International Journal of Advanced Manufacturing Technology

A new surface joining technique for the design of shoe lasts

--Manuscript Draft--

Manuscript Number:	JAMT-D-12-11052R1
Full Title:	A new surface joining technique for the design of shoe lasts
Article Type:	Original Research
Keywords:	Shoe last design; blending and morphing joining techniques; curve network.
Corresponding Author:	Antonio Jimeno-Morenilla, Ph.D. Alicante University San Vicente del Raspeig, Alicante SPAIN
Corresponding Author Secondary Information:	
Corresponding Author's Institution:	Alicante University
Corresponding Author's Secondary Institution:	
First Author:	Francisco Javier Amorós-González, Computer Engineer
First Author Secondary Information:	
Order of Authors:	Francisco Javier Amorós-González, Computer Engineer
	Antonio Jimeno-Morenilla, Ph.D.
	Faustino Salas-Pérez, Computer Engineer
Order of Authors Secondary Information:	
Abstract:	The footwear industry is a traditional craft sector, where technological advances are difficult to implement owing to the complexity of the processes being carried out, and the level of precision demanded by most of them. The shoe last joining operation is one clear example, where two halves from different lasts are put together, following a specifically traditional process, to create a new one. Existing surface joining techniques analysed in this paper are not well adapted to shoe last design and production processes, which makes their implementation in the industry difficult. This paper presents an alternative surface joining technique, inspired by the traditional work of lastmakers. This way, lastmakers will be able to easily adapt to the new tool and make the most out of their know-how. The technique is based on the use of curve networks that are created on the surfaces to be joined, instead of using discrete data. Finally, a series of joining tests are presented, in which real lasts were successfully joined using a commercial last design software. The method has shown to be valid, efficient and feasible within the sector.

Dear editor,
please, find below our answers to the reviewers.

Most of the changes that they suggest have been carried out.
I expect the changes are satisfactory and this version can be used for publication.

Sincerely,
Antonio Jimeno Morenilla

Reviewer #2:

1. You state in the conclusion that "G2 continuity is achieved across the whole surface and in joining areas"

How do you know that ? G2 continuity means continuity in curvature everywhere. For specific surface types (NURBS, Bicubic, Bezier...) you can aquire and prove this mathematically but you have not presented what mathematical representation you are using. As a reader you get the impression that what you mean with G2 continuity is that the result looks fine on the pictures. G2 is a mathematical requirement that must be proved or at least validated with zebra plots or similar.

I suggest that you either prove that you have G2 continity everywhere or change the text. I agree with you that your method is superior over methods using data from a single intersection but I don't think you have shown that the blend is G2 everywhere.

As the reviewer suggests, NURBS is the model used in this paper to represent the surfaces, which, therefore, ensures C2 continuity. The joining area is constructed using a Gordon Surface, which also guarantees C2 continuity. Following the reviewer's suggestion, the following text has been added at the end of section 1 (in the middle of page 2) to clarify the geometric model used:

> “The volume of a shoe last should resemble that of a natural object such as a human foot. It might be said that, there are two very important aspects of completely different nature that should be considered with regard to this: On the one hand, the object should be modelled using free surfaces, due to the fact that it derives from a natural form. On the other hand, the shoe will acquire the shape of the last with which it is made. Therefore, small errors in this component would detrimentally affect the foot that wears the shoe, in the form of grazes and general discomfort. Hence - as industrially produced objects - lasts require a high level of precision in design and manufacture. For these reasons, CAD / CAM software programmes aimed at last design and manufacture, use free surfaces with a high level of precision as geometric models, which require techniques used for their manipulation to be precise, thus avoiding approximate models.”

In section 3 (step 5, on page 11), the formulation of the Gordon surface, with regard to the proposed model, has been added including these important references:

[29] Gordon W.J., (1969), Spline-Blended Surface Interpolation Through Curve Net-works. *Journal of Mathematics and Mechanics*. Volume 18, No. 10.

[30] Smid J., (1991), Triangular Gordon Surface with Unique Normal. *TR EDS-GM.SIAM Conference on Geometric Modeling*, *Tampe, AZ*.

Furthermore, in the same section, zebra plots have also been added in order to prove G2 continuity between new and original surfaces. This proves continuity between the three areas concerned. Figure 26 on page 13 shows those images. As a result, the following text has been added in section 3 (step 5, on page 11):

> "The joining surface is constructed by using a Gordon Surface, which was created using the guide curves previously described. The basic formulation used to create the afore-mentioned surface based on the main and guide curves can be observed in the (1-4) expressions."
>
> "As it can be seen in the zebra plot in figure 26, both the main and auxiliary curves obtained from the original lasts guarantee C2 continuity in the transition area between surfaces."

2. I searched the Compendex scientific database for the phrase "shoe last" and I got 373 hits. Browsing through the titles I saw many papers related to shoe last design using CAD technology of different kinds. I'm sure your work is original but I still want to point out that there seems to be a lot of work done in this area and you have very few references to similar work.

This paper addresses a specific issue concerning the joining of shoe lasts. However, as the reviewer mentioned, a lot of work has already been done relating to other aspects of the design and production of this industrially produced object. For this reason, and to complete the scientific background, the following literature references corresponding to recent research have been added and commented on in the introduction section (at the end of page 2).

[1] Luximon A., Luximon Y., (2009), Shoe-last design innovation for better shoe fitting. *Computers in Industry*, Volume 60, Issue 8, pp 621-628.

[2] Raffaeli R., Germani M., (2011), Advanced computer aided technologies for design automation in footwear industry. *International Journal on Interactive Design and Manufacturing (IJIDeM)*. Volume 5, Issue 3, pp 137-149.

[3] Franciosa P., Gerbino S., Lanzotti A., Silvestri L., (2013), Improving comfort of shoe sole through experiments based on CAD-FEM modelling. *Medical Engineering & Physics*. Volume 35, Issue 1, pp 36-46.

[4] Wang C.-S., (2010), An analysis and evaluation of fitness for shoe lasts and human feet. *Computers in Industry*. Volume 61, Issue 6, pp 532-540.

[5] Wang J., Zhang H., Lu G., Liu Z., (2011), Rapid parametric design methods for shoe-last customization. *The International Journal of Advanced Manufacturing Technology*, Volume 54, Issue 1-4, pp 173-186.

[6] Muñoz P., Coronel J.L., Continuidad en superficies espaciales para diseño industrial. *Universidad de Buenos Aires*. (2004).

3. There is no page numbering but line 15 on what I think is page 3 should read "dealing" instead of "deal".

Corrected. Page numbering has been added.

4. Page 6 line 50 replace "those" with "that".

Corrected.

Reviewer #3:

The manuscript describes a practical application of CAD modeling in the shoe industry. The focus is on the design of a shoe last or mould around which various types of footwear can be constructed. Lasts are classified based on type of footwear type (boots, sandals, dress shoes) and toe shape. The target region of the shoe last for shape modeling is the toe area because it reflects shoe aesthetics and fashion.

The authors propose a method for surface joining based on continuous data instead of discrete data points. In essence, this is a very interesting industrial case-study with little theoretical contribution to arbitrary shape modeling. The proposed technique appears to be a derivative of many other manual approaches used to join surface segments and patches to create a complete 3D model.

We agree with the opinion of the reviewer that the proposed technique addresses an interesting problem, with little theoretical contribution. The authors would like to demonstrate that, as a matter of fact, the problem being dealt with, has a large impact on the design of this industrially produced object. In this way, a process that is usually carried out by hand in the vast majority of models is sped up, from an operation that took almost 6 hours to complete it only in seconds.

The authors believe that the success of a research work does not only rely on its theoretical contributions, but rather on the fact that the solutions brought forward are able to reach out society and have a real effect on the industry. Throughout the course of this research, we have also been able to prove that the expert lastmakers, who participated in the execution of the tests, not only used the CAD tool with little difficulty, but they were also capable of making the most out of the tool, even more than we were able to do: They were able to successfully join very different models, that we thought would be almost impossible to do. We believe that this is due to the fact that automated process is similar to the manual process, as the reviewer pointed out (one example being the selection of the joining planes, which derive from the areas where professionals used to physically cut the models). This has proved to be an important advantage for them. This way, the designers can take advantage of the "know-how" they gained from years of professional experience. In addition, this contributes to the implementation of CAD/CAM programmes into this typically traditional industry, which is frequently resistant to using new technology.

Some specific issues that must be addressed:

- The Abstract is very generic and does not clearly identify the methodology, novelty of the contribution, and/or specific outcomes of the analysis used to verify the proposed joining technique.

Following the reviewer's comment, the abstract has been modified in such a way that the methodology and novelty of the contribution can be clearly identified, as well as the impact of results obtained. The abstract will read as follows:

> "The footwear industry is a traditional craft sector, where technological advances are difficult to implement owing to the complexity of the processes being carried out, and the level of precision demanded by most of them. The shoe last joining operation is one clear example, where two halves from different lasts are put together, following a specifically traditional process, to create a new one. Existing surface joining techniques analysed in this paper are not well adapted to shoe last design and production processes, which makes their implementation in the industry difficult. This paper presents an alternative surface joining technique, inspired by the traditional work of lastmakers. This way, lastmakers will be able to easily adapt to the new tool and make the most out of their know-how. The technique is based on the use of curve networks that are created on the surfaces to be joined, instead of using discrete data. Finally, a series of joining tests are presented, in which real lasts were successfully joined using a commercial last design software. The method has shown to be valid, efficient and feasible within the sector."

- The Introduction section needs to be more focused with an update review of continuous curve and surface joining techniques. Over the last decade a lot of research has been performed on using B-splines, NURBs and T-splines for free form modeling. It would also strengthen the paper if the authors expanded the discussion on the role CAD modeling plays in the shoe industry.

For clearer reading, in section 1, aspects relating to the footwear design process, which are addressed in this paper, have been left as an introduction, with a focus on the importance of CAD systems within the footwear sector. In particular, the following paragraphs have been added:

> "The footwear industry is a sector dominated by small and medium-sized companies, where most of the processes are carried out by hand. This is the case of a traditional sector, where automation has been implemented in very specific operations, mainly due to the fact that footwear manufacture involves numerous complex tasks that are difficult to automate...
> Given the high percentage of manual and craft work carried out in this sector, every progress in automated design and production systems would help speeding up these processes, thus providing further reliability and precision. Furthermore, many of the craftworkers who participate in footwear production are considered as artists and, typically, changes in techniques or processes arising from the use of CAD/CAM techniques are rejected because they are thought to impose a possible limitation on their creativity."
> "This has been approached from the point of view of traditional lastmakers; with the aim of demonstrating that automation can positively affect the design process, without thereby bringing about rejection in the sector."

Also, at the end of section 1, on page 3, a paragraph explaining the paper structure has been added:

> "This paper is structured as follows: section 2 presents an overview of the different existing techniques allowing the joining or transition between three-dimensional surfaces. Furthermore,

> those processes specifically focused on shoe last joining are analysed. In section 3, a new methodology is presented that is inspired by the manual process, dealing with the accurate joining of lasts. In section 4, experiments conducted on real lasts to prove the validity of the new method are presented. Finally, section 5 outlines the main conclusions of this paper.”

The Conclusions section has also been restructured, in order to clarify to the reader the contributions of research. Specifically, the section reads as follows on pages 16 and 17:

> “The footwear sector is a very traditional industry, where automation has not been able to impose its presence. The reasons for the prevalence of manual or craft labour over CAD/CAM processes are of a very distinct nature. The replacement of any manual design or production task by a CAD/CAM-based technique is extremely important, not only because of the obvious speeding up of processes, but also because it represents one step towards the full automation in a sector reluctant to change.”
>
> “This paper presents a precise technique - based on NURBS surfaces - that is inspired by the manual process by improving it, and speeding it up, without causing rejection among users of this specific sector with little experience in the use of CAD tools.”
>
> “In relation to current techniques that specifically deal with shoe-last joining, this technique constitutes a step forward in considering continuous surfaces (NURBS) as joining element. However, techniques used so far deal with discretisations of the last surface. This feature provides greater precision in the joining process, and avoids limitations imposed by the use of a discrete set of sections, since the problem posed by wrongly cutting the sections defining the last for the algorithms that process the last joining process, does no longer exist. G2 continuity is achieved across the whole generated surface and in joining areas with the surfaces of useful back and toe parts, due to using interpolation curves that join the back and the toe makes the direction of final surface creation to be transverse, thus enhancing continuity.“
>
> “With regard to the existing generic techniques, which enable the joining of three-dimensional surfaces, this technique provides an improved interface to perform joining of shoe-lasts. The technique has been inspired by the way traditional lastmakers work, defining cutting planes (before areas to be saw-cut) and areas to be smoothed (in the past, areas to be bonded and sanded). This way, two important objectives are met: traditional lastmakers can make use of their pre-existing know-how, and they are not reluctant to use it, as the process sounds familiar to them.”
>
> “Another important aspect to be considered is that the generated solution can be easily edited by manually editing the guide curves and automatically reconstructing the surface. This feature provides flexibility to the method, and allows the easy adjustment of the automatically obtained solution.”
>
> “Finally, due to the fact that this is a generic method, joining together two lasts in any part is possible. This type of joining is less common but the automation of the process results in considerable time saving and provides improved precision. This means that the new method is also capable of working in the leg area, so as to be able to create a new last using the lower part of a last and the upper part of another one. This technique has been tested by experienced

> lastmakers and is successfully applied by a last-design software programme that is widely used worldwide.”
>
> “However, there are still some aspects that could be improved. Further research on this method will focus on improving the way in which control points are obtained, making them dependent on the section topology, so that said points concentrate on those areas of more complex surface. Another important aspect is to also take into account some of the original surface sections found in the joining area – and discarded by this method - since they can provide valuable information for the joining process. .”

- Although the application is interesting, the authors need to clearly explain the novelty of the proposed methodology and why the results are meaningful in the larger footwear and clothing industry.

As it has already been mentioned, in section 1, emphasis has been put on the importance that CAD/CAM has and could further have in the future of the footwear industry, characterised by very low levels of automation. In addition, in the “Experiments” section, on page 13, the following paragraph has been added, which demonstrates the importance of obtained results.

> “…Expert lastmakers took part in the execution of the tests, and they not only used the CAD tool with little difficulty, but they were able to successfully join models that seemed to be very different. This is due to the fact that the automated process greatly draws influence from the manual process. For example, the selection of the joining planes derives from the areas where professionals physically cut the models, which ultimately has been considered as a big advantage for them. This means that, this way, designers can benefit from their “know-how” gained from years of professional experience. In addition, this makes the implementation of CAD/CAM tools in this type of industries - frequently reluctant to new technologies - easier.
> Analysing the advantages that the technique presents, it is worth mentioning that the complete manual process takes an expert lastmaker about 5 to 6 hours of work. Thanks to the use of the technology, this can be now performed in 10 to 15 minutes. Additionally, considering that only a short learning curve is required for a person that is used to working with the computer, various lastmakers were able to be taught how to use the tool in a training course that took about 20 hours.”

- Section 2 is a "Literature Review" on the generic techniques and methods proposed by a variety of researchers to solve 3D surface joining problems. Unfortunately, most of the references are more than 5 to 10 years old. The CAD modeling industry has changed rapidly in the last few years. Newer approaches to connecting and blending free form surfaces have not been discussed.

In section 2, both the title and the structure have been modified in order to make two aspects clearer to the reader: first, generic joining techniques are analysed (new references and methodologies have been added, following the reviewer’s comment). Then, those techniques that are specially aimed at joining shoe-lasts are further reviewed.

The first paragraph in Section 2, on page 3, has been extended so as to point out that current methodologies able to solve generic problems in joining three-dimensional surfaces are considered.

> “Literature proves that plenty of research has been done regarding the geometric joining of surfaces. This section presents an overview of those techniques that are most widely used to join NURBS surfaces, such as those used in shoe last joining. Nevertheless, other approximations which are not based on free surfaces but on discrete representations are also given aiming at solving the problem of last joining. Most of the solutions to surface joining or transitions are related to the “blending” and “morphing” concepts.”

The following paragraph has been added after the analysis of blending and morphing techniques, thus pointing out the importance of providing “feasible” solutions, considering the target sector (at the end of page 4):

> “In addition to the above, it has to be considered that for a surface joining technique to succeed in an industrial sector, such as the footwear sector with prevalence of traditional and manual labour, automatic techniques to be used shall not cause rejection among users. In other words, a lastmaker has not CAD/CAM expertise or advance knowledge on 3D geometry. Consequently, even though the above-mentioned techniques are efficient and precise, they would be rarely used and with little success. Nevertheless, experienced lastmakers gained a lot of know-how throughout years of professional experience. Expertise passed from one generation to the next allows them to successfully join two apparently very different shoe lasts.”

At the end of section 2, on page 5, it is concluded that “This paper proposes a joining technique based on the use of surfaces and continuous curves that, on the one hand, allows continuity and editability of models used in the joining, and on the other, is inspired by the traditional process of manual joining of surfaces. This way, its use is intuitive for traditional lastmakers, who can make the best out of their “know-how”.

Newer approaches to connecting and blending free form surfaces have been discussed. Specifically, the following references have been added:

About BLENDING (at the end of page 3):

> “Recent works introduce an interactive surface interpolation method by spline surfaces that is not aimed at joining but is useful in controlling the blending surface as a result of joining [12]. The technique is based on linear blending and works for a large type of surfaces including bicubic Bézier, B-spline, NURBS surfaces and trigonometric surfaces. Shi et al. propose a method of G^n blending multiple parametric surfaces in polar coordinates [13]. It models the geometric continuity conditions of parametric surfaces in polar coordinates and presents a mechanism of converting a Cartesian parametric surface into its polar coordinate form. Performance is an important issue and it is analyzed in [14]. Here render experiments were carried out with trimmed blending surfaces on a GPU and results show an eighteen-fold to twenty-fold increase in rendering speed over a CPU version. Finally in [15], a study of geometric continuity C1G2 of blending surfaces by a shape-blending process was developed. This paper studies the continuity of the ruled surfaces constructed by linear interpolation between two pairs of C1G2 continuous curves.”

References:

[12] Juhásza I., Hoffmann M., (2009), Surface interpolation with local control by linear blending. *Annales Mathematicae et Informaticae*, Volume 36, pp 77-84.

[13] Shi k., Yong J-H., Sun J-G., Paul J-C., (2010), Gn blending multiple surfaces in polar coordinates. *Computer-Aided Design*, Volume 42 Issue 6, pp 479-494.

[14] Dae-Hyun K., Jieun L., Seong-Jae L., and Seung-Hyun Y., (2011), Construction and Rendering of Trimmed Blending Surfaces with Sharp Features on a GPU*. ETRI Journal*, Volume 33, No.1, pp 89-98.

[15] Kouibiaa A., Pasadasa M., Sbibihb D., Zidnac A., Belkhatirb B., (2013), Geometric continuity C1G2 of blending surfaces. *Computer-Aided Design.* Volume 45, Issue 3, pp 733–738.

About MORPHING (at the middle of page 4):

> "Zhang et al. [23] presented a multiresolution-based technique that can be used to create different levels of detail, thus preserving features in areas with large deformation. Afterwards, the same author developed a new technique based on Laplacian meshes [24], which combines traditional concepts such as Laplacian editing with skeleton-based and as-rigid-as-possible (ARAP) techniques. Li et al. [25] present a sketch-based technique that uses a skeleton to be deformed, and then this deformation is transferred to the entire geometry. This technique is frequently used for deformation of organic objects.
>
> Recently, space-based morphing techniques have been appearing. Huang et al. proposed a Free Form Deformation (FFD) technique applied to complex objects, using a coarse control mesh and a series of deformation iterations based on Barycentric coordinates [26]. Obtained results are good, but could be improved. Finally [27] provides a cage-based technique based on the combination of cage deformations and Plaplacian meshes."

References:

[23] Zhang S., Wu E., (2009), Multiresolution Animated Models Generation Based on Deformation Distance Analysis. *Proceeding ICCMS '09 Procedings of the 2009 International Conference on Computer Modeling and Simulation*. pp 73-77.

[24] Zhang S., Huang J., Metaxaz D., (2010), Robust mesh editing using Laplacian coordinates. *Journal Graphical Models archive* Volume 73, Issue 1, pp 10-19.

[25] Li M., ASharf G., (2011), Sketch Based 3D Character Deformation. *Book Transactions on edutainment V. Lecture Notes in Computer Science*, Volume 6530, pp 177-188. Springer-Verlag Berlin.

[26] Huang J., Chen L., Liu X., Bao H., (2009), Efficient Mesh Deformation Using Tetrahedron Control Mesh. *Computer Aided Geometric Design*, Volume 26, No. 6, pp 617-626.

[27] Savoye Y., Franco J-S., (2010), CagelK: Dual-Laplacian Cage-Based Inveres Kinematics. *Proceeding AMDO'10 Proceedings of the 6th international conference on Articulated motion and deformable objects*.

- It is not clear from the discussion as to whether the surface joining algorithm needs to be highly accurate or only approximate. If approximate free forms are acceptable then there are other techniques that can be used to obtain a closed geometry.

A shoe last is an industrial object derived from an organic object, that is, the human foot. For this reason, two aspects of different nature are combined in this component: on one hand, the object to be made is not well parameterised as it comes from an organic object, hence, free forms or NURBS is considered to be one of the best suited geometric models. On the other hand, a last – as an industrially produced object – demands high precision in its design and manufacture (estimated as ± 0.1 mm, according to the sector). The reason being that the shoe will acquire the shape of the last on which it is made, and small errors in this component would finally affect the wearer's foot with grazes or discomfort. That is why approximate techniques for geometry generation are avoided in last design and manufacture.

The following paragraph has been added to the Introduction in order to explain this twofold aspect of shoe lasts at the middle of page 2:

> "The volume of a shoe last should resemble that of a natural object such as a human foot. It might be said that, there are two very important aspects of completely different nature that should be considered with regard to this: On the one hand, the object should be modelled using free surfaces, due to the fact that it derives from a natural form. On the other hand, the shoe will acquire the shape of the last with which it is made. Therefore, small errors in this component would detrimentally affect the foot that wears the shoe, in the form of grazes and general discomfort. Hence - as industrially produced objects - lasts require a high level of precision in design and manufacture. For these reasons, CAD/CAM software aimed at last design and manufacture, use free surfaces with a high level of precision as geometric models, which requires techniques used for their manipulation to be precise, thus avoiding approximate models."

- The methodology is rather intuitive and straightforward (ie. technique appears largely ad hoc). The interpolation guide curves used to connect disjointed parts (Figure 13) is interesting but it is not clear from the text how this was determined.

The reviewer is right. Several paragraphs have been added to section 3 in order to clarify how guide and control curves are obtained, including the formulation used for the joining surface creation and zebra plots, which prove the continuity in the joining area. As it has already been mentioned, one of the advantages of this method is that it is inspired by the manual work carried out by traditional lastmakers.

In particular, the following comments have been added to section 3:

Step 2 – Following the first paragraph. (on page 7):

> "…The 5mm distance between the valid parts to be joined is used in order to ensure a distance which allows obtaining information about the area of the last where continuity and smoothness

> between the valid parts and the new joining surface is needed. This information is useful to obtain G2 continuity between the joining parts. Furthermore, the fact that it is a constant and homogeneous separation makes information intervals smooth and continuous. "

Step 3 – At the middle of page 9, just after the algorithm 3:

> "Depending on the distribution of points, guide curves from step 4 are going to create the homogeneous and smooth modelling of the filling area in one way or another. G2 continuity obtained at the end of the process is not only the same NURBS filling surface, but also the joining of valid parts thanks to the obtaining of interpolation points on the valid parts. Due to the importance of this parameter, a study regarding obtained points' quantity influence on the quality-speed relation of the entire process has been included in section 4."

Step 5 – On page 11.

NURBS is the model used in this paper to represent the surfaces, which, therefore, ensures C2 continuity. The joining area is constructed using a Gordon Surface, which also guarantees C2 continuity. In section 3 (step 5, on page 11), the formulation of the Gordon surface, with regard to the proposed model, has been added including these important references:

[29] Gordon W.J., (1969), Spline-Blended Surface Interpolation Through Curve Net-works. *Journal of Mathematics and Mechanics*. Volume 18, No. 10.

[30] Smid J., (1991), Triangular Gordon Surface with Unique Normal. *TR EDS-GM.SIAM Conference on Geometric Modeling*, *Tampe, AZ*.

Furthermore, in the same section, zebra plots have also been added in order to prove G2 continuity between new and original surfaces. This proves continuity between the three areas concerned. Figure 26 on page 13 shows those images. As a result, the following text has been added in section 3 (step 5, on page 11):

> "The joining surface is constructed by using a Gordon Surface, which was created using the guide curves previously described. The basic formulation used to create the afore-mentioned surface based on the main and guide curves can be observed in the (1-4) expressions."
>
> "As it can be seen in the zebra plot in figure 26, both the main and auxiliary curves obtained from the original lasts guarantee C2 continuity in the transition area between surfaces."

- A very detailed case-study is presented in Section 4 that describes the implementation of the technique for different shoe last styles. However, the differences are not always apparent in the figures. One recommendation is to reduce the number of unnecessary figures in the manuscript and focus on a more comprehensive discussion on the methodology.

Following the recommendation of the reviewer, non-representative figures have been deleted. In particular, figures: [27 - 29, 34, 36 the 2 figures on the left].

Reviewer #4:

1. A shoe last joining technique is proposed in this paper based on the surfaces divided into several functional areas and then blending these surfaces to complete the shoe model. According to the discussion, the proposed method provides great flexibility to accurately determine the functional areas to be preserved in the useful back and toe parts. Furthermore, G2 continuity is achieved across the whole generated surface and in joining areas with the surfaces of useful back and toe parts by using interpolated guide curves that join the back and the toe. I suggest to accept this paper as a full paper.

2. The algorithms shown in the paper are written in C language. I suggest to rewritten the algorithm in more generalized flow chart.

As the reviewer says, algorithms have been generalised in order to make them more readable, using a higher-level pseudocode that is less close to C.

A new surface joining technique for the design of shoe lasts

Francisco Javier Amorós-González[1], Antonio Jimeno-Morenilla[2*], Faustino Salas-Pérez[1]

[1] Footwear Technological Institute (INESCOP)
[2]Computer Technology and Computation department
University of Alicante

*Data of the corresponding author:

Postal address:
Campus de San Vicente del Raspeig s/n
03001 Alicante
Spain

Phone: +34 96 590 34 00 Ext. 2453
Fax: +34 96 590 96 43
email: jimeno@dtic.ua.es

Abstract

The footwear industry is a traditional craft sector, where technological advances are difficult to implement owing to the complexity of the processes being carried out, and the level of precision demanded by most of them. The shoe last joining operation is one clear example, where two halves from different lasts are put together, following a specifically traditional process, to create a new one. Existing surface joining techniques analysed in this paper are not well adapted to shoe last design and production processes, which makes their implementation in the industry difficult. This paper presents an alternative surface joining technique, inspired by the traditional work of lastmakers. This way, lastmakers will be able to easily adapt to the new tool and make the most out of their know-how. The technique is based on the use of curve networks that are created on the surfaces to be joined, instead of using discrete data. Finally, a series of joining tests are presented, in which real lasts were successfully joined using a commercial last design software. The method has shown to be valid, efficient and feasible within the sector.

Keywords. Shoe last design; blending and morphing joining techniques; curve network.

1 Introduction

The footwear industry has traditionally been a craft sector in which very few automatisms were used. However, necessary improvements have been gradually implemented in order to gain competitiveness by saving time and manufacturing costs, and enhancing the results. This sector is dominated by small and medium-sized companies, where most of the processes are carried out by hand. This is the case of a traditional sector, where automation has been implemented in very specific operations, mainly due to the fact that footwear manufacture involves numerous complex tasks that are difficult to automate. Furthermore, many of the craftworkers who participate in footwear production are considered as artists and, typically, changes in techniques or processes arising from the use of CAD/CAM techniques are rejected because they are thought to impose a possible limitation on their creativity.

In the footwear sector, the last is the mould around which whatever type of footwear is constructed, and it is characterised by its length, width and depth. There are different types of lasts, among which the most common ones are those used for trainers, shoes, boots, booties and sandals. Moreover they can also be classified according to their intended use (women’s, men’s or children’s footwear) or toe shape (pointed, square, round, etc.) (see figures 1 and 2). This paper focuses on the toe area since this is the most important part of the last from the aesthetic point of view and is determined by fashion.

Traditionally, the way of working for last joining consisted of making copies of the lasts to be joined, establishing the cut points on which the lasts seemed “more or less” to coincide in length and width, and then cutting them using a saw. The next step was to glue the parts to be joined and leaving them for 5 or 6 hours to dry and then work on the final last. Finally, in order to adjust the joint area and getting it as smooth as possible, the wooden last was manually sanded. The process was validated by trial and error, in such a way that the lastmaker, according to their experience, could determine when the joint was accurate at sight and touch.

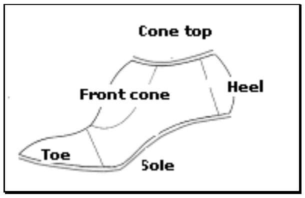


Figure 1: Main parts of the last.

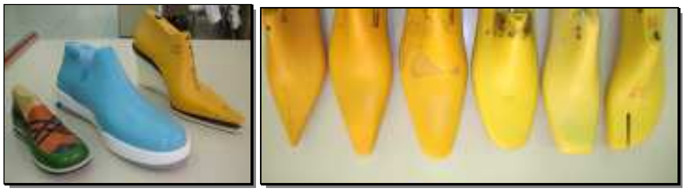


Figure 2: Different types of last according to their intended use and toe shape.

With regard to the materials used for their manufacture (see figure 3), wood was traditionally used due to its easy handling and flexibility, both in the prototyping and production process. However, wood is currently used in the prototyping process and plastic is used for production since it is more resistant. There is a third material, aluminium, which is quite unusual and is only used for certain operations in which the last can suffer and get damaged.

(a) (b) (c)

Figure 3: Different materials used for last manufacture: (a) wood, (b) plastic, (c) aluminium

The volume of a shoe last should resemble that of a natural object such as a human foot. It might be said that, there are two very important aspects of completely different nature that should be considered with regard to this: On the one hand, the object should be modelled using free surfaces, due to the fact that it derives from a natural form. On the other hand, the shoe will acquire the shape of the last with which it is made. Therefore, small errors in this component would detrimentally affect the foot that wears the shoe, in the form of grazes and general discomfort. Hence - as industrially produced objects - lasts require a high level of precision in design and manufacture. For these reasons, CAD/CAM software aimed at last design and manufacture, use free surfaces with a high level of precision as geometric models, which requires techniques used for their manipulation to be precise, thus avoiding approximate models.

Therefore, the motivation behind this study addresses the improvement of the joining of shoemaking last surfaces, which will result in time, quality and production cost improvements and will enhance the accuracy and possibilities for determining the cutting areas, all of this thanks to the use of computing-based solutions. This has been approached from the point of view of traditional lastmakers; with the aim of demonstrating that automation can positively affect the design process, without thereby bringing about rejection in the sector.

This paper addresses a specific issue concerning the joining of shoe lasts. However, a lot of work has already been done relating to other aspects of the design and production of this industrially produced object. Nowadays, different authors are researching on various aspects related to shoe lasts. The work of Ameersing and Yan Luximon [1] presents a new platform developed to gain comfort and obtain aesthetically comfortable shoes. This shoe last model is based on foot shape measurement data and foot biomechanics. The application of different CAD technologies in the sector is incipient, which is demonstrated by the progress shown in [2]. “Fitting” has been the most studied and pursued concept, always relating to comfort provided by the last and the materials the final shoe is made of. Concerning this idea, Franciosa et al. [3] present a study about the importance of pressures exerted on the foot sole in order to obtain good fitting and comfort results. In a parametric environment, the influence of geometry and materials in comfort is studied via a combination of real tests and CAD-FEM simulations. Another paper relating to the study of feet in order to obtain suitable last designs can be found in [4], where human feet were scanned, using reverse engineering based on Fuzzy theory. In addition, rapid prototyping techniques are widely used in several industrial sectors. The research presented in [5] proposes a CAD system for shoe-last rapid customized design based on the piecewise reconstruction to carry out the interactive deformation and separate/global shoe-last form reuse.

This paper is structured as follows: section 2 presents an overview of the different existing techniques allowing the joining or transition between three-dimensional surfaces. Furthermore, those processes specifically focused on shoe last joining are analysed. In section 3, a new methodology is presented that is inspired by the manual process, dealing with the accurate joining of lasts. In section 4, experiments conducted on real lasts to prove the validity of the new method are presented. Finally, section 5 outlines the main conclusions of this paper.

2 A review of joining techniques

Literature proves that plenty of research has been done regarding the geometric joining of surfaces. This section presents an overview of those techniques that are most widely used to join NURBS surfaces, such as those used in shoe last joining. Nevertheless, other approximations which are not based on free surfaces but on discrete representations are also given aiming at solving the problem of last joining. Most of the solutions to surface joining or transitions are related to the “blending” and “morphing” concepts.

Blending techniques

Blending is defined as the smooth joining of two surfaces that gives place to a third surface that presents certain continuity conditions. There are different types of blendings, among which are: Implicit, Parametric, Approximate and Exact blends, and in all of them, the continuity concept is closely linked to the validity of the blending process result, therefore information about continuity types can be found in [6], specifically about geometric, morphological and intentional continuity. Regarding this investigation, the geometric continuity is the most important of them, especially: G0 (position), G1 (tangency), and G2 (curvature). Belkhatir et al [7] provided information about G1 and G2 continuity in the blending of surfaces and curves, so they talked about the need of using G2 continuity and why G1 continuity caused surfaces or curves geometrically faulty, thus proposing a blending method that used Bézier curves and patches.

Several techniques have been developed to produce blending surfaces. Hoffman and Hopcroft [8] presented a method for blending implicit surfaces, which was guaranteed to produce blending surfaces of lowest possible degree. For this, they gave two paradigms: the first one view the joined surfaces as surfaces swept out by space curves; the second, more general paradigm, considered the surfaces as a result of deformation of a parameter space by substitution. This second paradigm is extended to blend blending surfaces at solid vertices without a degree penalty, under the assumption that the vertex valence has been reduced to three. It may also lead to a general solution for blending patches of algebraic surfaces that meet tangentially. This method works perfectly with quadrics surfaces, due to their low degree, but is more complicated with higher degree surfaces.

Wallner and Pottmann [9] presented a method that constructed exact G1 rational blends between quadrics, which are frequently used in solid modelling systems. For this purpose we use quadratic projections which have their origin in the theory of kinematic mappings [10] to define intrinsic control structures for NURBS curves [11] and surfaces on quadrics. This method has problems when dealing with the case of closed intersection curves and trimlines to manage the surface, because the curves do not have to be closed for their projections and to be of the same differentiability class. For the transition surfaces, however, the situation is different, as C1- or G1-boundary conditions for NURBS surfaces are rather complicated, it is desirable to have something closed in IR4 to project.

Recent works introduce an interactive surface interpolation method by spline surfaces that is not aimed at joining but is useful in controlling the blending surface as a result of joining [12]. The technique is based on linear blending and works for a large type of surfaces including bicubic Bézier, B-spline, NURBS surfaces and trigonometric surfaces. Shi et al. propose a method of G^n blending multiple parametric surfaces in polar coordinates [13]. It models the geometric continuity conditions of parametric surfaces in polar coordinates and presents a mechanism of converting a Cartesian parametric surface into its polar coordinate form. Performance is an important issue and it is analyzed in [14]. Here render experiments were carried out with trimmed blending surfaces on a GPU and results show an eighteen-fold to twenty-fold increase in rendering speed over a CPU version. Finally in [15], a study of geometric continuity

C1G2 of blending surfaces by a shape-blending process was developed. This paper studies the continuity of the ruled surfaces constructed by linear interpolation between two pairs of C1G2 continuous curves.

Morphing techniques

With regard to *morphing*, it is defined as the gradual and smooth deformation process suffered by an initial object to result in a final object. In most cases, it is applied to 2D images, but there is a growing increase in methods to study morphing on 3D objects. The most common application domains are industrial design [16], geometric modelling, medicine [17], visual effects and animation [18].

Castro and Ugail [19] showed a classification of morphing techniques based into two main groups that are essentially distinguished by the kind of approach employed in their development, as ‘volume-based’ and ‘boundary-based’ approaches. The first kind regarded the entire surface representing the object as a set of specific control points that can be modified. This technique provides excellent results when applied to objects represented by implicit surfaces, producing smooth transitions and keeping volume unaltered and offering fine-grain edition of the surface. The second approach was based on the modification of specific values of the boundaries describing the object. However, it was noticed that a small variation of the data describing the boundary may result in an invalid object, disrupting the smoothness of the sequence.

The studies in [18, 20, 21] provided more information about the application of morphing techniques, for instance, Turk and Obrien [20] introduced a method that allowed two objects to be morphed using variable interpolation, while Takahashi et al [21] proposed an alternative to morph two topologically non-equivalent objects creating a fourth dimension by means of a transition mesh. It is also possible to find in [18] a mapping mechanism between topologically similar surfaces, which by means of cross-parameterization and compatible remeshing processes manage to create a bijective correspondence between models thanks to the use of patches with identical connectivity. As regards to the main problems posed by morphing, in [22] is presented a study of the most common ones related to specification, deformation generation and transition control.

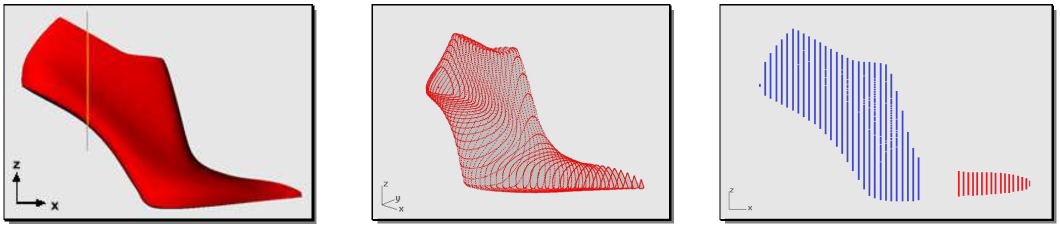
Zhang et al. [23] presented a multiresolution-based technique that can be used to create different levels of detail, thus preserving features in areas with large deformation. Afterwards, the same author developed a new technique based on Laplacian meshes [24], which combines traditional concepts such as Laplacian editing with skeleton-based and as-rigid-as-possible (ARAP) techniques. Li et al. [25] present a sketch-based technique that uses a skeleton to be deformed, and then this deformation is transferred to the entire geometry. This technique is frequently used for deformation of organic objects.

Recently, space-based morphing techniques have been appearing. Huang et al. proposed a Free Form Deformation (FFD) technique applied to complex objects, using a coarse control mesh and a series of deformation iterations based on Barycentric coordinates [26]. Obtained results are good, but could be improved. Finally [27] provides a cage-based technique based on the combination of cage deformations and Plaplacian meshes.

In short, once reviewed the main techniques of blending and morphing, the following conclusions can be drawn: blending methods provide continuity between two surfaces, but they lack precise control in the area of smoothing, so its use to produce industrial products such as shoe lasts is very limited. In the other hand, morphing techniques are defined to produce changes in whole objects but not in object parts; therefore, it is difficult to apply them directly for the replacement of parts.

Ad-hoc joining techniques for shoe lasts

It has to be considered that for a surface joining technique to succeed in an industrial sector, such as the footwear sector with prevalence of traditional and manual labour, automatic techniques to be used shall not cause rejection among users. In other words, a lastmaker has not CAD/CAM expertise or advance knowledge on 3D geometry. Consequently, even though the above-mentioned techniques are efficient and precise, they would be rarely used and with little success. Nevertheless, experienced lastmakers gained a lot of know-how throughout years of professional experience. Expertise passed from one generation to the next allows them to successfully join two apparently very different shoe lasts. For this reason, a search of the techniques applied to the footwear sector has been carried out without very successful results. More specifically, only a paper has been found directly dealing this issue. The reason for this must be the non-disclosure agreements between companies and the lack of automation in this sector. This paper was written by Li Guo and Ajay Joneja in 2005 [28], where they proposed a solution based on the interpolation of morphing and blending functions and using energy minimization in order to automatically obtain a smooth filling surface in a controlled way. The method inputs were two lasts as point clouds structured in sections having the same amount of points and being equidistant, one for the toe part and one for the back of the lasts to be joined, which were obtained from the intersection of both surfaces with an YZ plane (see figure 4).



(a) (b) (c)

Figure 4: Inputs. (a) Surface with cutting plane to obtain sections. (b) Point clouds structured in sections. (c) Valid sections from lasts to join.

To this initial situation blending and morphing techniques were separately applied and the results were evaluated. In the case of blending, a G2 blend surface was obtained, but in some cases the quality of the resulting surface was undesirable, since it tended to loose the shape characteristics of the surfaces being connected and it failed to give sufficient control. On the other hand, if a purely morphing approach was applied, the final filling surface exhibited poor continuity in the boundaries, where changes could be observed in respect of the initial parts of the joint.

Faced with the problems that blending and morphing posed separately, the basic idea developed in [14] consisted of filling the gap between the two parts by creating intermediate morphed discrete slices at a constant distance and extended slices to then create valid blended slices for the gap. For this, the neighbourhood was taken into account and the G1 and G2 continuity conditions were respected thanks to energy minimisation functions that avoided discontinuity by means of weights distribution.

That paper offered some general lines for a solution and concluded that more efficient computation could be achieved by refining the energy minimisation approach. However, it yielded useful outputs in real applications and it was hoped that further works on these techniques would solve this problem efficiently and result in a useful tool that could be applied to similar problems in other domains.

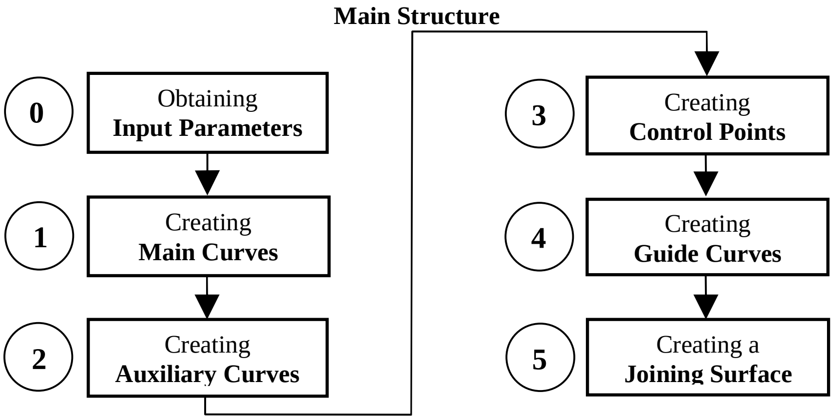
Concerning the limitations of this method, it can be improved as regards inputs data and accuracy and editability of result. With reference to inputs, the method starts from a set of slices contained in the YZ plane, which must be parallel to each other, contain an equal number of points per slice and be similarly aligned. Besides, the boundaries of the parts to be merged must be complete slices, i.e. it is not possible to establish a joining area by means of cutting planes or surfaces, and the area is delimited from a certain slice in the back part to a certain slice in the toe part. Finally, due to the fact that the slice structure cannot be broken, the method cannot be extended for use as a joining method for the leg part, which is also a quite usual operation. As regards to accuracy, there is a significant limitation due to the fact that the density of points that define the slices will determine the quality of the join, especially in maximum curvature areas, in that this method always works with discrete data. Regarding the editability, the method offer a poor mechanism to adjust the solution because the only way to change it is edit point by point the new sections generated, and if the quality is high, this work can be almost impossible.

However, there is also a positive aspect to be highlighted: in order to generate a filling surface that guarantees G2 continuity at both ends, the method uses previous and next extended slices to the boundary slices to help establishing the smoothest possible curvature, which manages solving the problem posed by merely applying a morphing operation. However, this method is based on discretized surfaces so its flexibility and ease of design is very low for an industrial use.

This paper proposes a joining technique based on the use of surfaces and continuous curves that, on the one hand, allows continuity and editability of models used in the joining, and on the other, is inspired by the traditional process of manual joining of surfaces. This way, its use is intuitive for traditional lastmakers, who can make the best out of their “know-how”.

3 Methodology

In this section the method to join two lasts in the toe part is described. This new method works with surfaces and curves to avoid the limitations derived from working with discrete data as the other hybrid techniques described in previous section. Next scheme shows the six stages of the algorithm:



Step 0: Obtaining Input Parameters

In this stage the two different lasts to be joined are properly positioned, and a set of parameters set up to define the join zone accurately.

- **Original lasts:** Selection of two lasts that are adequately positioned and orientated, i.e. they must be superimposed in a way that their maximum ball width and transverse axis coincide as much as possible, so that transitions are smooth and help achieving G2 continuity in the joining area (see figures 5 and 6).

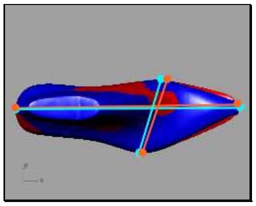


Figure 5: Lasts correctly positioned and orientated.

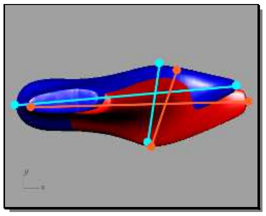


Figure 6: Lasts wrongly orientated.

- **3D cutting surfaces:** Two surfaces that cut the original lasts and delimit the joining area to be generated (see figure 7).

Figure 7: Cutting surfaces delimiting the joining area.

- **Number of auxiliary sections for the back part and the toe part:** This corresponds to the number of auxiliary curves to be created in step 2 to help achieving G2 continuity between the resulting joining surface and the parts connected thereby.
- **Number of points per section:** Number of points per section used to reconstruct the curves resulting from the intersections between the last surfaces and the cutting surfaces.

- **Rotation angle for circular sweep:** This refers to the rotation angle to be used in step 3 in order to create a series of planes that will intersect each of the sections to obtain the points that will be used in step 4 for the creation of interpolation guide curves necessary for the final network of curves.

Step 1: Creating Main Curves

Given the two original lasts and the two cutting surfaces, 2 curves are obtained, which result from the intersection of each last with the corresponding surface. These curves will define the start and end of the filling area for the creation of the joining surface (see figure 8).

Once the curves are obtained, they are reconstructed with the number of points set by the user; this way, new curves are generated, which replace the original ones with a certain amount of interpolation points (see algorithm 1). Reconstructing the curves makes them both have an equal number of points and be similarly aligned, which is essential when it comes to obtaining certain control points for guide curves in step 3.

```
Input
    B:  Back part, T: toe part
    $CS_B$: Cut surface over B $CS_T$: Cut surface over T
    NP: Number of points to normalize

Output $MC_B$: Main curve over B $MC_T$: Main curve over T

procedure CreateMainCurves
Start
    $MC_B$ ← INTERSECT( B, $CS_B$ ); $MC_B$ ← NORMALIZE( $C_B$, NP )
    $MC_T$ ← INTERSECT( T, $CS_T$ ); $MC_T$ ← NORMALIZE( $C_B$, NP )
    return $MC_B$, $MC_T$
End
```

Algorithm 1: Method for creating the main curves.

From this double division 4 surfaces are created, two of which will be rejected and the useful parts to be joined will be preserved. The two useful surfaces are those that, together with the intermediate filling surface, will make up the method’s solution.

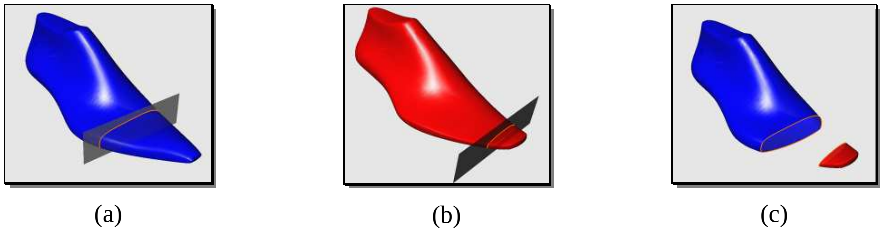


Figure 8: (a and b) Main curves over lasts. (c) Useful parts to be joined.

Step 2: Creating Auxiliary Curves

In this step, a set of curves that are parallel to the 2 main curves is created. The amount of auxiliary curves is set by the user and it can differ in the toe part and back part. The creation logic consists of progressively creating cutting surfaces parallel to the main curves at ±5 mm and intersecting the lasts to create new curves parallel to the main one (see figure 9). The 5mm distance between the valid parts to be joined is used in order to ensure a distance which allows obtaining information about the area of the last where continuity and smoothness between the valid parts and the new joining surface is needed. This information is useful to obtain G2 continuity between the joining parts. Furthermore, the fact that it is a constant and homogeneous separation makes information intervals smooth and continuous.

Similarly to main curves, these curves are reconstructed with the same number of points and so they are similarly aligned. The possibility of indicating different amounts for the back part and the toe part allows deciding to which side is the resulting surface to be balanced (see algorithm 2). This will determine the nature of the guide curves with which the final filling surface will be created. The usefulness of these curves will be proven in steps 3 and 4.

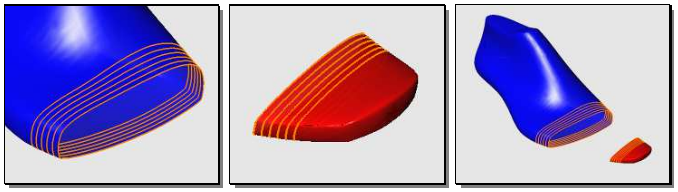


Figure 9: Auxiliary sections over useful parts.

```
Input
 B: back part, T: toe part, $CS_B$: Cut surface for B,$CS_T$: Cut surface for T
 $N_B$: Number of auxiliary curves for B $N_T$: Number of auxiliary curves for T
 NP: Number of points to normalize

Output $ACG_B$: Auxiliary curves for B, $ACG_T$: Auxiliary curves for T

procedure CreateAuxiliaryCurves
Start
    repeat
       $CS_B$ ← MOVE( $CS_B$, step * i )
       auxCurve ← INTERSECT( B, $CS_B$ ); auxCurve ← NORMALIZE (auxCurve, NP )
       $ACG_B$.ADD( auxCurve )
    until i > $N_B$

    repeat
       $CS_T$ ← MOVE( $CS_T$, -step * i )
       auxCurve ← INTERSECT( T, $CS_T$ ); auxCurve ← NORMALIZE (auxCurve, NP )
       $ACG_T$.ADD( auxCurve )
    until i > $N_T$

    return $ACG_B$, $ACG_T$
End
```

Algorithm 2: Method for creating the auxiliary curves.

Step 3: Creating Control Points

A transformation is applied to the obtained set of curves for obtaining parametric points to be used in step 4 for the creation of interpolation guide curves. This process will guarantee the correct connection between the back and toe parts. Using the rotation angle, a bundle of planes is created for each section passing through its centroid, thus obtaining point pairs resulting from the intersection with a given section. The circular sweep with the bundle of planes is obtained from a reference plane, which is created from a point and a normal, the point being the centroid of the section and the normal (0,0,1) (see figure 10).

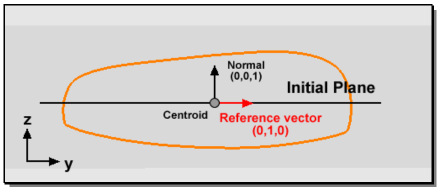


Figure 10: Initial state of the process to obtain points.

As algorithm 3 shows, the bundle of planes goes from 0º of the reference vector (0, 1, 0) to 180º, and the rotation angle determines how much the plane is to be rotated in each iteration. The rotation vector used to rotate the plane is the normal of the plane in which the section curve is contained. If the cutting surface is not a plane, then the vector (1, 0, 0) is used.

The resulting number of control points for each of the sections will depend on the rotation angle. The larger the rotation angle, the smaller the number of points and the lower the computation time and vice-versa. Applying this operation to each one of the sections with a 10º rotation angle, the control points showed in figures 11 and 12 are obtained.

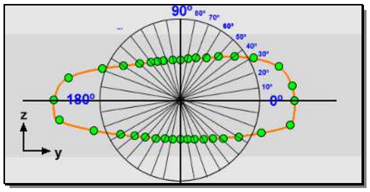


Figure 11: Example of the whole process with a 10º rotation angle.

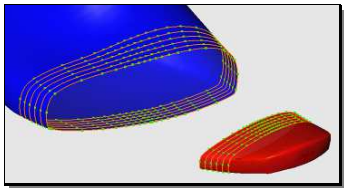


Figure 12: Control points over curves in useful parts.

```
This procedure is executed twice, one for back valid part over the first
last and two for toe valid part over the second last to join.

Input
    N: Number of auxiliary curves, CG: Group of auxiliary curves
    $\alpha$: Main angle to establish the step to rotate the intersect plane.

Output CPG: Group of N points groups with points over $C_{i=0...N}$.

procedure CreateControlPoints
Start
    repeat
        plane ← Plane( 0, 0, 1 ); $\beta$ ← $\alpha$;
        repeat
            pts ← INTERSECT( $CG_i$, plane );
            $PG_i$.ADD( pts );
            Plane ← ROTATE( plane, $\alpha$ );
            $\beta$ ← $\beta + \alpha$;
        until $\beta < 180$
        CPG.ADD( $PG_i$ );
    until i < N

    return CPG
End
```

Algorithm 3: Function for creating the control points.

Depending on the distribution of points, the future guide curves from next step are going to create the homogeneous and smooth modelling of the filling area in one way or another. G2 continuity obtained at the end of the process is not only the same NURBS filling surface, but also the joining of valid parts thanks to the obtaining of interpolation points on the valid parts. Due to the importance of this parameter, a study regarding obtained points' quantity influence on the quality-speed relation of the entire process has been included in section 4.

Step 4: Creating Guide Curves

In this step, a set of interpolation curves (see figure 13) that join the useful back and toe surfaces is created.

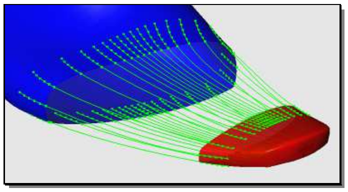


Figure 13: Interpolation guide curves that connect the useful parts.

The key issue of these curves is that they do not start exactly from the edge of the surfaces from which the filling area starts, but thanks to the auxiliary sections, certain points on the useful surfaces are used, which allow the curvature and continuity of these areas to be kept.

At this point, a slight anomaly was observed that showed a poor performance in creating guide curves (see figure 14 and 15). This happened sometimes, depending on input lasts, the position of cutting surfaces and, the rotation angles for obtaining control points. This was due to the fact that the main sections of the back and toe parts could differ a lot, so there was no adequate correspondence between the control point distribution in both parts and, therefore, the resulting curves in the sole area were twisted.

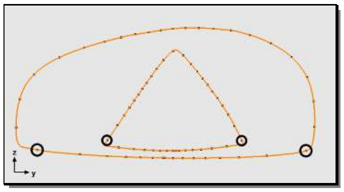


Figure 14: Sole points in main sections.

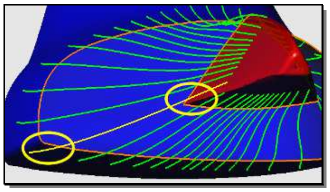


Figure 15: Guide curve with side torsion.

In order to solve this situation, a new complementary criterion for the definition of control points was needed. At first, the solution was focused to find in each section the maximum curvature points at the sides in order to generate new guide curves right on the sole area, but some reliability problems arose as a result of common digitising errors in the surfaces to be joined (see figure 16, 17, and 18).

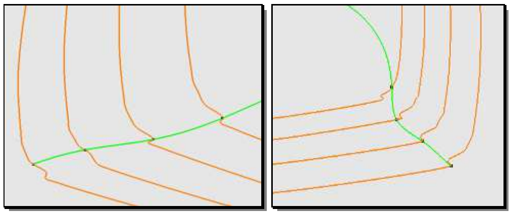


Figure 16: Faulty points on the sole due to section faults**.**

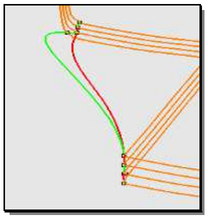


Figure 17: Expected guide curve (red) and obtained guide curve (green).

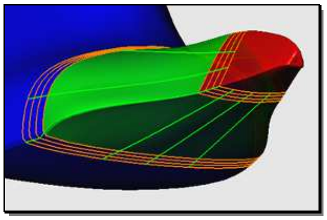


Figure 18: Joining surface obtained using faulty side guide curves.

The filling surface often suffered significant malformations in the sole area; therefore, the method looked for another alternative. Instead of generating a curve at each side right on the sole line from the points found in every section, two curves at each side was generated, one 5% over and the other one 5% below the sole point found on the main section and on the auxiliary sections at each side (see figure 19).

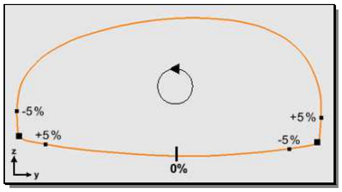
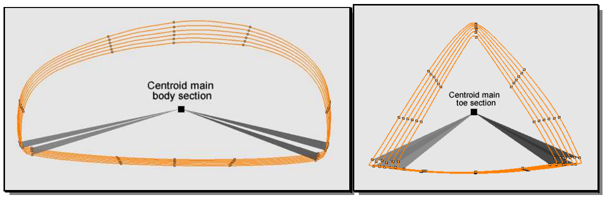


Figure 19: Control points around the sole points that are initially obtained.

A new complementary system was established for obtaining control points based on the intersection of a certain plane with the side area (see figure 20). This plane was defined by 3 points:

- The sole point found on the main section.
- The sole point found on the last auxiliary section.
- The centroid of the main section.

By creating 4 of these planes for the 4 concerned areas – left side and right side of the back part, and left side and right side of the toe part – and intersecting them with the adequate set of sections in the suitable area, new control points were obtained for the 4 new interpolation guide curves (see figure 21).



(a) (b)

Figure 20: Planes used for the intersection, and points obtained. (a) Body part. (b) Toe part.

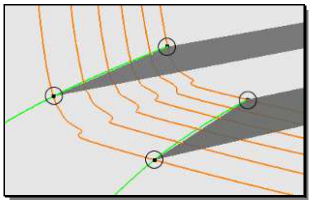


Figure 21: Side guide curves.

```
Input
 $N_B$: Number of auxiliary curves over B $N_T$: Number of auxiliary curves over T
 $CG_B$: Group of auxiliary curves over B $CG_T$: Group of auxiliary curves over T

Output SGCG: Group of 4 sole guide curves, 2 left side and 2 right side

procedure CreateSoleGuideCurves
Start
    $pts_0$    ← FIND_CORNERS_POINTS( $CG_{B\ 0}$ )
    $pts_{N-1}$  ← FIND_CORNERS_POINTS( $CG_{B\ NB\ -\ 1}$ )
    centroid      ← $CG_{B\ 0}$.CENTROID( )
    LeftUpPlane    ← CREATE_PLANE( $pts_{0\ 0}$, $pts_{N-1\ 0}$, centroid )
    LeftDownPlane  ← CREATE_PLANE( $pts_{0\ 1}$, $pts_{N-1\ 1}$, centroid )
    RightUpPlane   ← CREATE_PLANE( $pts_{0\ 2}$, $pts_{N-1\ 2}$, centroid )
    RightDownPlane ← CREATE_PLANE( $pts_{0\ 3}$, $pts_{N-1\ 3}$, centroid )

    repeat
        $PointsLeftUp_B$.ADD( INTERSECT( $CG_{B\ i}$, LeftUpPlane ) )
        $PointsLeftDown_B$.ADD( INTERSECT( $CG_{B\ i}$, LeftDownPlane ) )
        $PointsRightUp_B$.ADD( INTERSECT( $CG_{B\ i}$, RightUpPlane ) )
        $PointsRightDown_B$.ADD(INTERSECT( $CG_{B\ i}$, RightDownPlane ) )
    until i < $N_B$

    For Toe valid part is the same. When all points are available over back
    and toe valid parts, the procedure creates sole curves joining the
    interpolation points:

    SGCG.ADD( CREATE_GUIDE_CURVE( $PointsLeftUp_B$,    $PointsLeftUp_T$ ) )
    SGCG.ADD( CREATE_GUIDE_CURVE( $PointsLeftDown_B$,  $PointsLeftDown_T$ ) )
    SGCG.ADD( CREATE_GUIDE_CURVE( $PointsRightUp_B$,   $PointsRightUp_T$ ) )
    SGCG.ADD( CREATE_GUIDE_CURVE( $PointsRightDown_B$, $PointsRightDown_T$ ) )

    return SGCG
End
```

Algorithm 4: Method for creating the guide curves for the sole zone.

Using this new technique, the method prevented the side area from rotating, and the system is not so restrictive with such a sensitive area (see figure 22). The pseudocode of the function can be observed in algorithm 4.

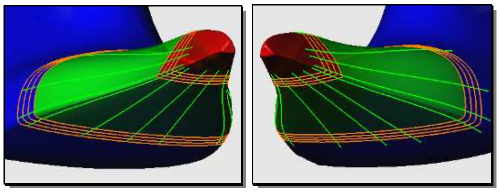


Figure 22: Surface without side torsion thanks to the 4 guide curves in the sole area.

Step 5: Creating a Joining Surface

This is the last step for the creation of a joining surface by means of a network of curves, using main curves and guide curves. Guide curves present a constraint in that they cannot intersect each other and must be in contact with the 2 main curves. Main curves can neither intersect each other.

The joining surface is constructed by using a Gordon Surface [29], which was created using the guide curves previously described. The basic formulation used to create the afore-mentioned surface based on the main and guide curves can be observed in (1-4) expressions, as a direct mapping of the Gordon surface [30].

$$G(s,t) = MC + CC + corrections \tag{1}$$

$$MC = \sum_{j=1}^{m+2} MC_j(s) * \psi_j(t) \tag{2}$$

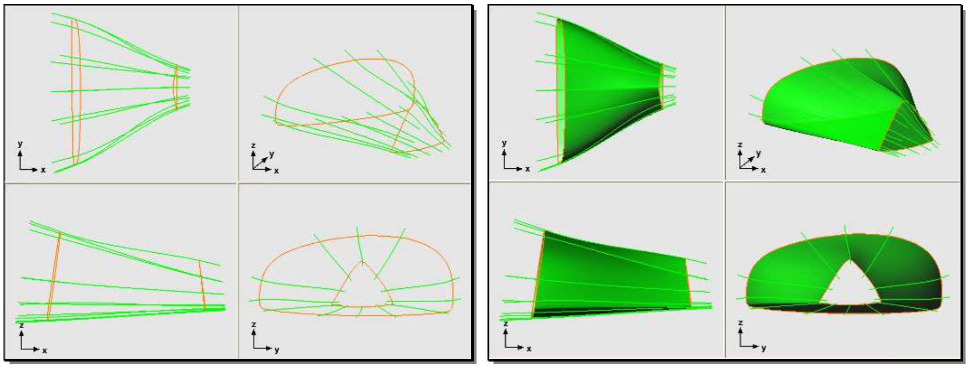
$$CC = \sum_{i=1}^{n+2} CC_i(t) * \phi_i(s) \tag{3}$$

$$corrections = -\sum_{i=1}^{n+2} \sum_{j=1}^{m+2} G_{ij} * \phi_i(s) * \psi_j(t) \tag{4}$$

from a rectangle $[s_1, s_n]x[t_1, t_m]$ in the (s,t) plane to $\Re^3$. We can assume that $s_1 = 0, s_n = 1, t_1 = 0, t_n = 1$.

A network of intersecting curves MC_j and CC_i in $\Re^3$ homeomorphic to a planar rectangular grid is given, where the MC group represents the 2 main curves over the valid parts, and CC group represents the cross curves that interconnect those valid parts. Each curve is parametrized on the interval $[0,1]$. Here, $\{CC_i\}_{i=1}^{n}$ and $\{MC_j\}_{j=1}^{m}$ are given 3-dimensional vector functions defining curves in $\Re^3$, while $\{CC_i\}_{i=n+1}^{n+2}$ and $\{MC_j\}_{j=n+1}^{m+2}$ are cross-boundary derivatives. The fixed vectors S_{ij} represent either the curve intersections or the derivatives at the boundary grid points. Scalar-valued blending functions are expressed in (5). An example of this network of curves and the surface generated is showed in figure 23.

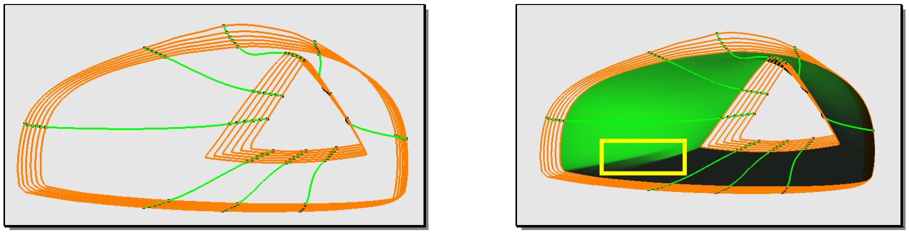
$$\psi_j(t), \phi_i(s) \qquad (i = 1,2,\ldots,n+2;\ j = 1,2,\ldots,m+2) \tag{5}$$



(a) (b)

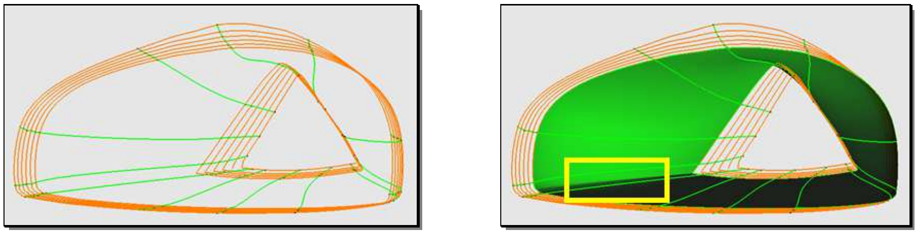
Figure 23: (a) Surface skeleton, main curves (orange) and guide curves (green). Join surface generated.

In the previous step, the need to obtain curves in the sole area so as to prevent guide curves from being distorted was highlighted, however, these 4 curves were necessary even if no torsion was observed in basic guide curves, since sometimes the resulting surface was distorted in spite of the fact that curves were fair (see figure 24).



(a) (b)

Figure 24: Torsion problem: (a) Surface skeleton. (b) Surface generated without side guide curves.



(a) (b)

Figure 25: Torsion problem fixed: (a) Surface skeleton. (b) Surface generated with side guide curves.

Thanks to the new guide curves in the sole, it can be observed that the resulting surface is more similar to a fair and torsion-free surface, since this forces a closer correspondence between the back and toe parts in such sensitive areas as the last feather line (see figure 25).

Zebra tests

As it can be seen in the zebra plot in figure 26, both the main and auxiliary curves obtained from the original lasts guarantee C2 continuity in the transition area between surfaces.

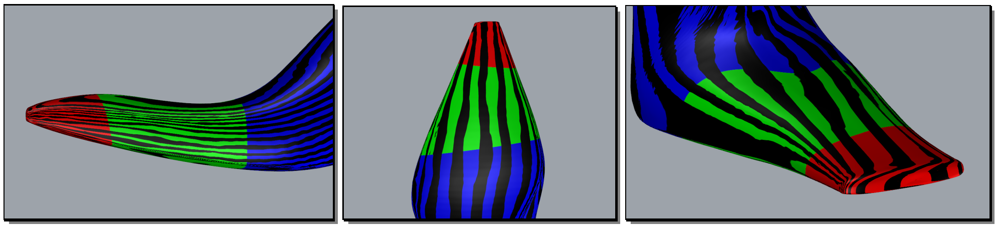


Figure 26: Zebra plots for three different joining examples.

4 Experiments

In this section, various types of experiments were prepared to evaluate and demonstrate the validity and possibilities that the new method can offer. There are 4 types of experiments: body-toe tests, setup tests, performance tests and generic tests.

Body-toe tests

The purpose of this study is to solve the joining of shoemaking lasts on replacing toes. Figures 27, 28, 29 ,and 30 show some pictures of the tests carried out with different last styles, different toe shapes and different positions of the cutting surfaces. Expert lastmakers took part in the execution of the tests, and they not only used the CAD tool with little difficulty, but they were able to successfully join models that seemed to be very different. This is due to the fact that the automated process greatly draws influence from the manual process. For example, the selection of the joining planes derives from the areas where professionals physically cut the models, which ultimately has been considered as a big advantage for them. This means that, this way, designers can benefit from their “know-how” gained from years of professional experience. In addition, this makes the implementation of CAD/CAM tools in this type of industries - frequently reluctant to new technologies - easier.

Analysing the advantages that the technique presents, it is worth mentioning that the complete manual process takes an expert lastmaker about 5 to 6 hours of work. Thanks to the use of the technology, this can be now performed in 10 to 15 minutes. Additionally, considering that only a short learning curve is required for a person that is used to working with the computer, various lastmakers were able to be taught how to use the tool in a training course that took about 20 hours.

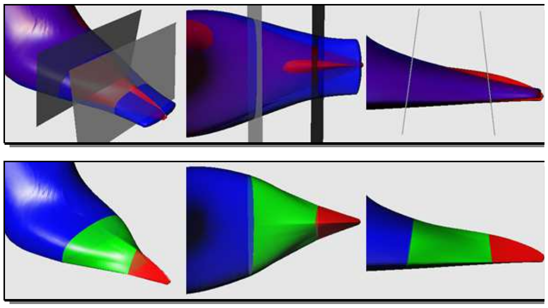


Figure 27: Women’s last transformation from square toe into pointed toe.

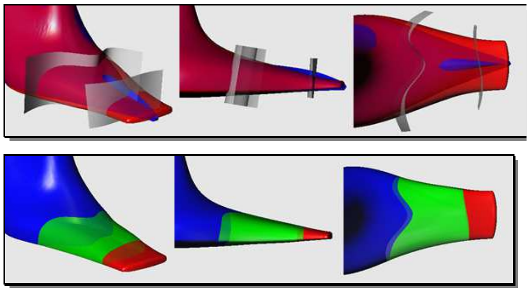


Figure 28: Women’s last transformation from pointed toe into square toe.

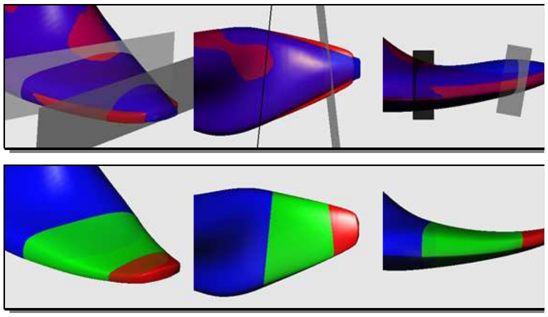


Figure 29: Men’s last transformation from pointed toe into round toe.

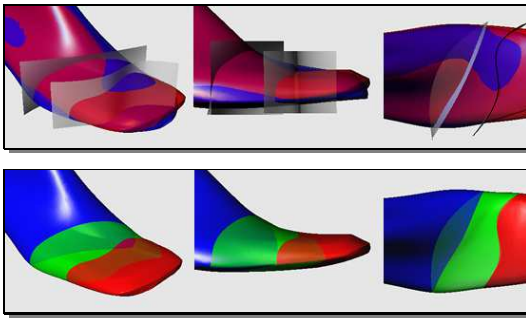


Figure 30: Booty last with increased toe width.

Setup tests

Each one of the input parameters are analysed below to see how they affect the outcome.

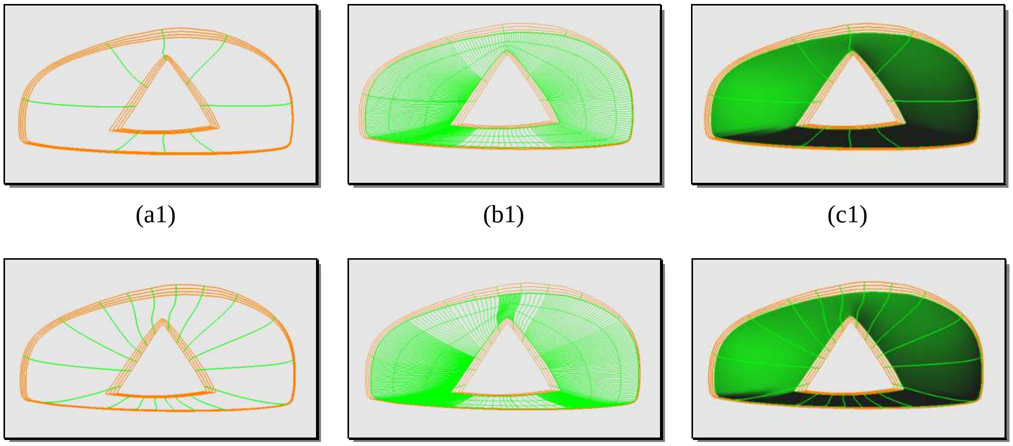
Rotation Angle

Depending on the rotation angle, more or less control points will be obtained on the sections, which will provide more or less guide curves between the back and toe parts. This relationship is inversely proportional, i.e. the larger the rotation angle, the smaller the number of control points and the smaller the number of guide curves, and vice versa (see figure 31). The examples show that with a high number of curves the generated surface is quite complex and somehow forced, while a larger angle provides a smaller number of guide curves, so the network of curves has more freedom and also provides valid results more efficiently.

Having more or less control curves directly affects the simplicity and accuracy of the surface, in that editing the obtained solution with less curves would be more simple but less accurate in turn. The user should find the balance between speed and accuracy.

Points per Section

This parameter affects the search of sole points and the continuity of the surface generated from the original back and toe parts. It is used to reconstruct the sections with a certain value, so the higher the number of points, the higher the total computation time. Figure 32 shows a last with a faulty side. Depending on the number of points used to reconstruct the sections, the search of sole points will be finer or coarser and the outcome will be different.



(a1) (b1) (c1)

(a2) (b2) (c2)

Figure 31: Guide curves using: (1) angle 45º. (2) Angle 30 º. (a) Skeleton. (b) Internal structure. (c) Solid surface.

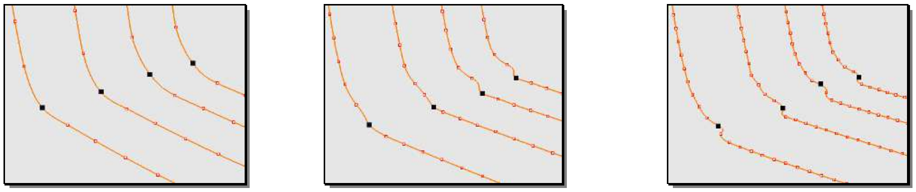

(a) (b) (c)

Figure 32: Sole points found on reconstructed sections with (a) 100, (b) 200 and (c) 500 points respectively.

As can be observed, the higher the number of points per section, the greater the control on the search area, and so the reconstructed surfaces will resemble more the original surface. Nevertheless, this high definition can pose a problem with lasts having faulty sides, in that it can lead to finding false sole points.

Furthermore, the pictures show how this parameter affects the smoothness of the joint between the original parts and the newly generated filling surface. The need for a higher number of points per section will be determined by the area on which the cuts with the main surfaces are applied. If the base lasts do not have strange irregularities in the toe area to work with, the reconstruction values can be low; however, if the area presents some irregularities, a higher value will be required in order to model the curves with more points and then create the smoothest and most accurate possible network of curves (see figures 33 and 34).

As shown in figures 33 and 34, the blue area in the back part shows some irregularities and 200 points per section seem to be insufficient to be able to model this section accurately. However, the other example shows that the sections reconstructed with 500 points per section provide better modelling of the area and the green filling surface is able to propagate the back part morphology and increase the smoothness. The toe part is smooth enough with 200 points per section. Therefore, the higher number of points per section, the fairer the filling surface.

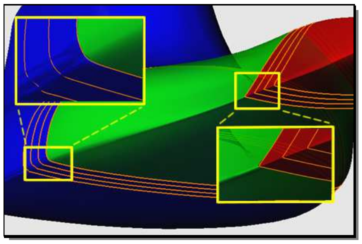


Figure 33: Surface generated by reconstructing the curves with 200 points. Problems with continuity.

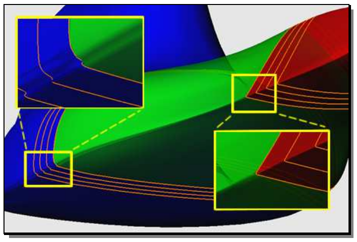


Figure 34: Surface generated by reconstructing the curves with 500 points. Correct continuity.

In short, there is some disagreement with regard to the most suitable value to be used for the correct application of this method. For this reason, different values are used, i.e. a low value is used to search the sole points and to smooth the curves and avoid digitising errors, while a higher value is used for the definitive reconstruction of curves that will result in the final network of curves to achieve the solution. It is up to the user to establish the suitable values in each case according to the lasts and areas to be joined.

Performance Tests

Once the input parameters had been independently analysed, a performance test on the whole method was carried out using the input parameters set at certain values for a specific toe joining example (see figure 35)

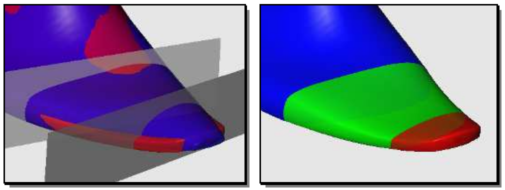
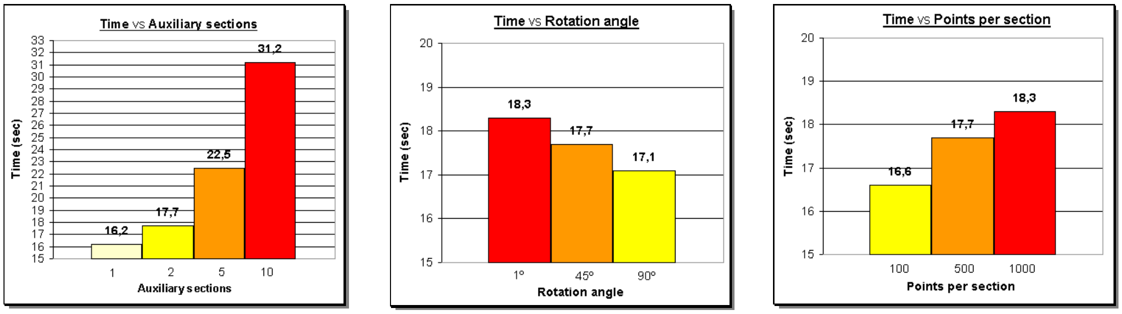

(a) (b)

Figure 35: Joining example used in performance tests on men's lasts: (a) Initial situation. (b) Final join result.

Performance tests were conducted on a computer with a processor Intel Core 2 Duo E7300 @ 2.66 GHz, 3 GB RAM, and a graphic card: ASUS EAH4670 de 1024 MB. To help the visualisation of results, the graphs below show the performance in respect of time. The fixed parameters were: 2 auxiliary sections, a rotation angle of 45º, and 500 points per section.



(a) (c) (b)

Figure 36: Performance tests based on: (a) number of auxiliary sections, (b) rotation angle. (c) points per section.

As can be observed in figure 36, the times taken prove the theoretical concepts that were previously discussed. The increase in the number of auxiliary sections and in the number of points per section increases the response time; however, the increase in the rotation angle to obtain control points makes this time decrease. It should also be highlighted that the most abrupt change is related to the number of auxiliary sections, where increasing the number of sections from 5 to 10 largely affects the response times mainly due to a higher number of intersections to be made.

Generic tests

The purpose of this study is to solve the joining of shoemaking lasts on replacing toes, but given the structure of the achieved solution, this can be easily extended to the process of joining lasts in the leg part. This is useful when working with boots or booties, since the back parts of existing lasts are to be kept, as when joining lasts in the toe area.

For the adaptation of this method to this new joining process, some fine-tuning was required in the internal structure for obtaining the control points. For instance, it was no longer necessary to find sole points, but the steps to be performed and the input parameters were the same. Figure 37 shows some examples showing the real practical application of this method for joining lasts in the leg part.

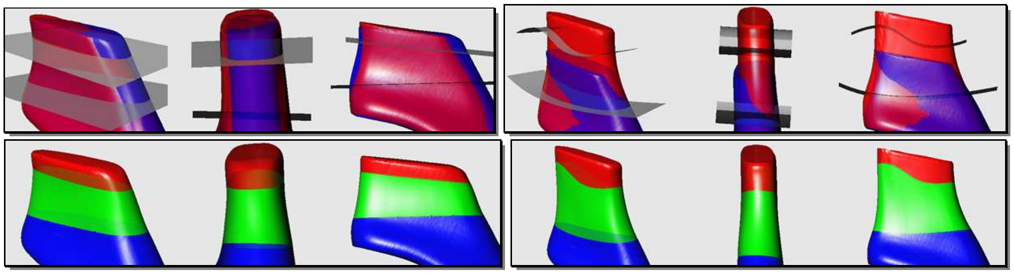


Figure 37: Examples of booties in which the leg parts have been replaced.

5 Conclusions

The footwear sector is a very traditional industry, where automation has not been able to impose its presence. The reasons for the prevalence of manual or craft labour over CAD/CAM processes are of a very distinct nature. The replacement of any manual design or production task by a CAD/CAM-based technique is extremely important, not only because of the obvious speeding up of processes, but also because it represents one step towards the full automation in a sector reluctant to change.

This paper presents a precise technique - based on NURBS surfaces - that is inspired by the manual process by improving it, and speeding it up, without causing rejection among users of this specific sector with little experience in the use of CAD tools.

In relation to current techniques that specifically deal with shoe-last joining, this technique constitutes a step forward in considering continuous surfaces (NURBS) as joining element. However, techniques used so far deal with discretisations of the last surface. This feature provides greater precision in the joining process, and avoids limitations imposed by the use of a discrete set of sections, since the problem posed by wrongly cutting the sections defining the last for the algorithms that process the last joining process, does no longer exist. G2 continuity is achieved across the whole generated surface and in joining areas with the surfaces of useful back and toe parts, due to using interpolation curves that join the back and the toe makes the direction of final surface creation to be transverse, thus enhancing continuity.

With regard to the existing generic techniques, which enable the joining of three-dimensional surfaces, this technique provides an improved interface to perform joining of shoe-lasts. The technique has been inspired by the way traditional lastmakers work, defining cutting planes (before areas to be saw-cut) and areas to be smoothed (in the past, areas to be bonded and sanded). This way, two important objectives are met: traditional lastmakers can make use of their pre-existing know-how, and they are not reluctant to use it, as the process sounds familiar to them.

Another important aspect to be considered is that the generated solution can be easily edited by manually editing the guide curves and automatically reconstructing the surface. This feature provides flexibility to the method, and allows the easy adjustment of the automatically obtained solution.

Finally, due to the fact that this is a generic method, joining together two lasts in any part is possible. This type of joining is less common but the automation of the process results in considerable time saving and provides improved precision. This means that the new method is also capable of working in the leg area, so as to be able to create a new last using the lower part of a last and the upper part of another one. This technique has been tested by experienced lastmakers and is successfully applied by a last-design software that is widely used worldwide.

However, there are still some aspects that could be improved. Further research on this method will focus on improving the way in which control points are obtained, making them dependent on the section topology, so that said points concentrate on those areas of more complex surface. Another important aspect is to also take into account some of the original surface sections found in the joining area – and discarded by this method - since they can provide valuable information for the joining process.

6 References

[1] Luximon A., Luximon Y., (2009), Shoe-last design innovation for better shoe fitting. *Computers in Industry*, Volume 60, Issue 8, pp 621-628.

[2] Raffaeli R., Germani M., (2011), Advanced computer aided technologies for design automation in footwear industry. *International Journal on Interactive Design and Manufacturing (IJIDeM)*. Volume 5, Issue 3, pp 137-149.

[3] Franciosa P., Gerbino S., Lanzotti A., Silvestri L., (2013), Improving comfort of shoe sole through experiments based on CAD-FEM modelling. *Medical Engineering & Physics*. Volume 35, Issue 1, pp 36-46.

[4] Wang C.-S., (2010), An analysis and evaluation of fitness for shoe lasts and human feet. *Computers in Industry*. Volume 61, Issue 6, pp 532-540.

[5] Wang J., Zhang H., Lu G., Liu Z., (2011), Rapid parametric design methods for shoe-last customization. *The International Journal of Advanced Manufacturing Technology*, Volume 54, Issue 1-4, pp 173-186.

[6] Muñoz P., Coronel J.L., Continuidad en superficies espaciales para diseño industrial. *Universidad de Buenos Aires*. (2004).

[7] Belkhatir B., Kouibia A., Pasadas M., Sbibih D. and Zidna A., (2008), Geometric Continuity C1G2 of Blending Curves. *Int. J. Contemp. Math. Sci.* 3 (29-32): 1451-1460

[8] Hoffman C., Hopcroft J., (1985) The Potential method for blending surfaces and corners. *Cornell University*.

[9] Wallner J., Pottmann H., (1997), Rational blending surfaces between quadrics. *Computer Aided Geometric Design*. 14(5): 407-419.

[10] Bottema - Roth, (1979), Theoretical kinematics. *North Holland, Amsterdam*.

[11] Piegl, L., Tiller, W., (1995), The NURBS Book. *Springer-Verlag, Berlin*.

[12] Juhásza I., Hoffmann M., (2009), Surface interpolation with local control by linear blending. *Annales Mathematicae et Informaticae*, Volume 36, pp 77-84.

[13] Shi k., Yong J-H., Sun J-G., Paul J-C., (2010), Gn blending multiple surfaces in polar coordinates. *Computer-Aided Design*, Volume 42 Issue 6, pp 479-494.

[14] Dae-Hyun K., Jieun L., Seong-Jae L., and Seung-Hyun Y., (2011), Construction and Rendering of Trimmed Blending Surfaces with Sharp Features on a GPU. *ETRI Journal*, Volume 33, No.1, pp 89-98.

[15] Kouibiaa A., Pasadasa M., Sbibihb D., Zidnac A., Belkhatirb B., (2013), Geometric continuity C1G2 of blending surfaces. *Computer-Aided Design.* Volume 45, Issue 3, pp 733–738.

[16] Hui K. C. and Li Y., (1998), A feature-based shape blending technique for industrial design. *Computer-Aided Design*, 30 (10): 823-834.

[17] Liu L., Wang G., Zhang B., Guo B., and Shum H., (2004), Perceptually based approach for planar shape morphing. Proceedings of 12th Pacific Conference on Computer Graphics and Applications, pp : 111–120 .

[18] Kraevoy V. and Sheffer A., (2004), Cross-parameterization and compatible remeshing of 3D models, *in Proc. SIGGRAPH '04*, 23: 861–869.

[19] Castro G. and Ugail H., (2007), Shape Morphing of Complex Geometries Using Partial Differential Equations. *University of Bradford. Journal of Multimedia*, 2(6): 15-25

[20] Turk G. and Obrien J.F., (2005), Shape transformation using variational implicit functions. *SIGGRAPH '05*. pp. 13-20.

[21] Takahashi S., Kokojima Y., and Ohbuchi R., (2001), Explicit control of topological transitions in morphing shapes of 3D meshes, *Proceedings of the 9th Pacific Conference on Computer Graphics and Applications*, pp.70– 79.

[22] Wolberg G., (1998), Image morphing: a survey. *The Visual Computer*. 14(8-9): 360-372.

[23] Zhang S., Wu E., (2009), Multiresolution Animated Models Generation Based on Deformation Distance Analysis. *Proceeding ICCMS '09 Procedings of the 2009 International Conference on Computer Modeling and Simulation*. pp 73-77.

[24] Zhang S., Huang J., Metaxaz D., (2010), Robust mesh editing using Laplacian coordinates. *Journal Graphical Models archive* Volume 73, Issue 1, pp 10-19.

[25] Li M., ASharf G., (2011), Sketch Based 3D Character Deformation. *Book Transactions on edutainment V. Lecture Notes in Computer Science*, Volume 6530, pp 177-188. Springer-Verlag Berlin.

[26] Huang J., Chen L., Liu X., Bao H., (2009), Efficient Mesh Deformation Using Tetrahedron Control Mesh. *Computer Aided Geometric Design*, Volume 26, No. 6, pp 617-626.

[27] Savoye Y., Franco J-S., (2010), CagelK: Dual-Laplacian Cage-Based Inveres Kinematics. *Proceeding AMDO'10 Proceedings of the 6th international conference on Articulated motion and deformable objects*.

[28] Guo Li, Joneja A., (2005), Gap filling using a blend of morphing and extension with application to footwear design. *Computer-Aided Design & Applications*, 2(1-4): 243-252.

[29] Gordon W.J., (1969), Spline-Blended Surface Interpolation Through Curve Net-works. *Journal of Mathematics and Mechanics*. Volume 18, No. 10.

[30] Smid J., (1991), Triangular Gordon Surface with Unique Normal. *TR EDS-GM. SIAM Conference on Geometric Modeling, Tampe, AZ.*