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UNIT SUMMARIES

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Unit 1. PHYSICAL PRINCIPLES OF SEMICONDUCTORS

• Types of solids

There are various types of bonds between atoms; however, in order to systematize their study two main types of bonds are considered: (a) ionic (formed between a metal and a non-metal) and (b) covalent (formed between non-metals). In practice, no substance forms totally ionic or totally covalent bonds and these two types are extreme cases.

From the macroscopic point of view a solid is a rigid or elastic substance, that is, a substance that has an elastic behaviour under the influence of either hydrostatic forces or strain and tension.

Considering their structure, solids may be divided into two types: amorphous and crystalline. Solids may also be classified based on the predominant type of bond between the atoms or ions making up the crystals. In this classification, five types may be considered: covalent, ionic, those with hydrogen bonds, molecular and metallic.

• Energy bands. Conductors, insulators and semi-conductors

The concept of energy band is very useful for understanding various properties of solids such as electric conductivity.

When atoms join together to form a solid, their external energy levels overlap giving rise to bands. At absolute zero, insulators and semi-conductors have a full valence band separated by a prohibited band of energy from an empty conduction band.

In conductors, the occupied energy bands –valence and conduction bands (allowed bands)– are separated by prohibited bands that cannot be occupied.

Only partially filled bands can give rise to electric currents in a solid when an electric field is applied. In the theory of bands, the difference between conductors and insulators thus resides in whether or not there are any partially occupied bands.

Silicon and germanium have the same external structure as carbon; thus they would be expected to have an energy band diagram similar to that of diamond. However, since the energy gap E_G between the valence and conduction bands is small, when the temperature is raised some electrons in the valence band may acquire sufficient energy to jump to the conduction band. This enables an electric current to be created when an electric field is applied, as in the case of a conductor. Therefore, such substances are called *semiconductors*.

The distinction between an insulator and a semiconductor resides solely in the value of E_G. At ambient temperature, E_G is 1.12 eV for Si and 0.72 eV for Ge. There are two types of current carriers in a semiconductor, electrons and holes, and the total current is the sum of the currents due to each type of carrier.

• Intrinsic and extrinsic semiconductors

In pure semiconductors (intrinsic) conduction takes place due to the electrons that are present simply because the material is purely crystalline (for example, Ge or Si) and not due to the presence of impurities. The only way in which conduction may take place is to provide the electrons with an energy equal to or greater than E_G –thus enabling them to jump to the conduction band. This may be done by light or thermal excitation.

When minuscule quantities, of the order of one part per million, of the right impurities (different atoms) are added to a pure semiconductor, the latter may exhibit electrical conductivity over a greater range of temperatures (extrinsic semiconductor). This is known as doping. Thus, the electrical properties of a semiconductor may change drastically when small concentrations of donor impurities (*n*-type semiconductor) or acceptor impurities (*p*-type semiconductor) are added.

• Semiconductor equation and electrical neutrality

Semiconductor equation

Let n and p are the concentrations of free electrons and holes in a semiconductor, respectively. In an intrinsic semiconductor, and due to thermal excitation, an electron moves out of the valence band and leaves behind a hole, the number of free electrons is equal to the number of holes

$$n = p = n_i = p_i$$

where n_i and p_i are the intrinsic concentrations of carriers. In general, for any semiconductor, the processes of electron-hole pair generation and recombination are continuous, and an equilibrium is reached at each temperature in which the product np is constant. Thus

$$n \cdot p = n_i^2 = p_i^2 = \textit{constant} \text{ (for fixed } T \textit{ and } E_{\mathrm{G}}\text{)}$$

This equation, known as the semiconductor equation or **mass action law**, is essential when studying semiconductors and semiconducting devices and is valid for both intrinsic and extrinsic semiconductors in thermal equilibrium. When there is no equilibrium, that is, when n and p are governed by external conditions, the above equation is not valid.

The condition of electrical neutrality

If a semiconductor is doped, the mass action law is not sufficient to determine the concentration of carriers. In addition, it is necessary to include an expression that links concentration and density of donor or acceptor impurities. This expression is given by the *electrical neutrality condition:*

[positive charges] = [negative charges]

If N_a denotes the concentration of acceptor impurities and N_d that of donor impurities (number of atoms per unit volume), the electrical neutrality condition is written as:

$$p + N_d = n + N_a$$

which is easy to understand considering the following points: (a) The semiconductor is electrically neutral if no external field acts upon it. (b) When donor and acceptor impurities are added, additional electrons and holes are present. (c) All the donor and acceptor impurities are ionized.

• Transport phenomena in semiconductors

Charge transport phenomena may occur in a semiconductor due to either application of electric fields or the existence of carrier concentration gradients, that is, when the concentration varies depending on the position in the semiconductor material.

There are numerous types of transport phenomena but only the following will be considered

(a) Electric conduction due to charge transport caused by applying a uniform electric field.
(b) Electric conduction due to diffusion of charge caused by the existence of a carrier gradient.
(c) The Hall effect, whereby application of a magnetic field gives rise to an electric field (that is, a potential difference).

Since there are two types of charge carriers in a semiconductor (electrons and holes), the expression for displacement (or drift) current has two terms, one for electrons and one for holes:

$$\mathbf{J} = e(n\mu_n + p\mu_p)\mathbf{E}$$

and so the conductivity of semiconductors is given by:

$$\sigma = e(n\mu_n + p\mu_p)$$

• Semiconductor devices

When a type *p* semiconductor is joined to a type *n* semiconductor a junction diode (*p-n* junction) is formed which is the basis for manufacturing semiconductor devices such as diodes or transistors. These devices play a fundamental role in modern electronics.

A diode contains a *p-n* junction, whereas a bipolar junction transistor contains two *p-n* junctions that may be *p-n-p* or *n-p-n.*

Unit 2. KYNEMATICS

• Introduction

Mechanics is the branch of physics concerned with the relationships between the motion of physical systems and the forces that cause this motion. Mechanics is divided into three parts: *kinematics* which studies motion without considering the forces that cause it; *dynamics* which studies motion and the forces that cause it; and *statics* which studies forces on bodies in static equilibrium.

• Position, velocity and acceleration

The first step in describing the motion of a particle is to establish a system of coordinates or reference system. The position vector **r** situates an object with respect to the origin of a reference system and, in general, is a function of time. In Cartesian coordinates:

$$\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}$$

When a particle moves, the end of vector **r** describes a curve called trajectory. If s is the space covered by the particle along its trajectory, s will be a function of time t and the function $s = f(t)$ is known as a *law of motion*.

The displacement vector $\Delta\mathbf{r}$ is the change in position vector between two points P_1 and P_2:

$$\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$$

The average velocity $\mathbf{v}_{av}$ of a particle is given by the displacement of a point during a time interval $\Delta t = t_2 - t_1$ divided by the time interval:

$$\mathbf{v}_{av} = \frac{\Delta\mathbf{r}}{\Delta t}$$

The instantaneous velocity **v** (a vector quantity) is the limiting value of the average velocity as the time interval Δt tends to zero. The following conditions are met:

$$\mathbf{v} = \lim_{\Delta t\to 0}\frac{\Delta\mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

The instantaneous velocity vector is tangential to the trajectory of the particle at each point of its trajectory.

The average acceleration $\mathbf{a}_{av}$ of a material point is the change in velocity during an interval of time Δt, divided by this time interval:

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t}$$

The instantaneous acceleration **a** is the limit of average acceleration as the time interval tends to zero:

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

• Intrinsic components of acceleration

The instantaneous acceleration vector, **a**, may be decomposed into two vectors, one perpendicular to the trajectory called normal or centripetal acceleration, $\mathbf{a}_N$, and the other, tangential to the trajectory, called tangential acceleration, $\mathbf{a}_T$. These components are known as intrinsic components of acceleration:

$$\mathbf{a} = \mathbf{a}_T + \mathbf{a}_N = a_T\,\mathbf{u}_T + a_N\,\mathbf{u}_N$$

$\mathbf{a}_T$ takes into account the change in modulus of velocity vector and $\mathbf{a}_N$ takes into account the change in direction of velocity vector:

$$a_T = \frac{dv}{dt}$$

$$a_N = \frac{v^2}{r}$$

where r is the radius of curvature of the trajectory of the particle at each point of the trajectory. The following condition is met:

$$a = \sqrt{a_T^2 + a_N^2}$$

• Linear motion

Linear motion is one in which the trajectory is a straight line and the space covered coincides with the modulus of the displacement vector. Moreover, the radius of curvature is infinite and there is no normal acceleration.

In uniform linear motion, the velocity is constant and so the acceleration is zero. If motion takes place along the x–axis, the following conditions are met:

$$a(t) = 0$$

$$v(t) = v = \text{constant}$$

$$x(t) = x_0 + vt$$

In the case of uniformly accelerated linear motion, the acceleration is constant and the following conditions are met:

$$a(t) = a = \text{constant}$$

$$v(t) = v_0 + at$$

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

• Circular motion

Circular motion is motion in a plane and the trajectory is a circumference of radius R. The space covered s may be written as a function of angle θ as follows:

$$\theta = \frac{s}{R}$$

The angular velocity ω is the variation in θ with time t:

$$\omega = \frac{d\theta}{dt}$$

The following condition is met:

$$\omega = \frac{v}{R}$$

The angular acceleration α is the variation in angular velocity ω with time t:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

The following condition is met:

$$\alpha = \frac{a_T}{R}$$

A vector $\boldsymbol{\omega}$ may be assigned to the angular velocity ω and another $\boldsymbol{\alpha}$ to the angular acceleration α. In both cases the vectors are perpendicular to the circular trajectory of the particle and fulfil the following conditions:

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$

$$\mathbf{a}_T = \boldsymbol{\alpha} \times \mathbf{r}$$

$$\mathbf{a}_N = \boldsymbol{\omega} \times \mathbf{v} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$$

where **r** is the position vector.

In uniform circular motion, the angular acceleration is zero and the angular velocity constant. This implies that there is no tangential acceleration (the modulus v is also constant) and the normal acceleration is constant since v and the radius R are constant. The following conditions are met:

$$\alpha(t) = 0$$

$$\omega(t) = \omega = \text{constant}$$

$$\theta(t) = \theta_0 + \omega t$$

In uniformly accelerated circular motion, the angular acceleration is constant. The tangential acceleration is constant but the normal acceleration is not. The following conditions are met:

$$\alpha(t) = \alpha = \text{constant}$$

$$\omega(t) = \omega_0 + \alpha t$$

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

• Combination of motions. Projectile motion

Another example of motion in a plane is the motion of a projectile launched at a constant velocity v_0 making an angle α with the x–axis and affected by the acceleration of gravity g along the y–axis. The trajectory is a parabola and the motion is composed of uniform linear motion along the x–axis and uniformly accelerated linear motion along the y–axis. Thus projectile motion is two-dimensional and the plane of motion is the xy–coordinate plane. The time of flight (the time when the projectile hits the ground), t, maximum height, h, and horizontal range, R, are:

$$t = \frac{2v_0 \sin\alpha}{g}$$

$$h = \frac{v_0^2 \sin^2\alpha}{2g}$$

$$R = \frac{v_0^2 \sin 2\alpha}{g}$$

The equation for the trajectory $y(x)$ is:

$$y = -\frac{g}{2v_0^2 \cos^2\alpha}x^2 + (\tan\alpha)x$$

Unit 3. DYNAMICS

• Introduction

Dynamics is the branch of mechanics concerned with motion and the forces that cause it. The motion of a body is a direct result of its interaction with other surrounding bodies and these interactions are conveniently described using the concept of force. The mass of a body is a measure of the object's resistance to changes in its velocity.

• Newton's laws

Newton's laws are fundamental laws of nature and form the basis of mechanics.

Newton's first law of motion (law of inertia): An object at rest remains at rest and an object in motion remains in motion at the same speed and in the same direction unless acted on by any kind of force.

Newton's second law of motion (fundamental equation of dynamics): the acceleration **a** of an object is directly dependent upon the net force **F** acting on the body:

$$\mathbf{F} = m\mathbf{a}$$

Newton's third law of motion (principle of action and reaction): If body *A* exerts a force $\mathbf{F}_{AB}$ (*action*) on body *B*, then body *B* exerts a force $\mathbf{F}_{BA}$ (*reaction*) on *A* of the same intensity but in the opposite direction. In other words, for every action there is an equal and opposite reaction:

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

The forces of action and reaction act on different bodies. Newton's laws are only valid in an inertial reference frame, that is, a reference system in which an object at rest remains at rest if no net force acts upon it. Any reference system that moves at constant velocity with respect to an inertial system is also an inertial reference system. The surface of the earth is an approximate inertial reference system.

• Gravitational force. Weight

The Law of Gravitation was stated by Newton and makes it possible to determine the force of attraction $\mathbf{F}_{12}$ between two bodies of masses m_2 and m_2 separated by a distance r:

$$\mathbf{F}_{12} = -G\frac{m_1 m_2}{r^2}\mathbf{u}_r$$

where $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2} is the gravitational constant and $\mathbf{u}_r$ is a unitary vector in the direction of vector $\mathbf{r}$ that joins the two masses ($\mathbf{u}_r = \mathbf{r}/r$). The force of gravity is always one of attraction. Mass characterizes two different properties of an object −its resistance to changes in its velocity (inertial mass) and its gravitational interaction with other objects (gravitational mass). Experiments show that the two are proportional and in the units system chosen they are the same.

Supposing the earth has radius R and mass M. Then, a body of mass m situated on the earth's surface will be attracted by a force:

$$F = G\frac{Mm}{R^2}$$

and according to Newton's second law, such a mass will be subject to an acceleration g:

$$g = G\frac{M}{R^2}$$

which is the acceleration due to gravity. The *weight* **P** of a body is the force exerted by the earth on the body:

$$\mathbf{P} = m\mathbf{g}$$

• Application of Newton's laws for solving dynamical problems

The procedure for solving a problem in mechanics is as follows:

(i) Sketch the system and identify the object (or objects) to which Newton's second law will be applied. Use vectors to represent the different forces.

(ii) Draw a diagram that includes the coordinate axes in order to decompose the vectors into their components. These diagrams should be drawn in such a way as to simplify the subsequent calculations. Normally this means including as many axes as possible along the directions of the forces, or placing an axis in the direction of acceleration, if this direction is known.

(iii) Using this diagram, write the components of Newton's second law as a function of known and unknown quantities and solve these equations for each unknown quantity. Finally, substitute the numerical values of the known quantities (including the units) in the equations and calculate each of the unknown quantities.

• Linear momentum and angular momentum

The *momentum* or *linear momentum* **p** of a particle of mass m moving at a velocity **v** is given by:

$$\mathbf{p} = m\mathbf{v}$$

Taking into account the relation $\mathbf{a} = d\mathbf{v}/dt$, Newton's second law may be written in terms of momentum as:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

According to the *law of conservation of linear momentum,* in an isolated system, that is, one that is not subjected to external forces, the linear momentum is conserved:

$$\mathbf{F} = 0 \quad \Rightarrow \quad \frac{d\mathbf{p}}{dt} = 0 \quad \Rightarrow \quad \mathbf{p} = \text{constant}$$

The *mechanical impulse* of a force **J** is defined as:

$$\mathbf{J} = \int_{t_1}^{t_2} \mathbf{F} dt$$

The change in the momentum of an object produced by a single impulsing force applied on the object is given by:

$$\Delta \mathbf{p} = \mathbf{p}_2 - \mathbf{p}_1 = \mathbf{J}$$

which is the impulse–momentum theorem.

The *angular momentum* **L** of a particle of mass *m* with respect to a point *O* is given by:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where **r** is the vector which originates at the point *O* and ends at the particle, and $\mathbf{p} = m\mathbf{v}$ is the linear momentum of the particle. It may also be written as:

$$\mathbf{L} = m\mathbf{r} \times \mathbf{v}$$

which shows that **L** is perpendicular to the velocity vector. Deriving the equation $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ with respect to time gives:

$$\frac{d\mathbf{L}}{dt} = \mathbf{r} \times \mathbf{F}$$

That is, the variation in angular momentum of a particle is equal to the momentum of the total force acting on the particle.

The *law of conservation of angular momentum* states that if the external torque (momentum of the total force acting on a particle) is zero ($\mathbf{r} \times \mathbf{F} = \mathbf{0}$), the angular momentum remains constant:

$$\frac{d\mathbf{L}}{dt} = 0 \quad \Rightarrow \quad \mathbf{L} = \text{constant}$$

If the angular momentum remains constant, the trajectory of the particle is confined to a plane.

In order for $\mathbf{r} \times \mathbf{F} = \mathbf{0}$ the following conditions must be met:

(i) **F** = 0 (free particle)

(ii) **F** and **r** are two parallel vectors (**F** is a central force).

Unit 4. WORK AND ENERGY

• Introduction

Work and energy are two of the most important concepts in physics and also in everyday life. In physics, a force performs work when it acts on an object which moves a distance and a component of the force acts along the line of motion of the object. The concept of energy is closely linked to that of work. When a system performs work on another system, energy is transferred between the two systems. There are many forms of energy. Kinetic energy is the energy an object possesses due to its motion. Potential energy is energy which results from position or configuration, such as the distance between a body and the earth. Thermal energy is the kinetic energy resulting from the random movement of atoms and molecules within a system and is closely related to temperature. One of the fundamental laws of nature is the law of conservation of energy. If the energy of a system is conserved, its total energy remains constant, but some of it may be transformed from one form to another.

• Work and power

The *work W* performed by a force **F** that acts on a body moving along a trajectory is defined by the integral:

$$W = \int_1^2 \mathbf{F} \cdot d\mathbf{r}$$

In the simple case of a constant force and distance $\Delta\mathbf{r}$ along a straight line, the work is given by the scalar product:

$$W = \mathbf{F} \cdot \Delta\mathbf{r}$$

For a force that varies along one dimension (for example, along the x-axis):

$$W = \int_{x_1}^{x_2} F_x(x)\,dx$$

The unit of work in the SI is the Joule (J).

The *power P* is the speed at which a force performs work:

$$P = \frac{dW}{dt}$$

The power of a force **F** performing work on an object moving at a velocity **v** is given by:

$$P = \mathbf{F} \cdot \mathbf{v}$$

In the SI, power is measured in watts (W).

• Kinetic energy. Kinetic energy theorem

The *kinetic energy* E_c of a body of mass m moving at a velocity v is:

$$E_c = \frac{1}{2}mv^2$$

Kinetic energy is the energy resulting from motion. The *kinetic energy theorem* states that the work done by a force acting on a body is equal to the change in kinetic energy of the body:

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = E_{c,f} - E_{c,i}$$

that is:

$$W = \Delta E_c$$

- **Conservative forces and potential energy**

A force is *conservative* if the work done in moving a particle around a closed trajectory is zero. In addition the work done by a conservative force is independent of the path taken and depends solely on the initial and final points.

The work done by the weight of a body near the earth's surface is:

$$W = -mg(y_2 - y_1)$$

and it is independent of the trajectory connecting the initial and final points. Such a force is conservative.

The *potential energy E_p* depends solely on position. Two examples of potential energy are gravitational potential energy:

$$E_p = mgy$$

and elastic potential energy due to compression or stretching of a spring:

$$E_p = \frac{1}{2}kx^2$$

In the case of a conservative force, the work W and potential energy E_p are related by the equation:

$$W = -\Delta E_p$$

and the force **F** and potential energy E_p by the equation:

$$\mathbf{F} = -\mathbf{grad}E_p = -\nabla E_p$$

which in the one-dimensional case may be written as:

$$F_x = -\frac{dE_p}{dx}$$

The motion of an object may be represented by a potential energy graph. The points of equilibrium may be identified on this graph.

• Law of conservation of mechanical energy

The sum of the kinetic and potential energy of a system is called mechanical energy *E*:

$$E = E_c + E_p$$

If there are no external forces acting on the system and all the internal forces are conservative, the total mechanical energy of the system remains constant:

$$E = E_c + E_p = \text{constant}$$

that is, between two states, initial 1 and final 2:

$$E_{c1} + E_{p1} = E_{c2} + E_{p2}$$

The total energy of the system E_{syst} is the sum of the different types of energy. One way of transferring the energy (absorbed or ceded) of a system is to exchange work with the surroundings. If this is the only source of transferred energy, the law of conservation of energy is written as:

$$W_{ext} = \Delta E_{syst}$$

W_{ext} is the work done on the system by external forces and ΔE_{syst} is the variation in total energy of the system. This is the work-energy theorem.

• Collisions

The theory of collisions is one of the most important applications of the conservation of momentum law. We only consider collisions between two bodies and two types of collisions: *elastic* and *inelastic.*

In a *perfectly elastic collision* kinetic energy is conserved (i.e., no energy is lost from the system). An inelastic collision is one in which kinetic energy is not conserved (e.g., some energy is lost to friction, crumpled fenders, or such). In a *perfectly inelastic collision* the colliding particles stick together. A perfectly inelastic collision occurs when the maximum amount of kinetic energy of a system is lost.

Unit 5. HEAT AND TEMPERATURE

• Introduction

Temperature is often said to be a measure of the degree of hotness or coldness of a body, but this definition is not valid from the point of view of physics. When a body heats up or cools down, some of its physical properties change. For example, most solids and liquids dilate when heated, when an electrical conductor is heated its resistance varies, etc. A physical property that varies with temperature is called a *thermometric property* and a change in this property indicates that the *temperature* of the object has changed.

• Thermal equilibrium and zeroth law of thermodynamics

Two systems in contact are in *thermal equilibrium* when their properties no longer change with time. For two systems to be in contact they must be separated by a *diathermal or diathermic wall* that facilitates their thermal interaction. An *adiabatic wall* does not allow such interaction: each system is isolated from the other and each may remain in its equilibrium state. The *zeroth law of thermodynamics* states that if two systems are in thermal equilibrium with a third system, they are also in thermal equilibrium with each other. The concept of temperature is related to the state of thermal equilibrium of two systems since they will be in thermal equilibrium if they have the same temperature.

• Thermometers and ideal gas temperature scale

In order to establish a temperature scale a thermometric property is used. Gas thermometers are characterized by the fact that they all register the same temperature provided that the density of the gas used in the thermometer is very low. The temperature of an ideal gas is defined using a limit with real gases diluted in a *constant volume gas thermometer*. The temperature scale is adjusted taking a temperature of 273.16 K as the triple point of water. In this state, the melting point, boiling point and sublimation point coincide and it occurs at a vapor pressure of 610 Pa and temperature of 0.01°C. The ideal gas temperature is defined as:

$$T = 273.16K \frac{p}{p_3}$$

where p is the pressure of the gas in the thermometer when it is in thermal equilibrium with the system whose temperature is to be measured, and p_3 is the pressure when the thermometer is in a ice-water-steam bath at its triple point.

• The ideal gas equation

The ideal gas equation is:

$$pV = nRT$$

where p is the pressure, V the volume, n the number of moles, T the absolute temperature and R the ideal gas constant, whose numerical value depends on the units of the other physical magnitudes in the equation. In SI units, p is in Pa, V in m^3, n in moles and T in K:

$$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$$

Taking into account that

$$1 \text{ cal} = 4.186 \text{ J} \qquad 1 \text{ J} = 0.24 \text{ cal}$$

It is possible to write

$$R = 1.99 \text{ cal mol}^{-1} \text{ K}^{-1} \approx 2 \text{ cal mol}^{-1} \text{ K}^{-1}$$

If p is expressed in atmospheres (atm) and V in liters (L)

$$R = 0.082 \text{ atm L mol}^{-1} \text{ K}^{-1}$$

• Thermal expansion

Most substances expand or dilate when their temperature rises and contract when it falls.

(i) *Linear expansion:* The change ΔL in length L_0 of an object due to a change in temperature ΔT is:

$$\Delta L = \alpha L_0 \Delta T$$

The constant α is the coefficient of linear expansion and describes the thermal expansion properties of a particular material.

(ii) *Surface expansion:* The change ΔS in the surface S_0 of an object due to a change in temperature ΔT is given by:

$$\Delta S = \gamma S_0 \Delta T$$

where γ is the coefficient of surface dilatation of the substance. For an isotropic substance $\gamma = 2\alpha$.

(iii) *Volume expansion:* The change ΔV in volume V_0 of an object due to a change in temperature ΔT is given by:

$$\Delta V = \beta V_0 \Delta T$$

where β is the coefficient of volume expansion. In the case of an isotropic substance, $\beta = 3\alpha$. Water exhibits an anomalous thermal expansion between 0° and 4°C, since it decreases in volume with increasing temperature.

• Quantity of heat: heat capacity and specific heat

When a cold spoon is put into a cup of hot coffee, the spoon heats up and the coffee cools down in order to approach thermal equilibrium. These changes in temperature are basically caused by a transfer of energy from one substance to the other. The transfer of energy that occurs solely due to a difference in temperature is called heat flow or heat transfer, and the energy transferred is called heat. The symbol Q is used to denote the quantity of heat.

A calorie is the quantity of heat necessary to raise the temperature of 1 g of water from 14.5ºC to 15.5ºC. The relation between a joule and a calorie is 1 cal = 4.186 J. This relation is called the *mechanical equivalent of heat*.

The quantity of heat dQ supplied to a system of mass m and the change in temperature produced dT are related by the specific heat c which depends on the material:

$$dQ = m\,c\,dT$$

$$c = \frac{1}{m}\frac{dQ}{dT}$$

The quantity of heat Q needed to raise the temperature of a mass m of a material from T_1 to T_2 is approximately proportional to the change in temperature $\Delta T = T_2 - T_1$ and mass m of the material

$$Q = m\,c\,\Delta T$$

If the number of moles n is known we can refer to the molar heat capacity C:

$$dQ = n\,C\,dT$$

$$C = \frac{1}{n}\frac{dQ}{dT}$$

where $n = m/M$, and M is the molecular mass. We can write

$$m\,c = nC \quad \Rightarrow \quad C = \frac{m}{n}c = Mc$$

and then:

$$Q = n\,C\,\Delta T$$

In the case of solids, we normally work with specific heat and molar heat capacity at constant pressure (c_p and C_p), whereas with gases, constant volume (c_V and C_V).

- **Calorimetry, phase changes and latent heat**

The heat necessary to melt a solid substance is given by:

$$Q_f = m\,L_f$$

where L_f is the latent heat of fusion. For water at atmospheric pressure L_f = 333.5 kJ/kg = 80 cal/g. The heat necessary to vaporize a liquid is given by:

$$Q_v = m\,L_v$$

where L_v is the latent heat of vaporization. For water at atmospheric pressure L_v = 2257 kJ/kg = 540 cal/g. Under the same conditions, each phase change occurs at a certain temperature, and while a phase change is taking place the temperature of the system remains constant.

• Heat transfer by conduction

The heat Q is the energy transferred between a system and its surroundings due solely to a difference in temperature between the system and a part of its surroundings. The heat flow continues until the temperatures are the same. When heat is propagated due to conduction, it is transmitted between two systems by direct contact. If the means separating the systems, whose temperatures are T_1 and T_2, has a length L and section S, in the stationary state (T no longer changes with time t) the heat that passes through a transversal section per unit of time (thermal current, $H = Q/t$) is given by:

$$H = kS\frac{T_2 - T_1}{L}$$

where k is the thermal conductivity of the medium. The thermal resistance of the medium R is given by:

$$R = \frac{L}{kS}$$

and

$$H = \frac{\Delta T}{R}$$

For a compound wall in the stationary state, its equivalent thermal resistance is the sum of the thermal resistances of the wall components if they have the same surface area.

The thermal current H in the case of non-stationary conditions and diverse geometries is given by:

$$H = -kS\frac{dT}{dx}$$

which is known as Fourier's law. dT/dx is the temperature gradient and H is the instantaneous heat current through an element of area S. The negative sign indicates that the heat flows from high to low temperatures.

• Heat transfer by convection and radiation

When heat is propagated by convection, heat is transferred from one place to another by movement of heated material giving rise to macroscopic convection currents, which may appear in liquids in a gravitational field whose density varies with temperature (natural convection). Convection may also be forced using ventilators.

When heat is transferred by radiation, the heat current H radiated by a surface of area S is given by the Stefan-Boltzmann's law:

$$H = e\sigma ST^4$$

where e is the emissivity and

$$\sigma = 5.67 \text{ x } 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$$

is the Stefan-Boltzmann constant. All objects emit energy from their surfaces when they are hot and thermal radiation is a type of electromagnetic radiation.

In all types of heat propagation, if the temperature difference between the body and its surroundings is small, the body's rate of cooling is approximately proportional to the temperature difference (Newton's law of cooling).

Unit 6. THERMODYNAMICS

• Work

Work is a transfer of energy between a system and its surroundings due to movement of part of the surroundings. The work done by a gas on its surroundings is given by:

$$W = \int_{V_1}^{V_2} p\,dV$$

Note that by the definition of work, if the gas does work by expanding, the work done is positive, whereas if the gas is compressed by the force on the piston, the work done by the gas is negative. The work done by a system is positive when energy is transferred from the system to the surroundings.

Work in gas systems (reversible processes):

(a) *Isochoric process* (constant volume, $dV = 0$): $W = 0$.

(b) *Isobaric process* (constant pressure, $dp = 0$):

$$W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1) = p\Delta V$$

(c) *Isothermal process* (constant temperature, $dT = 0$), for an ideal gas $pV = nRT$ and if T is constant $p_1V_1 = p_2V_2$:

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{nRT}{V}dV = nRT\int_{V_1}^{V_2} \frac{dV}{V} = nRT\ln\frac{V_2}{V_1} = p_1V_1\ln\frac{V_2}{V_1} = p_2V_2\ln\frac{V_2}{V_1} = p_1V_1\ln\frac{p_1}{p_2}$$

(d) *Adiabatic process* ($Q = 0$), for an ideal gas $pV^\gamma = \text{constant}$:

$$\gamma = \frac{C_p}{C_v} \qquad pV^\gamma = p_1V_1^\gamma \quad \Rightarrow \quad p = \frac{p_1V_1^\gamma}{V^\gamma}$$

$$W = \int_{V_1}^{V_2} p\,dV = \int_{V_1}^{V_2} \frac{p_1V_1^\gamma}{V^\gamma}dV = \frac{p_1V_1}{\gamma - 1}\left[1 - \left(\frac{V_1}{V_2}\right)^{\gamma - 1}\right]$$

• State functions and equations of state

The state variables p, V, T and n of a substance are interrelated by a mathematical equation called an *equation of state*. For example, for an ideal gas:

$$pV = nRT$$

where $R = 8.31\ \text{J·mol}^{-1}\text{·K}^{-1} = 2\ \text{cal·mol}^{-1}\text{·K}^{-1}$ es the universal gas constant. A system's state of equilibrium may be represented by a point on the p-V diagram. *Quasistatic* processes are those in which the system goes through a series of equilibrium states, and may be represented by curves on a p-V diagram.

• First law of thermodynamics. Internal energy

The relationship between the energy transferred between a system and its surroundings and change in *internal energy* U of the system is given by:

$$\Delta U = Q - W$$

The heat Q ceded to the system and the work W done by the system depends on each particular process. However, internal energy is a function of state: the change in internal energy $\Delta U = U_f - U_i$ depends solely on the initial (i) and final (f) states, and not on the process carried out to go from one to the other. In the equation describing the first law of thermodynamics, the sign convention is as follows: $Q > 0$ if heat is ceded to the system from its surroundings and $W > 0$ if the system does work on its surroundings.

• Some applications of the first law of thermodynamics

(a) *Isochoric process* (V = constant): The work done is zero ($W = 0$) and applying the first law:

$$\Delta U = Q - W = Q$$

(b) *Isobaric process* (p = constant): Both types of energy, heat Q and work W, may be transferred between the system and its surroundings.

(c) *Adiabatic process* ($Q = 0$): $\Delta U = Q - W = -W$

(d) *Isothermal process* (T = constant): An example of an isothermal process is a phase change.

(e) *Free expansion of an ideal gas:* Take a container with two compartments, one of which holds an ideal gas and the other a vacuum. The two compartments are separated from each other and the container is isolated from the surroundings. When the stopcock between the two compartments is opened the gas expands freely until the pressure in the two compartments is the same. In the case of free expansion there is no change in temperature of the gas and therefore $Q = 0$. The system does not do any work on its surroundings $W = 0$, so the internal energy does not change either $\Delta U = 0$. Since the pressure and volume of the gas changed, U is not a function of either p or V, but since the temperature remains constant it may be deduced that *the internal energy of an ideal gas depends only on the temperature*, $U(T)$. Since at constant volume $dU = dQ = nC_V dT$ and U only depends on T. At constant pressure it will be also $dU = nC_V dT$.

(f) *Cyclic process:* Initial and final states coincide. Since U is a function of state, in a cycle $\Delta U = 0$.

• Heat capacity of gases

Heat capacity of an *ideal monatomic gas*:

$$C_v = \frac{3}{2}R \qquad C_p = C_v + R = \frac{5}{2}R$$

Heat capacity of an *ideal diatomic gas*:

$$C_v = \frac{5}{2}R \qquad C_p = C_v + R = \frac{7}{2}R$$

The equation:

$$C_p - C_v = R$$

is known as *Meyer's relation*. An ideal gas that undergoes a quasistatic adiabatic process experiences changes in pressure and volume given by the equation:

$$pV^{\gamma} = \text{constant}$$

with $\gamma = C_p/C_V$. For ideal monatomic gases $\gamma = 5/3 = 1.67$, and for ideal diatomic gases $\gamma = 7/5 = 1.40$.

• Heat engines and the second law of thermodynamics

According to the *Kelvin-Planck* statement of the second law of thermodynamics, no cycle is possible in which heat is absorbed from a reservoir at a fixed temperature and completely converted into work. Any device that transforms heat partly intro work or mechanical energy is called a heat engine. A refrigerator is a *heat engine* operating in reverse.

• Thermal efficiency of heat engines and coefficient of performance of refrigerators

If a heat engine extracts heat, Q_C, from a hot reservoir, performs work, W, and cedes heat, Q_F, a cold reservoir, its thermal efficiency, η, is:

$$\eta = \frac{W}{Q_C} = \frac{Q_C + Q_F}{Q_C} = \frac{Q_C - |Q_F|}{Q_C} = 1 - \frac{|Q_F|}{Q_C} < 1$$

A refrigerator extracts heat, Q_C, from a cold reservoir, and cedes heat, Q_F, to a hot reservoir. The refrigerator requires a net input of mechanical work, W. The coefficient of performance of a refrigerator, e, is given by:

$$e = -\frac{Q_F}{W} = \frac{Q_F}{|W|}$$

According to the Clausius statement of the second law, no process is possible in which the sole result is the transfer of heat from a body of lower temperature to a body of higher temperature.

• The Carnot cicle

A reversible process is one which may be reversed by means of infinitesimal changes in the surroundings of the system. Any process that is not reversible is irreversible. A Carnot machine is a reversible machine working between two reservoirs at temperatures T_C and T_F, following a cycle made up of:

1. A quasistatic isothermal expansion that absorbs heat (Q_C) from the hot reservoir at temperature T_C.

2. A quasistatic adiabatic expansion to a lower temperature T_F.

3. A quasistatic isothermal compression ceding heat (Q_F) to a cold reservoir at temperature T_F.

4. A quasistatic adiabatic compression to the initial state (temperature T_C).

The efficiency of the Carnot cycle is given by:

$$\eta = \frac{W}{Q_C} = 1 + \frac{Q_F}{Q_C} = \frac{T_C - T_F}{T_C} = 1 - \frac{T_F}{T_C}$$

and it is the upper limit on efficieny of real heat engines operating between these two temperatures. According to *Carnot's theorem*, all reversible engines operating between two temperatures, T_C and T_F, are equally efficient and no engine operating between these temperatures can be more efficient than a Carnot engine operating between the same temperatures.

- **Thermodynamic temperature**

The quotient between the absolute temperatures of two reservoirs is defined by the quotient between the heat ceded and absorbed by the reservoirs when it is shown that an engine undergoes a Carnot cycle between the two reservoirs:

$$\frac{T_F}{T_C} = -\frac{Q_F}{Q_C} = \frac{|Q_F|}{Q_C}$$

The thermodynamic temperature T of a system is defined as:

$$T = 273.16 \frac{|Q|}{|Q_3|} K$$

where Q and Q_3 are the exchanges of heat that take place in a Carnot cycle operating between the system and water at its triple point.

- **Entropy**

Entropy (S) is a state variable. The infinitesimal entropy change dS during an infinitesimal reversible process at absolute temperature T is given by:

$$dS = \frac{dQ_{rev}}{T}$$

where dQ_{rev} is the heat ceded to the system in a reversible process connecting these states. The variation in entropy of a system may be positive or negative. The difference in entropy between two states of a system is given by:

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{dQ_{rev}}{T}$$

where the integral is valid for any reversible process connecting the two states. Entropy is a measure of the disorder in a system.

• Calculation of variations in entropy in various processes

When a system undergoes a process, the change in entropy depends solely on the initial and final states, since it is a function of state. Thus, in a real process, reversible or irreversible, a reversible process connecting the initial and final states is used to calculate the entropy, bearing in mind that this reversible process is used only for the purpose of the calculation.

(i) *Phase change:* For example, when a solid such as ice melts, the process is irreversible; however, we imagine a reversible process that consists in using a reservoir whose temperature is insignificantly higher than that of the solid and the heat is ceded reversibly at the melting point of ice. The heat absorbed by the ice is $Q_f = mL_f$, and since the temperature does not change:

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{dQ_{rev}}{T} = \frac{1}{T}\int_1^2 dQ_{rev} = \frac{Q_{rev}}{T} = \frac{mL_f}{T}$$

(ii) *Change in temperature:* For example, if water is heated at constant pressure, a reversible process is considered with reservoirs at temperatures insignificantly higher than that of the water:

$$\Delta S = \int_{T_i}^{T_f} \frac{dQ_{rev}}{T} = \int_{T_i}^{T_f} \frac{mc_p dT}{T} = mc_p \ln\frac{T_f}{T_i}$$

(iii) *Change in volume:* For example, in the case of free expansion (an irreversible process) of an ideal gas ($Q = 0$, $W = 0$, $\Delta U = 0$, $\Delta T = 0$), we imagine an isothermal reversible process between two states (V_i,T) and (V_f,T). According to the first law:

$$dQ = dU + pdV = 0 + pdV = pdV$$

and since $pV = nRT$,

$$\frac{dQ}{T} = \frac{pdV}{T} = \frac{nRT}{V}\frac{dV}{T} = \frac{nRdV}{V}$$

$$\Delta S = \int_{V_i}^{V_f} \frac{dQ_{rev}}{T} = \int_{V_i}^{V_f} \frac{nRdV}{V} = nR \ln\frac{V_f}{V_i}$$

• Entropy and the second law

The change in entropy of the universe is equal to the sum of the changes in entropy of a system and its surroundings. According to the expression of entropy found in the second law, in all processes the entropy of the universe either increases (if the process is irreversible) or remains constant (if the process is reversible).

The increase in entropy of an isolated system in an irreversible process is related to a lost opportunity to do work ($W_{lost} = T\Delta S$). Irreversible processes go from more ordered to less ordered states. Entropy is related to probability: a highly ordered system has low probability and low entropy.

Unit 7. ELECTRIC FIELD

• Introduction

The ancient Greeks were the first to observe the phenomenon of electrical attraction. They found that when amber was rubbed, it attracted small objects such as straw or feathers. In fact, the word "electric" comes from the Greek word for amber, *elektron.*

Electrical nature of matter. Electric charge

Electric charge is a fundamental property of matter and two types of charge exist: positive and negative. Two bodies with the same type of charge repulse each other, whereas if they are of opposite charge they attract each other.

Quantification and conservation of electric charge

The electric charge is always a multiple of the elementary charge or quantum of electric charge, whose value is given by:

$$e = 1.602177 \text{ x } 10^{-19} \text{ C}$$

which is the absolute value of the charge on an electron.

In all processes that take place in nature, the total or net charge of an isolated system remains constant.

• Coulomb's law. Electric force between point charges

Coulomb's law expresses the electric force **F** exerted by a point charge q on another point charge q':

$$\mathbf{F} = K_e \frac{qq'}{r^2} \mathbf{u}_r$$

where **r** is the vector starting at q and ending at q' and $\mathbf{u}_r = \mathbf{r}/r$. K_e is the constant:

$$K_e = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \, \text{N C}^{-2} \, \text{m}^2$$

where ε_0 is the *permittivity of free space* or *vacuum permittivity*:

$$\varepsilon_0 = 8.854 \times 10^{-12} \, \text{N}^{-1} \, \text{C}^2 \, \text{m}^{-2}$$

This force is inversely proportional to the square of the distance, and is a force of attraction between charges of opposite signs and of repulsion between charges of the same sign.

• Electric field

Electric field of a point charge

If an electric charge experiences a force due to the presence of other charges in the region, an electric field is created. The electric field **E** generated by a charge distribution is the force **F** exerted by the distribution on a test particle divided by the charge q of the test particle:

$$\mathbf{E} = \frac{\mathbf{F}}{q} \qquad \mathbf{F} = q\mathbf{E}$$

For a point charge:

$$\mathbf{E} = K_e \frac{q}{r^2}\mathbf{u}_r$$

where $\mathbf{u}_r$ is the unit vector that points along the line from the charge q to the point where the field **E** is evaluated.

Principle of superposition

The electric field created by a system of point charges is equal to the sum of the electric fields produced by each individual charge.

Electric field lines

The spatial characteristics of an electric field may be illustrated by drawing lines of force or electric field lines. The tangent of a line of force at each point indicates the direction of the field **E** at that point.

The electric field lines start at the positive charges and end at the negative charges. A uniform field has the same intensity and direction at all points in space and is represented by rectilinear field lines that are parallel and equidistant.

Electric field created by a continuous charge distribution

For a continuous charge distribution (in volume, surface or line) the electric field is calculated as follows:

$$\mathbf{E} = K_e \int \frac{dq}{r^2}\mathbf{u}_r$$

This may be applied, for example, to determine the field created by a ring or a disk at points on its axes or the field created by a rectilinear segment at points on its perpendicular bisector.

Movement of charge in an electric field

If an electric force is the only force acting on a particle of mass m and charge q, according to Newton's second law the acceleration **a** is given by

$$\mathbf{a} = \frac{q}{m}\mathbf{E}$$

When a particle moves in a uniform electric field, its motion is described by the kinematics of motion with constant acceleration. It is of particular interest to study the motion of a charge that enters a region where there is a uniform electric field perpendicular to the velocity vector of the charge.

• Potential electric energy and electric potential

The electric force between two point charges is directed along a line joining the two charges and is inversely proportional to the square of the distance between them, as is the force of gravity between two masses. Like the force of gravity, the electric force is conservative. Thus, there is a potential energy function associated with the electric force. If a test point charge is placed in an electric field, its potential energy is proportional to its charge. The electric potential energy per unit of charge is a function of the position of the charge in space and is called electric potential.

The electric force is conservative. The *electric potential energy* of a test particle in the field created by various fixed particles q_i is given by:

$$E_p = K_e q \sum_i \frac{q_i}{r_i}$$

(taking the origin of potential energy at infinity).

The *electric potential* of a charge q is defined as:

$$V = \frac{E_p}{q} \qquad E_p = qV$$

The SI unit of potential is called one volt (V):

$$1\ \text{V} = 1\ \text{J/C}$$

Electric potential of a point charge

For a point charge (with the origin of potential at infinity):

$$V = K_e \frac{q}{r}$$

For a system of point charges (with the origin of potential at infinity):

$$V = K_e \sum_i \frac{q_i}{r_i}$$

Electric potential of a continuous charge distribution

For a continuous charge distribution:

$$V = K_e \int \frac{dq}{r}$$

Applying this equation, it is possible to calculate the potential of, for example, a ring charged along its axis.

Potential difference

The difference in potential ΔV between two points 1 and 2 is related to the work W done by the electric field:

$$W = -\Delta E_p = E_{p1} - E_{p2} = q(V_1 - V_2) = -q\Delta V$$

Gives:

$$\Delta V = V_2 - V_1 = -\int_1^2 \mathbf{E} \cdot d\mathbf{l}$$

The potential difference $V_2 - V_1$ is the negative work per unit of charge done by the electric field on a positive test charge when the latter moves from point 1 to point 2. The potential difference ΔV is also the positive work per unit of charge that must be done against the electric field in order to move the charge from 1 to 2.

- **Relation between electric potential and electric field**

The relation between electric field E and electric potential V is given by:

$$dV = -\mathbf{E} \cdot d\mathbf{l}$$

The electric field lines point in the direction in which the potential decreases. If the expression for **E** is known, the potential V at a point P may be obtained by means of the line integral of **E**:

$$V = -\int_\infty^P \mathbf{E} \cdot d\mathbf{l}$$

If V is known, the field **E** may be found by means of the gradient of V:

$$\mathbf{E} = -\mathbf{grad}\,V = -\nabla V$$

If the electric field is constant along a direction (for example, along the x axis):

$$E_x = -\frac{dV}{dx}$$

If the potential depends solely on the module of **r** (that is, r):

$$E_r = -\frac{dV}{dr} \qquad \mathbf{E} = E_r \mathbf{u}_r$$

Equipotential surfaces

Surfaces that have the same electric potential at their points, that is V = constant, are known as *equipotential surfaces*. The field lines are perpendicular to equipotential surfaces. For a point charge, equipotential surfaces are spherical surfaces concentric with the charge.

• Electric flux. Gauss's law

The electric field flux Φ_E over a surface S is defined as the surface integral of the electric field vector **E** over the whole surface:

$$\Phi_E = \int_S \mathbf{E} \cdot d\mathbf{S}$$

When the flux across a closed surface is calculated, this surface is known as a gaussian surface. Field lines may be used to visualize the flux across a surface. The total flux may be positive, negative or zero. When it is positive, the flux is outgoing and when negative, incoming.

Gauss's law states that the total electric flux through any closed surface is equal to the total (net) electric charge inside the surface divided by ε_0:

$$\Phi_E = \frac{q}{\varepsilon_0} \qquad \int_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

• Calculation of electric field using Gauss's law

Gauss's law may be used to find the electric field produced by highly symmetrical charge distributions such as infinite lines, planes or spheres. The crucial step in this process is to select the gaussian surface.

Unit 8. CONDUCTORS, CAPACITORS AND DIELECTRICS

- **Conductors in electrostatic equilibrium**

Experimentally it is shown that certain substances have the property of allowing the movement of electric charges through them, whereas others prevent this flow. The former are called conductors and the latter insulators or dielectrics. Metals and their alloys, aqueous solutions of acids, bases and salts, etc. are conductors; however, only metallic conductors will be studied. These are made up of positive ions occupying fixed positions in a crystalline lattice and electrons that have become detached from the metal ions and circulate freely and randomly within the conductor, giving rise to a kind of "electron gas" which causes the phenomenon of electric conduction.

General properties of conductors in electrostatic equilibrium

(i) The electric field inside a conductor in electrostatic equilibrium is zero

(ii) The net electric charge of a conductor in electrostatic equilibrium is found on its surface.

(iii) The electric field on the surface of a conductor in electrostatic equilibrium is perpendicular to the surface.

(iv) The surface of a conductor in electrostatic equilibrium is an equipotential surface.

Electric field in the proximities of a conductor in electrostatic equilibrium

The electric field at points near the surface of a conductor is perpendicular to the surface and is given by:

$$E = \frac{\sigma}{\varepsilon_0}$$

which is known as *Coulomb's theorem*. This is the field created by the total charge of the conductor and may be considered to be the sum of the field created by a small disk of area *dS* and that created by the rest of the conductor. The field E_1 created by a small disk at nearby points is:

$$E_1 = \frac{\sigma}{2\varepsilon_0}$$

then the field E_2 created by the rest of the surface charges on the conductor is:

$$E_2 = \frac{\sigma}{2\varepsilon_0}$$

therefore $E = E_1 + E_2$.

Conductors in an electric field

When a conductor is placed in an electric field, the field inside the conductor must be cancelled in order for the conductor to be in electrostatic equilibrium. This results in the charges in the conductor being reordered to create an electric field inside the conductor to compensate the applied electric field.

Dielectric breakdown and the point effect

Many non-conducting materials are ionized in very high electric fields and become conductors. This phenomenon is known as *dielectric breakdown* and the dielectric limit of an insulator is the maximum electric-field magnitude, E_{max}, that can exist in this material without producing dielectric breakdown. This value of the field is also known as the dielectric strength. When dielectric breakdown takes place the molecules of the material are ionized and the material starts to conduct. In a gaseous material such as air $E_{\max} \approx 3\times10^{6}$ V/m and this effect is accompanied by the emission of light due to the recombination of electrons with the ionized molecules, a phenomenon known as *arc discharge*.

When a conductor is non-spherical in shape, its surface is equipotential but the surface charge density and the electric field just outside the conductor varies from one point to another. The electric field is stronger at the points near the regions of smaller radius of curvature of the conductor (*point effect*). If the conductor has very small radius of curvature points, the dielectric surrounding it may break down at relatively low potentials. Lightning conductors extract the charge from nearby clouds before the potential of the cloud reaches a destructively high value.

• Capacitance

A conductor with charge Q and potential V has a capacitance C given by:

$$C = \frac{Q}{V}$$

In the SI capacity is measured in farads (1 F = 1 C/V). A farad is a very large quantity, and the usual capacitor in an electric circuit is of the order or microfarads (1 μf = 10^{-6} F) or picofarads (1 pf = 10^{-12} F).

• Capacitors

A capacitor is an electric device used in circuits to store charge and electric energy. It is made up of two conductor plates with a potential difference ΔV between them. The capacitance of a capacitor is given by:

$$C = \frac{Q}{\Delta V}$$

The capacity depends on the geometric design of the capacitor and the nature of the dielectric between its plates or within its housing. For a *plane parallel capacitor* in vacuum (a capacitor with two plane parellel conducting plates, each with area *S*, separated by a distance *d*), the capacitance is:

$$C = \frac{\varepsilon_0 S}{d}$$

• Capacitors in series and parallel

The equivalent capacitance of various capacitors connected together is the capacitance of a single capacitor which, when used on its own, produces the same external effect. The equivalent capacitance C_e of various capacitors in *series* is:

$$\frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots = \sum_i \frac{1}{C_i}$$

When various capacitors are in *parallel* the equivalent capacitance C_e of the system is calculated using the equation:

$$C_e = C_1 + C_2 + \ldots = \sum_i C_i$$

• Electrostatic energy

The potential electric energy E_p (or U) stored in a charged capacitor is calculated as the work required to charge it:

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

When this energy is associated with an electric field **E**, the electric energy density u_E in the space occupied by the field (in a vacuum) is:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

In a material medium, ε_0 has to be substituted by ε. The total electric-field energy U in a volume V may be calculated using the integral:

$$U = \int_V u_E \, dV$$

• Dielectrics

Electrostatic properties of dielectrics

When a dielectric is placed within the housing of a capacitor in which there was a vacuum between the plates (capacitance, C_0), the capacitance C increases so that:

$$C = \varepsilon_r C_0$$

whereas the potential difference and electric field decrease:

$$V = \frac{V_0}{\varepsilon_r} \qquad E = \frac{E_0}{\varepsilon_r}$$

where ε_r is the relative permittivity defined as follows

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

There are *polar* and *non-polar* dielectrics. The molecules of the former have no electric dipolar moment, whereas those of the latter have a permanent electric dipolar moment.

When a non-polar dielectric is placed in an electric field, its atoms or molecules become electric dipoles oriented in the direction of the electric field. If the dielectric is polar, its permanent dipolar moments are oriented in parallel to the external field.

When the electric dipoles of a substance line up spontaneously (*ferroelectric* substances) or due to the action of an external electric field, the substance is said to be *polarized*.

A polarized dielectric has charges on its surface and, unless the polarization is uniform, in its volume as well. These polarization charges, however, are frozen in the sense that they are bound to atoms or molecules and are not free to move in the dielectric. In a conductor, the charges are capable of moving freely and are called *free charges*.

Unit 9. ELECTRIC CURRENT

• Introduction

A conductor is a material which allows charged particles to move freely through it and these particles carry the charge on the conductor. For example, a metal may be considered to be a structure of positive ions occupying fixed positions on a lattice with the free electrons moving between them. The charge of all the free electrons is equal and opposite to the charge of all the ions, thus giving rise to a neutral material. The free electrons can move freely within the lattice of ions and carry the charge through the metal.

• Electric current and electric charge motion. Current density

An *electric current* is a flow of charged particles. The intensity I of the electric current characterizes the charge flowing through a circuit element:

$$I = \frac{dQ}{dt}$$

The direction of the current is the same as the direction of the drift velocity $\mathbf{v}$ of the positive charge carriers. In the SI, the intensity I is expressed in amperes (A). The *current density* **J** is the charge flow at a point inside a conducting medium:

$$\mathbf{J} = nq\mathbf{v}$$

where n is the number of charge carriers per unit of volume (concentration of particles) and $\mathbf{v}$ is the drift velocity of the charge carriers. Successive collisions with the ions in the lattice cause the drift velocity of the charge carriers to be a constant average. If **J** is uniform, its module J is given by:

$$J = \frac{I}{S}$$

and in the general case:

$$I = \int_S \mathbf{J} \cdot d\mathbf{S}$$

Conventionally, the direction of the electric current is taken to be that of the positive charge carriers. When the charge carriers are negative, the direction of the current is contrary to the direction of movement of these negative charge carriers.

• Ohm’s law. Conductivity, resistivity and resistance

For a conductor at a constant temperature, Ohm’s law states that the quotient between the potential difference V between two points on a conductor and the electric current I circulating through the conductor is a constant known as *electrical resistance, R*:

$$R = \frac{V}{I} \qquad \Rightarrow \qquad V = RI$$

In this case the conductor is said to be ohmic. In the SI, the resistance R is expressed in ohms (Ω).

For a conductor of length l and section S, Ohm's law may be written as:

$$J = \frac{l}{RS} E = \frac{1}{\rho} E = \sigma E$$

where $\sigma = 1/\rho$ is the *conductivity* of the material and ρ its *resistivity*, which is expressed in $\Omega^{-1}\text{m}^{-1}$ and:

$$R = \frac{\rho l}{S}$$

The resistivity of metals increases with temperature. Ohm's law may be written in general as:

$$\mathbf{J} = \sigma \mathbf{E}$$

and the drift velocity of the electrons **v** may be written as:

$$\mathbf{v} = -\frac{\sigma}{en} \mathbf{E}$$

• Resistances in series and parallel

The equivalent resistance of resistances connected in *series* is given by:

$$R_e = R_1 + R_2 + \ldots = \sum_i R_i$$

and of resistances connected in *parallel*:

$$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots = \sum_i \frac{1}{R_i}$$

• Energy aspects of an electric current. Joule's law

A supply of energy is necessary to maintain an electric current since the charges must be accelerated by an electric field. The energy per unit of time, or power, required to maintain a current is given by $P = VI$. In the SI, power P is expressed in watts (W). For conductors that obey Ohm's law $V = RI$ and so:

$$P = VI = IR^2 = \frac{V^2}{R} \quad \text{(Joule's law)}$$

• Ammeters and voltmeters

Meters used for the measurement of current though a circuit are known by the general name of *ammeters* and they are connected in series with the circuit carrying the current to be measured.

An instrument to measure the voltage difference between two points is a circuit, said, two sides of a resistor, is called a *voltmeter* and it has to be connected in parallel with the circuit element whose voltage difference wants to be measured.

An *ohmmeter* is an electrical instrument that measures electrical resistance.