TOPOLOGICAL STRUCTURES OF COMPLEX BELIEF SYSTEMS (II): TEXTUAL MATERIALIZATION

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Abstract: Mythical and religious belief systems in a social context can be regarded as a conglomeration of sacrosanct rites which revolve around substantive values that involve an element of faith. Moreover, we can conclude that ideologies, myths and beliefs can all be analyzed in terms of systems within a cultural context. The significance of being able to define ideologies, myths and beliefs as systems is that they can figure in cultural explanations. This, in turn, means that such systems can figure in logic-mathematical analyses.

Keywords: Belief Systems, Ideology, Structuring Structure, Textual Materialization, Topology.

1. INTRODUCTION

Man, from his first artifacts to the present time, established and developed reflection, that is to say, the aptitude to interpret symbolically the material reality of the surrounding world. As hominids emerged in evolution at the time of the Neanderthals, drawings and tools show a progressive maturation of technical intelligence that is probably parallel to the evolution of language. An elementary property of language consists of creating, in parallel to the outer world, an all-powerful world of symbols without which intelligence would be deprived of an essential tool. Religious thinking had to follow the same course; the symbols formulated in words and actions reflect the feelings of fear and dominion that marks religious conscience. It seems extraordinary that we share a conception of the supernatural across time, though not in the sense in that we conceived it some millennia ago. Some millennia of rationalism have allowed western culture to refine ways to conceive the universal order, by means of mysticism, magic, dogma, law, philosophy, science, technique. In the conditions of paleolithic man, the anxiety to understand both well known unknown mysterious things were expressed in a belief system, that was different, but with its own system of reasons just as complex as the reasoning in Greek philosophy or Quantum Mechanics. This system was the key, for primitive man, for having security, in the lands of life and death.

Sensations perceived by man reflect a world without meaning, but due to his psychic system, he transforms them into significant and tactile objects. Man uses such objects to solve first his survival problems, and soon this cognitive activity led to the transcendental sense of his person. Signals arrive from the senses or memory, and are transformed like images being manipulated by abstract or associative cognitive routes. An abstract route is a mechanism that allows concept formation, whereas the associative route structures the representations by similarity, contiguity or contrasts. These phenomena take place in what we call a representational space that is a type of mental screen where images are projected and maintained by the senses, memory or imagination. Due to the connections between the world of the objects and the consciousness, we can communicate providing visual images. What we call an image may be an internal or external representation, of sensations structured by the conscience.
The natural "embodiment" of the perceived spatial relations is culturally encoded and thereby leads to a kind of iconic link between perception, language, and the perceived world. However, the codes representing space are not merely images of the surrounding world but instructions concerning its individual and social construction. We can define the existential space as a relatively stable system of images of the surrounding world of the man, and that it indicates that this surrounding space is a necessary part of the existential structure. A complete theory of the existential space must include abstract and concrete aspects:

1) The abstract aspects refer to general schemes of topological and geometric classes.
2) The concrete aspects refer to the physical environment, buildings, urban and rural landscapes.

Associated with the elementary properties of the existential space are the concepts of center and place, because man spontaneously acts in spaces. As opposed to the center, the place indicates certain dimension, but it is necessary to distinguish between one’s own place, that is the space that each organism protests like own, and the abstract image of well-known places. The existential space is therefore, a psychological concept, determined by the structure of the environment and the psychic character of the man.

**Definition 1:** Materialization is the conversion by means of certain mathematical correspondences of an abstract set whose elements are beliefs or ideas, in an impure set whose elements are material or energetic.

In the materialization process, we will distinguish two different although intimately united processes: symbolic materialization and textual materialization. We will divide the Primigenial Base into two parts, PB₁ containing the archetypes and PB₂ containing myths (Figure 1).

In order to establish patterns of materialization of the beliefs we are going to consider that these have defined mathematical structures. It will allow us to understand better the processes of textual, architectural, normative, educative, etc., materialization, of an ideology and we propose the following initial hypotheses (Usó-Doménech and Nescolarde-Selva, 2012; Nescolarde-Selva and Usó-Doménech, 2013):

**Hypothesis 1:** The beliefs are not the product of reason or of abstract and logical thought.

**Hypothesis 2:** In the origin of any belief system there is always a mythical system of beliefs.

Myths are not designed as analytical presentations of the kinds of social factors most likely to ensure a reasonable balance between individual liberty and supraindividual order. Neither are myths about the kinds of cultural circumstances that could lead to social upheaval and arbitrary abuse of power, or to a calcified order that benefits only the few. In mythical accounts, often there is conflict between good and evil – either freedom versus constraint or order versus chaos.
The notion that all we have to do is fight for freedom is closely linked to the belief that human beings are inherently good and that, given half a chance, they will behave with integrity. The notion that morality depends on a certain cultural order stems largely from the belief that human beings are egotistical and anarchistic, that they can behave decently only within a civil or religious framework. Faith in the noble savage can be regarded as a counter-faith to the doctrine of original sin.

2. TEXTUAL MATERIALIZATION

There is a textual space that is a materialization of the existential space. The textual space is also, a legacy that constrains the individual, is therefore prior to him since it reflects the existential spaces of his predecessors. And within this textual space we will
distinguish between the written textual space and the architectural spaces, the pictorial, etc. A special and very important case is the textual space of written texts.

Structurally, natural human conversation is an acoustic phenomenon without spatial extension: it is structured as a sequence in time. However, its product is the written text, which is referred to very often in terms of spatial metaphors. Therefore, in the phonetic chain only in terms of time, a spatial dimension is attributed to the linearity of signs: the geometry of the line. This is what Nöth (1994) calls a semiotic paradox, and he compiles an amazing number of examples for the linguistic expression of what he calls the geometry and topology of textual space. Nöth finds them especially in metaphors:

1) **First dimension**: the literal meaning of which refers to spatial structures as in points or lines.

2) **Second dimension**: metatextual topoi of space, levels, or surface structures.

3) **Third dimension**: an intratextual or intertextual reference, bodily topoi, or metatextual organization of units such as chapters, etc.

It may suggest a static concept of written textual space, but there are many examples of a dynamic spatial concept as well e.g., changes of (textual) space, movement within (textual) space, its limits and extension, and, of course, all the linguistic means of deictic reference. Indeed, cognitive semantics has attempted to explain the remarkable frequency of spatial metaphor in everyday language by the biological relevance of how humans perceive space and orient themselves within it in phases of prelinguistic language acquisition (Lakoff, 1987). All these findings open up a new perspective on the Saussurian notion of textual linearity. There is a focus on semiotic relationships between cognitive categorization of space and its textual representation in various sign systems (i.e. texts not only in the syntactical understanding of linguistics). Focusing on the problem of how to represent the complexity of three-dimensional space in the linearity of one-dimensional sign sequences (e.g., sentences), Wenz (1997) suggests that texts may develop their own (metaphorical) notion of space within which the reader will find his/her orientation during the process of reading. In other words, it is the reader who constructs 'textual space' based on interpretation of sign sequences functioning as a semiotic Gestalt. If we understand the linearity of texts as a projection of semiotic principles structured on a kind of ordo naturalis of language, we may logically also argue for a cultural convention or social order of space designed through texts: it is a matter of categorizing perception through shared knowledge, or, as Lakoff (1987) put it, seeing always means "seeing as". Everything in a text (T) looks for an equilibrium. There is a conscience of imbalance and a will of balance. Human groups (SB) operate by answers, and logically, T also does. That is to say, they respond before the perceived reality. But this perception is not the one of a harmonious reality, but a nontotalized reality. And this no-totalization of the reality is the one that mediates and determines the collective answers. Consciousness of the imbalance, of the not-totality, in its equilibrium seeking will respond of three different ways:

1) Prioritizing the equilibrium that is harmony (**classic materialization**). It is characterized by negation of the imbalance, in one first analysis, although in fact it does not happen in this way. Classic T is able to integrate itself, to balance all

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1 The Topoi are patterns of thought and expression and preset, rhetorical archetypes for persuasion, classic origin.
negation and imbalance. Its world vision perceives ruptures as provisional and one makes an effort to indicate the way of integration.

2) Prioritizing the perceived nonbalance (*revolutionary materialization*). This is the ruptural T. Ruptures are prioritized, breaking the styles and new forms are sought because previously the impossibility of satisfaction was acquired, inherited, integrated and part of the cultural balance. In these Ts, the newly obtained totalization is less rich and coherent than classic Ts. Nevertheless, from them arise the revolutions that allow dramatic leaps forward.

3) Denying all balance (*playful materialization*). It is the utopic conception, the mythical celebration of reality that may be playful and perhaps involve song. The playful thing, to exist, would not need any type of symbol, since it would perpetuate itself easily.

Through these three types of belief answers, any text (T) has to materialize itself with a certain equilibrium. Therefore, the Texts are the materialization of a peculiar world vision, a peculiar belief system and a peculiar ideology. They are simultaneously, its triumphant expression, its conservation, its transmission and its desire for eternity. In principle, a collective subject world vision generates its T’s problematic, mediating, inspiring and producing, but these mediations find a series of resistance: in content and form. Logically, a new world vision has to transform the inherited belief structures, ways of doing, behaviors, forms and contents. These mediations have to take place most of the times with damage to the T in their total unit, affecting even their coherence. When the world vision changes, the text accomplish a mission for which it had been conceived, and is left, forgotten and sometimes even destroyed, often with unusual violence, when a certain world vision has been replaced by another one with radically opposed substantive beliefs. Whole libraries have been burned, as they were considered heretical or blasphemous. Sanctuaries were destroyed or reused with a change in function. Old Gods were interpreted as demons or reused as new Gods (or saints). Forgetfulness is perhaps the kindest result after such a change. History has a multitude of examples. In this paper, we tried to give a logical and mathematical explanation to the materialization of a belief system through the mathematical structures that appear, as much in the belief system as in the text. For this we must formulate own previous concepts of our Text Theory (Usó-Domènech and Nescolarde-Selva, 2012).

Let WV be a determined world vision, T be a text and SA, SB be the author and reader respectively.

**Definition 2:** The content c of T is the materialization of a series of social relations produced in SB that take shape in a determined WV, and that uses previous materializations, considered historical, and produced in previous SB and within same culture C.

**Definition 3:** The form f is the materialization of a determined and specific WV.

The form f is a creation or social product that can appear at the same time as c. Nevertheless, we know by historical experience that f can take long periods of time (years and even centuries) in reaching a constant and unremovable structure.
**Consequence 1:** A content c of T is in relation to an inherited form f_{i-1}, constructed in advance of the materialization of c.

**Consequence 2:** Content c of T must find a form f to materialize itself, and that is preexisting to the c to which it is to materialize.

In according Ferreras (1980) a classification of forms f with respect to contents can be established:

1) *Adapted forms:* f offers minimum resistance to the materialization of c.

2) *Inadequate forms:* f offers maximum resistance to the materialization of c.

### 3. STRUCTURES OF MATERIALIZATION

In any text T we distinguish between the Structuring Structure (SS) and the Structured Structure (sS) of all text.

**Definition 4:** The Structuring Structure (SS) of T is the internal cause by means of which the different elements summoned in T are structured or organised.

1) SS is the self-regulating cause of the structure, since it is the source of the exclusion principle.

2) SS is bound deeply with ways of doing, thinking, and feeling in a T, with a determined WV.

3) This WV, or a collective, belonging to SB, generates a collective subject that materializes T.

4) In SS also are the organizational causes, or the self-regulating virtuality and the organizational virtuality, but these virtualities refer to a way of doing, not to content.

5) SS also is deeply bound with the WV of a social group.

6) In the concrete level of the structure of T, SS is its central core, the reason to be.

**Consequence 3:** There is a continuous movement by which the information coming from Structural Base (SB) modifies codes and ideologies and it is translated into new codes and ideologies.

SS comes from a collective or transpersonal subject, but this SS develops now, by means of an individual and specific action, with one sS that will be the form.

**Definition 5:** A structured Structure (sS) is the concrete materialization in SB of a Structuring Structure (SS).

A book, a church, a castle, a picture, a symphony, etc is determined by sS. A Structured Structure (sS) has the following characteristics:
1) A structuring or materialization of SS is a necessary condition for the T's existence.

2) sS has its own mediations. SS does not take automatically shape sS, but sS can even mediate its own SS, modifying it, by means of a feedback process.

3) sS is the primary connoted significance. SS "will be covered" by sS, like a secondary connoted significance.

The existence of both structures of T: SS and sS, have two characteristics:

1) These characteristics explain T in its relation with SB (subject group). The collective subject is the creator of the SS but not necessarily the producer of sS. The mediations of the S_\text{A} are decisive. Between content c of T and S_\text{A} there is an intimate connection. The "world of T" is the inner world of the S_\text{A}.

2) These characteristics allow valuing the internal coherence of T.

SS is the true motor and cause of T and comes from a collective or transpersonal subject. But this SS goes to face, by means of an individual and specific action with one sS to which we will denominate form. Therefore, the form f of a text T is the one structured structure sS of this text. A form f, or sS, is a produced social creation in SB, that is previously in the collective conscience (belonging to DS), and when the moment arrives for expressing a new c, sS already has been accepted and used. Nevertheless, sS is also a structure that has its own self-regulating internal laws that make it exist as f. While the new cs are not in contradiction with the internal laws of f, f will continue working as f. Nevertheless, by the law of historical evolution, f will be in opposition with the new cs. It will be necessary to consider not only the internal laws of SS that enters opposition, but also the existing differences between the new c and the old cs. The explanation has to be in the social evolution of SB, formed by groups and social classes that are those that construct f, use it and that at a certain moment, are incapable to use it without transforming or destroying it.

Example 1: Consider a very well-known literary form, the novel. The novel, as it forms an f or sS, it is a social form, one T, that has had its history, genesis and emergence. To find the social group, belonging to a certain historical SB, which created the novel, means, to know a mental structure, a way to think, to be related, to have values , a belief system, an ideology, and a peculiar WV. The form \( f = \text{novel} \) is of clear bourgeois origin. The Renaissance man was able to commit himself legally and economically and then the "modern man" emerged, and with him, the novel was born. This new man is a hero who lost his position in the great harmonious totality of an organized, totalized society and in the harmonic Cosmos, with a sense of the Divinity. It was a new SS that needed to materialize itself, and that when this happened it did so in a precise and specific way. The novel arose therefore, but as it is natural, these T could not emerge from anything; the Renaissance new man, had within the new field of his WV, two forms to consider: on the one hand, the memory and the conserved examples of old, Greek and Roman novels, and on the other hand, the inheritance of closed poems and national epic poems. And on these two inheritances, at least, the new novelist counted and operated. Cervantes evokes to Heliodoro and the cavalry books to
try to desacralize the closed, old world and to materialize for the first time, an individualized hero. The cavalry books represent the midpoint at which a form resists being used with a new WV; for that reason closed novel forms² are incapable of evolution. In the novel *Amadís de Gaula* (a popular Spanish cavalry book) the protagonist is already an individualized man, who is called Amadís, but his conduct fits a well-known table of values, that permits no evolution, and there is no way he can stop being the Amadís. Form $f = \text{epic}$ resist materializing the Renaissance subject in any way that is not representative of or of any table of values that are nationally accepted. To materialize himself as a new person, he has to break with one of the structures or internal laws of forms $f_1 = \text{epic}$ or $f_2 = \text{poem}$: that in which the hero is a collective hero, and representative of a whole society. The new SS, of bourgeois origin, is individualistic, fights to prevail and to materialize itself in the form that it finds and that is led by the historical mediations of its moment and space. Then it will transform this form $f$ until turning it into the form which at the moment we know as a novel.

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We have made a brief scheme, that is not absolutely complete, since we could consider another series of reports (images and projections) that would make the scheme much more complicated. Nevertheless, there is no way to understand the emergence of a form (novel in example 1), if do not consider the adjustments and nonadjustments of a series of inherited forms just prior to the necessities of materialization of a new transpersonal conscience, that is to say, of a new SS.

The sS is always necessary for the existence of the cultural unit, to avoid turnings into a pure conceptualization. In addition, it is necessary that ss responds to a certain coherence, or a minimum of coherence that we would agree to establish. A rigorous analysis of SS of a textual structure, would also give a WV and the collective subject that mediates and inspires the analyzed T.

**Consequence 4:** *All SS is consubstantial to the collective subject of the textual structure (TS).*

The mediation of the author $S_A$ usually is important and significant, nevertheless, the sS, usually is used collectively but also collective formed and created. $S_A$ has to be related to sS by means of a personal style and of a way of doing. This the *author’s psychological contexture*.

**Consequence 5:** *According to what we have defined previously the Structured Structure sS is the materialization of a world vision WV, is to say is the text T itself. Then $T \equiv sS$.*

4. **THE TOPOLOGICAL STRUCTURING STRUCTURE (SS)**

Let T be a text. In all text T, not concerning their material nature, certain elements exist (subtexts) containing their basic ideas. This subtext constitutes the Structuring Structure (SS). It is possible to form a subtext $SS \subseteq T$ containing others subtexts $SS = \{\tau_1, \tau_2, ..., \tau_m\}$ connoting the basic ideas of T. Topological structures are based on

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² Closed novel form, is one where there is a necessary relationship between the parts and the whole, so that each episode acquires its meaning in relation to the whole novel, and that it is not able or add or remove parts without harming against organizational equilibrium.

Hypothesis 3: When belonging to Structural Base (SB), the Structuring Structure (SS) will be "material", that is to say, a visual somehow as much for the believing as for the nonbelieving subjects.

Note 1: Being SS material then it will be a 3-dimensional Euclidean space \( R^3 \).

Let \( SS = \{\tau_1, \tau_2, \ldots, \tau_n\} \) be a set of structured substantive beliefs.

Definition 6: The Structuring Structure SS is a subtext formed by subtexts connoting the ideas or basic theses denoted by \( S_A \). Then \( SS = \{\tau_1, \tau_2, \ldots, \tau_m\} \).

Note 2: The subtext SS is defined as the Structuring Structure of all text T.

The Structuring Structure SS forms a finite sequence of materialized substantive or derived beliefs (Usó-Doménech and Nescolarde-Selva, 2012; Nescolarde-Selva and Usó-Doménech, 2013).

SS is a subset of Euclidean space \( R^3 \). Let \( \tau \) be a point of SS.

Definition 7: \( \tau \) is a point of closure of SS if every open ball centered at \( \tau \) contains a point of SS, being able to be \( \tau \) itself.

\( R^3 \) is a metric space with metric \( d \). \( \tau \) is a point of closure of SS if for every \( r > 0 \), there is a \( \nu \) in SS such that the distance \( d(\tau, \nu) < r \).

Definition 8: The accumulation point is a point which is the limit of this sequence.

As well as the set substantive beliefs there is one main belief, and SS does not have it. Its accumulation point will be different for each materialization, depending on the author (or authors) and on the world vision at the particular historical moment. What can be observed in a belief analysis of any text, written or other type of materialization that exists, is a convergence towards an idea or certain belief. This will be the accumulation point of text T.

Let SS’ be a subset of SS.

Definition 9: A structuring cover for SS’ is the collection of sets \( SC = \{SC_i\}_{i=1} \) such that \( \forall SC_i \subset SS \) and \( SS' \subset \bigcup_{i=1}^{\infty} SC_i \).

A subset \( SC' \subset SC \) is also a structuring cover of SS.
Definition 10: A structuring refinement \( SP \) of \( SC \) is a structuring cover of \( SS \) such that \( \forall V \in SP, \exists U \in SC \text{ such that } V \subset U \).

The neighborhood filter \( \Phi(\tau) \) for a point \( \tau \) is the collection of all neighborhoods for the point \( \tau \). The neighborhood basis \( B(\tau) \) for a point \( \tau \) is a filter base of the neighborhood filter \( B(\tau) \subset \Phi(\tau) \) such that \( \forall U \in \Phi(\tau), \exists B \in B(\tau), B \subset U \). The corresponding neighborhood filter is \( \Phi(\tau) = \{ U \ni B : B \in B(\tau) \} \). For each point \( \tau \) in \( SS_T \) there exists a sequence of open neighborhoods, \( U_1, U_2, \ldots \) of \( \tau \) such that for any open neighborhood, of \( \tau \), there exists an integer, \( i \), with \( U_i \) contained in \( V \). Therefore, \( SS_T \) is a first-countable because each point has a countable neighborhood basis.

Theorem 1: \( SS \) fulfills the Heine-Borel theorem; therefore \( SS \) is a compact space.

Proof:

1) \( SS \) is closed. Let \( \tau \) be an accumulation point of \( SS \), then any finite collection \( C \) of open sets, such that each open set \( U \) in the collection \( C \) is disjoint from some neighborhood \( V_U \) of \( \tau \), fails to be a structuring cover of \( SS \). Indeed, the intersection of the finite family of sets \( V_U \) is a neighborhood \( W \) of \( \tau \) in \( R^3 \), therefore \( W \) must contain a point \( \eta \) in \( SS \) (because \( \tau \) is an accumulation point of \( SS \)) and this \( \eta \in SS \) is not covered by the family \( C \) – because every \( U \) in \( C \) is disjoint from \( V_U \), hence disjoint from \( W \) that contains \( \eta \), so \( \eta \) is not in \( U \). If \( SS \) is compact but not closed, then it has an accumulation point \( \tau \) not in \( SS \). Consider a collection \( C' \) consisting of an open neighborhood \( N(\eta) \) for each \( \eta \in SS \), chosen small enough to not intersect some neighborhood \( V_\eta \) of \( \tau \). Then \( C' \) is an open structuring cover of \( SS \), but any finite subcollection of \( C' \) has the form of \( C \) discussed previously, and thus cannot be an open structuring subcover of \( SS \). This contradicts the compactness of \( SS \). Hence, every accumulation point of \( SS \) is in \( SS \), so \( SS \) is closed.

2) \( SS \) is bounded. Consider the open balls centered upon a common point \( \tau \), with any radius \( r \). This can cover any set, because all points in the set are some distance away from that point \( \tau \). Any finite structuring subcover of this structuring cover must be bounded, because all balls in the structuring subcover are contained in the largest open ball within that structuring subcover. Therefore, any set covered by this structuring subcover must also be bounded.

3) The closed subset \( SS' \) of \( SS \) is compact. Let \( C_{SS'} \) be an open structuring cover of \( SS' \). Then \( U = R^3 \setminus SS' \) is an open set and \( C_{SS} = C_{SS'} \cup \{ U \} \) is an open structuring cover of \( SS \). Since \( SS \) is compact, then \( C_{SS} \) has a finite subcover \( C'_{SS} \) that also covers the smaller set \( SS' \). Since \( U \) does not contain any point of \( SS' \), the set \( SS' \) is already covered by \( C''_{SS} = C'_{SS} \setminus \{ U \} \) that is a finite subcollection of the original collection \( C_{SS} \). It is thus possible to extract from any open structuring cover \( C_{SS'} \) of \( SS' \) a finite structuring subcover.

Then, if \( SS \) is closed and bounded, then it is compact.

Let \( SS \times SS \) be the cartesian product of \( SS \). The set \( \Delta = \{(\tau_1, \tau_2) \in SS \times SS : \tau_1 = \tau_2 \} \) is closed in the product topology of \( SS \times SS \).
Let \( SS = \{ \tau_1, \tau_2, \ldots, \tau_n \} \) be a set and \( T \) be a collection of subsets of \( SS \) as 
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T = \{ \{ \}, \{ \tau_1 \}, \{ \tau_2 \}, \ldots, \{ \tau_1, \tau_2, \ldots, \tau_n \} \}.
\]

**Note 3:** Subtext \( SS \) will form a topological textual space \( SS_T = (SS, T) \)

Let \( \tau_1, \tau_2 \) be two points in \( SS_T \). Points \( \tau_1, \tau_2 \) can be separated by neighborhoods since there is a neighborhood \( U \) of \( \tau_1 \) and a neighborhood \( V \) of \( \tau_2 \) such that \( U \cap V = \emptyset \) (Figure 2).

Figure 2: Empty intersection of two neighborhoods \( U \) and \( V \).

\( S \) and \( SS_T \) are locally small categories and \( \phi : S \rightarrow SS_T \) is a functor from \( S \) to \( SS_T \). The functor \( \phi \) induces a function \( \phi_{s_1,s_2} : Hom_S(s_1, s_2) \rightarrow Hom_SS(\phi(s_1), \phi(s_2)) \) for every pair of terms \( s_1 \) and \( s_2 \) in \( S \). The functor \( \phi \) is faithful functor because \( \phi_{s_1,s_2} \) is injective. Therefore \( \phi : S \rightarrow SS_T \) is a Top concrete category or **belief constructed** because at least one of its objects \( (SS_T) \) has topological structure and its morphisms are functions preserving this structure.

Let \( SS_T' \) be a subset of a topological space \( SS_T \).

**Definition 11:** Point \( \tau \) is a point of closure of \( SS_T' \) if every neighbourhood \( U \) of \( \tau \) contains a point of \( SS_T' \).

**Proposition 1:** Structuring Structure \( SS_T \) is a Kolmogorov space \( T_0 \).

**Proof:**
Two points \( \tau_1 \) and \( \tau_2 \) are **topologically distinguishable** because they have not exactly the same neighborhoods \( U \) and \( V \); that is, at least one of them has a neighborhood that is
not a neighborhood of the other. If \( x \) and \( y \) are topologically distinguishable points, then the intersection of singleton sets \( \{ \tau_1 \} \) and \( \{ \tau_2 \} \) must be disjoint \( \{ \tau_1 \} \cap \{ \tau_2 \} = \emptyset \).

Topological indistinguishability of points is an equivalence relation \( \sim \). We define a topology on the quotient set \( SS_T/\sim \) as follows: a set of equivalence classes in \( SS_T/\sim \) is open iff their union is open in \( SS_T \). This is the quotient topology on the quotient set \( SS_T/\sim \). Let \( f: SS_T \to SS_T/\sim \) be the projection map which sends each element of \( SS_T \) to its equivalence class. Then the quotient topology on \( SS_T/\sim \) is the finest topology for which \( f \) is continuous. The Kolmogorov quotient of \( SS_T \) \( KQ(SS_T) \) under this equivalence relation \( \sim \) is always \( T_0 \). \( KQ(SS_T) \) and \( SS_T \) are homeomorphic.

**Proposition 2:** The Structuring Structure \( SS_T \) is a symmetric space \( R_0 \).

*Proof:*

Two points \( \tau_1 \) and \( \tau_2 \) are separated because each of them has a neighborhood \( U \) and \( V \) that is not a neighborhood of the other. The \( \tau_1 \) and \( \tau_2 \) are separated iff if their singleton sets \( \{ \tau_1 \} \) and \( \{ \tau_2 \} \) are separated.

**Definition 12:** Point \( \tau \) is an accumulation point iff every open neighbourhood \( U \) of \( \tau \) contains a point of \( SS_T \) other than \( \tau \) itself.

**Proposition 3:** The Structuring Structure \( SS_T \) is a Frechet space \( T_1 \).

*Proof:*

\( SS_T \) is \( T_1 \) because is both \( T_0 \) and \( R_0 \).

**Proposition 4:** The Structuring Structure \( SS_T \) is a preregular space \( R_1 \).

*Proof:*

1) The two points \( \tau_1 \) and \( \tau_2 \) are distinguishables.
2) The two points \( \tau_1 \) and \( \tau_2 \) are separated by neighborhoods \( U \) and \( V \).
3) \( SS_T \) space is also be \( R_0 \).

**Proposition 5:** The Structuring Structure \( SS_T \) is a Hausdorff space \( T_2 \).

*Proof:*

Thus, \( SS_T \) is Hausdorff because it is \( T_0 \), \( R_1 \) and \( T_1 \).

Let \( SS_T' \) be a subspace of \( SS_T \).

**Theorem 2:** \( SS_T' \) is a Hausdorff space

*Proof:*

Let \( \tau_1, \tau_2 \in SS_T' \) where \( \tau_1 \neq \tau_2 \). Since \( SS_T \) is Hausdorff, there are disjoint neighborhoods \( U \) of \( \tau_1 \) and \( V \) of \( \tau_2 \). Then \( U \cap SS_T' \) is a neighborhood of \( \tau_1 \) in \( SS_T' \) and \( V \cap SS_T' \) is a neighborhood of \( \tau_2 \) in \( SS_T' \), and \( (U \cap SS_T') \cap (V \cap SS_T') = \emptyset \). Therefore, \( SS_T' \) is Hausdorff.
And ∀τ ∈ SS, we have

\[ \{ \tau \} = \bigcap_{\Gamma : \Gamma \subseteq SS} \{ \tau \in \text{open set } U \text{ such that } \tau \in U \subset \Gamma \} \]

Let U be a neighborhood of SS, let τ a point in U and let ζ a point in \( \overline{U} \). There are disjoints between open neighborhoods \( V_1 \) and \( V_2 \).

**Theorem 3:** \( U \subset V_1 \) and \( \zeta \in V_2 \).

**Proof:**
∀τ ∈ U, \( \exists V_{1\tau}, V_{2\tau}, V_{1\tau} \cap V_{2\tau} = \emptyset \) such that \( \tau \in V_{1\tau} \) and \( \zeta \in V_{2\tau} \). Then \( \{V_{1\tau}\}_{\tau \in U} \) is an open structuring cover for U. There exists a finite set \( U_0 \subset U \) such that \( \{V_{1\tau}\}_{\tau \in U_0} \) is a finite open structuring cover for U. Then \( V_1 = \bigcup_{\tau \in U_0} V_{1\tau} \) and \( V_2 = \bigcap_{\tau \in U_0} V_{2\tau} \). We suppose that \( \exists \nu \in V_1 \). Then the following conditions are satisfied:

a) \( V_1 \) and \( V_2 \) are open.
b) \( U \subset V_1 \) and \( \zeta \in V_2 \).
c) For some \( \tau \in U_0, z \in V_{1\tau} \)
d) \( V_{1\tau} \cap V_{2\tau} = \emptyset \)

Then \( \nu \notin V_2 \) and \( \nu \notin V_2 \).

**Proposition 6:** The Structuring Structure \( SS_T \) is a compact space.

**Proof:**
1) A compact space also be Hausdorff.
2) Let \( \{SC_i\}_{i \in I} \) be an arbitrary collection of open subsets (structuring covers) of \( SS_T \) such that \( \bigcup_{i \in I} SC_i \supseteq SS_T \) and there is a finite subset \( J \subset I \) such that

\[ \bigcup_{j \in J} SC_j \supseteq SS_T . \]

Then, \( SS_T \) is a compact topological space.

Let \( SS_T' \) be a subset of \( SS_T \).

**Theorem 4:** The following statements are equivalent:

a) \( SS_T' \) is compact.
b) Every open structuring cover of \( SS_T' \) has a finite structuring subcover.

**Proof:**
1) Suppose $SS_T'$ is compact, and $\{SC_i\}_{i \in I}$ is an arbitrary open structuring cover of $SS_T'$, where $SC_i$ are open sets in $SS_T$. $\{SC_i \cap SS_T'\}_{i \in J}$ is a collection of open sets in $SS_T$ with union $SS_T'$. Since $SS_T'$ is compact, there is a finite subset $J \subset I$ such that $SS_T' = \bigcup_{i \in J} (SC_i \cap SS_T') = \left( \bigcup_{i \in J} SC_i \right) \cap SS_T' \subset \bigcup_{i \in J} SC_i$, so $\{SC_i\}_{i \in J}$ is a finite open structuring cover of $SS_T'$.

2) Suppose every open structuring cover of $SS_T'$ has a finite structuring subcover, and $\{SC_i\}_{i \in I}$ is an arbitrary collection of open sets with union $SS_T'$. By definition of subspace topology, each $SC_i$ is of the form $T_{SC_i} \cap SS_T'$ for some open set $U_i$ in $SS_T$. Now $SC_i \subset U_i$, so $\{U_i\}_{i \in I}$ is a structuring cover of $SS_T'$ by open sets in $SS_T$. By assumption, it has a finite structuring subcover $\{U_i\}_{i \in J}$. Then $\{SC_i\}_{i \in J}$ covers $SS_T'$, and $SS_T'$ is compact.

5. STRUCTURES OF BELIEF MATERIALIZATION

One first meaning of a belief net in the genesis of $T$, would mean the equality and unity connecting the collective subject of $T$ and the collective subject of the society or of a determined social group. This means that the social group’s WV would be the same that is materialized in $T$: the ways to remember, to suffer, to understand, to reason, etc., of a social group are such ways that are materialized in $T$. One first meaning of correlation would consist of finding the materialization of $T$, with a certain sublanguage $L_i$, of its subjects, personages, situations, aesthetics, etc., corresponding to subjects, personages, situations, aesthetics, etc., of a certain social group. We cannot talk of a similarity, since the relations materialized in $T$ do not correspond with exactitude more or less, with the one produced in the Structural Base (SB) of a determined social group. We propose in a first approach, three mathematical structures of materialization: belief category, belief net and continuous materialization function.

5.1. First Hypothesis: Belief Category

The class concept is well known as a collection of sets or sometimes other mathematical objects which can be unambiguously defined by a property that all its members share. Every set is a class, no matter which foundation is chosen. A class that is a set is called a small class. If we have defined $S$ as a direct set, therefore $S$ is a small class. Let $SS = \{\tau_1, \tau_2, ..., \tau_m\}$ be the Structuring Structure, $sS = \{T_1, T_2, ..., T_m\}$ be the Structured Structure and $sS' = \{\Gamma_1, \Gamma_2, ..., \Gamma_m\}$ be other Structured Structure influenced by the first sS. Let bob($B$) be a class formed by $\{S, SS, sS, sS', ..., sS''\}$. We may establish a morphism $f$ between beliefs-objects of the class. Each morphism $f$ has a unique source object $s$ and target object $\tau$ where $s$ and $\tau$ are in bob($B$). We write $f : s \rightarrow \tau$ and we write $\text{hom}(s, \tau)$ to denote the hom-class of all morphisms from $s$ to $\tau$. For every three objects $S, \tau$ and $T$, there is a binary operation $\text{hom}(s, \tau) \times \text{hom}(\tau, T) \rightarrow \text{hom}(s, T)$ called the composition of morphisms; the composition of $f : s \rightarrow \tau$ and $f' : \tau \rightarrow T$ is written as $f' \circ f$.

Therefore,
**Definition 13:** There is a belief category $\mathcal{B}$ consisting of a class $bob(b)$ of belief objects, a class $hom(B)$ of morphisms between the belief objects and a binary operation of composition of morphisms $f \circ f$ such that the following axioms hold:

1) **Associativity:** If $f : s \to \tau$, $f' : \tau \to T$ and $f'' : T \to \Gamma$ then

   $f'' \circ (f \circ f) = (f'' \circ f) \circ f$

2) **Identity:** For every belief object $b$, there exists a morphism $I_b : b \to b$ called the identity morphism for $b$, such that for every morphism $f : s \to \tau$, we have $I_\tau \circ f = f = f \circ I_s$

**Definition 14:** Morphisms $f : s \to \tau$, $f' : \tau \to T$ and $f'' : T \to \Gamma$ are called function of connotation, function of materialization and function of influenced materialization respectively.

We will graphically express it in figure 3.

In the category of these sets, where morphisms are belief functions, two functions may be identical as sets of ordered pairs (may have the same range), while having different codomains. The two functions are distinct from the viewpoint of category theory.

Let $hom(B')$ a subclass formed by the morphisms $f'' : T \to \Gamma$, $f''' : T \to \Gamma^1$, ..., $f' : T \to \Gamma^\omega$.

**Definition 15:** Subclass $hom(B') \subset hom(B)$ constitutes a textual style.

**Note 4:** Morphisms $f' : \tau \to T$ constitutes the process of initiation of a textual style.

**Example 2:** In the middle of XII Century, in the monastic churches, as for example in both Cluny and Citaux, the gothic style prevailed. The Basilica Cathedral of Saint-Denis near Paris is considered the first monumental masterpiece of Gothic art, and later the Cistercian churches that gradually propagated the gothic style across Europe.

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### 5.2. Second Hypothesis: The Belief Net

We have defined $S$ like a direct set. We have demonstrated previously (Usó-Domènech et al., 2009; Usó-Doménech and Nescolarde-Selva, 2012) that texts have a topological space structure. Establishing a belief net between the SS of $T$ and an ideology or belief system belonging to Doxical Superstructure (DS) will give one first approach to the identifying the significance of $T$ (Usó-Doménech and Nescolarde-Selva, 2012; Nescolarde-Selva and Usó-Doménech, 2012). We are going to suppose a case limit: substantive beliefs of a belief system projects in a certain text. Let us take as a material example *The Nicene Creed* in theological medieval texts or we could take constructions like the roman or gothic cathedrals. In our theory, we will only work with substantive beliefs, but it can be extended to derived beliefs. In fact, thus it happens always. Any text reflects not only substantive beliefs but all the set of derived beliefs Nescolarde-Selva and Usó-Doménech, 2012), many of them incorporated and accumulated during the period of existence of the belief system.
Definition 16: If $SS_T$ is a textual topological space and $S$ a directed set, a belief net in $SS_T$ is a first materialization function $\phi$ from $S$ to $SS_T$, $\phi : S \rightarrow SS_T$.

Note 5: We write a net from $S$ to $SS_T$ in the form $(\tau_s)$, which expresses the fact that the term $s$ in $S$ is mapped to the subtext $\tau_s$ in $SS_T$.

Let $\Gamma$ be a subset of $SS$. Belief nets have the following properties:

1) If $(\tau_s)$ is a belief net from $S$ into $SS_T$, and if $\Gamma$ is a subset of $SS$, then we say that $(\tau_s)$ is residually in $\Gamma$ if $\exists s_j \in S, \forall s_i \in S, s_i \geq s_j$, the point $\tau_{s_j}$ lies in $\Gamma$.

Figure 3: Functions of connotation, materialization and influenced materialization.
2) If $\{\tau_i\}$ is a belief net in $SS_T$, and $\tau$ is an element of $SS_T$, we say that the belief net converges towards $\tau$ and write $\lim_{i \to \infty} \tau_i = \tau$ iff for every neighborhood $U$ of $\tau$, $(\tau_i)$ is eventually in $U$.

**Definition 17:** The belief net $\varphi$ is cofinally in $\Gamma$ if for every $s_j$ in $S$ there exists some $\tau_i \in S, \tau_i \geq s_j$, so that $\varphi(\tau_i)$ is in $\Gamma$.

**Definition 18:** A belief net $\varphi$ on $SS_T$ is called a belief ultranet if for every subset $\Gamma$ of $SS$, either $\varphi$ is eventually in $\Gamma$ or $\varphi$ is eventually in $SS - \Gamma$.

**Consequence 6:** Belief nets will always be belief ultranets.

Literary sacred texts, literary texts of political ideology, religious sanctuaries, etc, are examples of this class of belief ultranets.

A belief net is the truly mediating thing, that is to say, the way to think. A belief net puts in relation the WV of the collective subject with the SS of T. The ideological net corresponds to a way to relate.

Let $(\tau_i)$ be a belief net on $SS_T$ based on directed set $S$ and $\tau$ is a subtext of $SS$,

For Proposition 6 $SS_T$ is a compact space. Therefore every belief net $(\tau_i)$ in $SS_T$ has a belief subnet with a limit in $SS_T$.

### 5.3. Third Hypothesis: The continuous second materialization function

Let $B$ be a topological belief space $B = (S', bcl)$ such that $S' \subseteq S$ being $S'$ subset of the set of substantive beliefs $S$. Let $SS_T$ be a topological textual space $SS_T = (\tau, tcl)$ being $\tau$ a set of materialized substantive belief. $SS_T$ is contained in $B$ such that $SS_T \subseteq B$. That is, every element of $SS_T$ is also an element of $B$. Then the topology $SS_T$ is said to be a coarser belief topology than $B$, and $B$ is said to be a finer belief topology than $SS_T$. Because $B \neq SS_T$, we say $SS_T$ is strictly coarser than $B$ and $B$ is strictly finer than $SS_T$.

Let $B_1, B_2, ..., B_n, B$ be topological belief subspaces such that $B_1 \subseteq B_2 \subseteq ... \subseteq B_n \subseteq B$ and $SS_T \subseteq B_1 \subseteq B_2 \subseteq ... \subseteq B_n \subseteq B$. The binary relation $\subseteq$ defines a partial ordering relation on the set of all possible topologies.

We suppose we have a function $m : B \to SS_T$.

**Definition 19:** Function $m : B \to SS_T$ we call second materialization function.

**Definition 20:** We say that the materialization function $m : B \to SS_T$ is a continuous second materialization function at $s$ for some $s \in B$ if for any neighborhood $V$ of $m(s)$, there is a neighborhood $U$ of $s$ such that $m(U) \subseteq V$.

We will graphically express it in figure 4.
If \( m \) is continuous at every \( s \in B \), then we simply say \( m \) is continuous.

The first of the hypotheses will allow us to establish a class different from materialization: the mythical symbolic materialization.

6. REFLECTIONS: ON BELIEF SYSTEMS, IDEOLOGIES AND MYTHS

Culture is a symbolic system to be interpreted, understood, discussed, delineated, respected, and celebrated as the distinctive product of a particular group of people, of equal worth with all other such products. But it should never be used to explain anything about the people who produced it (Patterson, 2008). Not all cultural system building is compatible with sociological models for relationships between cultural and social forms. The semiotics of the linguistic disciplines can become so fixated on texts that references to an external and intersubjective – and potentially falsifiable – reality are weakened.

A recurring theme of ideologies is that they are designed to unite and stimulate collective action. Their credibility is often closely tied to the traits of specific leaders or parties, which are perceived as representing specific groups or life forms that are distinctive from other leaders, groups and life forms. An ideology is often skewed toward an antagonistic relationship to conflicting ideologies. This implies that an ideology’s credibility depends on an overarching justification of its attempt to express what is irrational, good or just (Usó-Doménech and Nescolarde-Selva, 2012; Nescolarde-Selva and Usó-Doménech, 2012a,b).

Social myths enter the picture at this point, and not only those with a clear political imprint. Nowadays, myths, from the Greek word *mythos*, are spread and interpreted in a variety of ways (Adams Perowne, 1990; Lévi-Strauss, Cl. 2001; Turner, 1968). By no means can it be taken for granted that the most important myths in contemporary
society are labelled and recognized as myths. As a rule, myths are perceived as concrete, often personified, accounts that provide us with dramatic pictures and explanations of the world order. Myths may be used to spread a religious message and to make the substance of politicized ideologies believable (Beaune, 1991). But myths are primarily tall tales, not ideological programs or religious beliefs.

Myths are often reminiscent of the sagas. A saga usually tells the story of a specific historical character, e.g. a central figure in a heroic battle. As opposed to fairy tales, sagas purport to relate actual historic events. We should guard against a hasty dichotomizing of the antithetical terms mythos/logos.

The question as to how true or false myths really are does not necessarily enter into an analytical delimitization of myths. If myths are automatically assumed to represent what is false – as is often the case – then an anti-myth can easily attain a status of being true because it is in opposition to the myth in question, which leads to an oversimplification. We expose a myth the same way that we expose an ideology – by using analytical criteria. As a working breakdown of myths, the following five hallmarks are typical for what are called myths:

1) Myths refer to familiar notions that purport to say something important about our lives.

2) Myths give shape to a universal struggle by reducing it to a conflict between two forces.

3) Myths incite our involvement by dramatizing these two forces as expressions of good and evil. There need not be equilibrium in the presentation of the two. As long as the one has been determined to be unequivocally good or bad, the other has thereby been defined.

4) Myths are archetypal or repetitive in character. It is up to alert guardians to remind us of their presence in shifting guises in ever changing situations. People can be taught to recognize the mythical drama in a given situation through specific codes or symbols.

5) Myths are usually geared more toward mobilizing the individual mind than toward inciting collective political action. They can be directed toward the cultural sphere rather than the political arena.

In light of the above, an approach to systems of belief will only be able to capture certain aspects of the collective faith that religion typifies. Without trying to list all the reasons why we can expect to find religion as a hallmark of human cultures, we will mention two of them.

One of these has to do with people’s need to feel a sense of security. From an emotional, ethical and even cognitive standpoint, people depend on mutual structure and on certainty. Ludwig Wittgenstein wrote at length about how certainty presupposes trust, without our being able – strictly speaking – to prove what we take for granted. (Wittgenstein, 1962). But for such a perception of reality to be meaningful and functional, there must first be a cultural institutionalization based on premises other than the strictly rationalistic. By extending this type of reasoning, we can claim that no unified perception of reality can be established apart from hypothetical premises, which in a certain sense contain an element of faith. Strictly speaking, the existence or non-
existence of God can no more be proved or disproved than historic determinism or indeterminism.

Dissimilar cultures are constructed around different but specific answers to these questions. Christian culture has maintained the existence of both a divine order and an individual accountability – that is, indeterminism in an essential field of reality. Islamic culture, by and large, has regarded the social sector as being determined by Allah's will. Buddhist culture has basically perceived reality as being determined by major repetitions, but without a personal God being behind the determinism. Humanists have usually taken a non-deistic view of reality, in which natural explanations are perceived as an adjustment to deterministic laws, while human values and individual freedom have been linked to an extreme emphasis on indeterminism.

The sociologist and moral philosopher Zygmunt Bauman (1992) has given an account of different cultures in relation to their choice of “life strategy” with respect to death and a consciousness of the risk of death. The religious response has consisted of identifying true life with the soul and immortality. What he calls “the modern strategy” has consisted of identifying with social groups and politico-cultural movements and fighting for their supraindividual survival. Furthermore, it consisted of an attempt to deconstruct the insolubility of death into specific problems that can be resolved at the individual level, such as health and accident insurance. Different life strategies predispose people to a predilection for widely different institutions and orientations. A “meaningful life” usually means one with purpose, a life where sorrows are counterbalanced by pleasures, a life where the overall goal gives direction to minor goals and purpose to major sacrifices. When people write and speak at length about a “cry for meaning,” they are thinking about something more than finding explanations for what is happening in people’s surroundings and for the need to have order in their daily lives (Frankl, 1978). In this context, “meaning” denotes something qualitatively important, something not necessarily found through mundane social interaction.

Even though the acquisition and development of meaning have important personal aspects, no civilization can relegate meaning to the status of a strictly private matter, with important social tasks reduced to supplying the necessities of life and maintaining a modicum of law and order. Communicative social bonds presuppose a common culture, in the sense of common frameworks of meaning. Without these frameworks, society's social bonds would ultimately disintegrate. From such a viewpoint, development and the maintenance of an adequate framework of meaning is imperative to ensure long-term viability of any society. Deductively, this can lead to the conclusion that in all cultures we can expect to find, not only a collective faith in axiomatic values, but an institutionalized religion as well. This jibes with Berelson and Steiner’s (1964) empirical inventory, that “all societies have religion(s).”

In cases where the socially supported religion is not transcendental in nature, we can expect political ideas to have a religious hue. This applies not just to ideologies in totalitarian societies. Even a belief in liberal humanity can assume religious forms. We will return later to the subject of the cult of human rights in our kind of society.

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