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Relative Concerns on Visible Consumption: A Source of Economic Distortions

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Abstract: Do relative concerns on visible consumption give rise to economic distortions? We re-examine the question posited by Arrow and Dasgupta (2009) building upon their general framework but recognizing that relative concerns can only apply to visible goods (e.g., cars, clothing, jewelry) and that households consume both visible and non-visible goods. Contrary to Arrow and Dasgupta (2009), the answer to this question turns to be always affirmative: the competitive equilibrium will always be different than the socially optimal one, since individuals do not take into account the negative externality they exert on others through the consumption of the visible good, while the social planner does. If one invokes separability assumptions, then the steady state competitive equilibrium consumption of non-visible goods will be strictly lower than the socially optimal one.

Keywords: visible goods, non-visible goods, conspicuous consumption, conspicuous leisure, labor supply, market distortions

JEL Classification Codes: D6, E2

1 Introduction

The thesis that economic agents care not only about their own consumption, but also about their consumption relative to others can be traced back to Adam Smith (1776), Veblen (1899) and Duesenberry (1949). Over the last years economists and sociologists have provided considerable empirical support for the notion that individuals care about their relative positions in their communities, often using subjective well-being evidence (Clark and Oswald 1996; Luttmer 2005; Clark, Frijters, and Shields 2007), but also

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linking consumption patterns to relative concerns (Charles, Hurst, and Roussanov 2009; Kuhn et al. 2011).¹

Charles, Hurst, and Roussanov (2009), using US data from the CEX 1985–2002, show that, controlling for differences in permanent income, Blacks and Hispanics devote larger shares of their expenditure bundles to visible goods (clothing, jewelry, and cars) than do comparable Whites.² Kuhn et al. (2011) find substantial social effects of lottery winnings: Dutch Postcode Lottery nonparticipants who live next door to winners have significantly higher levels of car consumption than other nonparticipants.³

Given these empirical findings, one natural question to ask is: Do relative concerns for visible consumption have negative welfare consequences?, where visible consumption can be understood as the consumption of goods that are readily observable in anonymous social interactions and that are portable across those interactions (Charles, Hurst, and Roussanov 2009).⁴ While the empirical apparatus is not well suited to answer this question, one can nevertheless, from a theoretical perspective, analyze the role of relative concerns regarding visible consumption on the allocation of resources in the economy, a direction followed for instance by Arrow and Dasgupta (2009).⁵ Here, we build upon their general framework but recognizing that not all goods are visible. If relative concerns are to be relevant, these concerns can only apply to visible goods. This basic insight

¹ There is also experimental evidence of social comparison in reward processing centers of the human brain (Fliessbach et al. 2007).
² Evidence from the developing world also reveals that poor people tend to spend large fractions of their budgets on conspicuous items such as phones, funerals and festivals (Banerjee and Duflo 2007).
³ In addition, they find that visible consumption is declining in reference group income, a crucial prediction of “demonstration effect” via status-signaling. In the economics literature, a standard explanation for why households might care about relative consumption of visible commodities is based on a signaling-by-consuming Veblen’s (1899) explanation: conspicuous consumption describes the advertisement of one’s income and wealth through lavish spending on visible commodities. Recently, Moav and Neeman (2012) construct a theoretical model and show that if human capital is observable and correlated with income, then a signaling equilibrium in which poor individuals tend to spend a large fraction of their income on conspicuous consumption can emerge. This would explain why poor educated people can kept locked in poverty and would be consistent with the expenditure patterns across the developing world.
⁴ Heffetz (2011) defines socio-cultural visibility of consumer expenditures as the speed with which members of a society notice a household’s expenditure on different commodities. It is important to highlight that the physical visibility of consumption is a necessary though not a sufficient condition for the existence of relative concerns on consumption.
⁵ Other theoretical contributions on relative concerns include the recent work by Aronsson and Johansson-Stenman (2014), and the references therein.
proves to have dramatic consequences, leading to crucial differences regarding our conclusions on the distortional power of relative concerns and those in Arrow and Dasgupta (2009).

We analyze a competitive economy where individual felicities depend not only on absolute consumption but also on relative consumption. Crucially, relevant concerns are only possible if consumption is visible (observable, or conspicuous in Veblen's terminology). Precisely, our twist and extension with respect to Arrow and Dasgupta (2009) is the inclusion of two types of consumption goods: visible and non-visible. Indeed, not all consumption is visible. In the US, for example, visible expenditures represent 12% of the mean quarterly expenditure (Charles, Hurst, and Roussanov 2009). In our model households felicities depend on visible consumption through its absolute and relative aspects, while depending only on the absolute aspect of non-visible consumption. Thus, while a household suffers a felicity loss when others' visible consumption levels rise, because his relative visible consumption now declines, others' non-visible consumption is immaterial to household satisfaction.

We characterize the competitive equilibrium and socially optimal paths, as well as, the steady state equilibrium, studying the effects of relative concerns of visible consumption on the mix of personal consumption of visible and non-visible commodities, leisure and saving in an inter-temporal economy to reexamine the question posited by Arrow and Dasgupta (2009): Do relative concerns on consumption give rise to economic distortions? Contrary to the finding in Arrow and Dasgupta (2009), we show that, when households derive utility from both visible and non-visible consumption goods and visible consumption of others enters the household utility, the answer to this question turns to be always affirmative: the competitive equilibrium is distorted. In addition, we show that the economic distortion must take the form of people consuming less of the non-visible good, no matter whether labor supply is endogenously determined and/or leisure is conspicuous, as long as the felicity function is separable in own consumption of visible and non-visible goods (and in own consumption of non-visible and others' consumption of visible goods).

As in Arrow and Dasgupta (2009), we work with a convex technology that involves a single type of labor and a single reproducible non-deteriorating capital good that serves also as a consumption (visible or non-visible to others) good. Proposition 1 establishes that, under relative concerns regarding visible consumption, as long as the felicity function includes also non-visible consumption, the market equilibrium and the socially optimal paths cannot coincide. In other words, there is no felicity function satisfying regularity conditions that avoids market distortions.
In addition, an important corollary of Proposition 1 allows us to characterize the steady state competitive equilibrium. We show that, if labor supply is exogenous, the steady state competitive equilibrium visible consumption is strictly larger than the socially optimal one, whereas the non-visible consumption is strictly lower than the socially optimal one. In addition, under separability assumptions, we can characterize the properties of the competitive equilibrium with endogenous labor supply. If leisure is inconspicuous, we obtain the same economic distortions characterized by Proposition 2 in Arrow and Dasgupta (2009) regarding visible consumption, capital and labor supply, plus an additional new qualitative result: the steady state competitive equilibrium level of non-visible consumption is lower than the socially optimal one. When leisure is conspicuous, the only unambiguous economic distortion is that, again, the steady state competitive equilibrium level of non-visible consumption is lower than the socially optimal one. These results are consistent with the recent findings by Charles, Hurst, and Roussanov (2009), who find that higher spending on conspicuous consumption is at the expense of inconspicuous consumption.

Our analysis applies to any context where relative concerns are important, including a world where conspicuous consumption is a signal of unobservable wealth, but more generally any environment where people feel bad if their consumption of visible goods is less than that of others. Section 2 contains our theoretical analysis. Conclusions are presented in Section 3.

2 An Economy with Relative Concerns

The economy consists of two different nonperishable goods: a visible good, $c_i$, and a non-visible one, $x_i$. Both goods are produced by perfectly competitive firms and sold to a continuum of infinitely lived and identical households, which are indexed by $i \in [0,1]$. Time is continuous and each household is endowed by one unit of total time to be employed in labor activities and leisure. Let $e_i(t)$ ($0 \leq e_i(t) \leq 1$) and $c_i(t)$ respectively denote labor supply and visible consumption rate of each households $i$ at time $t$, so that

$$C(t) = \int_0^1 c_i(t) \, di \quad \text{and} \quad E(t) = \int_0^1 e_i(t) \, di$$

represent the average visible consumption of the population, and the aggregate level of labor in the economy at $t$, respectively.
Assumption 1 The household felicity function is given by:

$$U(c_i, C, x_i) - \gamma V(e_i, \theta E)$$  \[1\]

where $\gamma \in \{0, 1\}$, controls for labor supply to be endogenous ($\gamma = 1$) or exogenous ($\gamma = 0$), and $\theta \in \{0, 1\}$ controls for leisure to be conspicuous ($\theta = 1$) or inconspicuous ($\theta = 0$). In addition, the felicity function of each household satisfies the following conditions:

(i) $U_c(c_i, C, x_i) > 0$, $U_C(c_i, C, x_i) < 0$, $U_x(c_i, C, x_i) > 0$, and $U_{ss}(c_i, C, x_i) < 0$, $s = \{c, x\}$

(ii) $U_c(\cdot)$ and $U_x(\cdot)$ are respectively strictly decreasing and strictly increasing when $c_i$, $C$ and $x_i$ move together. Moreover, $\lim_{c_i \to \infty} U_c(\cdot) = 0 \forall (C, x_i)$, and $\lim_{x_i \to \infty} U_x(\cdot) = 0 \forall (c_i, C)$.

(iii) $U_c(\cdot) + U_C(\cdot) > 0$ when $c_i$, $C$ and $x_i$ move together.

(iv) $V_e(\cdot) \geq 0$ (with strict inequality when $\theta = 1$), $V_{ee}(\cdot) > 0$, and $V_E(\cdot) \leq 0$ (with equality when $\theta = 0$).

(v) $V_e(E, E) + V_E(E, E) > 0 \forall E$.

(vi) $V_{ee}(E, E) + V_{ee}(E, E) > 0 \forall E$.

(vii) $\lim_{e_i \to 1} V_e(e_i, \theta E) = 0$ and $\lim_{e_i \to 0} V_e(e_i, \theta E) = \infty$, $\forall \theta \in \{0, 1\}$.

The interpretation of Assumption 1 is as follows. The specification of the felicity function [1] encompasses the cases of exogenous ($\gamma = 0$) and endogenous ($\gamma = 1$) labor supply. In this last case, leisure $(1 - E)$ can be conspicuous ($\theta = 1$) or inconspicuous ($\theta = 0$). Condition (i) describes regularity conditions of the utility function with respect to visible, $c_i$, and non-visible consumption, $x_i$, and implies that the average visible consumption of the population, $C$, is a negative externality for each household $i$ (i.e. $U_C(c_i, C, x_i) < 0$). Condition (iv) implies that the disutility of labor is a strictly convex function of $e_i$, and that when leisure is conspicuous ($\theta = 1$), the disutility of a given amount of labor is larger when the average leisure $(1 - E)$ in the society increases. This is the negative externality due to relative concerns regarding leisure. Finally, conditions (ii)-(iii) and (v)-(vii) guarantee that both the decentralized and the socially optimal equilibria exist and are unique.

According to eq. [1], household $i$ seeks to maximize the following utility function at $t = 0$

$$W_i = \int_0^\infty \exp(-\delta t)(U(c_i(t), C(t), x_i(t)) - \gamma V(e_i(t), \theta E(t))) \, dt, \ \delta > 0 \ \ [2]$$

and therefore social welfare at $t = 0$ can be defined as
utility function at \( t = 0 \). To close the model, we assume that output at \( t \) is produced by combining capital, \( K(t) \), and aggregate labor, \( E(t) \) with the production function \( F(K(t), E(t)) \), where \( F \) is concave, increasing and homogenous of degree 1 in \( K(t) \) and \( E(t) \).

Let us now characterize the market equilibrium and socially optimal paths. Assumption 1 and the concavity of \( F \) imply that households behave identically, so we can focus on the symmetrical equilibrium, that is \( c_i(t) = c(t) = C(t) \), \( e_i(t) = e(t) = E(t) \) and \( x_i(t) = x(t) \). Then, given that goods are nonperishable, the accumulation equation for the representative household can be expressed as

\[
\dot{K}(t) = F(K(t), E(t)) - c(t) - x(t), \quad \text{where } K(0) > 0 \text{ is given.}
\]

Hence, by deriving the first order optimality conditions for households and firms and imposing market clearing conditions, the decentralized equilibrium is fully characterized by the following system of equations

\[
U_c(C^m(t), C^m(t), x^m(t)) = p^m(t) \tag{5}
\]

\[
U_x(C^m(t), C^m(t), x^m(t)) = p^m(t) \tag{6}
\]

\[
\gamma V(e^m(t), \theta E^m(t)) = p^m(t) F_E(K^m(t), E^m(t)) \tag{7}
\]

\[
\frac{\dot{p}^m(t)}{p^m(t)} = \delta - F_K \tag{8}
\]

where the superscript \( m \) denotes market equilibrium and \( p^m(t) \) is the costate variable for the constraint [4]. As for the socially optimal path, this is characterized by

\[
U_c(C^o(t), C^o(t), x^o(t)) + U_c(C^o(t), C^o(t), x^o(t)) = p^o(t) \tag{9}
\]

\[
U_x(C^o(t), C^o(t), x^o(t)) = p^o(t) \tag{10}
\]

\[
\gamma[V(e^o(t), \theta E^o(t)) + \theta V_E(E^o(t), \theta E^o(t))] = p^o(t) F_E(K^o(t), E^o(t)) \tag{11}
\]

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6 As in Arrow and Dasgupta (2009), we do not study the existence of either equilibrium or optimal paths because the relevant theorems in Stokey and Lucas (1989) can be used to show that they do exist under the conditions we have placed on preferences and technology.
\[
\frac{\dot{p}^0(t)}{p^0(t)} = \delta - F_K
\]

The effects of relative concerns on visible consumption can be derived by comparing the equilibrium conditions in the decentralized economy with the socially optimal plan. The results of this comparison, as well as the characterization of the steady state equilibrium, are presented in the following proposition.

**Proposition 1** Assume that the felicity function of the representative household satisfies Assumption 1. Then the socially optimal and market equilibrium paths cannot coincide.

**Proof.** See the Appendix.

In addition, an important corollary of this proposition allows us to characterize the economic distortions of the steady state competitive equilibrium.

**Corollary 1** If labor supply is exogenous \((\gamma = 0)\), \(C^{m*} > C^o\) and \(x^{m*} < x^{o*}\). In addition, if the felicity function is separable in \(c_i\) and \(x_i\) \((U_{cx} = 0)\) and separable in \(x_i\) and \(C\) \((U_{xC} = 0)\), then the steady state equilibrium with endogenous labor supply \((\gamma = 1)\) is characterized by the following economic distortions:

(i) if leisure is inconspicuous \((\theta = 0)\), \(C^m > C^o\), \(x^m < x^o\), \(E^m > E^o\) and \(K^m > K^o\).

(ii) if leisure is conspicuous \((\theta = 1)\), \(x^m < x^o\).

**Proof.** See the Appendix.

In contrast with the results of Arrow and Dasgupta (2009) (Propositions 1 and 3), Proposition 1 establishes that, when we allow for the presence of both visible and non-visible consumption goods, and visible consumption is subject to relative concerns, there is no felicity function satisfying regularity conditions that avoids market distortions. This result holds independently of whether labor supply is a choice variable or not, or whether leisure is conspicuous or not. What is the intuition of this result? When the consumer chooses over a bundle of goods but only a subset of them is subject to relative concerns, for the decentralized equilibrium to be Pareto optimal, the marginal utility of each good not subject to relative concerns needs to be the same across the market and socially optimal paths. This requirement implies that also the shadow prices have to be the same across the two equilibria, a condition that commands equality between the marginal utilities of externality-producing goods. For this last requirement to

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7 When \(\gamma = 0\), then \(E(t) = 1\) and \(F_E = 0\), so that eqs [7] and [11] are trivially satisfied.
be true, however, households should behave exactly as the social planner, so that they internalize the effects of the externalities when choosing their consumption bundles. Thus, there is no felicity function that can avoid market distortions. Moreover, as summarized in Corollary 1, as long as separability assumptions hold, such economic distortions are the same as that of Proposition 2 of Arrow and Dasgupta (2009) regarding visible consumption, labor supply and saving. However, adding non-visible consumption allows us to highlight a new economic distortion: the steady state competitive equilibrium non-visible consumption is strictly lower than the socially optimal one.

3 Conclusion

Our article relies on a very simple idea: If relative concerns are to be relevant, which is consistent with a bulk of empirical studies, these concerns can only apply to visible goods (i.e., goods that are readily observable in anonymous social interactions and portable across those interactions). If households consume both visible and non-visible goods, and the former are subject to relative concerns, then it must be the case that the competitive equilibrium is distorted: the competitive equilibrium marginal rate of substitution between the visible and non-visible goods will always be different than the socially optimal one, since individuals do not take into account the negative externality they exert on others through the consumption of the visible good, while the social planner does.

In general, it is not possible to provide an unambiguous characterization of the levels of consumption (and other economic variables) in the steady state competitive equilibrium with respect to their socially optimal levels. Still, if one imposes separability assumptions in the utility function, the steady state competitive equilibrium consumption of non-visible goods will be strictly lower than the socially optimal one. These assumptions do not appear to be ill-suited, at least given the empirical evidence showing that higher spending on conspicuous consumption is at the expense of inconspicuous consumption, both in developed and in developing countries.

We conclude with an important remark. The existence result of a utility function satisfying regularity conditions such that the competitive equilibrium is socially optimal in Arrow and Dasgupta (2009) requires that all goods are visible, so that all goods are subject to relative concerns. This is, of course, a very strong assumption. Once we relax it, we are back to a world where relative concerns on visible consumption are a source of economic distortions. Given that more visible goods tend to be more positional (Alpizar, Carlsson, and Johansson-Stenman 2005;
Carlsson, Johansson-Stenman, and Martinsson 2007; Solnick and Hemenway 2005), extending our model to incorporate different degrees of *positionality*\(^8\) for different goods (including leisure) could be a fruitful avenue for future research. This would allow to quantify the welfare losses due to economic distortions arising from relative concerns.

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## Appendix

### Proof of proposition 1

Assume that \(C^m(t) = C^o(t)\) and \(x^m(t) = x^o(t)\). Then eqs [6] and [10] imply \(p^o(t)/p^m(t) = 1\). Given eqs [5] and [9], it follows that

\[
1 = \frac{U_C(C^o(t), C^o(t), x^o(t)) + U_C(C^o(t), C^o(t), x^o(t))}{U_C(C^m(t), C^m(t), x^m(t))}.
\]

This requires \(U_C(C^o(t), C^o(t), x^o(t)) = 0\), which is a contradiction given Assumption 1. Hence, the socially optimal and the market equilibrium paths cannot coincide.

### Proof of corollary 1

Suppose that labor supply is exogenous (\(\gamma = 0\)). Given eqs [8] and [12], at the steady state \(K^m = K^o = K^s\). Now, assume that \(C^m \leq C^o\). The resources constraint \(F(K^s) = C^s + x^s\), \(s = \{o, m\}\), implies \(x^m \geq x^o\). In addition, from eqs [5]–[6] and [9]–[10] it follows that \(U_x(C^m, C^m, x^m) = U_x(C^m, C^m, x^m)\) and \(U_C(C^o, C^o, x^o) + U_C(C^o, C^o, x^o) = U_x(C^o, C^o, x^o)\). Combining these two equations and using the resources constraint

\[
\left[ U_C(C^m, C^m, F(K^s) - C^m) - U_C(C^o, C^o, F(K^s) - C^o) \right] - U_C(C^o, C^o, F(K^s) - C^o) = \]

\[
U_x(C^m, C^m, F(K^s) - C^m) - U_x(C^o, C^o, F(K^s) - C^o)
\]

---

\(^8\) The marginal degree of *positionality* measures the fraction of the total utility change which comes from increased relative consumption from the last dollar spent (see Alpizar, Carlsson, and Johansson-Stenman 2005).
However, given Assumption 1, the right-hand side (RHS) and the left-hand side (LHS) of the previous expression have different signs: the LHS is strictly positive and the RHS is non-positive. This contradicts $C^{m^*} \leq C^o$. Therefore, $C^{m*} > C^o$ and $x^{m*} < x^{o*}$.

Suppose now that labor supply is endogenous ($\gamma = 1$), that the utility function is separable in $c_i$ and $x_i$, and in $x_i$ and $C$ (i.e. $U_{cx} = U_{xC} = 0$), and that leisure is inconspicuous ($\theta = 0$). Let $f(k)$ denote the production function in intensive form, where $k = K/E$. Given the assumptions on $F, f$ is an increasing and concave function of $k$, with $f(0) = 0$. In addition, $F_k(K,E) = f'(k)$ and $F_E(K,E) = f(k) - kf'(k)$. From eqs [8] and [12], it follows that $k^{o*} = k^{m*} = k^*$. Moreover, the resources constraint can be written as $E^s f(k) = C^s + x^s$ with $s = \{m, o\}$. Let us define $dS = S^{m*} - S^{o*}$ as the change in the variable $S$ between the two equilibria. We first consider the case with $dE = E^{m*} - E^{o*} = 0$. Given eqs [7] and [11] and given that $k^{o*} = k^{m*}$, the costate variable must be the same in both equilibria. This contradicts Proposition 1. Therefore, we must have $dE \neq 0$. Consider now the case with $dE < 0$. Since $V(E, 0)$ is a strictly convex function of $E$, this implies $V_e(E^{m*}, 0) < V_e(E^{o*}, 0)$. Therefore $P^{m*} < P^{o*}$. Given the first order conditions [5]–[12], this requires that the consumption allocations pairs $(C^{m*}, x^{m*})$ and $(C^{o*}, x^{o*})$ satisfy

$$U_c(C^{m*}, C^{m*}, x^{m*}) - U_c(C^{o*}, C^{o*}, x^{o*}) < U_c(C^{o*}, C^{o*}, x^{o*})$$  \[14\]

$$U_x(C^{m*}, C^{m*}, x^{m*}) < U_x(C^{o*}, C^{o*}, x^{o*})$$ \[15\]

To verify the aforementioned conditions, we proceed by totally differentiating $U_c, U_x$ and the resources constraint. In doing so, we make use of the separability assumptions. Hence,

$$dU_c = (U_{cc} + U_{cC})dC$$ \[16\]

$$dU_x = U_{xx}dx$$ \[17\]

$$dEf(k) = dC + dx$$ \[18\]

Given eq. [18], $dE = E^{m*} - E^{o*} < 0$ requires that $dC$ and $dx$ cannot be both non-negative at the same time. Moreover, in order to satisfy conditions [14] and [15], we must have $dU_x < 0$ and $dU_c < 0$. Because of item (ii) in Assumption 1 and $U_{cx} = 0, (U_{cc} + U_{cC}) < 0$ which implies $dC > 0$ by eq. [16]. In addition, because of item (i) in Assumption 1, $U_{cx} < 0$ which implies $dx > 0$ by eq. [17]. But this is a contradiction, since $dx$ and $dC$ cannot be both non-negative at the same time. Finally, let us consider the case when $dE = E^{m*} - E^{o*} > 0$. From the first order conditions, this requires
\[ U_C(C^m, C^o, x^m) - U_C(C^o, C^o, x^o) > U_C(C^o, C^o, x^o) \]  
\[ U_X(C^m, C^m, x^m) > U_X(C^o, C^o, x^o) \]

Now, in order to satisfy condition [20], in equilibrium we must have \( dU_x > 0 \). Hence, given that \( U_{xx} < 0 \), eq. [17] implies \( dx < 0 \). Moreover, given eq. [18], assuming \( dE > 0 \) requires that \( dC \) and \( dx \) cannot be both non-positive at the same time. Therefore, \( dC > 0 \), which implies \( dU_c < 0 \), and this is compatible with eq. [19]. Therefore, at the steady state \( E^m > E^o \), \( C^m > C^o \), \( x^m < x^o \), and since \( k^m = k^0 \), \( K^m > K^o \).

Suppose now that \( \theta = 1 \), so that leisure is conspicuous. In this case, the total differentials of \( U_c \), \( U_x \), \( V_e \) and the resources constraint are given by eqs [16]–[18] and in addition

\[ dV_e = (V_{ee} + V_{eE})dE \]

Suppose that \( dE = 0 \). From eq. [21], \( dV_e = 0 \), and from eq. [18], \( dC = -dx \). Hence, we can distinguish three cases: (i) \( dC > 0 \) and \( dx < 0 \), (ii) \( dC < 0 \) and \( dx > 0 \), and (iii) \( dC = dx = 0 \). Let us start with (i). From eq. [16], \( dU_c < 0 \). From eq. [17], \( dU_x > 0 \), which implies \( P^{m^r} > P^{m^e} \), because of eqs [6] and [10]. In addition, from conditions [5] and [9] and conditions [7] and [11], \( P^{m^r} > P^{m^e} \) requires that

\[ U_c(C^m, C^m, x^m) - U_c(C^o, C^o, x^o) > U_c(C^o, C^o, x^o) \]

\[ V_e(E^m, E^m) - V_e(E^o, E^o) > V_e(E^o, E^o) \]

which are compatible with \( dU_c < 0 \) and \( dV_e = 0 \). Consider now (ii): \( dC < 0 \) and \( dx > 0 \). From eq. [16], \( dU_c > 0 \). From eq. [17], \( dU_x < 0 \), which implies \( P^{m^r} < P^{m^e} \), because of eqs [6] and [10]. But then, from conditions [5] and [9], \( P^{m^r} < P^{m^e} \) requires that

\[ U_c(C^m, C^m, x^m) - U_c(C^o, C^o, x^o) < U_c(C^o, C^o, x^o) \]

which is a contradiction, since the LHS of eq. [24] is strictly positive (\( dU_c > 0 \)) and its RHS is strictly negative (\( U_c < 0 \)). Finally, consider case (iii). \( dE = dC = dx = 0 \) contradicts Proposition 1, and therefore cannot be an equilibrium. Hence, we conclude that if \( dE = 0 \), then \( x^{m^r} < x^{o^o} \).

Suppose now that \( dE > 0 \). From eq. [21], \( dV_e > 0 \), and from eq. [18], \( dC \) and \( dx \) cannot be both negative at the same time. Again, we can distinguish three cases: (i) \( dC > 0 \) and \( dx \geq 0 \), (ii) \( dC > 0 \) and \( dx < 0 \), and (iii) \( dC \leq 0 \) and \( dx > 0 \). Let us start with (i). From eq. [17], \( dU_x \leq 0 \), which implies \( P^{m^r} \leq P^{m^e} \), because of eqs [6] and [10]. But then, from conditions [7] and [11], this requires that

\[ V_e(E^{m^r}, E^{m^r}) - V_e(E^{o^o}, E^{o^o}) \leq V_e(E^{o^o}, E^{o^o}) \]
which is a contradiction, since the LHS of eq. [25] is strictly positive ($dV_e > 0$) and its RHS is strictly negative ($V_E < 0$). Consider now (ii): $dC > 0$ and $dx < 0$. From eq. (16), $dU_C < 0$. From eq. [17], $dU_x > 0$, which implies $P^{m^*} > P^{m^o}$, because of eqs [6] and [10]. In addition, from conditions [5] and [9] and conditions [7] and [11], $P^{m^*} > P^{m^o}$ requires conditions [22] and [23] which are compatible with $dU_c < 0$ and $dV_e > 0$. As for the case (iii), $dC \leq 0$ and $dx > 0$, from eq. [17], $dU_x < 0$, which implies $P^{m^*} < P^{m^o}$, because of eqs [6] and [10]. But then, from conditions [7] and [11], this requires that eq. [25] is satisfied with strict inequality which is again a contradiction, since $dV_e > 0$ and $V_E < 0$. Hence, we conclude that if $dE > 0$, then $x^{m^*} < x^{o^*}$. Finally, suppose that $dE < 0$. From eq. [21], $dV_e < 0$, and from eq. [18], $dC$ and $dx$ cannot be both positive at the same time. Hence, we can distinguish three cases: (i) $dC \geq 0$ and $dx < 0$, (ii) $dC < 0$ and $dx < 0$, and (iii) $dC < 0$ and $dx \geq 0$. Following the same strategy as in the previous two cases, it is easy to show that in equilibrium, if $dE < 0$, then $x^{m^*} < x^{o^*}$. Thus, in all of the feasible cases, $x^{m^*} < x^{o^*}$. This proves the statement.

References


