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Systematic approach for the life cycle multi-objective optimization of buildings combining objective reduction and surrogate modeling

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Graphical abstract
To determine the optimum insulation thickness according to Multi-objective optimization

Problem statement

Objective reduction

Simulation time (s)

Surrogate modelling

Surrogate model time (s)

Optimal building designs

Pareto optimal solutions

10 cm 11 cm 8 cm
Highlights

- We present a systematic approach to optimize the thermal insulation of a building.
- The optimization reduces simultaneously the cost and several environmental impacts.
- We resort to an objective reduction method to simplify the problem resolution.
- We built a surrogate model to expedite the search for Pareto optimal solutions.
- Significant improvements compared to the base case (no insulation) are achieved.

Abstract

With the recent trend of moving towards a more sustainable economy, the interest on designing buildings with lower cost and environmental impact has grown significantly. In this context, multi-objective optimization has attracted much attention in building design as a tool to study trade-off solutions (“cost” vs “environmental impact”) resulting from the optimization of conflicting objectives. One major limitation of this approach (as applied to building design) is that it is computationally demanding due to the need to optimize several objectives using complex models based on differential equations (which are required to model the energy required by a building). In this work, we propose a systematic framework for the design of buildings that combines a rigorous objective reduction method (which removes redundant objectives from the analysis) with a surrogate model (which simplifies the calculation of the energy requirements of the building), both of which expedite the identification of alternative designs leading to environmental improvements. The capabilities of our methodology are illustrated through a case study based on a thermal modelling of a house-like cubicle, in which we optimize the insulation thicknesses of the building envelope. Results show that significant economic and environmental improvements can be achieved compared to the base case (cubicle without insulation). Furthermore, it is clearly illustrated how the minimization of an aggregated environmental metric, like the Eco-Indicator 99, as unique environmental objective may overlook some Pareto solutions that may be appealing for decision-makers.
Keywords: Multi-objective optimization, Objective reduction, Surrogate model, Life cycle assessment (LCA), Modelling, Buildings, Insulation

Nomenclature

Abbreviations

ACH Air changes per hour
COP Coefficient of performance
EI99 Eco-indicator 99
GLO Average global impact
LCA Life cycle assessment
MILP Mixed-integer linear programming
moNLP Multi-objective non-linear programming
MOO Multi-objective optimization
NLP Nonlinear programming
OECD Organisation for Economic Co-operation and Development
PCA Principal component analysis
PDE Partial differential equations
PU Polyurethane

Indices

c Impact category
k Construction material
n Year
Sets

$C$ Set of impact categories

$I$ Set of solutions

$K$ Set of construction materials

$RSO$ Reduced set of objectives

$SOO$ Set of objectives to be optimized

Variables

$CONS^{EN}$ Energy consumption [kWh]

$COST^{EN}$ Energy cost [€]

$COST^{MAT}$ Materials cost [€]

$COST^{TOT}$ Total (material and energy) cost of the building [€]

$IMP^c_{EN}$ Energy impact in each impact category $c$ [Points]

$IMP^c_{MAT}$ Material impact in each impact category $c$ [Points]

$IMP^c_{TOT}$ Total (material and energy) impact of the building in each impact category $c$ [Points]

$M_k$ Mass of material $k$ [kg]

Parameters

$ir$ Yearly electricity inflation rate [%]

$UCOST^E_{EN}$ Cost per kWh of energy [€/kWh]

$UCOST^k_{MAT}$ Cost per kilogram of component $k$ [€/kg]
$UIMP_{c}^{EN}$ Impact in category $c$ per kWh of energy [Points/kWh]

$UIMP_{c}^{MAT}$ Impact in category $c$ per kilogram of component $k$ [Points/kg]

Other symbols

$g_{d}(-)$ Implicit inequality constraints (i.e., embedded in the building simulation)

$h_{e}(-)$ Explicit equality constraints (i.e., computed offline)

$h_{f}(-)$ Implicit equality constraints (i.e., embedded in the building simulation)

$I\lambda$ Iterations

$x_{D}$ Vector of decision variables

$z$ Vector of objective functions

1 Introduction

In both developed and developing countries, the building sector is responsible for approximately 40% of the total annual worldwide consumption of energy [1], and for one third of global greenhouse gas emissions [2]. Many OECD countries have dictated measures for minimizing energy consumption in the building sector. In March 2007, the European Parliament approved a binding legislation comprising several goals: i) to achieve a 20% reduction in EU greenhouse gas emissions from 1990 levels; ii) to increase the share of EU energy consumption produced from renewable resources to 20%; and iii) to improve the EU's energy efficiency by 20% [3]. To meet these targets, several energy strategies must be put in place. Among them, building insulation appears as a promising option, since it has the potential to decrease the cooling and heating demand without compromising comfort and can be applied in both, new and refurbished buildings [4–6].
Nowadays the current trend is to implement high insulation thicknesses, given the fact that a thicker insulation reduces energy consumption and therefore the associated environmental impact. This strategy might be suboptimal, as the cost and environmental impact embodied in the insulation materials can be quite large. Blengini et al. [7] analysed the impact produced in all the phases of the life of a low energy house, finding that the impact embodied in the construction materials represented the greatest contribution towards the total impact. Following a similar approach, Stephan et al. [8] concluded that up to 77% of the total energy (embodied and operational) used by a passive house over 100 years can correspond to the energy embodied in the construction materials. Hence, the impact embodied in the insulation materials needs to be accounted for a proper optimization of the whole system.

At present, multi-objective optimization (MOO) [4,9–14] has become the prevalent approach to solve problems with more than one objective function (e.g. economic cost and environmental impact). This mathematical approach is widely employed in many areas of science and engineering for studying trade-off solutions and for optimizing several objective functions simultaneously [15–18]. Unfortunately, MOO is rather sensitive to the number of objectives considered in the analysis, mainly because both the calculation of the Pareto solutions and their visualization and analysis become more complex as we increase the number of criteria. To overcome this problem, the optimization is typically restricted to two or three objectives [19] by either removing objectives or by aggregating some of them into a single indicator based on subjective weights [20–22]. Both approaches are inadequate; the former because it omits objectives that might be relevant, and the later because it alters the structure of the problem by eliminating Pareto solutions potentially appealing for decision-makers. These drawbacks can be bypassed by means of dimensionality reduction methods, which remove redundant objectives from the multi-objective model while still preserving its underlying structure. Several dimensionality reduction methods have been proposed in the literature. In a seminal work, Deb and Saxena [23] introduced a statistical method based on principal component analysis (PCA) for removing redundant objectives in MOO problems. Brockhoff and Zitzler [24] presented
another approach based on the minimization of an approximation error (i.e., delta error) resulting from the elimination of objectives. More recently, Guillén-Gosálbez [25] introduced a multi-dimensionality reduction method based on a mixed-integer linear program (MILP) that minimizes the delta error proposed by Brockhoff and Zitzler [24].

Unfortunately, applying multi-objective optimization to building design is further complicated by the fact that estimating the energy performance of a building through simulation is computationally challenging. That is, even if the optimization is performed in a reduced domain of objectives, it might yet be difficult to evaluate the objective functions, as this requires solving a system of partial differential equations (PDE). Some approaches have attempted to reduce the complexity of the PDE model by streamlining the simulation process [26–28]. Other authors have explored the use of surrogate models to accelerate the optimization process [29–31]. These methods simulate first a set of sample points, to then use the output to construct a surrogate model. This is a black box model fitted to data points (generated with the rigorous simulation), which is faster to solve than the original model (which requires solving a system of PDEs), yet it provides approximated results. The use of surrogate models is particularly appealing when they are coupled with an optimization algorithm, as the latter needs to interrogate the simulation model many times during the optimization task. Caballero et al. [29,32] presented a methodology for the rigorous optimization of nonlinear programming (NLP) problems in which the objective function and some constraints are represented by noisy implicit black box functions. The black box modules are replaced by kriging meta-models, an interpolating method based on basic functions with adjustable parameters. Costas et al. [31] applied a surrogate-based multi-objective optimization technique to car crashworthiness problems, while Eisenhower et al. [30] presented a method to optimize building energy models using a meta-model generated from a set of design and operation scenarios of the building around its baseline.

This work introduces a novel approach for the multi-objective optimization of buildings that integrates multidimensionality reduction and surrogate modelling. To the best of our knowledge, this is the first time that these two methodologies have been combined within a
single framework. We illustrate the capabilities of our approach through a case study based on a house-like cubicle where the goal is to determine the optimal insulation thickness (for the building envelope) according to economic and environmental criteria.

The article is structured as follows. The problem statement is presented in Section 2. The methodology, which includes the description of the objective functions and the solution procedure, is introduced in Section 3. Details of the case study are given in Section 4, whereas in Section 5, the results are presented and discussed. Finally, the conclusions of the work are drawn in Section 6.

2 Problem statement

The problem we aim to solve can be formally stated as follows. Given is a building (i.e., cubicle) that will be retrofitted through the installation of insulation materials. The detailed cubicle configuration, along with cost and environmental data associated with different insulation materials and energy demands are provided. The goal of the analysis is to determine the optimal insulation material and thickness of the insulation layer so as to optimize simultaneously the economic and the environmental performance of the overall system.

3 Methodology

Our approach is based on building a surrogate model of the building that is optimized in a reduced domain of objectives. The model of the building is described first before presenting in detail our algorithmic framework.

3.1 Mathematical model

The optimization of a building considering economic and environmental criteria can be mathematically posed as a multi-objective non-linear programming problem (moNLP) such as problem SIMMOD:
\[
\begin{align*}
(SIMMOD) \quad & \min_{x_D} \quad z = \{z_1(x,x_D), \ldots, z_p(x,x_D)\} \\
& \text{s.t.} \\
& h_I(x,x_D) = 0 \\
& g_I(x,x_D) \leq 0 \\
& h_E(x,x_D) = 0 \\
& g_E(x,x_D) \leq 0 \\
& x, x_D \in \mathcal{R}
\end{align*}
\]

Here, \(z_1\) corresponds to the economic objective whereas \(z_2\) to \(z_p\) are the \(p-1\) environmental objectives. Regarding the constraints, we can distinguish between implicit and explicit constraints. Implicit equality and inequality constraints, denoted by \(h_I(\cdot)\) and \(g_I(\cdot)\) respectively, are the equations implemented in the building simulator to describe the energy balances through the building walls and roof (refer to the next section for further details). Conversely, explicit constraints, referred to by \(h_E(\cdot)\) and \(g_E(\cdot)\), are equations computed externally (i.e., outside of the building simulator), and which are mainly used to evaluate the objective functions in the point determined by the simulator as well as to establish bounds on the variables. Finally, \(x_D\) are the independent decision variables of the problem (i.e., the insulation thicknesses of the external surfaces of the building), whereas \(x\) account for the remaining dependent variables. That is, we distinguish between independent decision variables \(x_D\) (independent variables) whose values must be optimized, and dependent variables \(x\) whose values are given once the decision variables (corresponding to the degrees of freedom of the problem) are fixed.

3.1.1 Simulation software encoded equations

The energy loads of the building are calculated using EnergyPlus v.8 [33–35], which is a commercial simulator that models energy and water use in buildings. EnergyPlus includes a set of simulation properties, calculated via user-configurable modular systems, that are integrated with a heat and mass balance-based simulation environment that considers variable time steps and input/output data structures oriented to facilitate third party module and interface development [34]. In mathematical terms, EnergyPlus contains a system of partial differential equations (PDE) that describe a set of energy balances. These PDEs model the energy consumption during a given time horizon.
The simulator requires the decision variables \( x_D \) to be fixed to a given value and then runs the calculations to provide as output the value of the remaining variables \( x \) (mainly, the energy consumed). Note that the simulator does not perform the optimization, but rather determines the value of \( x \) for a given value of \( x_D \).

### 3.1.2 Objective function equations

In the ensuing sections, we describe each block of objective function equations in detail. Note that the objective functions considered in this study are encoded externally (i.e., outside of the simulation program), which provides more flexibility to the approach.

### 3.1.3 Economic indicators

The economic performance of each building design alternative is quantified through the cost of the construction materials and the cost of the energy consumed for heating and cooling over the operational phase of the building. Hence, the final goal is to minimize the total cost (\( \text{COST}^{\text{TOT}} \)) [36–39], which is calculated as in Eq. (2).

\[
\text{COST}^{\text{TOT}} = \text{COST}^{\text{MAT}} + \text{COST}^{\text{EN}}
\]

(2)

Here, \( \text{COST}^{\text{MAT}} \) denotes the cost of the materials, whereas \( \text{COST}^{\text{EN}} \) accounts for the cost of the energy consumed over the operational phase of the building:

The cost of the construction materials, which is assumed to be paid the first year of the time horizon, is given by Eq. (3).

\[
\text{COST}^{\text{MAT}} = \sum_{k \in K} UCOST_k^{\text{MAT}} \cdot M_k
\]

(3)

Here \( UCOST_k \) is the unitary cost of raw material \( k \) (belonging to the set of raw materials \( K \)) and \( M_k \) is the corresponding mass of raw material \( k \).
The total economic cost of the energy required to cover the heating and cooling requirements of the building is given by:

\[
COST^{EN} = \sum_{n \in N} CONS_n \times UCOST^{EN} \times (1+ir)^n
\]

where \(CONS_n\) is the energy consumed for heating and cooling (which is considered to be constant for all the years) in year \(n\) (belonging to the set of years \(N\)), \(UCOST^{EN}\) is the current unitary energy cost (i.e., the unitary cost of energy at the start of the simulated time horizon) and \(ir\) is the yearly increase in the energy cost.

### 3.1.4 Environmental indicators

The environmental impact caused by the energy consumed and the construction materials is assessed through the Eco-indicator 99 (EI99) methodology [40,41], which is based on LCA principles. The EI99 covers three different damage categories (human health, ecosystem quality and resources), which include a total of 10 specific impact indicators. In this study, we consider individual indicators according to the EI99 report [40], which carry less uncertainty than the aggregated indicator. This is because the aggregated indicator suffers from the added uncertainty resulting from the weighting process of converting the individual indicators into an aggregated metric. We also report the values of the aggregated impact calculated according to the average weighting set and the hierarchic perspective. Particularly, the following impacts are considered: acidification & eutrophication, ecotoxicity, land occupation, carcinogenics, climate change, ionising radiation, ozone layer depletion, respiratory effects, fossil fuel extraction and mineral extraction. The total impact of the building in each impact category \(c\) (e.g. carcinogenics belonging to the set of categories \(C\)), denoted by \(IMP^{TOT}_c\), is calculated from the impact in category \(c\) associated to the construction materials of the building, which is given by \(IMP^{MAT}_c\), and the impact of the energy consumed over the operational phase, which is represented by \(IMP^{EN}_c\):
\begin{equation}
IMP_{c}^{\text{TOT}} = IMP_{c}^{\text{MAT}} + IMP_{c}^{\text{EN}} \\
\forall c \in \mathcal{C} \tag{5}
\end{equation}

The total impact of the building materials in impact category $c$ is determined via Eq. (6),

\begin{equation}
IMP_{c}^{\text{MAT}} = \sum_{k \in \mathcal{K}} UIMP_{kc}^{\text{MAT}} \cdot M_k \\
\forall c \in \mathcal{C} \tag{6}
\end{equation}

where $UIMP_{kc}^{\text{MAT}}$ is the impact in category $c$ per kilogram of component $k$ (an information available in environmental databases, such as the ecoinvent database version 3 [42]), and $M_k$ is the mass of material $k$.

The impact of heating and cooling is calculated using the following equation:

\begin{equation}
IMP_{c}^{\text{EN}} = UIMP_{c}^{\text{EN}} \cdot \sum_{n \in \mathcal{N}} CONS_n \\
\forall c \in \mathcal{C} \tag{7}
\end{equation}

Here $UIMP_{c}^{\text{EN}}$ is the impact in category $c$ per kWh of energy and $CONS_n$ is the energy consumed in the building in year $n$ for heating and cooling requirements.

3.2 Solution procedure

We solve problem SIMMOD combining dimensionality reduction and surrogate modelling. First, we apply sampling techniques to generate an initial set of solutions. This initial sample serves two main purposes, as it is used to: (i) apply the dimensionality reduction method, which will reduce the number of objectives in the original model; and (ii) build a surrogate model, which will expedite the optimization task. Finally, the surrogate model is optimized in the reduced set of objectives, yielding a set of optimal building designs (Pareto solutions). These Pareto points can be used in turn to improve the performance of the dimensionality reduction algorithm and the quality of the surrogate model, thereby leading to better solutions.

The algorithm (see Fig. 1) we propose is summarized next. Let $SOO$ be the set of objectives to be optimized.
0) Initialize the reduced set of objectives $RSO = \emptyset$, and the iteration counter $it = 0$.

1) Simulate a given number of building designs. Let $I$ be the set of solutions resulting from these simulations.

2) If $|RSO| = |SOO|$, stop: further reductions in the number of objectives are not possible and hence $I$ is the final set of optimal building designs. Else:

   1) If $it \neq 0$, make $SOO = RSO$, $it = 0$ and return to 2.1. Else, make $it = 1$ and:

      1) Apply the objective reduction method to set $I$. Update $RSO$ eliminating the redundant objectives.

      2) Build a surrogate model $SURMOD$ from solutions in set $I$.

      3) Use a MOO method to optimize the surrogate model $SURMOD$ considering objectives in $RSO$ (i.e., optimize model $RSUMOD$). Update $I$ so that it contains the resulting set of optimal solutions.

   2) End if.

3) End if.

Note that steps 2.1.1 and 2.1.2 can be applied in parallel. Each of the steps of the previous approach is explained in detail in the ensuing sections.

### 3.2.1 Generation of an initial sample

A set of solutions $I$ is generated by running different simulations with EnergyPlus using a parametric tool called JEPlus [43]. Specifically, JEPlus is used to generate a sample composed of $|I|$ different combinations of values of the decision variables $x_D$. These values of the decision variables are then fixed in EnergyPlus, which simulates the corresponding building designs and provides the values of the remaining variables $x$ (note that this is accomplished by solving the energy balances implemented in the simulator). Finally, the values of the objective functions $z_1(x, x_D)$ to $z_p(x, x_D)$ are determined from the values of the variables.
The samples serve two different purposes: (i) reduce the dimensionality of the problem; and (ii) construct a surrogate model that approximates the PDEs implemented in the building simulator.

### 3.2.2 Dimensionality reduction method

A dimensionality reduction analysis is carried out to eliminate redundant objectives. The model objectives are different in nature and their values may differ in several orders of magnitude, thereby causing numerical problems during dimensionality reduction. To overcome this, the solutions in the set $I$ are first normalized so they fall in the range 0-1. Then, a dimensionality reduction method is applied to eliminate redundant objectives. The overall strategy presented in section 3.2. can work with any dimensionality reduction method available in the literature [23,25]. Without loss of generality, however, we apply here an exhaustive exploration based on the work by Brockhoff and Zitzler [24]. This method seeks to replace the original set of objectives $SOO$ by a reduced subset of objectives $RSO$ that shows minimum delta approximation error ($\delta$). This concept is further clarified by means of Fig. 2, which depicts 4 Pareto optimal solutions (A,B,C,D) (i.e., no solution is dominated by any of the others). Assume that objective 4 is removed from the original set of objectives ($SOO = \{1, 2, 3, 4\}$), thus yielding a new reduced set of objectives ($RSO = \{1, 2, 3\}$). If we do this, the original dominance structure of the problem will be modified (i.e., solution C is dominated by solution B in the reduced set of objectives $RSO$, whereas in the original one this does not happen). In this context, it is possible to define a delta error associated with the approximation made (when removing subsets of objectives), which is given by the largest difference between the objective values (before and after removing objectives) that would prevent a change in the dominance structure (i.e., that would prevent that a Pareto optimal solution in the original set of objectives is dominated in the reduced set). In the case of $RSO$, the delta error is given by the difference between the value of objective 4 in solution B, and the value required to dominate solution C in the original space of objectives (i.e., $\delta = 0.25$). Now consider the reduced set resulting from removing objectives 2 and 3, while maintaining objectives 1 and 4 ($RSO' = \{1, 4\}$). As seen, this reduced set does not modify the dominance structure, since all the solutions are also Pareto
optimal in the reduced domain $RSO'$. In this case, we say that the reduced objective set ($RSO' = \{1, 4\}$) is non-conflicting with the original one ($SOO = \{1, 2, 3, 4\}$). Hence, the goal of objective reduction is to identify the minimum number of objectives entailing a zero delta error, or the minimum delta error for a given number of objectives.

3.2.3 Building the surrogate model

The PDE model $SIMMOD$ is complex and leads to large CPU times associated with the solution of the PDEs. Furthermore, when this model is coupled with an optimization algorithm, we need to calculate its derivatives. This is a very time consuming task that can show inherent numerical noise, thus leading to poor numerical performance [44]. In order to simplify the calculations and enhance the robustness of the optimization algorithm, we build a surrogate model $SURMOD$ to approximate the original model $SIMMOD$ and to estimate the $p$ explicit objective functions. Hence, the optimization algorithm minimizes the decision variables by interrogating the surrogate model (rather than the original model) as follows:

$$(SURMOD) \quad \min_{\mathbf{x}_D} \quad z = \{f_1^{SUR}(\mathbf{x}_D), \ldots, f_p^{SUR}(\mathbf{x}_D)\}$$

The functions of the surrogate model, $f_{ob}^{SUR}(\cdot)$, are obtained from the initial sample $I$ generated, as described in Section 3.2.2. In particular, and without loss of generality, the interpolated value at a query point is based on a cubic spline interpolation (using not-a-knot end conditions) of the values at neighbouring grid points in each respective dimension. Interpolation by cubic splines ensures $C^2$ continuity, which is very important when optimizing the resulting model. Other interpolation approaches (i.e. linear interpolation is just $C^0$, nearest point is discontinuous and cubic only ensure $C^1$ continuity) could be also applied in this step of the method.

In order to get an accurate interpolation, it is necessary to generate a 5-dimensional grid. A sufficient number of points are required to ensure a satisfactory level of accuracy in the predictions while at the same time improving the numerical performance of the optimization algorithm by avoiding the direct use of the simulation model.
3.2.4 MOO of the surrogate model in the reduced domain

In this step of the algorithm, we aim to identify the optimal building designs that minimize simultaneously the objective functions in vector $z$. For this, the MOO problem $SURMOD$ is solved in a reduced domain of objectives $RSO \subseteq SOO$, thus giving rise to problem $RSUMOD$:

\[
(RSUMOD) \quad \min_{z'} = \{ f_{ob}^{SUR}(x_D) \mid ob \in RSO \} 
\]

(9)

Note that model $RSUMOD$ makes use of both, the surrogate model $SURMOD$ obtained in step 2.1.2 of the algorithm and the reduced set of objectives $RSO$ identified in step 2.1.1.

The solution of multi-objective optimization problems like $RSUMOD$ is given by a set of Pareto points representing the optimal trade-off between conflicting objectives [9,45]. These Pareto solutions feature the property that it is not possible to find another solution that improves any of them in one objective without worsening at least one of the others. In mathematical terms, $x^* \in X$ is a Pareto optimal solution if there does not exist any $x' \in X$ such that $f_{ob}^{SUR}(x') \leq f_{ob}^{SUR}(x^*)$ for all $ob \in RSO$, and $f_{ob'}^{SUR}(x^*) < f_{ob'}^{SUR}(x^*)$ for some $ob' \in RSO$. If $x^*$ is Pareto optimal, then $z'(x^*)$ is called non-dominated point or efficient point.

In order to solve problem $RSUMOD$ and obtain a set of Pareto optimal solutions, one can use any MOO method available in the literature [46–49]. Without loss of generality, here we use the epsilon constraint method [50,51], which consists of calculating a set of auxiliary single-objective problems in which one objective is kept as main criterion while the others are transferred to auxiliary constraints and limited within allowable bounds.

3.2.5 Remarks

- The initial sample $I$ is not the result of any optimization process, but rather the outcome of evaluating model $SIMMOD$ in different points of the space of the decision variables.
• The CPU time of the objective reduction approach is rather sensitive to the number of solutions, but the outcome itself does not change significantly with an increasing number of points (i.e., sample size).

• Different surrogate models might be used to approximate the solution of the simulation model SIMMOD, including kriging or linear, thin-plate and splines interpolations [52,53].

4 Case study

The capabilities of the proposed approach are illustrated through the optimization of the insulation thickness of a house-like cubicle considering both economic and environmental concerns. The decision variables of the problem are the insulation thicknesses of the external surfaces of the building.

4.1 Cubicle description

The model of the cubicle is based on real life cubicles built by the research group GREA in Puigverd, (Lleida, Spain). Several studies before are based on these cubicle models [20,54,55]. The cubicles considered in the present study show identical dimensions (five plane walls with 2.4×2.4×0.15m), and the same construction systems, but differ in the insulation thickness implemented (polyurethane in this case study, see Table 1 for its physical properties).

The cubicles show a conventional Mediterranean construction system (Fig. 3). Four mortar pillars with reinforcing bars allocated in each corner of the building configure the structure of the cubicle. The walls of the cubicle, which are identical from one model to the other except for the insulation thickness, are configured with 6 layers of different materials: an exterior cement mortar cover (0.1m), a hollow bricks structure (0.07m), a 0.05m air chamber, the polyurethane layer (insulation) whose thickness varies depending on the case, a perforated bricks structure (0.14m) and the interior cover, which is a plaster plastering layer (0.01m). A concrete base of 3×3m with reinforcing bars configure the floor, which is in contact with the ground. On the
other hand, the roof contains a structure of concrete precast beams (0.05m) and 0.05m of concrete slab. The internal finish is a plaster plastering layer (0.01m). The insulation material is placed over the concrete, and it is protected with a cement mortar layer (0.1m) with a slope of 3% and a double asphalt membrane (0.05m). The construction materials of the cubicles are displayed in Table 2. Data for the case study were retrieved from the LIDER [56] and ITeC [57] databases. A reference cubicle with no insulation is also considered [54,58] for comparison purposes.

Heating and cooling demands are supplied by a heat pump with a COP of 3. The electricity consumed is calculated by dividing the demand by the COP of the heat pump.

4.2 Model specifications

For the physical modelling EnergyPlus is implemented. This software mainly requires four modelling modules. The first one includes the building physical description (construction system, materials, geometry and internal distribution), and for the energy simulations the operational spaces can be defined as thermal units. The second module defines the HVAC systems including the selection of the equipment, power, efficiency and the operation scheduling for the set points. The third module defines the internal loads (people occupation and activity, electronic devises and miscellaneous). Finally, module four allows to define the weather conditions including temperature, solar radiation, wind speed and direction and humidity (defined using time steps per hour). For more details see [34].

For the cubicle simulation, the following specifications are used. The construction system is the one defined in Section 4.1. The range of insulation thickness considered varies from 0.01 to 0.21m of insulation. As will be later discussed in more detail, the insulation thickness is first varied uniformly (i.e., all the walls with the same thickness), and then considering different thicknesses for the five external surfaces of the cubicles. Heating and cooling demands are supplied by a heat pump with a COP of 3, and an internal set point temperature of 24°C is fixed
for the whole year [54,55]. Neither doors nor windows are included in the model. No mechanical or natural ventilation is used, but a fixed infiltration rate of 0.12 ACH (air changes per hour) [59] is assumed. There is no internal mass, and no human occupancy is considered. A building lifetime of 20 years is assumed [60,61]. The investment in construction materials is paid the first year of the time horizon. As for the electricity, a cost of 0.16 €/kWh [62] is considered with a yearly increase in cost of 5%.

The weather conditions of the simulations are given by the location of the cubicles, which corresponds to a continental Mediterranean climate characterized by moderate cold winters, dry hot summers and significant daily temperature oscillations between day and night [63].

The environmental impact of each cubicle alternative, quantified via LCA principles, takes into account the manufacturing, operational and dismantling phases. In particular, the 10 impact categories considered in the EI99 methodology, along with the EI99 itself, are studied. Table 3 summarizes the impact per kilogram of material used, whereas Table 4 presents environmental data of the Spanish electricity market. This information has been retrieved from the ecoinvent database [42].

5 Results and discussions

5.1 Initial simulation results
An initial sample of solutions is first obtained by simulating different cubicle designs. We define 6 insulation thicknesses (i.e. 0.01, 0.03, 0.06, 0.09, 0.15 and 0.21m) and generate 7776 points by means of JEPlus (number of alternatives raised to the number of walls, that is, 6^5), each with a different combination of external building surfaces. We then simulate the resulting cubicles in EnergyPlus to obtain sample I containing 7776 solutions. Note that for these solutions the building properties and weather conditions are the same, but the insulation thicknesses and consequently the energy consumption and objective functions values are different.

Fig. 4 shows a parallel coordinates plot corresponding to the solutions (belonging to I) with the same insulation thicknesses in all their external surfaces (i.e., that is, the solution with all the thickness values equal to 0.01 m, the one with all of them equal to 0.03 m, and so on). Each line in the plot represents a different solution. As seen in the figure, impacts related with ecotoxicity, land occupation, ionizing radiation, ozone layer depletion and mineral extraction tend to decrease with the insulation thickness of the cubicles, while the other impacts behave in an opposite manner. This suggests the existence of objectives showing similar behavior and which might be removed from the pool without altering the dominance structure of the problem.

5.2 Objective reduction

The cubicle solutions generated in the previous step (i.e., solutions in the set I) are normalized and then used to identify redundant objectives by means of the exhaustive exploration dimensionality reduction approach presented in Section 3.2.2. In this particular case, we force the economic performance to be always part of the reduced set of objectives RSO. The approach was implemented in GAMS in a computer HP Compaq Pro 6300 SFF with an Intel Core Processor 3.30 GHz and 3.88 GB of RAM. The required CPU time was around 120 seconds.

Fig. 5 shows the minimum delta error achieved for a decreasing number of objectives retained. Note that different combinations of objectives can be removed for a given reduction in size (for
a given cardinality of the set \(|RSO|\), and each such combination will lead to a different delta error. As seen, 3 objectives suffice to keep the original Pareto structure unaltered (i.e., delta error = 0).

Table 5 displays the delta error (expressed in \(\%\)) for all possible sets of three objectives kept sorted in lexicographic order. As seen, two out of 55 combinations (i.e., the triples: economic objective, carcinogenics, ionising radiation; and economic objective, carcinogenics, ozone layer depletion) present a delta error of 0. These results are consistent with Fig. 4, where we already observed that several indicators behave similarly.

Carcinogenics and ionising radiation are finally selected along with the cost as the reduced set of objectives to be minimized (i.e., \(RSO = \{\text{Cost, Carcinogenics, Ionising radiation}\}\)). Note that the delta error of the couple “EI99 - Economic cost” is 10.64. Hence, it is clear that the use of the aggregated EI99 as unique environmental objective may leave Pareto points out of the analysis. This is an important finding that highlights the need to avoid aggregated metrics and work instead with disaggregated environmental metrics in the optimization. In fact, even when considering a third environmental indicator along with the EI99 and cost, the delta error is still above zero (Table 6).
5.3 Optimization with a surrogate model

The surrogate model SURMOD is implemented in Matlab R2015a [62] using the 7,776 cubicle solutions of sample I generated in the first step. A multivariate cubic spline interpolation, which uses piecewise cubic polynomials, is applied to build this surrogate model, for which analytical derivatives can be obtained. The use of low-order polynomials is especially attractive for surface fitting because they reduce the numerical instabilities that arise with higher degree polynomials. The most compelling reason for their use is their C2 continuity, which guarantees continuous first and second derivatives across all polynomial segments. To optimize the surrogate we access the state-of-the-art NLP solvers through the MATLAB-TOMLAB [63] optimization environment. TOMLAB allows us to standardize the model definition and interfaces with the main optimization solvers regardless of the different syntax (i.e., it is not required a specific inter-face routine for each optimization solver). In addition, for the definition of the optimization problem we have developed a homemade modeling system with indexing capacities and interfaced with the MATLAB-TOMLAB optimization environment. Building the SURMOD takes approximately 77,760 seconds in a computer HP Compaq Pro 6300 SFF with an Intel Core Processor 3.30 GHz and 3.88 GB of RAM. Some of the objectives in SURMOD are eliminated according to the output of the objective reduction algorithm. This gives rise to multi-objective surrogate model RSUMOD, which is then solved using the epsilon-constraint method. 25 epsilon parameters values were defined for each objective, leading to 625 NLPs (i.e., 25|RSO|-1 = 252), which were solved by CONOPT version 3.10. The algorithm takes 2,500 seconds to solve the 625 NLPs, which leads to a total CPU time of 80,260 seconds (around 1 day), considering also the time required for the construction of the surrogate model. Note that the time required to optimize the system using EnergyPlus would be much higher than the one associated to the surrogate model. More precisely, using CONOPT, each NLP requires on average 17 iterations to be solved, each of which needs 6 evaluations of the objective functions. If we consider 625 NLPs, 17 iterations per NLP, 6 evaluations per iteration and a
simulation time of 10 seconds for each simulation in EnergyPlus, the whole process would take 637,500 seconds (around 1 week). Hence, the CPU time is reduced more than 7 times (i.e., approximately 8 times), compared to the direct optimization of the simulation software. Moreover, this reduction in time in the optimization task might be much more significant for more complex building models. Note also that in addition to the reduction in time, we benefit from a simplified analysis of the Pareto solutions that focuses on key environmental metrics, thereby avoiding the need to study all of them simultaneously.

At this point of the overall algorithm, the Pareto solutions obtained can be used in both, the dimensionality reduction and the construction of the surrogate model, in an attempt to further improve the quality of the final set of solutions. However, in this case study this step is not required, since a significant reduction in the number of objectives is achieved in the first iteration (i.e. RSO contains only 3 objectives).

5.4 MOO solutions

After conducting the optimization with the surrogate model we obtain 19 different Pareto solutions (Fig.6) (we solve 625 NLPs, 48 render feasible, and within this group of solutions there are 29 repeated solutions and 19 unrepeated points). In these solutions, the insulation thickness of North, East and West walls vary from 0.06 to 0.21 m, that of the South from 0.04 to 0.2 m and that of the roof from 0.07 to 0.21 m.

The minimum cost solution has 0.08 m of insulation thickness in the North, East and West walls, and 0.07 and 0.09 m in the South and roof, respectively. The optimal solution from the perspective of carcinogenic effects on humans has thinner insulation thicknesses in all of the external surfaces (0.06 m in the North, East and West and 0.04 and 0.07m in the South and the roof, respectively). The solution with minimum impact on human health caused by ionizing radiation shows thicker insulation thicknesses (i.e., 0.20 m in the South facade and 0.21 m in all the other surfaces). This solution is the worst from the standpoints of impact in carcinogenics and economic performance.
For a better understanding of the tradeoff between the objectives, Fig. 6 shows the 19 optimal solutions of the problem in a three dimensional space along with the two dimensional projections onto 2-D subspaces. When solutions are projected onto the bi-criteria space considering objectives “carcinogens” and “cost”, only 4 of them keep their Pareto optimality condition (i.e., the remaining 15 solutions that are Pareto optimal in the 3 dimensional space are dominated when only these two objectives are considered). In the bi-criteria space “cost” vs “ionising radiation”, 16 solutions keep their Pareto optimality condition and 3 become dominated. Finally, the original 19 Pareto optimal solutions (in the 3 dimensional space) are also Pareto optimal in the space of the two environmental impacts (i.e., “carcinogens” and “ionising radiation”). These results reinforce the idea that selecting a proper set of objectives in the objective reduction step is crucial to avoid losing potential Pareto optimal solutions.

Table 7 shows the different extreme optimal solutions and their improvements with respect to the base case (without insulation). For instance, the use of insulation can lead to savings between 800 and 1400€ (i.e., between 16 and 26%) in total cost. This means that the cost of the insulation material is compensated by the savings in the energy consumed. Regarding the impact in ionising radiation, the use of appropriate insulation allows for an improvement between 38 and 51%. In our case study, this impact is strongly dependant on the electricity consumption, and thus, on the electricity mix of the country. Consequently, the minimum ionising radiation solution (which consumes less electricity) reduces more than twice this indicator compared to the base case solution (with high electricity consumption). Conversely, not all the extreme solutions improve the base case in terms of carcinogens impact. In particular, the minimum ionising radiation solution involves an impact 9% higher than that of the base case in this category. The carcinogenic impact caused by the polyurethane is relatively important. Thus, when considering cubicles with thick insulation like this one (i.e., between 0.2 and 0.21 m in each external surface), the carcinogens impact increases when compared to the
base case. Despite this, the results reinforce the general idea that selecting a proper insulation thickness leads to significant reductions in economic cost and environmental impact.

The recommended insulation values of the regulatory framework about buildings basic requirements of safety and habitability are not close to the optimal results obtained in the present study [7]. In the location of Lleida, the Spanish law requires a thermal transmittance of 0.66 W/m²·K for the external facade walls and 0.38 W/m²·K for the roof. However, the results of the present study suggest lower thermal transmittance values of between 0.33 and 0.26 W/m²·K for the best economic solution in the facades and 0.285 W/m²·K in the roof. The solution showing better environmental performance from the point of view of ionising radiation suggests an insulation with a thermal transmittance of 0.133 W/m²·K in facades and roofs. To attain the solution with lower values of carcinogenics, the results of the present study suggest thermal transmittances of between 0.37 to 0.44 W/m²·K in facades and 0.33 in the roof.

A cubicle constructed according to the Spanish law requirements and evaluated through the sated methodology presents a higher price compared to the optimal solutions attained (between a 3% and 10% higher depending on the solution). This cubicle also presents higher values of ionizing radiation compared to the optimal solutions of the present study (between a 10 and a 24% higher depending on the solution) and also higher values of carcinogenics (between a 2% and a 7%).

6 Conclusions

In this work we have presented a systematic tool to effectively identify optimal building designs according to economic and environmental criteria that combines: (i) an objective reduction method that identifies redundant environmental metrics; and (ii) a surrogate modelling approach that expedites the optimization task by reducing the time required to estimate the energy consumed by the building.
The tool presented, which can be easily adapted to solve other MOO problem with similar features, was applied to a case study of a house-like cubicle where the insulation thicknesses of the external surfaces were optimized in order to minimize the cost and several environmental impacts assessed through LCA principles. Numerical results show that 3 objectives suffice to optimize the system while keeping its original dominance structure. We showed as well that the bi-objective optimization of the cost together with the widely used aggregated EI99 might change the problem’s structure, with the associated potential risk of losing solutions that are Pareto optimal in the original space of objectives.

Results also demonstrate that the surrogate model notably reduces the computational burden of the optimization task, thereby expediting the overall solution time (i.e., 8 times). This reduction in time may become more significant as the complexity of the building model considered increases.

The results of the case study illustrate how significant improvements can be achieved with respect to the base case (cubicle without insulation), when the appropriate insulation is used. In particular, the cost can be reduced by 26%, the carcinogenics impact can be mitigated by 17%, and the ionising radiation impact can be decreased by 51%.

The methodology presented here is intended to promote optimal economic solutions for energy efficiency in buildings, while also minimizing their environmental impact. This tool can guide decision-makers towards the adoption of more sustainable designs as well as policy-makers during the development of more effective regulations for improving the economic and environmental performance in the building sector.

7 Acknowledgments

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8 References


Fig. 1.

Algorithm summarizing the proposed optimization strategy.
Fig. 2. Dominance structure for the original set of objectives SOO. No solution dominates any of the others in the space of all the objectives, thus they are weakly efficient. RSO modifies the dominance structure ($\delta = 0.25$), however RSO' does not (all the solutions are still optimal in the reduced set of objectives).
Fig. 3.

Construction profile of the experimental cubicles in Puigverd de Lleida (Spain).
Fig. 4.

Parallel coordinate plot where the different objectives are presented in the horizontal axis and in the vertical one there are the normalized values of each solution in each objective. Only solutions of sample I entailing the same insulation thickness in all the external surfaces are depicted.
Fig. 5.

Minimum delta error achieved by sets with a given number of objectives.

Fig. 6

Pareto optimal solutions in the three dimensional space (3 objectives) and their corresponding projections on the different two dimensional spaces (2 objectives). As the insulation thickness of the optimal solutions increases, the
cost and the impact of carcinogens on human health tend to decrease, while the impact of ionising radiation on human health tends to increase.

Table 1.

Properties of the insulation material.

<table>
<thead>
<tr>
<th>Insulation material</th>
<th>Density (kg/m³)</th>
<th>Thermal conductivity (W/(m·K))</th>
<th>Specific heat (J/(kg·K))</th>
<th>Cost (€/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polyurethane</td>
<td>45</td>
<td>0.027</td>
<td>1,000</td>
<td>175</td>
</tr>
</tbody>
</table>

Table 2.

Inventory list of the materials and quantities used for the building construction and their corresponding cost. Since the amount of polyurethane (insulation material) varies from one case to another as a result of the value of the decision variables, a cubicle with 0.01m of polyurethane in all exterior surfaces is considered and included in the inventory list for illustrative purposes.

<table>
<thead>
<tr>
<th>Component</th>
<th>Used Mass (kg)</th>
<th>Cost (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick</td>
<td>5,456</td>
<td>287</td>
</tr>
<tr>
<td>Base plaster</td>
<td>518</td>
<td>43</td>
</tr>
<tr>
<td>Cement mortar</td>
<td>608</td>
<td>30</td>
</tr>
<tr>
<td>Steel bars</td>
<td>262</td>
<td>157</td>
</tr>
<tr>
<td>Concrete</td>
<td>1,240</td>
<td>44</td>
</tr>
<tr>
<td>In-floor bricks</td>
<td>1,770</td>
<td>62</td>
</tr>
<tr>
<td>Asphalt</td>
<td>153</td>
<td>317</td>
</tr>
<tr>
<td>PU (0.01m)</td>
<td>20</td>
<td>79</td>
</tr>
</tbody>
</table>
Table 3.

Inventory list of the materials and quantities used for the building construction and their corresponding environmental impacts. As an illustrative example, the amount of polyurethane (PU) used in a cubicle with 1cm of insulation thickness in all of their surfaces is also displayed.

<table>
<thead>
<tr>
<th>Component</th>
<th>Name in the data base Eco Invent</th>
<th>Ecosystem quality (PDF<em>m2</em>yr/kg)</th>
<th>Human Health (Daly/kg)</th>
<th>Resources (MJ/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Acidification &amp; eutrophication</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Ecotoxicity</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>Land occupation</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Carcinogens</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td>Climate change</td>
<td></td>
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<td></td>
<td></td>
<td>Ionising radiation</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Ozone layer depletion</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Respiratory effects</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>Fossil fuels</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mineral extraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brick</td>
<td>market for brick, at plant, GLO [kg]</td>
<td>3.73·10^5</td>
<td>2.1·10^5</td>
<td>3.6·10^5</td>
</tr>
<tr>
<td>Base plaster</td>
<td>market for base plaster, GLO [kg]</td>
<td>5.3·10^5</td>
<td>4.9·10^5</td>
<td>7.1·10^5</td>
</tr>
<tr>
<td>Cement mortar</td>
<td>market for cement mortar, GLO [kg]</td>
<td>6.0·10^5</td>
<td>6.5·10^5</td>
<td>8.3·10^5</td>
</tr>
<tr>
<td>Steel bars</td>
<td>market for section bar rolling, steel, GLO [kg]</td>
<td>3.1·10^5</td>
<td>1.3·10^4</td>
<td>5.1·10^4</td>
</tr>
<tr>
<td>Concrete</td>
<td>market for concrete, normal, GLO [m3]</td>
<td>7.5·10^2</td>
<td>7.8·10^2</td>
<td>5.5·10^2</td>
</tr>
<tr>
<td>In-floor bricks</td>
<td>market for concrete roof tile, GLO [kg]</td>
<td>5.8·10^5</td>
<td>8.6·10^5</td>
<td>5.8·10^5</td>
</tr>
<tr>
<td>Asphalt</td>
<td>market for mastic asphalt, GLO [kg]</td>
<td>7.4·10^5</td>
<td>8.0·10^5</td>
<td>1.4·10^5</td>
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<tr>
<td>Polyurethane</td>
<td>market for polyurethane, rigid foam, GLO [kg]</td>
<td>8.9·10^4</td>
<td>8.4·10^4</td>
<td>2.4·10^4</td>
</tr>
<tr>
<td>Disposal bricks</td>
<td>market for waste brick, GLO [kg]</td>
<td>9.3·10^6</td>
<td>2.4·10^6</td>
<td>-4.9·10^6</td>
</tr>
<tr>
<td>Disposal plaster</td>
<td>market for waste mineral plaster, GLO [kg]</td>
<td>6.7·10^6</td>
<td>8.0·10^6</td>
<td>-1.1·10^7</td>
</tr>
</tbody>
</table>
Disposal mortar | market for waste cement in concrete and mortar, GLO [kg] | $1.1 \cdot 10^5$ | $3.5 \cdot 10^3$ | $1.4 \cdot 10^5$ | $1.5 \cdot 10^9$ | $5.1 \cdot 10^{11}$ | $6.0 \cdot 10^{13}$ | $5.3 \cdot 10^{14}$ | $8.0 \cdot 10^{10}$ | $6.5 \cdot 10^{10}$ | $7.4 \cdot 10^{12}$
Disposal concrete + steel bars | market for waste reinforced concrete, GLO [kg] | $9.4 \cdot 10^6$ | $3.5 \cdot 10^4$ | $5.8 \cdot 10^6$ | $3.3 \cdot 10^{10}$ | $3.9 \cdot 10^{11}$ | $5.0 \cdot 10^{13}$ | $4.6 \cdot 10^{14}$ | $7.3 \cdot 10^{10}$ | $4.6 \cdot 10^{10}$ | $6.2 \cdot 10^{12}$
Disposal in-floor bricks | market for waste concrete, not reinforced, GLO [kg] | $7.9 \cdot 10^6$ | $1.1 \cdot 10^5$ | $4.0 \cdot 10^6$ | $2.6 \cdot 10^{10}$ | $3.2 \cdot 10^{11}$ | $4.3 \cdot 10^{13}$ | $3.4 \cdot 10^{14}$ | $6.8 \cdot 10^{10}$ | $4.0 \cdot 10^{10}$ | $4.0 \cdot 10^{12}$
Disposal asphalt | market for waste asphalt, GLO [kg] | $7.9 \cdot 10^6$ | $1.8 \cdot 10^5$ | $2.7 \cdot 10^5$ | $5.6 \cdot 10^{11}$ | $5.0 \cdot 10^{13}$ | $4.7 \cdot 10^{13}$ | $4.4 \cdot 10^{14}$ | $2.5 \cdot 10^{10}$ | $5.6 \cdot 10^{10}$ | $8.1 \cdot 10^{12}$
Disposal PU | market for waste polyurethane, GLO [kg] | $1.0 \cdot 10^4$ | $7.1 \cdot 10^4$ | $3.7 \cdot 10^5$ | $2.7 \cdot 10^8$ | $2.8 \cdot 10^9$ | $1.6 \cdot 10^{12}$ | $1.6 \cdot 10^{13}$ | $2.0 \cdot 10^9$ | $2.1 \cdot 10^9$ | $2.7 \cdot 10^{11}$

Table 4.

Environmental data per kWh of electricity in Spain (this dataset has been extrapolated from year 2008 to the year 2014).

<table>
<thead>
<tr>
<th>Component</th>
<th>Ecosystem quality (PDF<em>m²</em>yr/kWh)</th>
<th>Human Health (Daly/kWh)</th>
<th>Resources (MJ/kWh)</th>
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<tbody>
<tr>
<td></td>
<td>Acidification &amp; eutrophication</td>
<td>Ecotoxicity</td>
<td>Land occupation</td>
</tr>
<tr>
<td>Electricity (Spain)</td>
<td>$1.133 \cdot 10^4$</td>
<td>$4.03 \cdot 10^4$</td>
<td>$9.47 \cdot 10^5$</td>
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</table>
Delta error for all possible combinations of three objectives. These combinations are always formed by the economic objective (i.e., cost, Obj. 1) and two environmental objectives (Obj. 2 and Obj. 3). Here, 1 is total cost, 2 is acidification & eutrophication, 3 is ecotoxicity, 4 is land occupation, 5 is carcinogens, 6 is climate change, 7 is ionising radiation, 8 is ozone layer depletion, 9 is respiratory effects, 10 is fossil fuels, 11 is mineral extraction and 12 is the EI99 aggregated.

<table>
<thead>
<tr>
<th>Obj 1</th>
<th>Obj 2</th>
<th>Obj 3</th>
<th>Delta error [%]</th>
<th>Obj 1</th>
<th>Obj 2</th>
<th>Obj 3</th>
<th>Delta error [%]</th>
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Table 6.

Delta error for all combinations of three objectives considering cost (Obj. 1) and the EI99 (Obj. 2) along with different environmental midpoint indicators (Obj. 3). Here, 1 is cost, 2 is EI99, 3 is acidification & eutrophication, 4 is ecotoxicity, 5 is land occupation, 6 is carcinogens, 7 is climate change, 8 is ionising radiation, 9 is ozone layer depletion, 10 is respiratory effects, 11 is fossil fuels and 12 is mineral extraction.

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Table 7.

Comparison of the base case and the extreme optimal solutions. In the table, E, N, S, W, R are East, North, South, West and Roof and the attached numbers denote the thickness of insulation of the corresponding surface in cm (i.e. E8 is 0.08m of polyurethane in the East wall).

| Cubicle model | Economic cost (€) | Carcinogenics (DALYS) | Ionising radiation (DALYS) | Improvement (%) | Economic Carcinogenics Ionising radiation |
|---------------|------------------|------------------------|---------------------------|----------------|-----------------|------------------|
| Base case     | 5,485.24         | 2.53·10^-5            | 1.08·10^-6                | 0              | 0               | 0                |
| Economic      | 4,067.27         | 2.13·10^-5            | 6.21·10^-7                | 25.9           | 15.7            | 42.4             |
| Carcinogenics | 4,123.63         | 2.09·10^-5            | 6.68·10^-7                | 24.8           | 17.3            | 38.0             |
| Ionising radiation | 4,625.71 | 2.76·10^-5            | 5.24·10^-7                | 15.7           | -9.2            | 51.4             |