_1$ and $R_2$. Each $M_i$, with $i = 1, 2$, produces initially a single-product at zero marginal cost. However, each $M_i$ can increase the number of its products by investing in new product introduction. In particular, it can introduce an additional product variety after incurring a fixed cost, $F > 0$, for the establishment of a new production line. Depending on the manufacturers product introduction decisions, the total number of products in the market, $N$, will be two, three or four, i.e., $N \in \{2, 3, 4\}$.

Each $R_k$, with $k = 1, 2$, sells to the final consumers all the products of the two manufacturers after paying the per unit wholesale price, $w_n$, for each product $n$, with $n \in \{1, 2, 3, 4\}$, to its respective manufacturer.\footnote{We treat the distribution system as exogenous and assume implicitly that retailers are multiproduct and distribute all the products of all the manufacturers - there is no exclusive distribution. This assumption, as we mentioned in footnote 4, is satisfied in various markets. Moreover, this assumption is justified by the theoretical findings of the literature that endogenizes the distribution systems when there is market power at both the manufacturing and the retailing level (see e.g., O’Brien and Shaffer, 1997, Bernheim and Whinston, 1998, Lin, 1990, O’Brien and Shaffer, 1993, Chang, 1992, Moner-Colonques et al., 2004 and 2011, Maulen et al., 2011, Rey and Vergé, 2016). More specifically, Moner-Colonques (2004) and Rey and Vergé (2016) show, that as long as product substitutability is not too high, manufacturers choose to distribute their products through the same competing retailers. Similarly, Moner-Colonques et al. (2011) show, that retailers choose to overlap their product lines whenever their retail margins are not too asymmetric.} Moreover, $R_k$ and $R_l$ face a continuum of identical consumers. Following Singh and Vives (1984), we assume that the representative consumer has a quadratic utility function which is separable in income and is given by:
\[ u(Q_1, \ldots, Q_N, I) = a + \sum_{n=1}^{N} Q_n - \frac{1}{2} \left( \sum_{n=1}^{N} Q_n^2 + 2\gamma \sum_{n \neq j} Q_n Q_j \right) + I, \]  

(1)

where \( Q_n \) denotes the total quantity of good \( n \) sold in the market and \( I \) stands for consumer’s income. From the utility maximization program we obtain the following (inverse) demand function faced by \( R_k \) for each product \( n \):

\[ p_{nk}(q_{nk}, q_{nl}, Q_{-n}) = a - q_{nk} - q_{nl} - \gamma(Q_{-n}), \]  

(2)

where \( p_{nk} \) and \( q_{nk} \) are the price and the quantity respectively of product \( n \) sold by \( R_k \), \( q_{nl} \) is the quantity of the same product sold by its rival \( R_l \), with \( k, l = 1, 2 \) and \( k \neq l \), and \( Q_{-n} \) is the quantity of the rest of the product(s).\(^{11}\) The parameter \( \gamma \), with \( 0 < \gamma < 1 \), denotes the degree of product substitutability; namely, the higher \( \gamma \) is, the closer substitutes the products of the manufacturers are.\(^{12}\)

Before deciding on their investments in new product introduction, the manufacturers decide whether to merge among them. When they merge, a monopolist is created in the upstream market, denoted by \( M \). The upstream monopolist can also invest in product introduction. More specifically, \( M \) can increase the number of its products from two to three or four incurring the respective fixed cost \( F \) for each additional product. Alternatively, \( M \) can restrict its product range by withdrawing one of its already existing products. However, the fixed costs of introducing the two initial varieties are already sunk.

Firms play a four-stage game. In stage one, \( M_1 \) and \( M_2 \) decide whether or not they will merge. In stage two, if the merger has not taken place, each \( M_i \) decides whether it will introduce a new product into the market. If, instead, the merger has taken place, the merged entity \( M \) decides respectively how many products it will offer in the market. In stage three, the manufacturer(s) set the wholesale prices, \( w_n \), for each of its(there) products. Finally, in stage four, the retailers choose the quantities of the products.

Our notational convention for the rest of the paper will be as follows. The first superscript, \( M \) or \( S \), will denote respectively whether the manufacturers have merged or remained separated and the second superscript will denote the total number of products in the market.

\(^{11}\)It follows that \( Q_n = q_{nk} + q_{nl} \). Note that from the first order conditions of the maximization program we obtain: \( p_n(Q_n, Q_{-n}) = a - Q_n - \gamma(Q_{-n}) \). However, in order to be able to solve the last stage of the game (output decisions by the downstream firms) we need to distinguish explicitly the amount of each particular good sold by each retailer.

\(^{12}\)This specification corresponds to the case in which the two retailers are not differentiated and the products of the two manufacturers differ among them as much as the products of the same manufacturer. When we allow for retailers’ differentiation, the computations are very messy, but, as we discuss in Section 7, our main conclusions are qualitatively similar.
3 Equilibrium Product Variety

In the last stage of the game, independently of whether the manufacturers have merged or not, each $R_k$ solves the following maximization problem:

$$\max_{q_{nk}} \Pi_{R_k} = \sum_{n=1}^{N} [p_{nk}(q_{nk}, q_{nk}, Q-n) - w_n]q_{nk}. \quad (3)$$

Solving the system of first order conditions, we obtain the equilibrium quantities of each product $n$ sold by $R_k$: $q_{nk}(w_n, w_{-n})$.\(^{13}\) It follows that the total quantity of product $n$ is $Q_n = q_{n1}(w_n, w_{-n}) + q_{n2}(w_n, w_{-n})$.

3.1 Upstream Separated Firms and New Product Introduction

When $M_1$ and $M_2$ remain separated, there are three possible third-stage subgames depending on the number of their products: (i) the ‘no product introduction’ subgame where both $M_1$ and $M_2$ have one product each, (ii) the ‘partial product introduction’ subgame where $M_1$ has two products and $M_2$ one, and (iii) the ‘full product introduction’ subgame where both $M_1$ and $M_2$ have two products each.\(^{14}\) Clearly, in each of these subgames, there is both intrabranded competition, since the same product is distributed by different retailers, and interbrand competition, since a retailer distributes the distinct products of the different manufacturers. Next, we analyze each subgame separately.

In the ‘no product introduction’ subgame, $M_1$ and $M_2$ face the following maximization problems:

$$\max_{w_1} \Pi_{M_1} = w_1Q_1(w_1, w_2) \quad \text{and} \quad \max_{w_2} \Pi_{M_2} = w_2Q_2(w_1, w_2). \quad (4)$$

Solving (4), we obtain the equilibrium wholesale prices:

$$w_1^{S2} = w_2^{S2} = \frac{a(1-\gamma)}{2-\gamma}. \quad (5)$$

Substituting (5) into $q_{nk}(w_n, w_{-n})$ and (4), we obtain the equilibrium quantities and the equilibrium manufacturers’ profits included in Table 1.

<table>
<thead>
<tr>
<th>Product Introduction</th>
<th>Quantities</th>
<th>Manufacturers’ Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>$q_{nk}^{S2} = \frac{a}{6+3\gamma-3\gamma^2}$</td>
<td>$\Pi_{M_1}^{S2} = \frac{2a^2(1-\gamma)}{3(1+\gamma)(2-\gamma)^2}$</td>
</tr>
<tr>
<td>Partial</td>
<td>$q_{nk}^{S3} = \frac{a}{6(2-\gamma)+6(1+3-2\gamma)}$</td>
<td>$\Pi_{M_1}^{S3} = \frac{2a^2(1-\gamma)(\gamma)^2}{3(1+2\gamma)(2-\gamma)^2}$ - $F$</td>
</tr>
<tr>
<td></td>
<td>$q_{nk}^{S3} = \frac{a}{6+3\gamma(6+3+2\gamma)}$</td>
<td>$\Pi_{M_2}^{S3} = \frac{a^2(1-\gamma)(1+\gamma)^3}{3(1+2\gamma)(2-\gamma)^2}$</td>
</tr>
<tr>
<td>Full</td>
<td>$q_{nk}^{S4} = \frac{a(1-\gamma)}{6+18\gamma}$</td>
<td>$\Pi_{M_1}^{S4} = \frac{2a^2(1-\gamma)}{3+2\gamma} - F$</td>
</tr>
</tbody>
</table>

\(^{13}\) The exact expressions of the third-stage equilibrium quantities are included in Table 3 of the Appendix.

\(^{14}\) There is a fourth subgame in which $M_1$ has one product and $M_2$ two products. However, the analysis of this subgame is the same as the one of subgame (ii), with the roles of $M_1$ and $M_2$ reversed.
In the ‘partial product introduction’ subgame, the maximization problems of $M_1$ and $M_2$ are:

$$Max_{w_1, w_3} \Pi_{M_1} = w_1 Q_1(w_1, w_2, w_3) + w_3 Q_3(w_1, w_2, w_3) - F$$

and

$$Max_{w_2} \Pi_{M_2} = w_2 Q_2(w_1, w_2, w_3).$$

(6)

The equilibrium wholesale prices are:

$$w_1^{S3} = w_3^{S3} = \frac{a(2 + \gamma - 3\gamma^2)}{4 + 2\gamma(2 - \gamma)} \quad \text{and} \quad w_2^{S3} = \frac{a(1 - \gamma^2)}{2 + \gamma(2 - \gamma)}.$$  

(7)

It is easy to see that $w_1^{S3} > w_2^{S3}$. That is, the multi-product manufacturer charges higher wholesale prices than its single-product rival. The resulting equilibrium quantities and equilibrium manufacturers’ profits can be found again in Table 1.

Finally, in the ‘full product introduction’ subgame, the manufacturers solve the following:

$$Max_{w_1, w_3} \Pi_{M_1} = w_1 Q_1(w_1, w_2, w_3, w_4) + w_3 Q_3(w_1, w_2, w_3, w_4) - F;$$

(8)

$$Max_{w_2, w_4} \Pi_{M_2} = w_2 Q_2(w_1, w_2, w_3, w_4) + w_4 Q_4(w_1, w_2, w_3, w_4) - F.$$  

(9)

>From the solution of (8) and (9), we have:

$$w_1^{S4} = w_3^{S4} = w_2^{S4} = w_4^{S4} = \frac{a(1 - \gamma)}{2}.$$  

(10)

Substituting (10) into $q_{nk}(w_1, w_{-k})$, (8) and (9), we also obtain the equilibrium outputs and the equilibrium manufacturers’ profits, included in Table 1.

One can easily conclude, looking at (5), (7) and (10), that in all the subgames, the equilibrium wholesale prices exceed the manufacturer’s marginal cost of production. Hence, as standard in vertically related markets with linear contracts, double marginalization is present. A straightforward implication of this is, first, that the total output in the market, and in turn, the total industry profits, are restricted. And second, that the manufacturers are not able to extract all the industry profits - the retailers get part of the profits. Moreover, comparing the equilibrium wholesale prices in the three different subgames, we find:

**Remark 1** When the upstream manufactures are separated, the equilibrium wholesale prices decrease with the total number of products in the market.

When there is upstream competition, the higher the number of products, the lower are the wholesale prices. In other words, when the upstream manufacturers are separated, product introduction has a negative impact on the wholesale prices. In what follows, we refer to this as the *wholesale pricing effect* of product introduction. The driving force of the *wholesale pricing effect* is as follows. Both upstream manufacturers have incentives to

---

15 We implicitly assume in this case that products 1 and 3 are manufactured by $M_1$ and product 2 by $M_2$. 

8
behave aggressively and reduce their wholesale prices in order to favour the sales of their own products relative to the sales of their rival’s product(s). Clearly, their incentives to do so are stronger when competition is more intense and competition is indeed more intense when there are more products in the market; both intrabrand competition and interbrand competition are stronger then. Therefore, product introduction can induce a larger total output (since the retailers face lower marginal costs) and higher industry profits. At the same time though, product introduction can make the manufacturers extract less of the profits generated by each product.

Concerning the role of product differentiation, it is direct to see that as products become more differentiated (γ decreases), the equilibrium wholesale prices increase since the competition intensity decreases. The impact of γ on the equilibrium output is more complicated because a trade-off is in place. On the one hand, for given wholesale prices, a lower γ shifts the reaction functions of the retailers outwards, leading to higher output. But a lower γ also increases the wholesale prices charged by the manufacturers, increasing the retailers’ marginal costs, and, consequently, reducing their output. A consequence of this trade-off is that the relationship between γ and the equilibrium output is not monotone. The same holds for the relationship between γ and the size of the wholesale pricing effect.

Next, we analyze each manufacturer’s product introduction decision in stage two. There are three candidate second-stage equilibria, each of them corresponding to one of the subgames analyzed above. We start by noting that in the ‘full product introduction’ subgame, the manufacturer’s profits are negative unless product introduction is not too costly (see Table 1). Specifically, unless \( F < \overline{F}^S(\gamma) = \frac{\alpha^2(1-\gamma)(2+3\gamma)^2}{3(1+2\gamma)(2+2\gamma)(2-\gamma)^2} \). Taking this into account, we check for possible deviations from each of the candidate equilibrium, and we find the following:

**Proposition 1** When the upstream manufacturers are separated and \( F < \overline{F}^S(\gamma) \), there exist \( F_1^S(\gamma) \) and \( F_2^S(\gamma) \), with \( F_1^S(\gamma) > F_2^S(\gamma) > F_2^S(\gamma) \), such that the unique equilibrium is

(i) no product introduction (\( N = 2 \)) when \( \gamma > 0.9164 \) for all \( F \), as well as when \( \gamma < 0.9164 \) if \( F > F_2^S(\gamma) \),

(ii) partial product introduction (\( N = 3 \)) when \( 0.8346 < \gamma < 0.9164 \) if \( F < F_1^S(\gamma) \), as well as when \( \gamma < 0.8346 \) if \( F_2^S(\gamma) < F < F_1^S(\gamma) \),

(iii) full product introduction (\( N = 4 \)) when \( \gamma < 0.8346 \) if \( F < F_2^S(\gamma) \).

The introduction of a new product into the market gives rise to four main effects. First, there is an expansion effect which corresponds to the increase in the market demand caused by the introduction of an additional product variety. Clearly, this effect encourages product introduction. Second, there is a cannibalization effect. This refers to the fact that when a manufacturer extends its product range, its already existing product faces additional competition; the new product partially cannibalizes the demand for its existing product. This effect, in contrast to the expansion effect, discourages product introduction. The third effect is a competition effect, which corresponds to the intensification of competition with the rival manufacturer caused by the increase in the number of competing products.
Certainly, this effect also discourages product introduction. Finally, the fourth effect is the \textit{wholesale pricing effect} that we identified above. Since one of its consequences is that the manufacturers extract a smaller share of the industry profits when they introduce new products into the market, this effect can also discourage product introduction.

It follows from the above that when a manufacturer considers investing in product introduction, it has to compare the following: the gains from increasing output (due to the \textit{expansion effect} and the \textit{wholesale pricing effect}) with the losses from competing more fiercely (due to the \textit{cannibalization effect} and the \textit{competition effect}) and from extracting a smaller share of the industry profits (due to the \textit{wholesale pricing effect}). When products are sufficiently close substitutes and, thus, competition is tough, then as Proposition 1(i) informs us, there is no product introduction in equilibrium. This is so because when product differentiation is low, the market environment is already very competitive. In such an environment, the \textit{expansion effect} is weak and it is dominated by the \textit{cannibalization effect} and the \textit{competition effect}, which are strong. When the environment is not too competitive, the \textit{expansion effect} can dominate and result in product introduction if the investments are not too costly. In fact, when neither products are too close substitutes nor product introduction is too costly, both upstream firms extend their product lines in equilibrium (Proposition 1(iii)). It is important to note that when they do so, they are trapped into a prisoner’s dilemma situation (i.e., they would be better off if none of them innovated), unless products are sufficiently differentiated and the cost of product introduction is very low. The prisoner’s dilemma situation arises because in the ‘partial product introduction’ case, the non-innovating upstream manufacturer has unilateral incentives to deviate and innovate in order to expand its own market and steal away market share from its rival.

### 3.2 Upstream Merger and New Product Introduction

When the upstream manufacturers merge, the newly formed upstream monopolist solves the following:

\[
\text{Max}_{w_1, \ldots, w_N} \Pi_M = \sum_{n=1}^{N} w_n Q_n(w_n, w_{-n}) - (N - 2)F, \tag{11}
\]

where \(N \in \{2, 3, 4\}\) depending on its product introduction decision. Note that in this specification, we exclude the case in which \(N = 1\) because it never arises in equilibrium. The reason is twofold. The withdrawal of a product, first, would not result in cost savings given that the product’s introduction cost is already sunk. Second, the resulting positive effect of a reduction in intrabrands competition would not compensate the merged firms for the negative market \textit{contraction effect} from withdrawing one of their already existing products.

>From the first order conditions of (11), we find the equilibrium wholesale prices: \(w_n^{MN} = \frac{2}{3}\) for \(n \in \{1, 2, 3, 4\}\) and \(N \in \{2, 3, 4\}\). Three important observations are in place. First, the wholesale prices exceed the marginal cost of production - double marginalization is again present. Second, they are independent of the degree of product differentiation. And
third, they are also independent of the total number of products in the market. Thus, the wholesale pricing effect is absent here. In fact, there is, instead, an ‘indifference result’: the upstream multiproduct monopolist’s pricing decisions do not depend on the number of its products.\footnote{This result is somehow similar to the indifferenc result identified in the literature (Petrikas and Dhillon, 2002) regarding the independence of the wholesale prices charged by a single-product upstream monopolist from the number of downstream firms.}

**Remark 2** When the upstream manufactures are merged, the equilibrium wholesale prices are independent of the total number of products in the market.

Substituting $w^M_{\gamma}$ into $q_{\pi}(w_{\gamma}, w_{-\gamma})$, and then, into (11), we obtain the equilibrium quantities and the equilibrium profits of the merged manufacturers, which we include in Table 2.

<table>
<thead>
<tr>
<th>Product Introduction</th>
<th>Quantities</th>
<th>Manufacturer’s Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>$q^{M_2}_{\lambda_k} = \frac{a}{6+3\gamma}$</td>
<td>$\Pi^{M_2}_M = \frac{a^2}{3+3\gamma}$</td>
</tr>
<tr>
<td>One product</td>
<td>$q^{M_3}_{\lambda_k} = \frac{a}{4+1.5\gamma}$</td>
<td>$\Pi^{M_3}_M = \frac{a^2}{2+1.5\gamma} - F$</td>
</tr>
<tr>
<td>Two products</td>
<td>$q^{M_4}_{\lambda_k} = \frac{a}{6+3\gamma}$</td>
<td>$\Pi^{M_4}_M = \frac{2a^2}{3+3\gamma} - 2F$</td>
</tr>
</tbody>
</table>

Table 2: Equilibrium Output and Equilibrium Manufacturer’s Profits in the Merger Case

Note that given that the equilibrium wholesale prices do not change with the degree of product differentiation, there is a monotonic relationship between the equilibrium output and $\gamma$; in particular, as $\gamma$ decreases, the equilibrium output increases.

Concerning the manufacturers’ profits, one can easily note that they are positive in all cases under consideration if and only if $F < F^M(\gamma)$. Comparing $\Pi^{M_2}_M$, $\Pi^{M_3}_M$ and $\Pi^{M_4}_M$, we reach the following conclusion:

**Proposition 2** When the upstream manufacturers are merged and $F < F^M(\gamma)$, there exist $F^M_1(\gamma)$ and $F^M_2(\gamma)$, which decrease in $\gamma$ and satisfy $F^M_1(\gamma) > F^M_2(\gamma)$, such that the unique equilibrium is

(i) no product introduction ($N = 2$) if $F > F^M_1(\gamma)$,
(ii) partial product introduction ($N = 3$) if $F^M_2(\gamma) < F < F^M_1(\gamma)$, and
(iii) full product introduction ($N = 4$) if $F < F^M_2(\gamma)$.

As Proposition 2 states, the upstream monopolist has incentives to invest in enlarging its product line unless the cost of doing so is too high ($F > F^M_1(\gamma)$). In fact, when the cost of product introduction is sufficiently low ($F < F^M_2(\gamma)$), $M$ will introduce two additional products in the market. Intuitively, when the manufacturers merge, the competition effect of product introduction disappears. The same holds for the wholesale pricing effect. Hence, when the merged manufacturers decide regarding product introduction, they are faced only with the following trade-off: by investing in product introduction, they expand their total demand (expansion effect), but they cannibalize the demand for their already
existing products (cannibalization effect). It turns out that the expansion effect outweighs the competition effect. Therefore, when product introduction is not too costly (regardless of $\gamma$), the merged manufacturers invest in product introduction.

As Proposition 2 also states, the critical values of $F$ for product introduction decrease, when product substitutability increases. Thus, not surprisingly and in line with the non-merger case, the merged manufacturers have weaker product introduction incentives when the products are less differentiated. This is a straightforward implication of the fact that when products are close substitutes, the cannibalization effect is strong, while the expansion effect is weak.

4 Merger Implications on Wholesale Prices and Product Variety

In this section, we examine the implications of a manufacturers merger on the wholesale prices and on product variety.

Starting with the wholesale prices, our main conclusion, included in Proposition 3 below, is that the manufacturers set higher wholesale prices when they are merged than when they are separated. This result holds independently of the number of products in the market. In other words, it holds even when there is less product introduction in the non-merger case. Intuitively, once the manufacturers merge, they stop competing among them; they are able to internalize the externality that they would otherwise impose on each other by offering lower wholesale price(s) for their own product(s) so that their downstream sales are increased. Clearly, an implication of the larger double marginalization externality is that in the merger case, the manufacturers extract a larger share of the lower industry profits.

**Proposition 3** The wholesale prices are higher when the upstream firms are merged than when they are separated.

Combining Propositions 1 and 2, we are now able to evaluate the impact of an upstream merger on product variety.

**Proposition 4** When the upstream firms are merged, product variety is higher than when they are separated (i) if $\gamma > 0.8346$ and $F$ is not too high, (ii) if $0.693 < \gamma < 0.8346$ and $F$ is sufficiently low but not too low. Otherwise, when the upstream firms are merged, product variety is either lower or equal to the one when they are separated.

A manufacturers merger affects product variety. In fact, product variety can be higher when the manufacturers merge than when they remain separated. This holds when products are not too differentiated and product introduction is not too costly.\(^{17}\)

\(^{17}\) Actually, there are cases in which, while we have no product innovation in the non-merger case, we can have full innovation, and not only partial innovation, in the merger case and the reverse. For instance, when $\gamma > 0.9164$, the total number of products without the merger is always 2 while with the merger it is 3 if $F$
The intuition for the above result is as follows. Both in the merger and in the non-merger case, a manufacturer has an incentive to introduce a new product in order to expand the demand and a disincentive to do so in order to avoid the cannibalization of its already existing product(s). In the non-merger case though, two additional disincentives are present: a manufacturer does not want to introduce a new product to avoid the increase in interbrand competition as well as the decrease in the share of the industry profits that it extracts due to the resulting lower wholesale prices. In light of this, it is not surprising, that product variety can be higher when the merger materializes. However, this does not always hold for a number of reasons. One reason has to do with the fact that the cannibalization effect is stronger when the manufacturers are merged. This is so because product introduction increases the merged manufacturers’ own products from two to three or even to four. Instead, when the manufacturers are separated, product introduction can increase the products of a single manufacturer only from one to two. Another reason is the prisoner’s dilemma situation in which the manufacturers are trapped in the non-merger case when products are not too differentiated and product introduction is neither too costly nor too cheap, resulting in overinvestment in product introduction.

5 Merger Incentives and Welfare Implications

Next, we analyze the merger’s impact on firms’ profits, consumers’ surplus and total welfare.

The merger allows the manufacturers to eliminate interbrand competition; hence, it removes the negative externality that they would otherwise impose on each other. This, as we saw in Proposition 3, allows them to charge higher wholesale prices. Further, it allows them to coordinate their product introduction decisions, avoiding the prisoner’s dilemma situation. In light of this, it is not surprising that, as Proposition 5(i) below states, an upstream merger always has a positive impact on the upstream profits. In other words, merger incentives are always present.

Proposition 5 An upstream merger:

(i) increases the profits of the upstream firms,

(ii) decreases the profits of the downstream firms, consumers’ surplus and total welfare.

In contrast to the upstream profits, the profits of the downstream retailers decrease when the merger materializes. Two forces drive this result. The first one is the positive impact of product variety on downstream profits. This holds both in the merger and in the non-merger case and it is quite intuitive: when there are more products in the market, the market size increases; hence, the sales and the profits of the retailers also increase. In the non-merger case, the downstream profits are higher when there is more product variety for an additional reason: the wholesale pricing effect which translates into higher downstream efficiency. The

\[ \text{takes intermediate values and } 4 \text{ if } F \text{ is sufficiently low. The detailed description for all the cases that arise in equilibrium and the specific conditions under which they hold are all available in the proof of Proposition 4 in the Appendix.} \]
second force is the merger’s impact on the wholesale prices: as we saw in Proposition 3, the
downstream firms pay higher wholesale prices following a manufacturers merger. This force,
clearly, decreases the merger’s desirability from the downstream firms’ viewpoint. The first
force though works in the opposite direction, i.e., it works in favor of the upstream merger,
when the goods are sufficiently close substitutes since the merger results then into more
product variety (Proposition 4). But when the goods are close substitutes, competition
downstream is strong; hence, the positive expansion effect of product introduction is weak
and the second force dominates.

What about the merger’s impact on consumers’ surplus? According to Proposition 5(ii),
the merger always harms consumers. This result is driven by similar forces to the ones
analyzed above. More specifically, an increase in product variety benefits the consumers
since their utility function is characterized by a preference for variety feature. At the same
time though, an increase in the wholesale prices hurts the consumers since it translates into
higher final prices (double marginalization). As we saw above, an upstream merger always
leads to higher wholesale prices and hurts the consumers through this channel. This effect
is stronger than the merger’s positive impact on product variety, which is relatively weak
because it emerges only when products are close substitutes.

The merger’s negative impact on consumers’ surplus and downstream profits outsets its
positive impact on the upstream profits. As a result, the merger is always detrimental for
total welfare (Proposition 5(ii)). Stated in other words, the anticompetitive unilateral effect
of a "merger to monopoly" is so pronounced that it cannot be compensated by the potential
merger generated efficiency gains when the latter corresponds to enhanced product variety.
On this basis, we could claim that our paper provides a theoretical justification for the
view expressed in the recent U.S. Horizontal Merger Guidelines (2010) according to which
"efficiencies almost never justify a merger to monopoly or near-monopoly. Just as adverse
competitive effects can arise along multiple dimensions of conduct, such as pricing and new
product development, so too can efficiencies operate along multiple dimensions".

6 The Role of Vertical Relations

In this section, we explore the role of vertical relations for the merger’s implications. In
order to do so, first, we analyze briefly what happens in the absence of vertical relations,
i.e., in a one-tier market where product manufacturers sell directly their products to the
consumers, and then, we compare our results with the ones in a vertically related market.

In a one-tier market, stage three does not exist, i.e., the wholesale prices are zero by
assumption, and the quantities are chosen directly by the product manufacturer(s). In such
a market, similarly to what happens in a vertically related market, the merged firm never
withdraws one of its already existing products from the market. More importantly, a merger
in a one-tier market does not enhance product variety (Proposition 6(iii)). This finding is
mainly driven by the fact that in a one-tier market, the manufacturers set the quantities of
their products and not the wholesale prices. As a result, the wholesale pricing effect, which
decreases the incentives of the separated manufacturers to introduce additional products into the market is present only in a vertically related market and not in a one-tier market. An additional implication of the absence of wholesale prices is that the manufacturers do not extract a larger share of the industry profits in the merger case; hence, in a one-tier market they do not have an additional incentive to introduce more products when they merge.18

**Proposition 6** In a one-tier market, a merger:

(i) decreases product introduction when $F$ takes intermediate values; otherwise, it does not have any effect on product introduction,

(ii) increases profits, and

(iii) decreases consumers’ surplus and total welfare.

Independently of whether a merger in a one-tier market has no effect or a negative effect on product variety, since the merger results in monopoly, it is not surprising that it is always profitable (Proposition 6(ii)). A straightforward implication of the decreased product variety and increased market power is that the merger is again harmful to consumers and total welfare (Proposition 6(iii)). Therefore, a manufacturers merger to monopoly, either in a one-tier market or in a vertically related market, is welfare-detrimental.

**Proposition 7** Product variety in a one-tier market is either higher or equal to product variety in a two-tier market. This holds when the manufacturers remain separated as well as when they merge.

Proposition 7 informs us that in markets with vertical relations, there is less or equal product variety than in markets without vertical relations. This holds when there is just one product manufacturer in the market as well as when there are two of them. The key driver of this finding is again the fact that the double marginalization externality is absent—there are no additional distortions created through vertical trading. This has two important implications. First, that the increase in total output and, in turn, in industry profits, caused by product introduction, is larger. And second, that the product manufacturers extract the whole surplus that it is generated by the production activity and by product introduction. Clearly, this is not the case in a vertically related market, where the retailers get part of the surplus and double marginalization is present. Given this, it follows that the return of an investment in product introduction is higher for the manufacturer in a one-tier market than in a vertically related market.

It follows from the above that the consideration of vertical relations is of significant importance. The presence of vertical relations can crucially affect the incentives for new

---

18Note though that when firms compete in prices, competition is more intense than when they compete in quantities. As a result, the competition effect is stronger under price competition and can dominate when products are too close substitutes, by eliminating the product introduction incentives in the non-merger case, and result in more product variety in the merger case. The analysis of the one-tier market under price competition is available from the authors upon request.
product introduction, and in turn, the merger’s implications. If we did not take vertical relations into account we would reach the conclusion that a merger restricts product variety. In contrast, by allowing for such relations, we can conclude that, under certain conditions, a merger can enhance, instead of decrease, product variety.

7 Concluding Remarks

We have investigated the incentives of two competing final product manufacturers to invest in new product introduction and how they are affected by a merger among them. We have taken into account the fact that most product manufacturers instead of selling their products to consumers directly, they sell them through multiproduct retailers.

We have found that the wholesale prices can be affected both by the number of products in the market and by an upstream merger. Specifically, as the number of products in the market increases and, thus, as market competition intensifies, the wholesale prices decrease when the manufacturers are separated but not when they are merged. Furthermore, when the manufacturers merge, they internalize the negative interbrand competition externality that they impose on each other in the absence of the merger, and, as a result, they charge higher wholesale prices than when they are separated.

We have also found that the equilibrium number of products and, thus, product variety in the market can be affected both by a manufacturers merge and by the existence of vertical relations. More specifically, a manufacturers merger can cause an increase, instead of a decrease, in product variety. This holds when product introduction is not too costly and products are not too differentiated. This is so because, in the merger case, product introduction does not lead to the intensification of interbrand competition. Moreover, it is so because in the merger case, the manufacturers, through the higher wholesale prices, extract a larger share of the industry profits generated by product introduction. Importantly, the merger generated product variety efficiencies arise only when vertical relations are present. When, instead, the manufacturers sell directly their products to consumers, extracting all the industry profits, then, as long as they compete in quantities, a merger among them does not result in more product variety.

Although a manufacturers merger can give rise to efficiency gains, we have shown that its efficiency gains are not strong enough to overturn its negative impact on consumers and total welfare, caused by the increased wholesale prices. Stated in different words, we have provided a theoretical justification for the view expressed in the recent U.S. Horizontal Merger Guidelines (2010) according to which "efficiencies almost never justify a merger to monopoly or near-monopoly. Just as adverse competitive effects can arise along multiple dimensions of conduct, such as pricing and new product development, so too can efficiencies operate along multiple dimensions."

Driven by the above, one might wonder whether our conclusions would still hold if the manufacturers merger did not create a monopoly in the upstream market. Extending our analysis to the case in which there are initially three firms in the upstream market, we find
that when there is more competition among manufacturers, product variety can be higher
when two of them merge only when products are very differentiated and the fixed cost takes
intermediate values. Interestingly, product variety increases then not because the merged
manufacturers introduce new products, but because the outsider reacts aggressively and
adds a new product variety. As a consequence, the manufacturers merger is not profitable
then. Whenever the merger is profitable, it decreases product variety and in turn consumer
and total welfare. Therefore, a profitable manufacturers merger, even when it is not a
merger to monopoly, can be welfare-decreasing.

Our results have been obtained under the assumption that retailers are not differentiated
and the "distance" between any pair of goods is the same. Given this, one might also wonder
whether they might depend on this symmetry assumption. Extending our analysis, we find
that, for any degree of retailers' differentiation, the result that an upstream merger reduces
consumers' surplus and total welfare still holds. This is so because in order for a merger
to be welfare-improving, product variety should be higher with than without the merger.
This holds only when products are sufficiently close substitutes. But, in this case, there
is tough competition in the non-merger case, and in turn, the wholesale prices are lower
and total output is larger. So an upstream merger, by eliminating this intense competition,
leads to a drop in welfare that cannot be compensated by the positive welfare effects of
increased product variety. This reasoning prevails independently of whether retailers are
differentiated or not.

We should stress still that our paper constitutes just a first step in the direction of un-
derstanding the relation between manufacturer mergers and product variety. In a following
step, one could examine the role of alternative contract types, such as two-part tariffs. Moreover, one could endogenize the distribution system. That is, determine whether
manufacturers wish to sell all their product(s) through multiple and/or multiproduct retailers.

\footnote{In particular, for the simplest case with two goods (m and n) we consider the following demand functions:}
\footnote{Serious complications arise in situations in which rival upstream firms trade through non-linear contracts
with the same competing downstream firms. As mentioned in a review article by Molkos-Thal et al. (2010, p.345) "The formal modeling of such "interlocking" vertical relations has proved difficult... and we still
know relatively little about many basic questions... Interlocking relationships cause modeling issues such as
either the inexistence or a large multiplicity of equilibria even in simple competition games." Also, Inderst
(2010, p. 343) states that "... the benchmark model where competing upstream firms simultaneously make
take-it-or-leave-it offers to competing downstream firms, may fail to have an equilibrium in pure strategies." This
is why interlocking relationships, under two-part tariffs, have not been analyzed by the literature yet. A
recent exception is Rey and Vergé (2016) but only under the assumption that the retailers do not observe the
terms of trade offered to their rivals as well as under the assumption that manufacturers are single-product.
Clearly, there are even more complications when manufacturers are multiproduct.

\footnote{This gives rise to six different distribution systems in the non-merger case with two single-product man-
ufacturers and two possible retailers alone. Clearly, the number of the possible distribution systems that one
has to consider is much larger when the manufacturer(s) produce multiple products. While one might think
initially that manufacturers would always opt for exclusive distribution in order to avoid intrabrand com-
petition, when products are not too close substitutes, they might opt instead for a non-exclusive distribution
and purchasing in order to enjoy the gains of increased output. For more on this, see Moner-Colonques et
al. (2004) for the case of two single-product manufacturers and retailers, and Rey and Vergé (2016) for the}
Finally, one could also explore the role of alternative product introduction technologies, such as ones characterized by economies of scope. These extensions are beyond the scope of this paper and are left for future research.

8 Appendix

<table>
<thead>
<tr>
<th># of Products</th>
<th>Quantities</th>
</tr>
</thead>
</table>
| 2             | \( q_{1k}(w_1, w_2) = \frac{a(1-\gamma)-w_1+w_2}{3(1-\gamma)} \)  \\
|               | \( q_{2k}(w_1, w_2) = \frac{a(1-\gamma)-w_1-w_2}{3(1-\gamma)} \) |
| 3             | \( q_{1k}(w_1, w_2, w_3) = \frac{a(1-\gamma)-w_1+w_2+w_3}{3(1-\gamma)-6\gamma} \)  \\
|               | \( q_{2k}(w_1, w_2, w_3) = \frac{a(1-\gamma)-w_2+w_3+w_1}{3(1-\gamma)-6\gamma} \)  \\
|               | \( q_{3k}(w_1, w_2, w_3) = \frac{a(1-\gamma)-w_1+w_2+w_3}{3(1-\gamma)-6\gamma} \) |
| 4             | \( q_{1k}(w_1, w_2, w_3, w_4) = \frac{a(1-\gamma)-w_1+w_2+w_3+w_4}{3(1-\gamma)-6\gamma} \)  \\
|               | \( q_{2k}(w_1, w_2, w_3, w_4) = \frac{a(1-\gamma)-w_1+w_2+w_3+w_4}{3(1-\gamma)-6\gamma} \)  \\
|               | \( q_{3k}(w_1, w_2, w_3, w_4) = \frac{a(1-\gamma)-w_1+w_2+w_3+w_4}{3(1-\gamma)-6\gamma} \)  \\
|               | \( q_{4k}(w_1, w_2, w_3, w_4) = \frac{a(1-\gamma)-w_1+w_2+w_3+w_4}{3(1-\gamma)-6\gamma} \) |

Table 3: Third-stage Equilibrium Quantities

Proof of Proposition 1

From Table 1, it follows that a necessary condition for ‘partial product introduction’ and for ‘full product introduction’ to be an equilibrium is that \( F \leq F^S(\gamma) = \frac{a^2(1-\gamma)(2+3\gamma)^2}{3(1+2\gamma)(2+2(2-\gamma)\gamma)^2} \) and \( F \leq F^S(\gamma) = \frac{a^2(1-\gamma)^2}{3+9\gamma} \), respectively.

- ‘No product introduction’: The only possible deviation by \( M_1 \) is to 2 goods. If \( F > F^S(\gamma) \), such incentives do not exist. If \( F \leq F^S(\gamma) \), \( M_1 \) deviates if and only if \( \Pi^{S_1}_{M_1} \geq \Pi^{S_2}_{M_1} \), which holds when \( F \leq F^S(\gamma) = \frac{a^2(1-\gamma)^2}{3(1+2\gamma)(2+2(2-\gamma)\gamma)^2} \). Thus, \( F^S(\gamma) < F^S(\gamma) < F^S(\gamma) \) and one can see that \( F^S(\gamma) < 0 \) when \( \gamma > 0.9164 \). Thus, if \( \gamma > 0.9164 \), the deviation is not profitable and ‘no product introduction’ is an equilibrium. If, instead, \( \gamma \leq 0.9164 \), ‘no product introduction’ is again an equilibrium but only if \( F > F^S(\gamma) \).

- ‘Partial product introduction’: We know that it is not an equilibrium when \( F > F^S(\gamma) \) as well as when \( F \leq F^S(\gamma) \) and \( \gamma > 0.9164 \) or when \( \gamma \leq 0.9164 \) and \( F > F^S(\gamma) \). When \( \gamma \leq 0.9164 \) and \( F \leq F^S(\gamma) \), the only possible deviation is to 2 goods. This deviation is profitable if and only if \( \Pi^{S_2}_{M_2} \geq \Pi^{S_1}_{M_2} \), which holds if \( F \leq F^S(\gamma) = \frac{a^2(1-\gamma)(2+3\gamma)^2}{3(1+2\gamma)(2+2(2-\gamma)\gamma)^2} \), with \( F^S(\gamma) > F^S(\gamma) \). However, one can see that \( F^S(\gamma) < 0 \) when \( \gamma > 0.8346 \). Thus, if \( 0.8346 < \gamma \leq 0.9164 \), this deviation is not profitable and ‘partial product introduction’ is an equilibrium. If, instead, \( \gamma \leq 0.8346 \), ‘partial product introduction’ is again an equilibrium but only if \( F > F^S(\gamma) \).

- ‘Full product introduction’: It is not an equilibrium when \( F > F^S(\gamma) \). When \( F \leq F^S(\gamma) \), the only possible deviation is to 1 good. We know that this deviation is profitable for \( M_2 \).
when $\gamma > 0.8346$ or when $\gamma \leq 0.8346$ and $F > F_2^S(\gamma)$. Thus, ‘full product introduction’ is an equilibrium if and only if $\gamma \leq 0.8346$ and $F \leq F_2^S(\gamma)$.

**Proof of Proposition 2**

From Table 2, it follows that a necessary condition for ‘partial product introduction’ and for ‘full product introduction’ to be an equilibrium is that $F \leq \overline{F}^M(\gamma) = \frac{a^2}{2r+\gamma}$ and $F \leq \overline{F}^M(\gamma) = \frac{2a^2}{3r+\gamma}$, respectively, with $\overline{F}^M(\gamma) < \overline{F}^M(\gamma)$.

- ‘No product introduction’: It is always equilibrium when $F > \overline{F}^M(\gamma)$. When $\overline{F}^M(\gamma) < F \leq \overline{F}^M(\gamma)$, the only possible deviation is to 4 goods. If, instead $F \leq \overline{F}^M(\gamma)$, $M$ deviates to 3 goods if and only if $\Pi^M_M \geq \Pi^M_3$, which holds if $F \leq F_1^M(\gamma) = \frac{a^2(1-\gamma)}{6(1+3\gamma+2r)}$, with $F_1^M(\gamma) < \overline{F}^M(\gamma)$. $M$ deviates to 4 goods if and only if $\Pi^M_4 \geq \Pi^M_3$, which holds if $F \leq F_3^M(\gamma) = \frac{a^2(1-\gamma)}{6(1+4\gamma+3r)}$, with $F_1^M(\gamma) > F_3^M(\gamma)$. Hence, ‘no product introduction’ is an equilibrium if and only if $F > F_1^M(\gamma)$.

- ‘Partial product introduction’: We know that it is not an equilibrium when $F > \overline{F}^M(\gamma)$. When $F \leq \overline{F}^M(\gamma)$, the possible deviation is for $M$ to 2 or 4 goods. We know that a deviation to 2 goods is profitable if and only if $F > F_1^M(\gamma)$. $M$ deviates to 4 goods if and only if $\Pi^M_4 \geq \Pi^M_M$, which holds if $F \leq F_2^M(\gamma) = \frac{a^2(1-\gamma)}{6(1+\gamma(3+5r))}$, with $F_1^M(\gamma) > F_3^M(\gamma)$, $F_3^M(\gamma)$. Hence, ‘partial product introduction’ is an equilibrium if and only if $F_1^M(\gamma) < F \leq F_1^M(\gamma)$.

- ‘Full product introduction’: It is not an equilibrium when $F > \overline{F}^M(\gamma)$. When $\overline{F}^M(\gamma) < F \leq \overline{F}^M(\gamma)$, the only possible deviation is to 2 goods, while if $F \leq \overline{F}^M(\gamma)$, the possible deviations are to 2 or 3 goods. Thus, ‘full product introduction’ is an equilibrium if and only if $F \leq F_2^M(\gamma)$ (recall that $F_2^M(\gamma) < F_3^M(\gamma) < F_1^M(\gamma) < \overline{F}^M(\gamma) < \overline{F}^M(\gamma)$ and so $F \leq F_2^M(\gamma)$ also implies $F \leq F_3^M(\gamma)$).

Finally, it is direct to see that $F_1^M(\gamma)$, $F_2^M(\gamma)$, and $F_3^M(\gamma)$ are all decreasing in $\gamma$.

**Proof of Proposition 3**

The result follow immediately from the comparison of $w_n^{MN}$ with (10), (7) and (5).

**Proof of Proposition 4**

Combining Propositions 1 and 2, we can rank the different threshold values of $F$ and compare the equilibrium number of products with and without the merger:

- When $\gamma > 0.9164$, we have $F_2^S(\gamma) < F_1^{M}(\gamma) < 0 < F_2^M(\gamma) < F_1^M(\gamma)$. This implies that when $F < F_1^M(\gamma)$ the merger leads to more products, while when $F \geq F_1^M(\gamma)$ the merger has no impact on the number of products.

- When $0.8415 < \gamma < 0.9164$, we have $F_2^S(\gamma) < 0 < F_2^S(\gamma) < F_2^M(\gamma) < F_1^M(\gamma)$. This implies that when $F < F_1^M(\gamma)$ the merger leads to more product, while when $F \geq F_1^M(\gamma)$, it has no impact.

- When $0.8346 < \gamma < 0.8415$, we have $F_2^S(\gamma) < 0 < F_2^M(\gamma) < F_1^{M}(\gamma) < F_2^M(\gamma)$, meaning that when $F < F_2^M(\gamma)$ or $F_2^M(\gamma) < F < F_1^M(\gamma)$, the merger leads to more products, while when $F_2^M(\gamma) \leq F < F_1^M(\gamma)$ or $F \geq F_1^M(\gamma)$, it has no impact.

- When $0.7261 < \gamma < 0.8346$, we have $0 < F_2^S(\gamma) < F_2^M(\gamma) < F_1^M(\gamma)$, which implies that when $F < F_2^S(\gamma)$, or $F_2^M(\gamma) < F < F_1^M(\gamma)$, or $F \geq F_1^M(\gamma)$, the merger has
no impact, while when \( F^S(x) \leq F < F^M(x) \) or \( F^I(x) \leq F < F^I(x) \), it leads to more products.

- When \( 0.6931 < \gamma < 0.7261 \), we have \( 0 < F^S(x) < F^M(x) < F^I(x) \), which implies that when \( F < F^S(x) \), or \( F^M(x) \leq F < F^I(x) \), or \( F \geq F^S(x) \), the merger has no impact, while when \( F^S(x) \leq F < F^M(x) \), it leads to more products, and when \( F^M(x) \leq F < F^S(x) \), it leads to less products.

- When \( 0.4046 < \gamma < 0.6931 \), we have \( 0 < F^M(x) < F^S(x) < F^I(x) \), which implies that when \( F < F^M(x) \), or \( F^S(x) \leq F < F^I(x) \), or \( F \geq F^S(x) \), the merger has no impact, while when \( F^S(x) \leq F < F^M(x) \), or \( F^I(x) \leq F < F^S(x) \), it leads to less products.

- When \( \gamma < 0.4046 \), we have \( 0 < F^S(x) < F^I(x) < F^S(x) \), which implies that when \( F < F^S(x) \), or \( F^I(x) \leq F < F^S(x) \), or \( F \geq F^S(x) \), the merger has no impact, while when \( F^S(x) \leq F < F^S(x) \), or \( F^S(x) \leq F < F^I(x) \), it leads to less products.

**Proof of Proposition 5**

**Merger Profitability:** The possible equilibria, depending on \( \gamma \), are: (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3) and (4, 4). Comparing the upstream profits with and without the merger, in each case, we conclude that the latter are always higher.

**Merger impact on downstream profits:** The equilibrium downstream profits are: \( \Pi^S_{R_k} = \frac{2\gamma^2}{2(\gamma-\gamma^2)(\gamma+1)} \); \( \Pi^S_{R_k} = \frac{2\gamma^2(\gamma+1)(\gamma+2)}{18(1+\gamma)(-2+\gamma)(-2+\gamma)} \); \( \Pi^S_{R_k} = \frac{2\gamma^2}{18(1+\gamma)(\gamma+2)} \); \( \Pi^S_{M_k} = \frac{2\gamma^2}{12(1+\gamma)} \); \( \Pi^S_{M_k} = \frac{2\gamma^2}{12(1+\gamma)} \). We have to compare the downstream profits in the merger and non-merger case for all the possible combinations that arise in equilibrium: (2,2), (3,3), (4,4), (3,2), (4,2), (4,3), (2,3), (2,4), (3,4). It is straightforward to show that the merger reduces the downstream firms’ profits in all the cases.

**Merger impact on consumers’ surplus:** We need to compare the consumers’ surplus in the merger case and non-merger case when the merger increases variety since only then the merger can increase the consumers’ surplus.

- **Equilibrium (4,3):** It arises if \( 0.6931 < \gamma < 0.9164 \). We have: \( u(q^{M_k}_n, q^{M_k}_n, q^{M_k}_n, q^{M_k}_n, q^{M_k}_n) - \Pi^M_{M_k} + 2F - 2\Pi^S_{R_k} - [u(q^{S_k}_k, q^{S_k}_k, q^{S_k}_k, q^{S_k}_k, q^{S_k}_k) - \Pi^S_{M_k} - \Pi^S_{M_k} + 2F - 2\Pi^S_{R_k}] = \frac{2\gamma^2(\gamma+1)(\gamma+2)}{18(1+\gamma)(\gamma+2)} \). This expression is negative if \( \gamma > 0.1388 \). Hence, in this region the merger reduces consumers’ surplus.

- **Equilibrium (4,2):** It arises if \( 0.8415 < \gamma \). We have: \( u(q^{M_k}_n, q^{M_k}_n, q^{M_k}_n, q^{M_k}_n, q^{M_k}_n) - \Pi^M_{M_k} - 2F - 2\Pi^S_{R_k} - [u(q^{S_k}_k, q^{S_k}_k, q^{S_k}_k, q^{S_k}_k, q^{S_k}_k) - \Pi^S_{M_k} - \Pi^S_{M_k} + 2F - 2\Pi^S_{R_k}] = \frac{2\gamma^2(\gamma+1)(\gamma+2)}{18(1+\gamma)(\gamma+2)} \). This expression is negative if \( \gamma > 0.2942 \). Hence, in this region the merger reduces consumers’ surplus.

- **Equilibrium (3,2):** It arises if \( 0.7261 < \gamma \). We have: \( u(q^{M_k}_n, q^{M_k}_n, q^{M_k}_n, q^{M_k}_n, q^{M_k}_n) - \Pi^M_{M_k} - F - 2\Pi^S_{R_k} - [u(q^{S_k}_k, q^{S_k}_k, q^{S_k}_k, q^{S_k}_k, q^{S_k}_k) - \Pi^S_{M_k} - \Pi^S_{M_k} + 2F - 2\Pi^S_{R_k}] = \frac{2\gamma^2(\gamma+1)(\gamma+2)}{18(1+\gamma)(\gamma+2)} \). This expression is negative if \( \gamma > 0.2293 \). Hence, in this region the merger reduces consumers’ surplus.

**Merger impact on total welfare:** Since the marginal production cost is zero, we can define the (gross from any product introduction costs) total welfare as: \( W(Q_1, ..., Q_N) = \)

---

22The first number represents the number of varieties post-merger and the second one represents the number of varieties pre-merger.
We compare total welfare in the merger and non-merger case in all possible combinations that arise in equilibrium.

We start with the region $\gamma < 0.4046$. In this region we can have equilibria (2,2), (2,3), (2,4), (3,4), (4,4). We check each of them:

- Equilibrium (2,2): $u(q^{M2}_{nk}, q^{M2}_{nk}, 0, 0) - u(q^{S2}_{nk}, q^{S2}_{nk}, 0, 0) = \frac{a^{2} \gamma (-8 + 5 \gamma)}{9(2 - \gamma)(1 + \gamma)} < 0$, which implies that the merger reduces total welfare.

- Equilibrium (2,3): $u(q^{M2}_{nk}, q^{M2}_{nk}, 0, 0) - u(q^{S3}_{nk}, q^{S3}_{nk}, 0, 0) + 2F < 0$ if and only if $F < F_{W23}(\gamma) \equiv \frac{a^{2}(10 + 3(26 + 3(29 + 4\gamma(7 + 4\gamma)^{2})))}{9(1 + \gamma)(1 + 8\gamma)(2 - 3 + (2 - 3\gamma))^{5}}$. But we are in equilibrium (2,3) if $F_{2}(\gamma) \leq F < F_{3}(\gamma)$. It is direct to check that $F_{W23}(\gamma) > F_{3}(\gamma)$. Hence, the merger reduces total welfare.

- Equilibrium (2,4): $u(q^{M2}_{nk}, q^{M2}_{nk}, 0, 0) - u(q^{S4}_{nk}, q^{S4}_{nk}, q^{S4}_{nk}, 0) + 2F < 0$ if and only if $F < F_{W24}(\gamma) \equiv \frac{a^{2}(1 + (3 - 2 + (2 - 3\gamma)))}{8(1 + \gamma)(1 + 3\gamma)}$. But we are in equilibrium (2,4) if $F_{1}(\gamma) \leq F < F_{2}(\gamma)$. It is direct to check that $F_{W24}(\gamma) > F_{2}(\gamma)$. Hence, the merger reduces total welfare.

- Equilibrium (3,4): $u(q^{M3}_{nk}, q^{M3}_{nk}, 0, 0) - u(q^{S4}_{nk}, q^{S4}_{nk}, q^{S4}_{nk}, q^{S4}_{nk}) + 2F < 0$ if and only if $F < F_{W34}(\gamma) \equiv \frac{a^{2}(5 + 14\gamma(2 - 2\gamma))}{8(1 + \gamma)(1 + 3\gamma)}$. But we are in equilibrium (3,4) if $F_{3}(\gamma) \leq F < F_{4}(\gamma)$ and $F_{W34}(\gamma) > F_{4}(\gamma)$. Hence, the merger reduces total welfare.

- Equilibrium (4,4): $u(q^{M4}_{nk}, q^{M4}_{nk}, q^{M4}_{nk}, q^{M4}_{nk}) - u(q^{S4}_{nk}, q^{S4}_{nk}, q^{S4}_{nk}, q^{S4}_{nk}) = \frac{a^{2} \gamma (-4 + \gamma)}{9(2 + 2\gamma)} < 0$, which implies that the merger reduces total welfare.

Proceeding in exactly the same way, it is direct to show that the merger reduces total welfare also in the rest of the regions.

**Proof of Proposition 6**

In the non-merger case, it is straightforward to find the equilibrium quantities and (gross) profits:

- $\hat{q}_{M1}^{S} = \frac{a}{2 + \gamma}$; $\hat{q}_{M1}^{S} = \frac{a}{2 + (2 - \gamma)\gamma}$; $\hat{q}_{M1}^{S} = \frac{a}{2 + (2 - \gamma)\gamma}$; $\hat{M}_{M1}^{S} = \frac{a^{2}}{2(1 + \gamma)}$; $\hat{M}_{M1}^{S} = \frac{a^{2}}{2(1 + \gamma)}$; $\hat{M}_{M1}^{S} = \frac{a^{2}}{2(1 + \gamma)}$.
- $\hat{M}_{M1}^{S} = \frac{a^{2}}{2(1 + \gamma)}$; $\hat{M}_{M1}^{S} = \frac{a^{2}}{2(1 + \gamma)}$; $\hat{M}_{M1}^{S} = \frac{a^{2}}{2(1 + \gamma)}$; $\hat{M}_{M1}^{S} = \frac{a^{2}}{2(1 + \gamma)}$; $\hat{M}_{M1}^{S} = \frac{a^{2}}{2(1 + \gamma)}$.

- ‘No product introduction’: The only possible deviation by $M_{1}$ is to 2 goods. If $F > \hat{F}_{S}(\gamma)$, such incentives do not exist. If $F \leq \hat{F}_{S}(\gamma)$, $M_{1}$ deviates if and only if $\hat{M}_{M1}^{S} > \hat{M}_{M1}^{S}$, which holds if $F \leq \hat{F}_{1}(\gamma) = \frac{a^{2}(1 + \gamma)(8 - 8\gamma - 4\gamma)}{2(8 + 8\gamma - 4\gamma)}$, with $\hat{F}_{1}(\gamma) < \hat{F}_{S}(\gamma)$.

- ‘Partial product introduction’: We know that it is not an equilibrium when $F > \hat{F}_{S}(\gamma)$. When $F \leq \hat{F}_{1}(\gamma)$, the possible deviation is to 2 goods. This deviation is profitable for $M_{2}$ if and only if $\hat{M}_{M2}^{S} > \hat{M}_{M2}^{S}$, which holds if $F \leq \hat{F}_{2}(\gamma) = \frac{a^{2}}{2} \left( \frac{1 + \gamma}{(1 + 2\gamma)^{2}} - \frac{2}{2(2 - 3\gamma)\gamma} \right)$, with $\hat{F}_{2}(\gamma) > \hat{F}_{S}(\gamma)$.

- ‘Full introduction’: When $F \leq \hat{F}_{S}(\gamma)$, the only possible deviation is to 1 good. We know that this deviation is profitable for $M_{2}$ if and only if $F > \hat{F}_{S}(\gamma)$. Thus, ‘full introduction’ is an equilibrium if and only if $F \leq \hat{F}_{S}(\gamma)$.

Next, we turn to the merger case. It is straightforward to obtain the equilibrium quantities and (gross) profits:

- $\hat{q}_{M2}^{M2} = \frac{a}{2(1 + \gamma)}$; $\hat{q}_{M2}^{M2} = \frac{a}{2(1 + 2\gamma)}$; $\hat{q}_{M2}^{M2} = \frac{a}{2(1 + 3\gamma)}$; $\hat{M}_{M2}^{M2} = \frac{a^{2}}{2(1 + \gamma)}$; $\hat{M}_{M2}^{M2} = \frac{a^{2}}{2(1 + \gamma)}$.
- $\hat{M}_{M2}^{M2} = \frac{a^{2}}{2(1 + \gamma)}$; $\hat{M}_{M2}^{M2} = \frac{a^{2}}{2(1 + \gamma)}$.

A necessary condition for ‘partial product introduction’ and for ‘full product introduction’ to be an equilibrium is that $F \leq \hat{F}_{M}(\gamma) \equiv \hat{M}_{M}^{M2}$ and $F \leq \hat{F}_{M}(\gamma) \equiv \hat{M}_{M}^{M4}$, respectively.
- ‘No product introduction’: It is always equilibrium when \( F > \tilde{F}^M(\gamma) \). When \( \tilde{F}^M(\gamma) < F \leq \bar{F}^M(\gamma) \), the only possible deviation is to 4 goods. If, instead, \( F \leq \tilde{F}^M(\gamma) \), there could be deviations to both 3 and 4 goods. If \( F \leq \tilde{F}^M(\gamma) \), \( M \) deviates to 3 goods if and only if \( \tilde{F}_1^M - F \geq \tilde{F}_4^M \), which holds if \( F \leq \bar{F}_1^M(\gamma) = \frac{a^2(1-\gamma)}{4+12\gamma+18\gamma^2} \), with \( \tilde{F}_1^M(\gamma) < \tilde{F}_4^M(\gamma) \). \( M \) deviates to 4 goods if and only if \( \tilde{F}_1^M - 3F \geq \tilde{F}_4^M \), which holds if \( F \leq \tilde{F}_4^M(\gamma) = \frac{a^2(1-\gamma)}{2(2-9\gamma-6\gamma^2)} \), with \( \tilde{F}_1^M(\gamma) > \tilde{F}_3^M(\gamma) \). Thus, ‘no product introduction’ is an equilibrium if and only if \( F > \tilde{F}_1^M(\gamma) \).

- ‘Partial product introduction’: We know that it is not an equilibrium when \( F > \tilde{F}^M(\gamma) \). When \( F \leq \tilde{F}_1^M(\gamma) \), the possible deviation is to 2 or 4 goods. We know that a deviation to 2 goods is profitable if and only if \( F > \tilde{F}_1^M(\gamma) \). On the other hand, \( M \) deviates to 4 goods if and only if \( \tilde{F}_2^M - F \geq \tilde{F}_4^M \), which holds if \( F \leq \tilde{F}_2^M(\gamma) = \frac{a^2(1-\gamma)}{4+8\gamma} \), with \( \tilde{F}_1^M(\gamma) > \tilde{F}_3^M(\gamma) \). Hence, ‘partial product introduction’ is an equilibrium if and only if \( F > \tilde{F}_1^M(\gamma) \).

- ‘Full product introduction’: When \( \tilde{F}^M(\gamma) < F \leq \tilde{F}_1^M(\gamma) \), the only possible deviation is to 2 goods, whereas if \( F \leq \tilde{F}^M(\gamma) \), the possible deviations are to 2 or 3 goods. Thus, ‘full product introduction’ is an equilibrium if and only if \( F \leq \tilde{F}_2^M(\gamma) \) (recall that \( \tilde{F}_2^M(\gamma) < \tilde{F}_3^M(\gamma) \)).

(i) Comparing the critical values, we find that \( \tilde{F}_2^M(\gamma) < \tilde{F}_3^M(\gamma) < \tilde{F}_2^S(\gamma) < \tilde{F}_1^S(\gamma) \). Therefore, the merger leads to less product variety when \( \tilde{F}_2^M < F < \tilde{F}_1^S \). In all other cases, product variety is the same with and without the merger.

(ii) Comparing the equilibrium profits in all the possible areas that arise from the second stage equilibrium, we find that the profits of the merged manufacturers are higher than the sum of the profits of the two separated manufacturers.

(iii) Following similar steps as in the proof of Proposition 5(ii), we conclude that both consumers’ surplus and total welfare are lower with than without the merger.

**Proof of Proposition 7**

We have to compare the equilibrium number of products in a one-tier market and in a vertically related market. In the non-merger case, it is direct to see that \( \tilde{F}_2^S(\gamma) < \tilde{F}_2^M(\gamma) < \tilde{F}_2^S(\gamma) < \tilde{F}_1^S(\gamma) \), which implies that (i) for \( F < \tilde{F}_2^S(\gamma) \), we have 4 products in both markets, (ii) for \( \tilde{F}_2^S(\gamma) < F < \tilde{F}_2^M(\gamma) \), we have 4 products in the one-tier market and 3 in the vertically related market, (iii) for \( \tilde{F}_2^S(\gamma) < F < \tilde{F}_1^S(\gamma) \), we have 4 products in the one-tier market and 2 in the vertically related market, (iv) for \( \tilde{F}_2^S(\gamma) < F < \tilde{F}_2^S(\gamma) \), we have 3 products in the one-tier market and 2 in the vertically related market and, (v) for \( F > \tilde{F}_2^S(\gamma) \), we have 2 products in both markets.

In the merger case, it is easy to see that when \( \gamma < 0.33 \), we have \( \tilde{F}_2^M(\gamma) < \tilde{F}_1^M(\gamma) < \tilde{F}_2^M(\gamma) < \tilde{F}_1^M(\gamma) \), which, following the same reasoning as above, implies that product variety is always larger or equal in an one-tier market. When, instead, \( \gamma > 0.33 \), we have \( \tilde{F}_2^M(\gamma) < \tilde{F}_2^M(\gamma) < \tilde{F}_1^M(\gamma) < \tilde{F}_1^M(\gamma) \), which implies that (i) for \( F < \tilde{F}_2^M(\gamma) \), we have 4 products in both markets, (ii) for \( \tilde{F}_2^M(\gamma) < F < \tilde{F}_2^M(\gamma) \), we have 4 products in the one-tier market and 3 in the vertically related market, (iii) for \( \tilde{F}_2^M(\gamma) < F < \tilde{F}_1^M(\gamma) \), we have 3
products in both markets, (iv) for $F^M(\gamma) < F < \hat{F}_1^M(\gamma)$, we have 3 products in the one-tier market and 2 in the vertically related market and, (v) for $F > \hat{F}_1^M(\gamma)$, we have 2 products in both markets.

9 References


U.S. Department of Justice and FTC, 2010, Horizontal Merger Guidelines, Washington, DC.