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Abstract: Parallel aligned liquid crystal (PA-LC) devices are widely used in many optics and photonics applications to control the amplitude, phase and/or state of polarization (SOP) of light beams. Simplified models yet with a good predictive capability are extremely useful in the optimal application of these devices. In this paper we propose and demonstrate the validity of a novel model enabling to calculate the voltage dependent retardance provided by parallel-aligned liquid crystal (PA-LC) devices for a very wide range of incidence angles and any wavelength in the visible. We derive the theoretical expressions, and both experimental and theoretical retardance results are obtained showing a very good agreement. The proposed model is robust and well adapted to a reverse-engineering approach for the calibration of its parameters, whose values are obtained without ambiguities. The model is based on only three physically related magnitudes: two off-state parameters per wavelength and one global voltage dependent parameter, the tilt angle of the LC molecules. To our knowledge it represents the most simplified model available for PA-LC devices yet showing predictive capability. Not only eases the design of experiments dealing with unconventional polarization states or complex amplitude modulation, but it also serves to analyze the physics and dynamics of PA-LC cells since we have estimation for their voltage dependent tilt angle within the device.

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Alicante, 26 of February 2015

Dear Editor,

I have submitted through the online system for publication in Optics and Lasers in Engineering the manuscript “*Effective angular and wavelength modelling of parallel aligned liquid crystal devices*” with its corresponding figure files. A highlight file has been also submitted.

Yours sincerely,

Andrés Márquez

Highlights

- A highly usable model for parallel-aligned liquid crystal (PA-LC) devices is demonstrated.
- It represents the most simplified model available yet showing excellent predictive capability.
- It enables the calculation of the retardance for a very wide range of incidence angles and any wavelength in the visible.
- Estimation of the optical path difference and the tilt angle of the LC director are obtained without ambiguities.

Effective angular and wavelength modelling of parallel aligned liquid crystal devices

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ABSTRACT

Parallel aligned liquid crystal (PA-LC) devices are widely used in many optics and photonics applications to control the amplitude, phase and/or state of polarization (SOP) of light beams. Simplified models yet with a good predictive capability are extremely useful in the optimal application of these devices. In this paper we propose and demonstrate the validity of a novel model enabling to calculate the voltage dependent retardance provided by parallel-aligned liquid crystal (PA-LC) devices for a very wide range of incidence angles and any wavelength in the visible. We derive the theoretical expressions, and both experimental and theoretical retardance results are obtained showing a very good agreement. The proposed model is robust and well adapted to a reverse-engineering approach for the calibration of its parameters, whose values are obtained without ambiguities. The model is based on only three physically related magnitudes: two off-state parameters per wavelength and one global voltage dependent parameter, the tilt angle of the LC molecules. To our knowledge it represents the most simplified model available for PA-LC devices yet showing predictive capability. Not only eases the design of experiments dealing with unconventional polarization states or complex amplitude modulation, but it also serves to analyze the physics and dynamics of PA-LC cells since we have estimation for their voltage dependent tilt angle within the device.

Keywords: Liquid-crystal devices, Parallel-aligned, Birefringence, Spatial light modulators, Displays.

1. INTRODUCTION

Liquid crystal (LC) devices are commonly used in optics and photonics both in display [1] and in non-display applications [2]. In the latter, parallel-aligned cell geometry is usually the LC technology of choice since it enables the more energetic efficient phase-only operation without amplitude coupling [3,4]. Parallel-aligned liquid crystal (PA-LC) devices can be assimilated to linear variable retarders, and as such are characterized by their linear retardance. Then a number of methods typically used in the characterization of waveplates become available [5]. Recently, we demonstrated a novel method based on time-average Stokes polarimetry [6], able to provide robust and precise measurement of the linear retardance value even in the presence of flicker, exhibited by electrooptic devices such as liquid crystal on silicon (LCoS) displays [7-9].

Accurate calculation of the performance of liquid-crystal cells is possible when the different parameters characterizing the LC material and the LC cell are known [2,10]: such parameters are ordinary and extraordinary refractive indices, cell gap, pre-tilt angle, index of refraction of the glass window, viscosity and elastic coefficients, electrode structure, among others. In a first step, accurate calculation of the actual orientation of the LC director across the LC layer as a function of applied voltage applied is performed by minimizing the total free-energy of the LC cell [2,10]. In a second step, precise numerical

methods are used to calculate the electromagnetic propagation of radiation across the cell. Among these propagation methods are the exact Berreman's 4x4 matrix approach [11] or the very precise extended Jones matrix calculation methods, proposed by Yeh, Gu or Lien [12-14], appropriate to be applied to oblique incidence in the general case of inhomogenous director axis orientation, such as in twisted-nematic LC cells [15,16]. Full advantage of this rigorous approach is possible for LC designers and manufacturers with access to all details to optimize the electro-optical properties of LC devices.

Most of the time users of LC devices have not access to this detailed information or the precision required in their application does not justify the use of the more complex rigorous approach. More simplified models and/or reverse-engineering approaches, enabling analytical expressions, are then highly desirable, as it was the case with transmissive twisted-nematic liquid crystal displays (TN-LCD) [17,18] which were the devices mostly used in spatial light modulation applications [2-4] until the appearance of modern parallel-aligned LCoS (PA-LCoS) panels. These models enabled the calculation of the complex amplitude transmittance at normal incidence, which is the typical working geometry used with transmissive LCDs in non-display applications. In LCoS panels, Lizana et al. analysed experimentally the wavelength dependence of the phase-shift [19] and also its magnitude under oblique incidence [20], which is a fairly common working geometry in reflective devices such as LCoS devices. Recently, in the case of PA-LCoS panels [21] we showed angular dependence of retardance and flicker amplitude. A theoretical and analytical modelling of this dependence with a simplified approach compatible with a reverse-engineering strategy has not yet been attempted. If successful this provides the benefit of a deeper insight on the physics of the device together with a predictive capability useful to optimize the working conditions of PA-LC devices with angle and wavelength according to the specific application in mind.

In the present work we propose and demonstrate a novel physical model which, through a reverse-engineering approach, is able to provide with a good accuracy the linear retardance value versus applied voltage as a function of both the incidence angle and the illuminating wavelength for PA-LC devices. The model is based on only three physical parameters whose values are obtained without ambiguities by fitting a limited amount of calibration measurements. Experimental and simulated results will be provided, using a PA-LCoS microdisplay as the device under test. An excellent agreement is obtained under a wide range of situations, which is especially remarkable since the performance of such a complex device can be predicted with a highly reduced physical model. For the sake of comparison a rigorous model describing the retardance in a homogenous PA-LC cell with arbitrary director axis orientation [16] is presented and used as a reference. We show that it offers no better agreement with experiment, and furthermore is not able to provide the values for its parameters without ambiguities, which highlights the usefulness of our proposal. To our knowledge our proposal represents the most simplified model available for PA-LC devices yet showing a high predictive capability.

2. THEORETICAL MODEL

A rigorous physical model for the phase retardance introduced by a homogeneous uniaxial anisotropic plate can be obtained by direct application of the Maxwell equations as it is for example developed in pp. 326-328 by Gu and Yeh [10] or also by Lien [16]. Next we show the basic expressions resulting from this rigorous approach, whose diagram showing the meaning of the various magnitudes is presented in Fig. 1, where α and θ_{inc} are respectively the tilt angle for the LC director and the angle of incidence for the light beam, ϕ_d and ϕ_{inc} are the corresponding azimuth angles, and n_o and n_e are respectively the ordinary and the extraordinary refractive index in the LC layer of thickness d . First, let us note that double refraction is produced for a beam of light transmitted into a uniaxial medium, thus both an extraordinary and an ordinary wave are produced. The phase retardation of a uniaxial medium of thickness d is then given by [10,16],

$$\Gamma = (k_{e,z} - k_{o,z})d \quad (1)$$

, where $k_{e,z}$ and $k_{o,z}$ are the z axis components of the wavevectors of the extraordinary and ordinary waves, respectively,

$$k_{e,z} = \frac{2\pi}{\lambda} \frac{n_e n_o}{\varepsilon_{zz}} \left[\sqrt{\varepsilon_{zz} - \left(1 - \frac{n_e^2 - n_o^2}{n_e^2} \cos^2 \alpha \sin^2 (\phi_d - \phi_{inc}) \right) \sin^2 \theta_{inc}} - \frac{\varepsilon_{xz}}{\varepsilon_{zz}} \sin \theta_{inc} \right] \quad (2)$$

$$k_{o,z} = \frac{2\pi}{\lambda} \sqrt{n_o^2 - \sin^2 \theta_{inc}} \quad (3)$$

, and ε_{xz} and ε_{zz} are given by,

$$\varepsilon_{xz} = (n_e^2 - n_o^2) \sin \alpha \cos \alpha \cos (\phi_d - \phi_{inc}) \quad (4)$$

$$\varepsilon_{zz} = n_o^2 + (n_e^2 - n_o^2) \sin^2 \alpha \quad (5)$$

In the paper we restrict our attention to the basic configuration of interest [20,21] in many photonic applications with PA-LC devices where the LC director axis lays along or perpendicular to the incidence plane: typical illumination with a light beam linearly polarized parallel or perpendicular to the incidence plane stays with the SOP unchanged and providing phase-only operation when parallel to the LC director. In the simulations we consider the specific situation where $\phi_{inc} = \phi_d = 0^\circ$, i.e. LC director along XZ which is the incidence plane. Expressions (1)-(5) will be used as the reference against both the experimental results and the simulated results provided by the calculations with the simplified model we propose.

[location of Figure 1]

Figure 1. Diagram for arbitrary light impinging on a homogeneous uniaxial medium in accordance with the rigorous approach used as a reference model for the PA-LC cell.

Let us now introduce the simplified model we propose in the paper for PA-LC devices. Its general diagram is presented in Fig. 2, where we explicitly consider a reflective cell with a cell gap d . We note that incidence plane and LC director are along the XZ plane, which is the situation under analysis already commented with the rigorous reference model. Reflective geometry is given since the experimental results in the paper are obtained with a PA-LCoS microdisplay, which are reflective devices. Transmissive PA-LC devices can be considered as a specific case where only one passage through the LC layer is produced. Notation for the light incidence and LC director tilt angles are the same as in Fig. 1 for the reference model, i.e. angles θ_{inc} and α . LC molecules have their director axis (optical axis) aligned at an angle φ with respect to the traversing light beam direction. θ_{LC} is the refraction angle in the LC medium. When a voltage V is applied the director axis tilts an angle α with respect to the entrance face. This is the only voltage dependent magnitude, i.e. $\alpha(V)$. At the backplane the light beam is reflected and a second passage is produced across the LC layer whose effect is equivalent to a forward propagation at an angle $-\theta_{inc}$. We note that angles are taken positive in the counter-clockwise sense, thus in the figure θ_{inc} , θ_{LC} and φ are positive, whereas α and θ_{ref} are negative. The same convention is also valid for Fig. 1 previously presented. Some simplifications have been introduced in the diagram: we consider that the LC director orientation is homogenous across the LC cell (even when voltage is applied), no pretilt angle is taken into account, and no double refraction is considered at the interface with the LC layer. Furthermore, no Fresnel coefficients at the interfaces are taken into account, since we do not have a priori information about their values. We note that despite these approximations, the good agreement between experimental and simulated results which will be later presented for the visible spectrum range and for a large range of angles of incidence from 0° to 45° , proves that the straightforward and reduced physical model that we propose still contains the most essential physics of PA-LC devices, enabling predictive capability in the calculations.

[location of Figure 2]

Figure 2. Diagram for the PA-LC cell considered in the simplified model proposed.

Let us derive the expressions for the simplified model. For a uniaxially anisotropic material, as it is the case in most of liquid crystals, we know that the effective extraordinary index n_{eff} depends on the angle φ between director axis and light beam as follows,

$$\frac{1}{n_{eff}^2(\varphi)} = \frac{\sin^2 \varphi}{n_e^2} + \frac{\cos^2 \varphi}{n_o^2} \quad (6)$$

The phase-shift introduced between the extraordinary and ordinary components, i.e. the retardance Γ , is given by,

$$\Gamma = \frac{2\pi}{\lambda} \frac{d}{\cos \theta_{LC}} [n_{eff}(\varphi) - n_o] \quad (7)$$

, where λ is the wavelength of the light beam and $d/\cos \theta_{LC}$ is the length of the trajectory across the LC layer. Substitution of Eq. (6) into (7) provides,

$$\Gamma = \frac{2\pi}{\lambda} \frac{d}{\cos \theta_{LC}} \left[\frac{n_e n_o}{\sqrt{n_e^2 \cos^2 \varphi + n_o^2 \sin^2 \varphi}} - n_o \right] \quad (8)$$

This simplified expression can be found in textbooks (see p. 309 in Ref. [1] or p. 140 in Ref. [22]). Initially we used this expression to calculate the linear retardance provided by our PA-LC device under the wide range of working conditions already mentioned: wavelengths covering the visible spectrum and oblique incidence ranging from 0° to 45° . Fitting experimentally measured values produced multiple sets of solutions for n_o , n_e and d providing equally good fits. This ambiguity did not enable to establish a more precise set of values, which was not satisfying. Additionally, the solutions obtained for these parameters were only partly consistent with physically meaningful values in the literature. These reasons motivated our goal to go one step further and search for an even more minimalistic model, yet with the capability to provide a good predictive capability in the calculations. To reduce the number of parameters in the model let us introduce the following substitution in Eq. (8), $n_e = n_o(1 + \Delta n/n_o)$, where $\Delta n = n_e - n_o$ is the birefringence. Then for n_e^2 in the denominator we apply the following approximation $n_o \gg \Delta n$ which renders, $n_e^2 \cong n_o^2(1 + 2\Delta n/n_o)$ when only the first order term in $\Delta n/n_o$ is left. Then we obtain,

$$\Gamma = \frac{2\pi}{\lambda} \frac{d}{\cos \theta_{LC}} \left[\frac{n_o^2(1 + \Delta n/n_o)}{n_o \sqrt{(1 + 2\Delta n/n_o)\cos^2 \varphi + \sin^2 \varphi}} - n_o \right] \quad (9)$$

After some algebraic manipulation it can be simplified into,

$$\Gamma = \frac{2\pi}{\lambda} \frac{dn_o}{\cos \theta_{LC}} \left[\frac{1 + \Delta n/n_o}{\sqrt{1 + (2\Delta n/n_o)\cos^2 \varphi}} - n_o \right] \quad (10)$$

Taking into account the approximation

$$\sqrt{1 + (2\Delta n/n_o)\cos^2 \varphi} \cong 1 + (\Delta n/n_o)\cos^2 \varphi \quad (11)$$

, where we apply $n_o \gg \Delta n$ once again, and with the following substitutions $OPL = dn_o$ and $OPD = d\Delta n$, which correspond respectively to the magnitudes of the optical path length for the ordinary component and the optical path difference between extraordinary and ordinary components, we eventually obtain,

$$\Gamma = \frac{2\pi}{\lambda} \frac{OPL}{\cos \theta_{LC}} \left[\frac{1 + (OPD/OPL)}{1 + (OPD/OPL) \cos^2 \varphi} - 1 \right] \quad (12)$$

, which is the central expression for the simplified model proposed in the paper. According to Fig. 2, angle φ is given by,

$$\varphi(\theta_{inc}, V) = \frac{\pi}{2} + \alpha(V) \mp \theta_{LC}(\theta_{inc}) \quad (13)$$

where the minus (plus) sign applies for the forward (backward) passage. Then the total retardance in a reflective device is simply the addition of the forward and backward retardances. We note that in the case of normal incidence and LC director axis parallel to the entrance face, then Eq. (12) simplifies into the well known expression $\Gamma = 2\pi d \Delta n / \lambda$.

The only voltage dependent (on-state) parameter, both in the reference model (Eq. (1)-(5)) and in the proposed model (Eq. (12)), is the tilt angle $\alpha(V)$. With regard to the off-state parameters, they have been reduced from three (n_o , n_e and d) to two (OPL and OPD), also possessing a direct physical sense. We note that the refracted angle θ_{LC} in the proposed model is obtained through Snell law where the bulk index for the LC layer is considered to be equal to the one of the glass, i.e. $n_{LC} = n_{glass} = 1.5$, which is a reasonable approximation. Further comparison between theoretical and experimental results in the rest of the paper will validate to which extent the approximations undertaken are acceptable.

Once the rigorous and the simplified models have been presented, next we have to find the values for their parameters by fitting the theoretical expressions to experimental retardance measurements. We divide this calibration process in two steps. In the first one we consider the measurements with the LC device switched off to obtain the values for the off-state parameters. In the second one, the device is switched on and the value for the on-state parameter $\alpha(V)$ is obtained for each voltage applied. In the second step the off-state parameters, already obtained in the first step, are applied as constants in the theoretical expressions in the fitting procedure.

3. EXPERIMENTAL RESULTS

3.1. CALIBRATION

The specific PA-LC device considered in this work is a commercially available PA-LCoS microdisplay, model PLUTO distributed by the company HOLOEYE. It is filled with a nematic liquid crystal, with 1920x1080 pixels and 0.7" diagonal. The retardance measurements are obtained applying the time-average Stokes polarimetric technique [6]. This technique enables the measurement of the retardance both in stable and in flickering devices, as it happens in electrooptic devices such as the LCoS microdisplay used in this work [21]. Both off and on-state retardance measurements have been taken at various angles of incidence (3°, 23°, 35° and 45°) and for three wavelengths (473, 532 and 633 nm) sampling the visible spectrum. We will use the measurements taken at 3° and 35° for calibration, and then the measurements at 23° and 45° will be used to analyse the predictive capability of both the reference and the proposed model.

In Table 1 and 2 we show various solution sets obtained for the calibration of the off-state parameters respectively with the reference and with the proposed model. The values measured in the experiments for the linear retardance at the angle of incidence of 3° (35°) are correspondingly 987° (900°), 803° (719°) and 600° (538°) respectively for the wavelengths 473, 532 and 633 nm. These are the experimental values fed into the fitting algorithm for the off-state parameters. The figure of merit χ^2 to be minimized combines two squared differences: on one hand between theoretical and experimental retardance values normalized by the experimental value, and on the other hand between the theoretical and experimental ratios of the retardance values for the pair of incidence angles considered (3° and 35°) normalized by the experimental ratio. These two normalized squared differences are added up for the three wavelengths and the resultant minimized value for each solution set is shown in row χ^2 (second row).

Table 1. Some of the solution sets using the reference model. Fitting off-state retardance measurements for $\lambda=473, 532$ and 633 nm, and for $\theta_{mc}=3^\circ$ and 45° .

	Set#1	Set#2	Set#3	Set#4
χ^2	2.7×10^{-30}	1.0×10^{-29}	2.3×10^{-30}	2.9×10^{-30}
d (μm)	1.40	2.52	1.89	2.08
n_e (633 nm)	1.68	1.51	1.58	1.55
n_o (633 nm)	1.30	1.30	1.30	1.30
n_e (532 nm)	1.71	1.52	1.60	1.57
n_o (532 nm)	1.28	1.28	1.28	1.28
n_e (473 nm)	1.86	1.65	1.74	1.71
n_o (473 nm)	1.39	1.39	1.39	1.39

In Table 1 we show a small set of the possible combinations of values returned when applying the reference model. Since the theoretical expressions are nonlinear, the optimization process is leading to a different solution set depending on the starting values assigned to the parameters. In this sense we have selected to show in the paper some solution sets which were obtained when applying as starting values the refractive indices at 25° for the commercial LC mixture E7, one of the classical LC mixtures in the literature [23]. Then, these starting values are: $n_e(633\text{nm}) = 1.7305$, $n_o(633\text{nm}) = 1.5189$, $n_e(532\text{nm}) = 1.7512$, $n_o(532\text{nm}) = 1.5268$, $n_e(473\text{nm}) = 1.7763$, $n_o(473\text{nm}) = 1.5353$. Each column in Table 1 corresponds to the solution set obtained for different starting values where only the cell gap d is varied: in the second column (“Set#1”) the starting value for d was $2 \mu\text{m}$, and then incremented to 3, 4 and $5 \mu\text{m}$ for each of the solution sets that we show in the successive columns. We see that, even though we have only varied one of the parameters, the resultant solutions are very different: the reference model is leading to ambiguity in the parameter evaluation. Additionally, we appreciate that the various solution sets are partly out of the range of physically feasible values: for example, the ordinary refractive indices are too low when compared with typical values in the literature [23]. This deviation from physical values may be partly due to the various effects, such as multiple internal reflection effects due to the layered structure of the device [24,25], pretilt angle, or Fresnel coefficients at the interfaces not taken into account. However, the fitting is still very good in all the cases since we obtain very small values for the figure of merit χ^2 (second row). In the case of the proposed model, Table 2, no such ambiguities exist and basically equal values for the parameters are obtained irrespectively of the starting values for *OPL* and *OPD* considered. To this end three solution sets are shown in Table 2 which were obtained using very different starting values for *OPL* and *OPD*. This existence of a non-ambiguous solution is indicative that the two off- state parameters in our model are highly decoupled from each other, and they further summarize the core physics lying behind when modeling the device.

Table 2. Some of the solution sets using the proposed model. Fitting off-state retardance measurements for $\lambda=473, 532$ and 633 nm, and for $\theta_{mc}=3^\circ$ and 45° .

	Set#1	Set#2	Set#3
χ^2	3.8×10^{-28}	5.3×10^{-28}	5.5×10^{-28}
OPD (633 nm)	$0.528 \mu\text{m}$	$0.528 \mu\text{m}$	$0.528 \mu\text{m}$
OPL (633 nm)	$2.576 \mu\text{m}$	$2.576 \mu\text{m}$	$2.576 \mu\text{m}$
OPD (532 nm)	$0.594 \mu\text{m}$	$0.594 \mu\text{m}$	$0.594 \mu\text{m}$
OPL (532 nm)	$2.606 \mu\text{m}$	$2.606 \mu\text{m}$	$2.606 \mu\text{m}$
OPD (473 nm)	$0.649 \mu\text{m}$	$0.649 \mu\text{m}$	$0.649 \mu\text{m}$
OPL (473 nm)	$6.893 \mu\text{m}$	$6.893 \mu\text{m}$	$6.893 \mu\text{m}$

In Table 3 we show the values for OPD and OPL calculated from the data in Table 1 for each of the four solution sets. We note that OPD is equal in the four sets, whereas OPL varies. If we compare OPD and OPL in Table 2 with the corresponding results in Table 3, we find that OPD is the same in both tables whereas OPL shows some differences. This comparison suggests that OPD is a more fundamental magnitude which stays unaffected by the approximations which led to Eq. (12). This is an important result from this work. It is very reasonable since retardance comes out from the combination of d and Δn , which are the two parameters composing the optical path difference parameter OPD . However, a priori we did not expect to be such a robust and invariant magnitude. The other parameter in the model, the optical path length OPL , is probably more affected by the approximations in the proposed model and its value cannot be regarded as the true physical value for the LC cell.

Table 3. OPD and OPL calculated from the values of each of the four solution sets given by the reference model in Table 1.

	Set#1	Set#2	Set#3	Set#4
OPD (633 nm)	0.528 μm	0.528 μm	0.528 μm	0.528 μm
OPL (633 nm)	1.812 μm	3.271 μm	2.457 μm	2.696 μm
OPD (532 nm)	0.594 μm	0.594 μm	0.594 μm	0.594 μm
OPL (532 nm)	1.789 μm	3.228 μm	2.424 μm	2.661 μm
OPD (473 nm)	0.649 μm	0.649 μm	0.649 μm	0.649 μm
OPL (473 nm)	1.946 μm	3.512 μm	2.638 μm	2.894 μm

Once the values for the off-state parameters have been obtained they are treated as constants in the theoretical expressions to fit the tilt angle at each applied voltage (gray level, GL, sent from the graphics card). In particular we will use the off-state solution “Set#3” in Table 1 and the solution “Set#1” in Table 2, respectively for the reference and the proposed models. The same figure of merit function χ^2 is now used for the on-state fitting procedure, and the optimization is run gray level by gray level independently. The experimental values applied correspond to the voltage (gray level) dependent retardance measurements taken for incidence angles at 3° and 35° . In Fig. 3 (a) and (b) we show the retardance versus gray level plots for the theoretical fitting using the reference model (continuous line), the proposed model (dashed line) and the experimental data (dots) respectively for the incidence angles at 3° and 35° , and for the three wavelengths. We note that simulated and experimental results agree very well with each other at both incidences and for the three wavelengths. We also see that the fitting is equally good both with the reference and with the proposed models. If we analyse further the results in Fig. 3 we see that the retardance decreases with the gray level, which tells us that in our device the voltage applied increases (forcing the tilting of the LC molecules) with the increase in gray level. Within each of the two graphs we see that retardance values increase with the decrease in wavelength, with maximum values (at 0 GL) rather close to the experimental values presented in the off-state fitting step. The retardance dynamic range, which is given by the difference between the maximum (at 0 GL) and minimum (at 255 GL) retardance, increases for shorter wavelengths and is also larger at quasi-perpendicular incidence.

[location of Figure 3]

Figure 3. Experiment (dots) and theoretical fitting with the reference (continuous line) and the proposed (dashed line) models for the wavelengths 473, 532 and 633 nm and for incidence at: (a) 3° ; (b) 35° .

Now in Fig. 4 we show the tilt angle as a function of gray level resulting from the on-state fitting with the reference and the proposed models, whose retardance simulated curves are shown in Fig. 3 (a) and (b). Results obtained with either model are very similar with differences smaller than 2° between them. The tilt angle varies between 5° (0 GL) to about 50° (255 GL), thus LC molecules are still far from being totally tilted. Let us note that the tilt angle obtained can be thought as an effective average value since actually tilt angle varies inhomogeneously across the cell with LC molecules in the vicinity of the

windows not able to tilt due to adherence to the alignment layer (see p. 137 in Ref. [22]). We observe that the monotonic increase of the tilt angle with the gray level shows a highly nonlinear variation at lower gray levels which, however, produces a linear relation between retardance and gray level in the plots in Fig. 3.

[location of Figure 4]

Figure 4. Tilt angle as a function of gray level obtained when applying the reference and the proposed model.

3.2. PREDICTIVE CAPABILITY AND DISCUSSION

In Fig. 5 (a) and (b) measurements at 23° and 45° respectively and for the three wavelengths, indicated on the curves, are used to analyse the predictive capability of the proposed model. To see if the more complete approach provides a better result we also analyse the predictive capability of the reference model. We note the good agreement between the theoretical prediction with either model (continuous and dashed lines) and experimental values (dots). No superior performance is obtained with the reference model: probably due to the various effects not taken into account, some of them already commented (multiple internal reflection effects, pretilt angle, or Fresnel coefficients at the interfaces), with the addition of the inhomogenous LC director orientation when in the on-state. We note that we have applied the curve in Fig. 4 to replace the X-axis in the plots for the tilt angle instead of the applied gray level. This helps to make more explicit that the actual degree of freedom behind the retardance variation with the gray level is actually the tilt angle of the LC director. We see the nonlinear dependence of retardance with tilt angle. We specially note the small sensitivity of retardance to the tilt angle at low tilt angles. This is compensated via driver electronics with an appropriate nonlinear relation with the tilt angle of the gray level addressed (Fig. 4) in order to generate the linear curves plotted in Fig. 3.

[location of Figure 5]

Figure 5. Experiment (dots) and prediction with the reference (continuous line) and proposed (dashed line) models for wavelengths 473, 532 and 633 nm and for incidence at: (a) 23°; (b) 45°.

For a closer examination of the accuracy of the proposed model, and also the reference model, we have used the results in Fig. 5 to calculate the difference between predicted and experimental retardance values normalized by the predicted value. This normalized retardance difference for the three wavelengths is plotted for incidence at 23° in Fig. 6 (a) and (b) respectively for the reference and the proposed models, and then equivalently for incidence at 45° in Fig. 6 (c) and (d). We see for the proposed model (Fig. 6 (b) and (d)) that deviation is below ± 0.05 , i.e. $\pm 5\%$, only exceeded by the wavelength 633 nm and only at the larger tilt angles. Something more or less similar happens with the reference model. Then, the model proposed predicts the retardance with relative uncertainties about 5% or less under most of the circumstances, which is accurate enough when evaluating the capabilities offered by the PA-LC device to be used in a certain application.

[location of Figure 6]

Figure 6. Difference between predicted and experimental retardance normalized by the predicted value for wavelengths 473, 532 and 633 nm and for incidence: at 23° with the reference (a) and proposed (b) model; at 45° with the reference (c) and proposed (d) model.

Once we have verified the predictive capability of the model proposed, let us investigate some of the possibilities it offers. We have fitted the extended Cauchy [23] relation $A + B/\lambda^2 + C/\lambda^4$ to the *OPD* values for the three wavelengths (Table

2). In the case of the *OPL* values simply a linear interpolation has been applied. In this way we are able to obtain the retardance not only for a wide range of incidences (till 45°) but also for the wavelengths in the visible region of the spectrum. In Fig. 7 (a) we show the maximum and minimum values of retardance versus wavelength and for three angles of incidence, indicated in the legend. Maximum (minimum) value is given by GL 0 (255). These sorts of curves could be exploited for example for applications dealing with broadband illumination as in Ref. [26]. Both the angle of incidence and the applied gray level can be used as tuning parameters to obtain the required retardance values across the broadband illumination spectrum. From results in Fig. 7(a) we calculate the retardance dynamic range, difference between the maximum and minimum retardance for a specific wavelength at a specific incidence angle: this is presented in Fig. 7(b). Retardance dynamic range is usually the magnitude of interest in spatial light modulation applications [3], e.g. to display phase-only blazed gratings or diffractive lenses a 360° retardance dynamic range is needed [27] even though larger retardance ranges are also interesting as shown recently by Alberio et al. [28]. We note that in Fig. 7(b) retardance range decreases significantly for larger wavelengths and larger angles of incidence. It is worth mentioning that Martínez et al. [25] take advantage of secondary beams produced by multiple internal reflections to have an increased phase dynamic range at 45° incidence angle.

[location of Figure 7]

Figure 7. Theoretical simulations of the retardance versus wavelength and for three incidences, with the proposed model. (a) Maximum and minimum retardance values; (b) Retardance dynamic range.

For the sake of completeness of the basic calculations provided by the proposed model, in Fig. 8(a) and (b) we show respectively the maximum and minimum values of retardance and the retardance dynamic range versus the angle of incidence and for three wavelengths. In Fig. 8(a) the maximum and minimum retardance follow opposite tendencies since the former decreases with the angle of incidence whereas the latter increases. As a result, the dynamic range in Fig. 8(b) decreases significantly with the angle of incidence. The calculations are restricted to the range of incidence angles between 0° and 45° , which is the range where we have demonstrated the agreement between model and experiment. It can be expected that for larger incidence angles some effects not included in the proposed model, such as double refraction and difference of Fresnel coefficients for the TE (transverse electric) and TM (transverse magnetic) components of the light beam at the interfaces, will produce larger deviations between the model and the experiment.

[location of Figure 8]

Figure 8. Theoretical simulations of the retardance versus incident angle and for the three wavelengths, with the proposed model. (a) Maximum and minimum retardance values; (b) Retardance dynamic range.

4. CONCLUSIONS

We have proven the validity of both the reference and the proposed model, the latter more minimalistic and robust, better adapted to a reverse-engineering approach since it provides the values for the parameters with no ambiguities. Simply with two off-state parameters, *OPD* and *OPL* for red, green and blue wavelength components, and with one voltage dependent parameter, $\alpha(V)$, the retardance can be obtained for a wide range of incidence angles and for any wavelength in the visible. This is helpful to propose experiments dealing with unconventional polarization states [29,30] or complex amplitude modulation, and also to analyse the physics and dynamics of liquid crystals since we have estimation for the tilt angle $\alpha(V)$ within the device and also for *OPD*, which we have found to be an invariant and robust parameter whose physical true value can be obtained with the proposed model.

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