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A SIMPLE METHOD TO PREDICT ELASTIC SETTLEMENTS IN FOUNDATIONS RESTING ON TWO SOILS OF DIFFERING DEFORMABILITY

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Abstract

This paper shows the analysis results obtained from more than 200 finite element method (FEM) models used to calculate the settlement of a foundation resting on two soils of differing deformability. The analysis considers such different parameters as the foundation geometry, the percentage of each soil in contact with the foundation base and the ratio of the soils' elastic moduli. From the described analysis it is concluded that the maximum settlement of the foundation, calculated by assuming that the foundation is completely resting on the most deformable soil, can be correlated with the settlement calculated by FEM models through a correction coefficient named "settlement reduction factor" (α). As a consequence, a novel expression is proposed for calculating the real settlement of a foundation resting on two soils of different deformability with maximum errors lower than 1.57%, as demonstrated by the statistical analysis carried out. A guide for the application of the proposed simple method is also explained in the paper. Finally, the proposed methodology has been validated using settlement data from an instrumented foundation, indicating that this is a simple, reliable and quick method which allows the computation of the maximum elastic settlement of a raft foundation, evaluate its suitability and optimize its selection process.

Keywords: foundations; soil/structure interaction; settlement; elasticity; finite-element modelling.

1. Introduction.

One of the main causes of structural damage and/or failure is excessive deformation of the soil when the ground surface is loaded, which manifests itself as foundation settlement. When the foundation rests on two different soils it may be affected by differential settlements which, under similar conditions, are more pronounced when the difference in soil deformability is greater. If maximum allowable settlements and angular distortions are exceeded, damage and loss of functionality in the structures occurs. As a consequence, a good understanding of these settlements and the accurate determination of their magnitude is required for optimizing the foundation selection process.

During the design or construction stages it is common to find soils with different stress-strain behaviour at the base of the foundation. This can be mainly caused, among other situations, by: a) a simple stratigraphic contact between two different lithologies (e.g. in subvertical dipping formations); b) mechanical contacts caused by faults; c) foundations resting on hillside embankments (partially founded on the fill material and partially on the in situ excavated material); and d) local changes in the deformational properties of soil induced by water content variation or a partial soil improvement under the foundation. In these cases, a deep foundation, or a raft foundation (reinforced concrete slab), which is able to absorb the ground heterogeneities and to produce uniform ground settlements, can be adopted as a solution instead of isolated footings. However, if finite element method (FEM) tools are not available, the manual calculation of settlements is difficult. As a consequence, the raft foundation solution is usually discarded based on qualitative justifications, with the consequent adoption of a deep foundation solution obviously resulting in an important cost increase.

Several authors have developed approximate solutions to this complex geotechnical problem. Votyakov (1964); Onopa *et al.* (1983); Shadunts & Marinichev (2003) studied the behaviour of particular structures founded on soils of differing deformability. Other authors (Malikova, 1979; Sheinin *et al.*, 2006) have proposed numerical approaches to solving this problem, allowing the computation of ground settlement at any point on the foundation. These algorithms are based on the modulus of soil reaction (also referred to as the subgrade modulus, K_s) and as a consequence their manual application is not straightforward. Bezvolev (2002) developed a procedure for calculating the deformations under a foundation resting on a multilayer terrain assuming nonlinear elasto-plastic soil behavior. Gazetas (1980) also proposed an analytical-numerical formulation base for the evaluation of settlements in multilayer soils, where the heterogeneity is vertical and the soil at the base of the foundation is homogenous. Denis *et al.* (2011) analysed the effect of longitudinal soil variability on a continuous spread footing from a qualitative point of view, with a method based on the modulus of soil reaction (K_s). Although more sophisticated methods that attempt to address the weaknesses of the method of modulus of soil reaction have been developed (e.g. (Shukla & Chandra, 1996)), unfortunately, their use is not as widespread as the latter.

It is therefore of interest to study this problem using alternative methods to those based on the modulus of soil reaction (K_s), and to analyse a significant number of cases in order to obtain more realistic results. Furthermore, since sophisticated modelling techniques such as FEM are sometimes not available, for a variety of reasons, for situations in which the suitability of raft foundations resting on two soils of differing deformability has to be checked, a simpler alternative method is proposed in this work.

In this paper, the suitability of raft foundations (i.e. slabs) in cases where two different soils are found at the base of the foundation is analysed using ANSYS+CIVILFEM v.11 finite-element method software. This program allows the settlements that affect a foundation structure to be modelled. The results obtained allowed a global settlement calculation method to be proposed for calculating and checking the suitability of reinforced concrete slab foundations under different situations, based only on settlement values.

The paper is organised as follows. Section 2 includes a short introduction to settlement calculation in these cases. Basic concepts, parameters used and the main results of the modelling of settlements in a raft foundation placed over two soils of differing deformability are included in section 3. Then, Section 4 is devoted to the discussion of the main results and the presentation of the proposed method for settlement prediction. Two special cases are explored in section 5. Section 6 shows a validation of the proposed formulas through the analysis of a real case study. Finally, the main conclusions are summarised in Section 7.

2. Calculation of settlements under foundations resting on two soils of differing deformability

Settlement analysis of a slab founded on soils with different stress-strain behaviour is not a simple problem to resolve, except for the cases in which it is modelled using FEM. Although methods based on the modulus of soil reaction (K_s) are used worldwide, they do not provide entirely satisfactory results, mainly due to difficulties in choosing the proper value for each case. In this method, subsoil is replaced by fictitious springs whose stiffness is equal to K_s. This is not an intrinsic soil parameter because it depends on the geometry of the foundation and the problem being considered. The most important deficiencies of methods based on the modulus of soil reaction are that a displacement discontinuity appears between the loaded and the unloaded part of the foundation surface (Imanzadeh *et al.*, 2013), and that the model cannot transmit the shear stresses which are derived from the lack of spring coupling (Brown, 1969a; Daloglu & Vallabhan, 2000; Stavridis, 2002; Avramidis & Morfidis, 2006; Imanzadeh *et al.*, 2013). Terzaghi (1955) even indicated that these methods should not be used for the purpose of estimating settlements.

FEM software is now used profusely, but unfortunately it is a tool which is not present or not applied to foundation design by a high percentage of companies or professionals belonging to the engineering and building industries. Traditional analytical settlement calculation methods based on elastic models (referred to as “manual methods”, from now on) allow the easy calculation of settlement in a homogeneous, isotropic and perfectly elastic layer and are only valid when the foundation rests on a single soil type, but not in the case of a foundation resting on two soils of different deformability. However, these manual methods (e.g. Schleicher, 1926; Mayne & Poulos, 1999) could be conveniently modified based on the results provided by more exact modelling using FEM, and a more accurate method may be proposed for calculating settlements in foundations resting on two soils of differing deformability. When the foundation rests on two different soils the maximum true settlement will be lower than that calculated considering only the most deformable soil’s parameters. As a consequence, one of the main aims of this paper is to state if both parameters (i.e. the true maximum settlements and the settlement calculated considering the whole foundation resting over the most deformable soil) are related, and then to propose an expression for calculating the true settlement in these cases, based on the aforementioned classic settlement calculation methods.

3. Settlement modelling

3.1. Basis of the modelling

Based on elastic methods, the elastic settlement (s) in the corner of a rectangular foundation of dimensions L (length) x l (width) resting on the Boussinesq's half-space can be computed following next expressions (Schleicher, 1926):

$$s = \frac{1-\upsilon^2}{E} \cdot l \cdot q \cdot I_s \tag{1}$$

Where q is the uniform contact pressure, E is the elastic modulus of the soil, ν is the Poisson's ratio of the soil and I_s is the influence coefficient given by next expression:

$$I_s = 1/\pi \left[Ln\left(\xi_s + \sqrt{1+\xi_s^2}\right) + \xi_s \cdot Ln \frac{1+\sqrt{1+\xi_s^2}}{\xi_s} \right] \tag{2}$$

and:

$$\xi_s = L/l \tag{3}$$

The aim of this work is to find a relationship (α) between the settlement (s) of the foundation resting on two soils of differing deformability (with elastic parameters E_1, E_2, ν_1 and ν_2) and the settlement (s_2) of the same foundation resting over the more deformable soil (E_2 and ν_2). Therefore, since the uniform contact pressure (q) and the geometrical dimensions of the foundations (L and l) are the same for both cases, the parameter α can be expressed as:

$$\alpha = \frac{s}{s_2} = \frac{\frac{1-\upsilon^2}{E} \cdot l \cdot q \cdot I_s}{\frac{1-\upsilon_2^2}{E_2} \cdot l \cdot q \cdot I_s} = \frac{\frac{1-\upsilon^2}{E}}{\frac{1-\upsilon_2^2}{E_2}} = \frac{E_2(1-\upsilon^2)}{E(1-\upsilon_2^2)} \tag{4}$$

where E and ν, which depend on the elastic parameters from the two soils, represent the elastic modulus and the Poisson's coefficient for the case in which the foundation rests on two soils of differing deformability and E_2 and ν_2 are the elastic modulus and the Poisson's coefficient of the most deformable soil. Therefore, the parameter α only depends on the elasticity modulus and the Poisson's ratios from both soils. To reduce the number of variables we have adopted a constant value for ν_1 and ν_2 equal to 0.3. In order to validate this assumption, we have performed a sensibility analysis of the relationship between the terms of eq. (4) involving the Poisson's coefficients. The sensibility analysis shows that, for all possible ν_1 and ν_2 combinations, the settlement computed using a constant Poisson's coefficient equal to 0.3 for both soils is, in all cases, lower than 3.5% of the settlement computed considering different coefficients. Thus, the influence of the Poisson's coefficient in the present computation is very small and as a consequence a constant value can be assumed for this parameter with very small error.

Consequently, based on the elasticity theory and the above mentioned assumptions we can state that the relationship between the settlement (s) of the foundation resting on two soils of differing deformability and the settlement (s_2) of the same foundation resting over the more deformable soil mainly depends on the elastic modulus of both soils (i.e. E_1 and E_2) and is independent of the dimensions (L and l) of the foundation and the uniform contact pressure (q) transmitted to the ground.

3.2. Geometry and parameters

ANSYS+CIVILFEM v.11 (Ansys_Inc, 2007a, 2007b, 2007c) software was used for modelling the settlements of the spread foundations. Plane42 elements, which are defined by four nodes having two degrees of freedom (i.e. translations in the nodal X and Y directions), were used for meshing. Previously, models with higher order elements (e.g. Plane82 and Plane182) were performed, with identical results. The models allowed the evaluation of the suitability of the foundation when considering settlements, but not considering the suitability from a stress perspective, because this second problem has a simpler manual resolution.

Elastic behaviour was considered and a Mohr-Coulomb failure criterion was adopted for modelling, more advanced soil behaviour models (e.g. plasticity) were not considered because the aim was to provide an easy tool to calculate settlements based on classic elastic manual methods. Considering that the parameter α that we are going to compute from the FEM is not dependent from the dimensions (l and L) of the foundation and the contact pressure (q) (see section 3.1), we have modelled a slab 20 m width (l) and 1 m in height (h) resting one meter below the ground surface and built using a concrete type material conforming to Eurocode 2 (CEN, 1991), with a compressive strength of 25 MPa (C25/30) (Figure 1).

The mesh sizing was performed using 1 m quadrilateral-shaped elements in agreement with the scope of the study and the dimensions of the modelled elements. Note that the validity of the mesh was verified by implementing models with different mesh sizes, gradually reducing the size of the mesh and increasing the limits of the model to find the values for which the calculated settlements in consecutive models were constant. Thereby, the lower and lateral model boundaries were set at 200 m and 65 m from the ground surface and the slab edge respectively (Figure 1) in order not to influence the stress distribution (infinite half-space). Subsequently, the model was verified. For this purpose, the settlements obtained from the FEM models and from Schleicher´s manual methods (Schleicher, 1926) were compared considering a unique homogeneous and perfectly elastic soil. From the comparison of both sets of data, it may be observed that the results provided by both methods were very similar and the differences irrelevant, providing a mean error of 3.92% (±1.06%). Consequently, it may be concluded that the selected mesh sizes and model boundaries are valid for the purpose of this research and the settlements calculated using FEM for a single elastic soil are equivalent to those calculated from classic manual methods, verifying the validity of the model that was used for analysing the complex problem proposed in this study.

No construction process was considered in the FEM analysis and therefore unloading processes due to the foundation excavation were disregarded. As a consequence, the manual calculation of settlements (i.e. those calculated using classic elastic expressions) had to be performed considering gross contact pressure (q_{gross}) instead of net contact pressure (q_{net}).

Due to the large number of variables in the problem, a prior sensitivity analysis allowed the identification of the parameters with the least influence on the final result. To this end a number of models were calculated using different foundation depth to width ratios, chosen considering normal constructive practice. The modelled foundations present a flexibility factor (Brown, 1969b) higher than 0.01 (rigid and intermediate rigidity foundations) and consequently the proposed method is not valid for lower flexibility factors.

Similar settlement values were obtained for all the ratios considered, showing a low sensitivity to this parameter. As a consequence, the analyses were performed for all models considering a uniform gross contact pressure (q_{gross}) of 40 kN/m^2, which is a typical working load for slabs with these dimensions. Note that the chosen dimensions provide a certain rigidity and hence an adequate behaviour with respect to differential settlements (CEN, 1994, 2004) and has a foundation flexibility factor higher than 0.01 (lower limit for the application of the methodology proposed in this paper).

The proposed models consider the existence of two different soils with perfectly elastic stress-strain behaviour, continuous in depth and with a vertical contact (Figure 1). Three different cases were considered for modelling the foundation, according to the extension of the less compressible soil below the foundation base (relative to the width, *l*):

- Case I: Less compressible soil covers 25% of the slab width *l*.
- Case II: Less compressible soil covers 50% of the slab width *l*.
- Case III: Less compressible soil covers 75% of the slab width *l*.

Furthermore, a number of elastic modulus ratios (E_1/E_2) were considered for Cases I to III, where E_1 is the highest elastic modulus (i.e. the least compressible) and E_2 is the lowest elastic modulus (i.e. the most compressible). The elastic modulus ratios (E_1/E_2) considered in the modelling were 2, 3, 5, 10, 100 and 1000. For each of these ratios, at least ten different cases were modelled using typical elastic modulus values for a variety of ground types, covering most plausible cases. For all these cases the foundation settlement distribution was calculated and the maximum value (located at the edge of the foundation resting over the most deformable soil (E_2), in all cases) was taken as the reference value. The statistical reliability of the results was evaluated based on the standard deviation and coefficient of variation of each data series (i.e. at least 10 different combinations of values for each E_1/E_2 ratio and every case of study reaching a total of over 200 models). Settlement serviceability limit states were evaluated considering a total maximum settlement of 5.0 cm, usually adopted for the geotechnical design of slabs (Terzaghi *et al.*, 1948; Groth & Chapman, 1969; Burland & Wroth, 1974a, 1974b;

Burland *et al.*, 1977; Zhang & Ng, 2005), and for buildings with columns, a maximum angular distortion of 1/300 according to Groth & Chapman (1969) recommendations. Note that, assuming a clear span between columns of 5-6 m for the angular distortion considered, the differential settlement would reach 1.67-2.00 cm between adjacent columns. These values were also used for identifying situations in which serviceability limit states are reached.

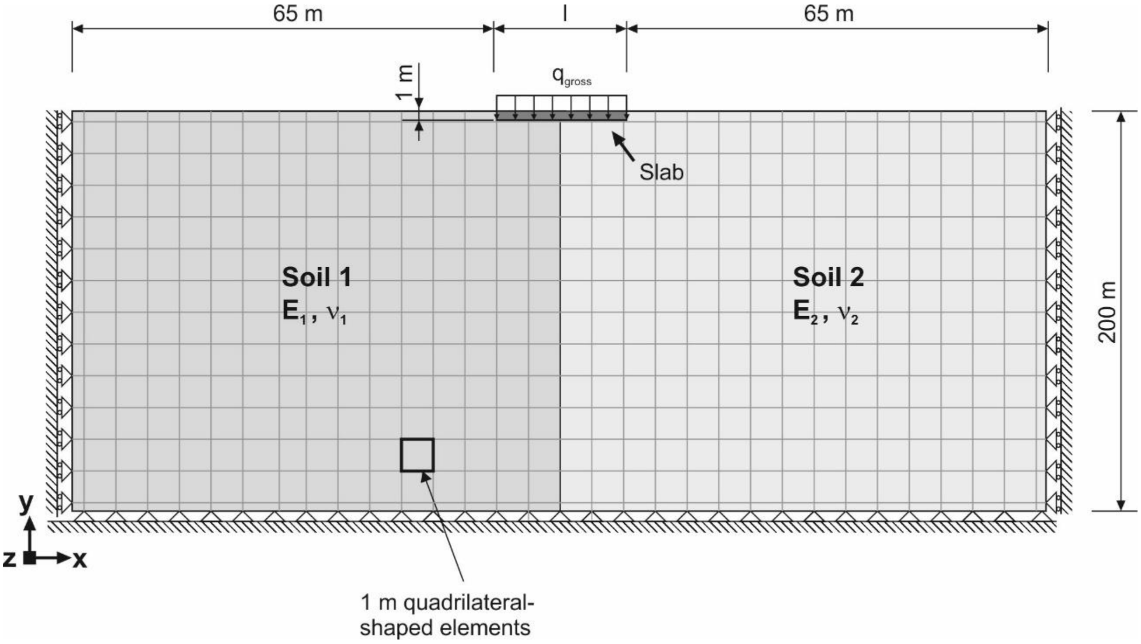

Figure 1. Diagram (not to scale) of the mesh and boundary conditions for the analysed models. Case II is shown (less compressible soil covers 50% of the slab width l).

3.3. Results

Considering the methods described in the previous section, up to 200 different models were analysed. These models are grouped according to the elastic modulus ratio of the two different soils (E_1/E_2 equal to 2, 3, 5, 10, 100 and 1000) and, in turn, considering the percentage of the foundation base that each soil occupies (Cases I, II and III). The results show that, for all E_1/E_2 ratios, the settlement values decreased as the values of E_1 and E_2 increased (Figure 2).

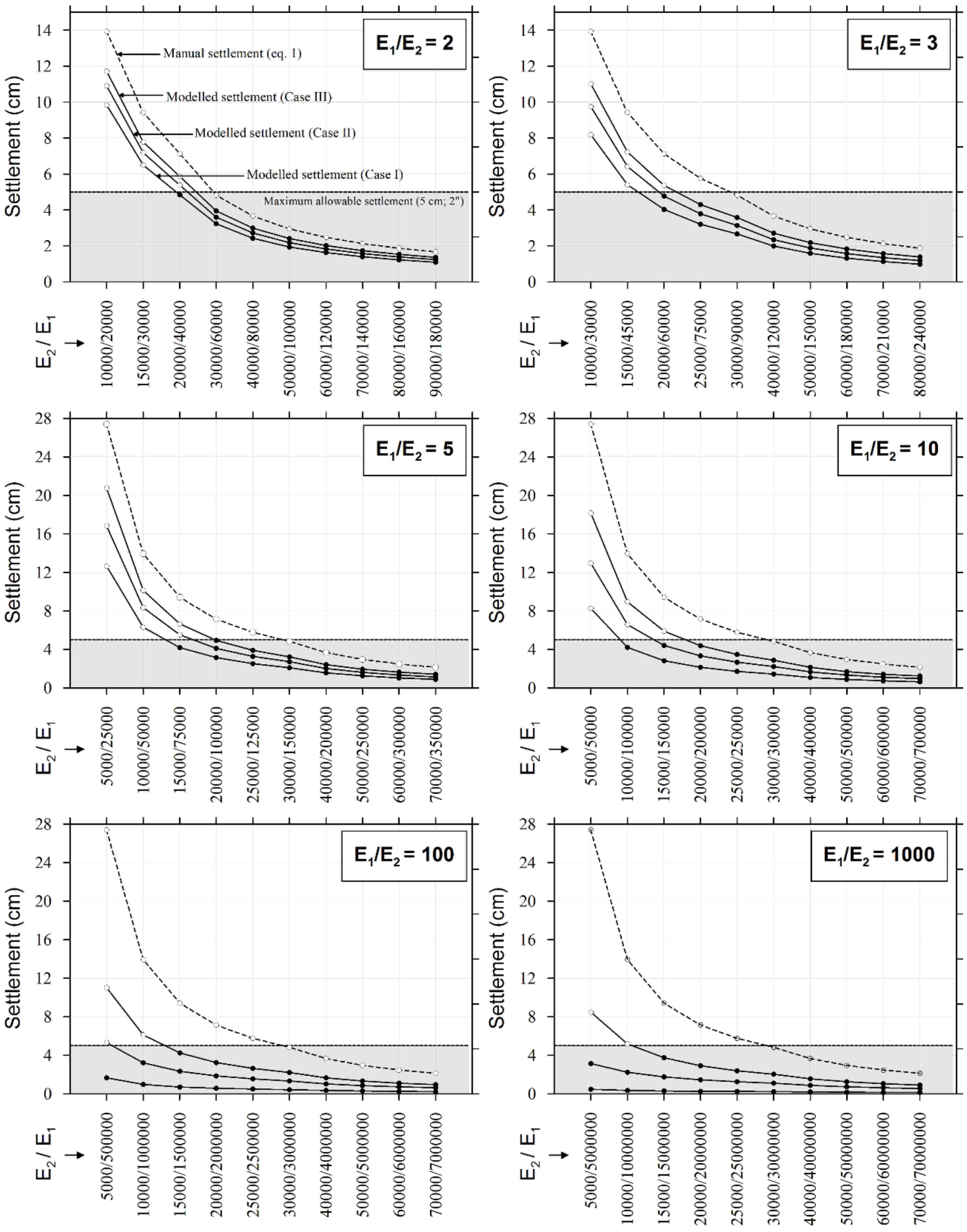

Figure 2. Comparison of settlements calculated for different E_1/E_2 ratios (black and white points) using FEM (continuous lines) and manual (dashed line) methods. Note that black dots correspond to elastic modulus combinations for which the angular

distortion was lower than 1/300. The shaded area contains allowable settlements lower than 5 cm.

A summary of the mean values (S_R), standard deviations (σ) and coefficients of variation (C_v) of the percentage reduction in settlement when comparing the manual and the FEM methods for the situations analysed is presented in Table 1. This data was obtained for the different E_1/E_2 ratios considered, and for each ratio includes the three cases shown in Figure 2. A very low variability may be observed in the analysed data. The standard deviations and coefficients of variation of the settlement reduction percentage (between the manual and the FEM methods) were always lower than 4.33% and 10.36%, respectively, for all the situations modelled. As a consequence, the foundation settlements calculated by FEM can be related with those derived from manual methods without large errors.

E_1/E_2	**Case**	**Nr**	**$S_R \pm \sigma$ (%)**	**C_V (%)**
2	**I**	12	17,87±0,66	3,71
	II	12	25,15±1,36	5,38
	III	12	33,17±1,68	5,08
3	**I**	12	25,07±1,60	6,38
	II	12	34,57±2,14	6,20
	III	12	44,96±1,94	4,32
5	**I**	12	31,44±3,26	10,36
	II	12	43,29±2,48	5,73
	III	12	56,55±1,39	2,46
10	**I**	11	39,63±2,99	7,56
	II	11	54,2±1,05	1,93
	III	11	70,22±0,23	0,33
100	**I**	10	55,46±1,53	2,76
	II	10	73,81±2,90	3,92
	III	10	91,41±1,41	1,54
1000	**I**	10	59,88±3,42	5,72
	II	10	78,95±4,33	5,49
	III	10	95,65±1,60	1,67

Table 1. Summary of the statistical parameters used for determining the variability of the data series. Nr: Number of models calculated using different E_1 and E_2 values for each case and E_1/E_2 ratio. S_R: Average settlement reduction between the manual and the FEM methods; σ: standard deviation; C_V: coefficient of variation.

Figure 2 also includes the situations in which maximum allowable angular distortion was not reached; they are plotted as black dots. Furthermore, the allowable settlements lower than 5 cm are those contained in the shaded area. These results indicate that the situations in which the maximum settlement is allowable (lower than 5.0 cm) also satisfy the allowable angular distortion criteria of 1/300 and therefore the differential settlement conditions (Groth & Chapman, 1969). As a consequence, for these cases it may be stated that if the foundation's maximum settlement is lower than 5 cm the slab will also satisfy the angular distortion and differential settlement conditions.

Figure 2 shows that of all analysis cases, only those where the elastic modulus (E_2) is lower than 10000 KN/m^2 and those in which the less deformable soil covers 25% of the slab width (Case I) are susceptible to exceed the allowable values (Figure 2).

As an example, Figure 3 shows the results of analysis of a particular case (E_1/E_2=10 and E_1=100.000 KN/m^2). Note that for cases I and II, the value of total settlement usually adopted for the geotechnical design of slabs (5 cm) (Terzaghi *et al.*, 1948; Groth & Chapman, 1969; Burland & Wroth, 1974a, 1974b; Burland *et al.*, 1977; Zhang & Ng, 2005) is exceeded, so that the settlement serviceability limit state is also exceeded. Case III shows a value of total settlement less than 5 cm, and for an ordinary clear span between columns of 5-6 m, angular distortion limit (1/300) is also satisfied. As it has been already commented, this happens in all cases studied. Moreover as expected, the maximum settlement always coincides with the edge of the foundation located on the more deformable soil.

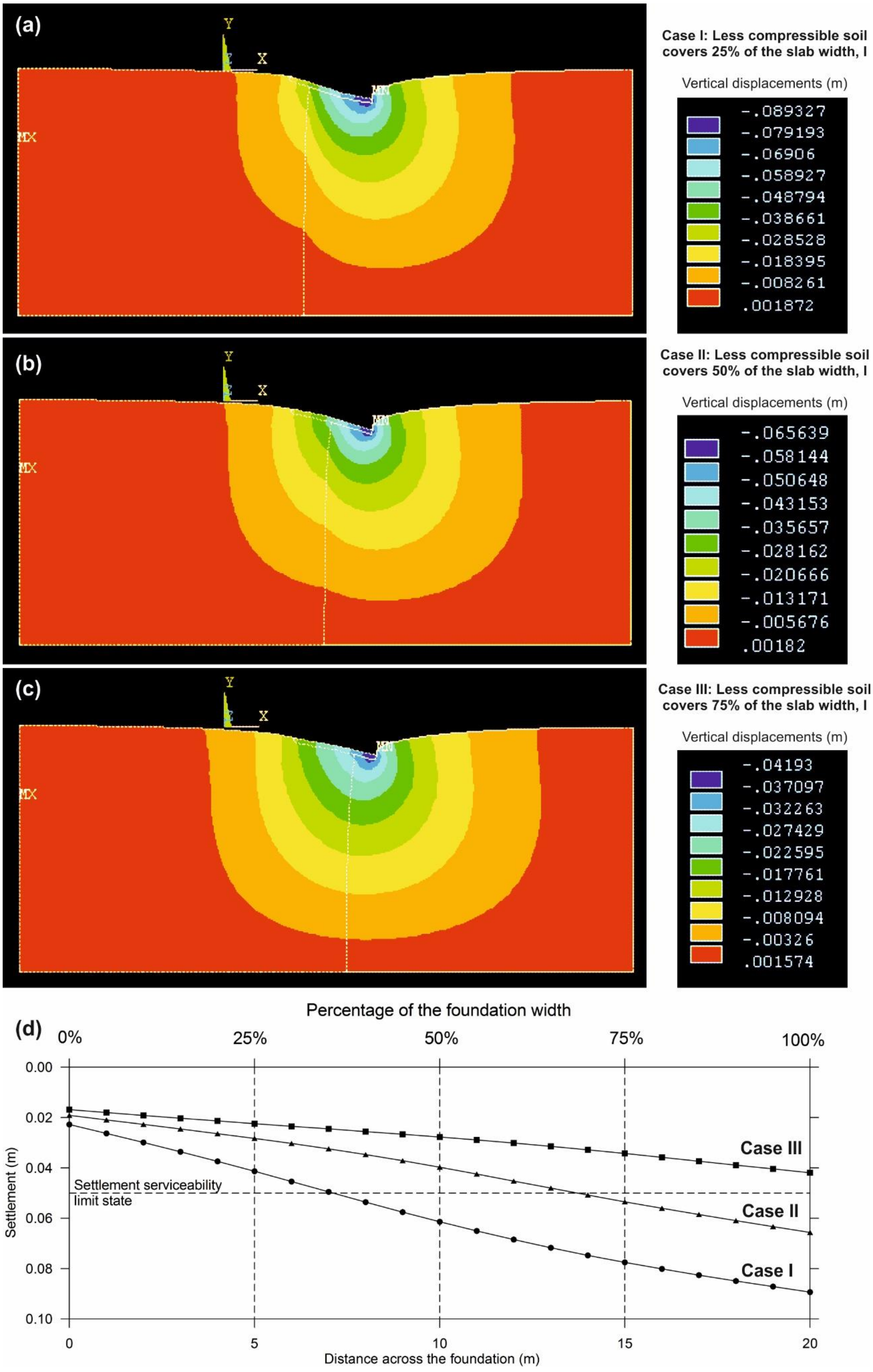
(a)
Y
X
MN
MX
Case I: Less compressible soil covers 25% of the slab width, l
Vertical displacements (m)
-.089327
-.079193
-.06906
-.058927
-.048794
-.038661
-.028528
-.018395
-.008261
.001872
(b)
Y
X
MN
MX
Case II: Less compressible soil covers 50% of the slab width, l
Vertical displacements (m)
-.065639
-.058144
-.050648
-.043153
-.035657
-.028162
-.020666
-.013171
-.005676
.00182
(c)
Y
X
MN
MX
Case III: Less compressible soil covers 75% of the slab width, l
Vertical displacements (m)
-.04193
-.037097
-.032263
-.027429
-.022595
-.017761
-.012928
-.008094
-.00326
.001574
(d)
Percentage of the foundation width
0%
25%
50%
75%
100%
Settlement (m)
0.00
0.02
0.04
0.06
0.08
0.10
Settlement serviceability limit state
Case III
Case II
Case I
0
5
10
15
20
Distance across the foundation (m)

Figure 3. Analysis final results of case E_1/E_2=10 and E_1=100.000 kN/m^2, (a) for percentage of the slab width occupied by the less deformable soil equal to 25% (Case I), (b) 50% (Case II), and (c) 75% (Case III). (d) distribution of settlements across the foundation for all three cases.

4. Proposed method for calculating settlements in foundations resting on soils of differing deformability

Considering the relationships discussed in the previous section, in this section a novel method is proposed for evaluating the settlement of a reinforced concrete slab foundation when it is founded on two soils of differing deformability. The proposed method can be applied as follows:

- Step 0. Check if the foundation flexibility factor (Brown, 1969b) is higher than 0.01 and if our case study is a special case from those described in section 5.
- Step 1. The maximum settlement (s_{m1}) of the slab or raft foundation is calculated using manual methods (e.g. (Schleicher, 1926)) assuming that is fully resting on the more rigid soil (E_1). The admissibility of the settlement must be checked. If this settlement (s_{m1}) is higher than the maximum allowable value adopted in this work (5.0 cm), the foundation resting on two soils of differing deformability will not be suitable because the maximum settlement and the angular distortion exceed the allowable values, and as such a new foundation must be designed.
- Step 2. The maximum settlement (s_{m2}) of the slab is calculated using manual elastic methods (e.g. (Schleicher, 1926)) assuming that is only resting on the more deformable soil (E_2).

- Step 3. The percentage of the slab width (*l*) occupied by the more deformable soil (E_2), p% is determined, in order to identify the specific case (I, II or III).
- Step 4. The elastic modulus ratio (E_1/E_2) is calculated.
- Step 5. Computation of the maximum reduction factor (α_{max}) by means of the provided table (Table 2), graphically (Figure 4) or by means of eq. (7).
- Step 6. The settlement obtained in step 2 is multiplied (s_{m2}) by the maximum reduction factor (α_{max}) computed in the previous step:

$$\mathrm{s} = \alpha_{\mathrm{max}} \cdot \mathrm{s}_{\mathrm{m2}} \tag{5}$$

The obtained settlement, s, is the real maximum settlement of the foundation resting on two soils of differing deformability. Note that the maximum reduction factor (α_{max}) can be obtained from Table 2 or alternatively from Equation 6 as a function of the elastic modulus ratio (E_1/E_2) determined in step 4, and the case (I, II or III) identified in step 3:

$$\alpha_{max} = A \cdot \left(\frac{E_1}{E_2}\right)^C + B \tag{6}$$

where the constants A, B and C are shown in Figure 4 for each different case (I, II or III). If the percentage of the slab width (*l*) occupied by the more deformable soil is different to 25, 50 or 75%, α_{max} may be calculated by interpolating between the closest values. Note that, this expression is only valid for elastic modulus ratios (E_1/E_2) higher than 1.

In order to provide a more general expression which covers all possible combinations of the elastic modulus ratio (E_1/E_2) and the percentage (p%) of less compressible soil compared to the total width (l), a hyperplane was fitted to the available data, obtaining a coefficient of determination (r^2) of 0.9996 and mean errors for the fitting points of 1.57%:

$$\alpha_{max} = 0.59 + 1.10 \cdot min\left[\left(\frac{E_1}{E_2}\right)^{-\frac{2}{3}}, 6.40\right] - 0.87 \cdot \log_{10}\left(min\left[\left(\frac{E_1}{E_2}\right),\ 6.40\right]\right) \cdot \frac{p}{100} \qquad (7)$$

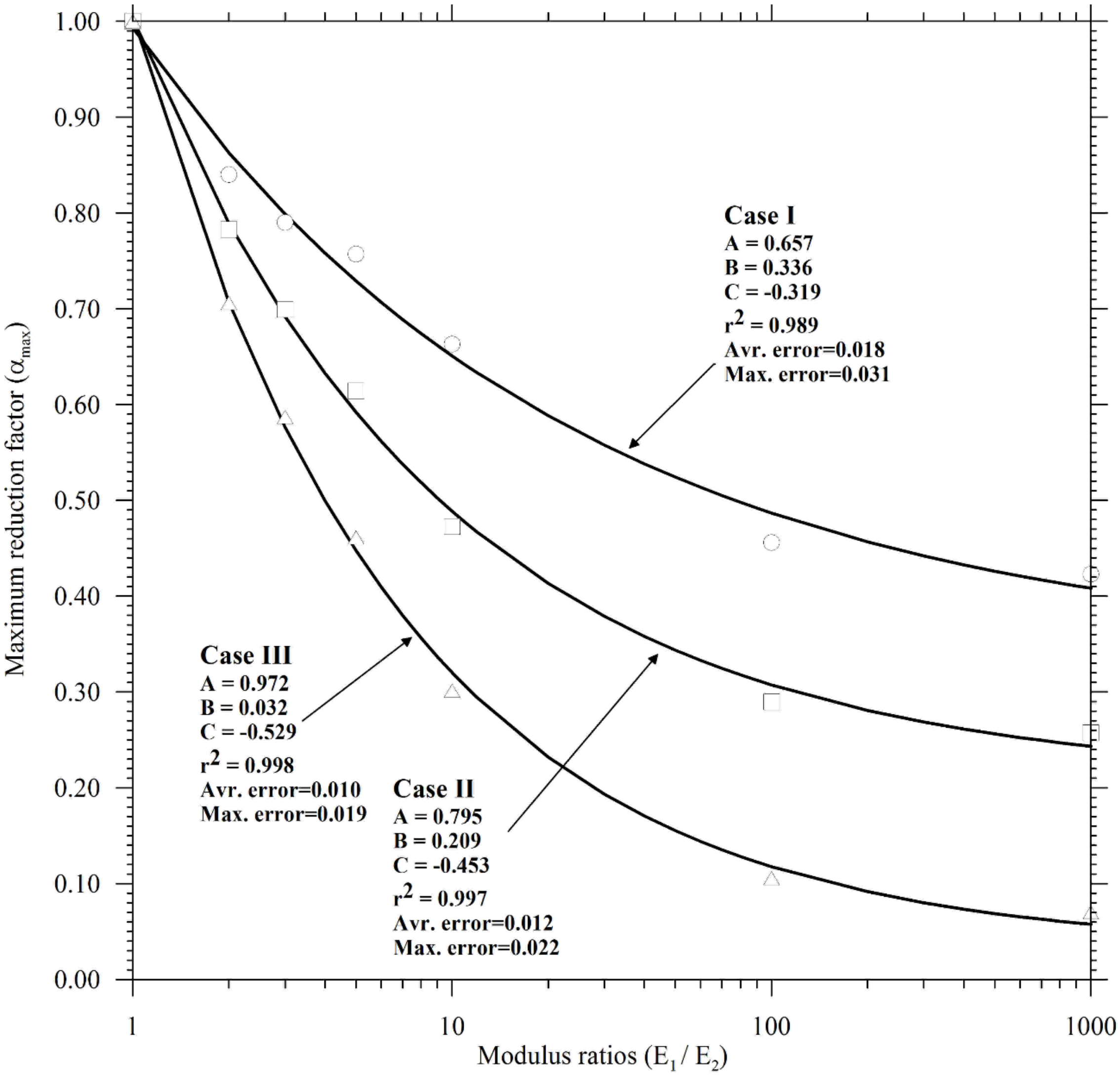

Figure 4. Relationships between the elastic modulus ratios (E_1/E_2) and the maximum reduction factors (α_{max}) for the different cases (I, II and III). A, B and C are the fitting values of Equation (6) and depend on the case in question (I, II or III). r^2, Av. Error and Max. Error, are the coefficient of determination, the average error and the maximum error, respectively.

E_1/E_2	**Case**	**Mean percentage reduction (%)**	**Maximum percentage reduction (%)**	**Error (%)**	**Mean reduction factor** (α_{mean})	**Maximum reduction factor** (α_{max})
2	**I**	82.13	84.00	1.87	0.82	**0.84**
	II	74.85	78.26	3.41	0.75	**0.78**
	III	66.83	70.61	3.78	0.67	**0.71**
3	**I**	74.93	79.01	4.08	0.75	**0.79**
	II	65.43	69.95	4.52	0.65	**0.70**
	III	55.04	58.71	3.67	0.55	**0.59**
5	**I**	68.56	75.71	7.15	0.69	**0.76**
	II	56.71	61.43	4.72	0.57	**0.61**
	III	43.45	46.15	2.70	0.43	**0.46**
10	**I**	60.37	66.31	5.94	0.60	**0.66**
	II	45.80	47.27	1.47	0.46	**0.47**
	III	29.78	30.13	0.35	0.30	**0.30**
100	**I**	44.54	45.60	1.06	0.45	**0.46**
	II	26.19	28.98	2.79	0.26	**0.29**
	III	8.59	10.59	2.00	0.09	**0.11**
1000	**I**	40.12	42.30	2.18	0.40	**0.42**
	II	21.05	25.73	4.68	0.21	**0.26**
	III	4.35	6.79	2.44	0.04	**0.07**

Table 2. Reduction factor (α_{max}) corresponding to the different elastic modulus ratios (E_1/E_2) and percentage of the foundation width (l) occupied by the more deformable soil.

- Step 7. The settlement calculated in step 5 (s) is checked against the maximum allowable settlement (5 cm). If it is below this value, the foundation differential settlements and distortion will also be allowable (see Figure 2). Note that this settlement (s) is the maximum suffered by the slab and occurs in the slab edge located in the more deformable soil.

Although Equation (7) provides a more conservative value of the real settlement (s) of the slab resting on two soils of differing deformability, the designer could also adopt the mean reduction factor (α_{mean}) for calculating settlements using Equation (5). The mean percentage error (i.e. the difference between the mean and maximum percentage settlement reduction) for all situations was 3.32%. This value indicates that although α_{max} provides a more conservative value, the settlements calculated using α_{mean} were quite similar.

5. Special cases

This paper covers most cases which may occur in the calculation of the settlement of a foundation which rests on two different soils. However, there are two special cases which were not explicitly analysed in the previous sections: non-vertical contact between the two soils, and a contact line between the layers parallel to the shorter side of the slab (l). For the case in which the contact plane between the two soils of differing deformability is non-vertical, two different situations can be defined:

- Situation 1 (Figure 5a). The less deformable layer (i.e. with an elastic modulus E_1) increases in thickness with depth. For this case, the analysis performed according to the method proposed in this paper (considering a vertical contact) will give conservative values.
- Situation 2 (Figure 5b). The more deformable layer (i.e. with an elastic modulus E_2) increases in thickness with depth. For this case the point of the contact between the two soils (P), located at a depth of 1.5 times the width of the slab (*l*) where the increase in vertical stress is negligible, must be projected onto the foundation base (point P’). Then, a vertical contact between the two soils passing through the point P’ has to be assumed. The calculated settlements will also be conservative values.

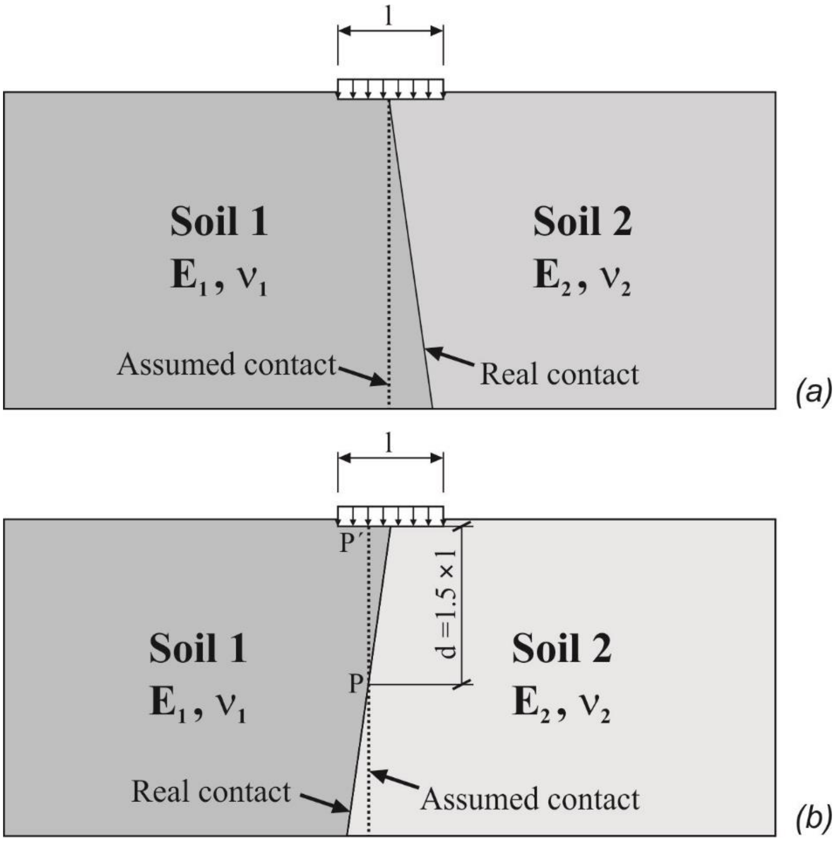

Figure 5. Transformation of the inclined contact into a vertical contact when the less deformable layer (a) increases in thickness with depth and (b) reduces in thickness with depth.

For the case in which the contact between the two layers is parallel to the shorter side (*l*) of the rectangular foundation (Figure 6a and b) rather than perpendicular (Figure 1), the settlement (s) of the original foundation (P in Figure 6b) may be calculated by expanding the width (*l*) to be equal to the longer length (L). The settlement (s_2) under the corner of this square equivalent foundation (P in Figure 6c), which can be calculated using the originally proposed method (contact parallel to the width, *l*), is related to the original settlement, based on the elasticity theory, by the following expression:

$$s = \frac{l}{L} \cdot \frac{I_s}{I_{s2}} \cdot s_2 \qquad (8)$$

where I_{s1} and I_{s2} are the influence coefficients of the original and the square equivalent foundation, respectively, calculated by means of the Schleicher's equations (Schleicher, 1926).

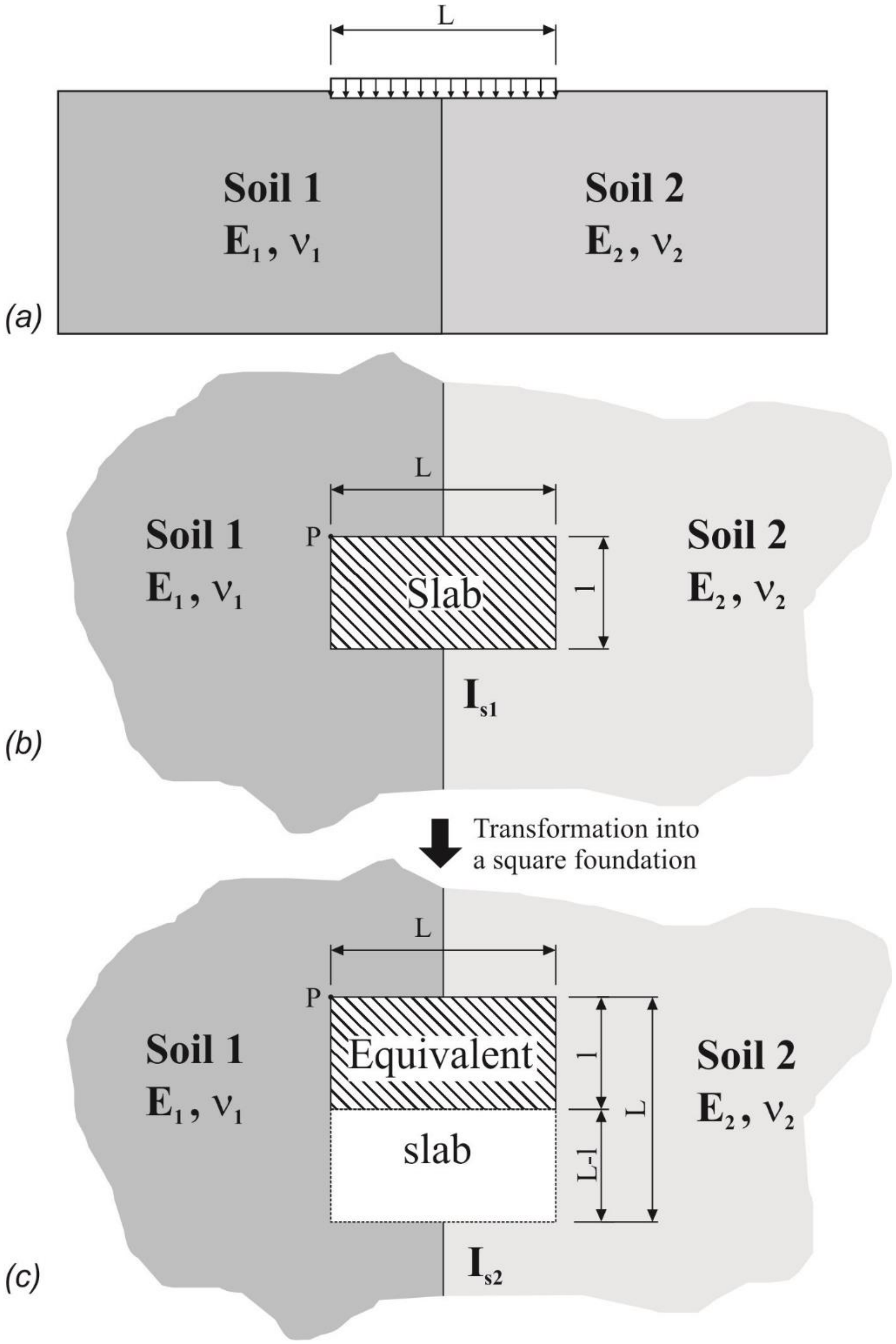

Figure 6. Transformation of a rectangular slab with a contact perpendicular to the length (L) into a square slab for the calculation of settlements.

6. Case study.

In this section we illustrate the applicability and limitations of the proposed method trough a case study of the slab foundation of a 5-storey residential building (Figure 7) placed at south of Madrid (Spain). The building suffered serious structural damage (mainly joints opening and cracks) and consequently it was monitored by means of a precise levelling of settlements of the façades on 26 benchmarks (Figure 7a). The monitoring data showed that the building was affected by a tilting process towards the SE. Later studies conducted on the building indicated that it was founded by means of a reinforced concrete slab foundation with a depth of 0.7, 15 m width and 40 m long resting on two soils of differing deformability (Figure 7b). The geotechnical investigation indicated that the existing gypsiferous marls and silty clays had deformation modulus of 70000 and 6500 kN/m^2, respectively. The rigidity of the foundation computed considering Brown's (Brown, 1969b) criteria, is higher that lower boundary proposed for the application of the method. Then we can apply it as indicated in section 4:

0. As it is a rectangular foundation with the lithological contact parallel to the width of the foundation, this is as a special case (see section 5) in which we have to compute the settlement of an equivalent square foundation to obtain the settlement (s) of the true rectangular foundation. This true settlement is related with the settlement of the square equivalent foundation (s_2) through eq. (8):

   $$s = \frac{15}{40} \cdot \frac{0.855}{0.561} \cdot s_2 = 0.571 \cdot s_2 \tag{9}$$

1. Computation of the maximum settlement (s_{m1}) of the foundation using eq. (1), considering that the whole foundation is resting on the less deformable terrain

(E$_1$). In this case, the maximum settlement is located in the centre of the foundation and is computed as:

$$s_{m1} = \frac{1-0.3^2}{70000} \cdot 15 \cdot 60 \cdot 0.855 \cdot 2 = 2.0\ cm \tag{10}$$

Note that this value is lower than 5 cm and then we can continue to apply the method (see section 3).

2. Computation of the maximum settlement (s$_{m2}$) of the foundation (in this case, square equivalent foundation) using eq. (1), considering that the whole foundation is resting on the most deformable terrain (E$_2$):

$$s_{m2} = \frac{1-0.3^2}{6500} \cdot 40 \cdot 60 \cdot 0.561 \cdot 2 = 37.7\ cm \tag{11}$$

3. Computation of the percentage of the slab width (l) occupied by the more deformable soil (p%):

$$p = \frac{12}{40} \cdot 100 = 30\% \tag{12}$$

4. Calculation of E$_1$/E$_2$ ratio:

$$\frac{E_1}{E_2} = \frac{70000}{6500} = 10.77 \tag{13}$$

5. Calculation of α_{max} using eq. (7):

$$\alpha_{max} = 0.59 + 1.10 \cdot 0.20 - 0.87 \cdot \log_{10}(6.40) \cdot \frac{30}{100} = 0.6 \tag{14}$$

Note that although in this example α_{max} has been calculated using eq. (7), a reasonably approximation of α_{max} can be achieved using interpolated values derived from Figure 2 or equation (6).

6. Finally, we can calculate the total maximum settlement of the true rectangular foundation (s) considering eq. (9):

$$s = 0.571 \cdot\ s_2 = 0.571 \cdot (0.6 \cdot 37.7) = 12.91\ cm \tag{15}$$

Note that the calculated settlement of the real foundation is equal to 12.91 cm. This value is higher than the usually assumed allowable settlement (i.e. 5 cm) and agrees with the observed damage and the tilt of the building. A simple calculation of the maximum settlement of the foundation trough the proposed methodology would have allowed to state the convenience of founding the building using a deep foundation. Additionally, we can conclude that the calculated value (12.91 cm) is very similar to the maximum measured at field (12.49 cm) with a relative error of 3.36%. Notice that, alternatively, a quick evaluation of the maximum settlements using Figure 2 would have allowed the recognition of the unsuitability of founding the building by means of a slab foundation.

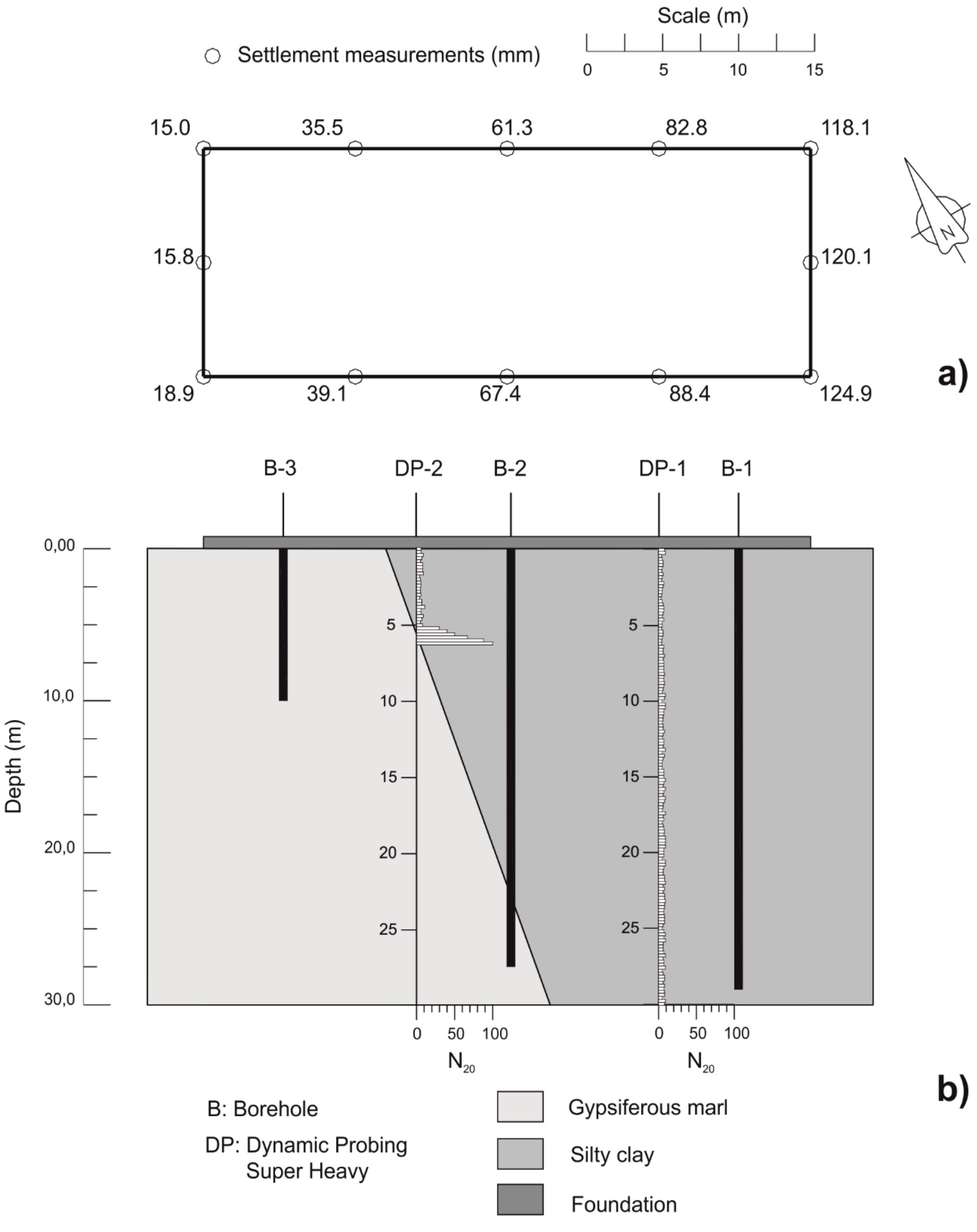

Figure 7. a) Plan view of the building with the labels of the measured settlements in the most representative benchmarks. b) Geotechnical cross section.

7. Conclusions

The existence of two different lithologies at the base of a foundation is a common problem not satisfactorily solved in geotechnics thus far. In this study, a novel simple method was proposed for the analysis of settlements in spread foundations resting on soils of differing deformability with a foundation flexibility factor (Brown, 1969b) higher than 0.01. The proposed method is based on commonly used analytical elastic formulas (e.g. (Schleicher, 1926)). The real settlement of the foundation can be calculated using the classic manual elastic methods, assuming that the slab is fully resting on the most deformable soil, and then correcting the obtained settlement by means of a reduction factor (α_{max}) which depends on the elastic modulus ratio (E_1/E_2) and the percentage of the foundation base covered by the less compressible soil (Cases I, II and III). It has been demonstrated that when settlement is lower than 5 cm, this assures that the foundation differential settlements and distortion will also be allowable. The proposed method can also be applied (after some minor transformations) to some special cases, namely those where there is a non-vertical contact or the contact is perpendicular to the longer length (L) of the foundation.

The data analysed exhibited a high consistency and a low dispersion, based on the calculated standard deviations and coefficients of variability, and as a consequence the proposed correlations are reliable. The calculated settlement corresponded to the maximum settlement of the slab, and was obtained with a maximum error of up to 1.57% of the real settlement derived from FEM.

As a consequence, in this paper, a novel simple method for evaluating the settlements of a slab foundation resting on two different soils is proposed. Thus, the proposed method allows the resolution of some common field situations, such as those in which a simple stratigraphic or mechanical contact between two different lithologies exists on the foundation plane or the foundation rests on embankments, partially founded on the fill material and partially on the in situ excavated material.

The proposed methodology has been also validated trough a real case study providing errors lower than 3.5% and demonstrating that this method allows a preliminary calculation of settlements in order to assess the validity of a slab foundation or the need of a deep foundation in early stages of foundation design. Summarizing, the analysed cases are usually resolved considering deep foundation solutions resulting in an important cost increase. As a consequence, the proposed methodology can be used for evaluating in a simple and fast way the serviceability limit states of the foundation during the engineering design process.

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References

Ansys_Inc. (2007a). Elements Reference.

Ansys_Inc. (2007b). Theory Reference.

Ansys_Inc. (2007c). CivilFEM Theory Manual.

Avramidis, I. E., & Morfidis, K. (2006). Bending of beams on three-parameter elastic foundation. *International Journal of Solids and Structures, 43*(2), 357-375. doi: http://dx.doi.org/10.1016/j.ijsolstr.2005.03.033

Bezvolev, S. G. (2002). Method of accounting for the deformability of an inhomogeneous elastoplastic bed in analyzing foundation slabs. *Soil Mechanics and Foundation Engineering, 39*(5), 162-170.

Brown, P. T. (1969a). Numerical analyses of uniformly loaded circular rafts on elastic layers of finite depth. *Géotechnique, 19*, 301-306.

Brown, P. T. (1969b). Numerical analyses of uniformly loaded circular rafts on deep elastic foundations. *Géotechnique, 19*, 399-404. Retrieved from http://www.icevirtuallibrary.com/content/article/10.1680/geot.1969.19.3.399

Burland, J. B., Broms, B. B., & De Mello, V. F. B. (1977). Behaviour of foundations and structures. *In: Proceedings of the 9th international conference on soil mechanics and foundation engineering, Japanese Society of Soil Mechanics and Foundation Engineering*, 495–546.

Burland, J. B., & Wroth, C. P. (1974a). Allowable and differential settlement of structures, including damage and soil structure interaction. *In: Proceedings of the conference on settlement of structures, Cambridge University, Cambridge*.

Burland, J. B., & Wroth, C. P. (1974b). Settlement of buildings and associated damage. *Proc. Conference on "settlements of structures", Pentech press London, England*, 611–654.

CEN. (1991). Eurocode 2: Design of concrete structures, part 1: General rules and rules for buildings. *ENV 1992-1-1 (1991), CEN, Brussels*.

CEN. (1994). Eurocode 1: Basis of design and actions on structures, part 1: Basis of design. *ENV 1991-1 (1994), CEN, Brussels*.

CEN. (2004). Eurocode 7: Geotechnical design. *ENV 1997:2004, CEN, Brussels*.

Daloglu, A., & Vallabhan, C. (2000). Values of k for slab on Winkler foundation. *Journal of Geotechnical and Geoenvironmental Engineering, 126*(5), 463-471. doi: doi:10.1061/(ASCE)1090-0241(2000)126:5(463)

Denis, A., Elachachi, S. M., & Niandou, H. (2011). Effects of longitudinal variability of soil on a continuous spread footing. *Engineering Geology, 122*(3), 179-190.

Gazetas, G. (1980). Static and dynamic displacements of foundations on heterogeneous multilayered soils. *Géotechnique, 30*, 159-177.

Groth, N. N., & Chapman, C. R. (1969). *Computer evaluation of deformations due to subsurface loads in a semi-infinite elastic medium*. Bachelor thesis: University of Sydney. Sidney.

Imanzadeh, S., Denis, A., & Marache, A. (2013). Simplified uncertainties analysis of continuous buried steel pipes on an elastic foundation in the presence of low stiffness zones. *Computers and Geotechnics, 48*(0), 62-71. doi: http://dx.doi.org/10.1016/j.compgeo.2012.09.015

Malikova, T. A. (1979). Calculation of foundation slabs on a base with variable rigidity. *Soil Mechanics and Foundation Engineering, 16*(6), 342-346.

Mayne, P., & Poulos, H. (1999). Approximate displacement influence factors for elastic shallow foundations. *Journal of Geotechnical and Geoenvironmental Engineering, 125*(6), 453-460. doi: doi:10.1061/(ASCE)1090-0241(1999)125:6(453)

Onopa, I. A., Skopintseva, E. V., & Dokhnyanskii, M. P. (1983). Settlement of buildings on heterogeneous bases. *Soil Mechanics and Foundation Engineering, 20*(6), 227-230.

Schleicher, F. (1926). Zur theorie des baugrundes. *Bauingenieur Band, 7*, 931-935, 949-952.

Shadunts, K. S., & Marinichev, M. B. (2003). Analysis of buildings and structures on complex nonuniformly compressible foundation beds. *Soil Mechanics and Foundation Engineering, 40*(2), 42-47.

Sheinin, V. I., Sarana, E. P., Artemov, S. A., & Favorov, A. V. (2006). Algorithm and program for engineering analysis of foundation-slab settlement with allowance for nonuniform bed loading and heterogeneity of the mass. *Soil Mechanics and Foundation Engineering, 43*(5), 153-160.

Shukla, S. K., & Chandra, S. (1996). A study on a new mechanical model for foundations and its elastic settlement response. *International Journal for Numerical and Analytical Methods in Geomechanics, 20*(8), 595-604. doi: 10.1002/(sici)1096-9853(199608)20:8<595::aid-nag835>3.0.co;2-9

Stavridis, L. (2002). Simplified Analysis of Layered Soil-Structure Interaction. *Journal of Structural Engineering, 128*(2), 224-230. doi: doi:10.1061/(ASCE)0733-9445(2002)128:2(224)

Terzaghi, K. (1955). Evaluation of coefficients of subgrade reaction. *Géotechnique, 5*, 297-326. Retrieved from http://www.icevirtuallibrary.com/content/article/10.1680/geot.1955.5.4.297

Terzaghi, K., Peck, R. B., & Mesri, G. (1948). *Soil mechanics in engineering practice*. New York: John Wiley and Sons.

Votyakov, I. F. (1964). Settlement of a building with an inclined stratification of severely compressible soils in the foundation bed. *Soil Mechanics and Foundation Engineering, 1*(3), 171-174.

Zhang, L. M., & Ng, A. M. Y. (2005). Probabilistic limiting tolerable displacements for serviceability limit state design of foundations. *Géotechnique, 55*, 151-161.