Optimal two-part tariff licensing contracts with differentiated goods and endogenous R&D*

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Abstract

In this paper we derive the optimal two-part tariff contract for the licensing of a cost-reducing innovation to a differentiated goods industry of a general size. We analyze the cases where the patentee is an independent laboratory or an incumbent firm. We show that regardless of the number of firms, the degree of product differentiation and the type of patentee, the innovation is licensed to all firms. Moreover, we endogenize R&D investment and obtain that an internal patentee invests more (less) in R&D when the technological opportunity is low (high), which is supported by an empirical test using data on R&D expenditures of Spanish manufacturing firms.

Key words: *patent licensing, two-part tariff contracts, R&D, product differentiation.*

JEL codes: L11, L13, L14.

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1 Introduction

Innovation is recognized by today's economists as a crucial ingredient of growth. This recognition has motivated the development of a huge literature analyzing the determinants of innovation. In particular, one of the main issues is whether the market provides firms with the right incentives to invest in research and development (R&D). Since Schumpeter's seminal work, there have been many empirical and theoretical papers addressing the effect of competition on R&D investment; however, the results are inconclusive. One common characteristic of many of these papers is that they do not consider the existence of a market for technology. In other words, the return to R&D investment comes from the final goods market.

In this paper we add to the literature by allowing for the existence of a market for technology. We consider the case of a research laboratory that owns a process innovation that would allow firms in a given industry to reduce their production costs. We consider two cases. If the laboratory remains independent as an external patentee, then it obtains revenues by licensing the patented innovation to the firms producing in this industry. Another possibility is that the laboratory integrates vertically with one of the firms in the industry, thus becoming an internal patentee. In this case, it may obtain revenues not only from licensing but also from participating in the final goods market.

As the literature on patent licensing has pointed out, the profitability of licensing depends on the type of licensing contracts that are available to the patentee. The earlier papers (Kamien and Tauman (1984,1986), Katz and Shapiro (1986) and Kamien et al. (1992)) show that licensing through a fixed fee or an auction is more profitable for an independent laboratory than is licensing through a royalty. In practice, however, licensing contracts often include both a fixed fee and a royalty (for example, Rostocker (1984) and Yanagawa and Wada (2000) show that this is the case for about half of all licensing contracts).

In this paper, we obtain the optimal two-part tariff licensing contract for the general case of an n -firm oligopolistic industry producing differentiated goods. We analyze the case where the patentee is an independent laboratory and also the case of an incumbent patentee.¹ We show that in both cases and regardless of the size of the innovation, the number of firms in the industry, and the degree of product differentiation the innovation is licensed to all firms in the industry.

In the second part of the paper, we endogenize the size of the innovation by allowing the patentee to invest in cost-reducing R&D. This allows us to compare the optimal R&D investment of the laboratory when it is an outsider patentee and when it is vertically integrated with one of the firms in the industry. We find that a vertically integrated laboratory invests more in R&D than an independent laboratory when the R&D investment is sufficiently costly. This result has a nice empirical implication regarding the internal organization of leading innovative firms. When the technological opportunity in an industry is high, which means that the cost of achieving a given cost reduction is relatively low, we can expect that innovation is dominated by independent research laboratories. In contrast, when the technological opportunity is low, we expect vertically integrated firms to be the leaders in innovation activities.

In the last part of the paper, we present some empirical evidence to test the result concerning our comparison of an internal versus an external patentee's incentives to innovate. We use data from the Survey on Firm's Innovation conducted by the Spanish Statistic Institute (INE), which are the official figures on R&D in Spain. The results of the empirical analysis seem to support our theoretical prediction.

¹There are some other papers in the literature that analyze patent licensing under differentiated goods, such as Muto (1993), Wang and Yang (1999), Mukherjee and Balasubramanian (2001), Poddar and Sinha (2004), Stamatopoulos and Tauman (2005). However they are confined either to pure upfront fee or pure royalty, or they consider only the duopoly case.

Apart from the present paper, the analysis of two-part tariff licensing contracts has been addressed, among others, by Fauli-Oller and Sandonis (2002), Sandonis and Fauli-Oller (2006), Sen and Tauman (2007), and Erutku and Richelle (2007). Fauli-Oller and Sandonis (2002) characterize situations where licensing a cost-reducing innovation to a rival firm in a Bertrand setting reduces social welfare. Sandonis and Fauli-Oller (2006) analyze whether a research laboratory prefers to license a cost-reducing innovation as an outsider patentee or to merge with one of the industry firms and license the innovation as an internal patentee. On the other hand, the papers by Sen and Tauman (2007) and Erutku and Richelle (2007) are closely related to this one. Yet they both focus on the case of homogenous goods, whereas we consider differentiated goods. Erutku and Richelle (2007) study the optimal two-part tariff contract to license a cost-reducing innovation for the case of an independent laboratory. They show that, regardless of the number of firms in the industry, the innovation is licensed to all firms. In this paper we show that their result extends to the case of differentiated goods and also to the case of a vertically integrated laboratory.

Sen and Tauman (2007) study the optimal *auction plus royalty* licensing policy for a general size oligopoly and for the cases of an internal and an external patentee. The difference with respect to a two-part tariff contract is that, in an auction plus royalty contract, the fixed part is determined through an auction. The authors show that, in this case, the innovation is licensed to all firms. They also study the incentives to innovate and show that the difference between post-innovation and pre-innovation profits is always higher for an external patentee. The intuition is that whereas an independent laboratory earns no profit in the absence of the innovation, an incumbent patentee earns the market profits. Because the R&D investment is presumed to increase the probability of obtaining a given cost-reducing innovation, Sen and Tauman (2007) obtain that the external patentee invests more in R&D.

In this paper, we adopt a different modelling strategy because R&D, in our model, determines the cost reduction. Therefore, what matters in this setting is not the difference between post-innovation and pre-innovation profits but rather the marginal profitability of R&D investment. We do not claim that our approach is more realistic than Sen and Tauman's. We only want to stress that, in a framework where R&D reduces production costs in a continuous way, an incumbent patentee may have more incentive to innovate than an independent laboratory. We see our result as being complementary to the one in Sen and Tauman (2007).

The rest of the paper is organized as follows. Section 2 presents the model for the cases of an independent laboratory and an incumbent patentee. In Section 3, we compare their incentives to innovate and then test our conclusions empirically. Finally, we conclude in Section 4. All proofs that are omitted from the text are relegated to the Appendix.

2 The model

We consider n symmetric firms competing in quantities and selling differentiated goods ($i = 1, \dots, n$). Firm i sells good i . The inverse demand of good i is given by:

$$\begin{aligned} p_i &= a - q_i - \gamma \sum_{j \neq i} q_j \\ i &= 1, \dots, n \end{aligned}$$

where q_i is the quantity sold of good i . All firms produce with marginal cost $c < a$.

We analyze two different settings. In the first, we assume there is an independent research laboratory that owns a patented process innovation. In the second setting, the laboratory is vertically integrated with one of the competing firms in the industry. The innovation enables the firms to reduce their cost of production to $c - \varepsilon$. We aim to derive the optimal two-part tariff licensing contract (F, r) for both an external and an internal patentee, where F denotes a

non negative fixed fee and r a linear per-unit royalty.

Although most papers in the literature impose nonnegative royalties (exceptions include are Liao and Sen 2005; and Erutku and Richelle 2007), for simplicity we solve the model without taking this constraint into account.²

We start by analyzing the case of an external patentee.

2.1 The case of an external patentee

In this case, the structure of the game is as follows. In the first stage, the patentee offers a two-part tariff contract (F, r) to the n competing firms. In the second stage, firms decide whether or not to accept the contract. The ones that do accept pay F to the patentee. Finally, the firms compete à la Cournot with a cost inherited from the licensing stage.

Assume that k firms have accepted a licensing contract (F, r) . In equilibrium, firms that have not accepted the contract produce:

$$q_N(k, r) = \begin{cases} \frac{(a - c)(2 - \gamma) - \gamma k(\varepsilon - r)}{(2 - \gamma)(2 + \gamma(n - 1))} & \text{if } r > \varepsilon - \frac{(a - c)(2 - \gamma)}{\gamma k}, \\ 0 & \text{otherwise.} \end{cases}$$

Observe that, if r is very low, then firms that do not accept the contract are driven out of the market. Meanwhile, the firms that accept the contract produce, in equilibrium,

$$q(k, r) = \begin{cases} \frac{(a - c)(2 - \gamma) - (-2 + \gamma(1 + k) - \gamma n)(\varepsilon - r)}{(2 - \gamma)(2 + \gamma(n - 1))} & \text{if } r > \varepsilon - \frac{(a - c)(2 - \gamma)}{\gamma k}, \\ \frac{a - c + \varepsilon - r}{k + 1} & \text{otherwise.} \end{cases}$$

Profits of non-accepting and accepting firms are given, respectively, by $\Pi_N(k, r) = (q_N(k, r))^2$ and $\Pi(k, r) = (q(k, r))^2$.

In the second stage, given that $k - 1$ firms accept the contract, the k th firm accepts the contract whenever $F \leq \Pi(k, r) - \Pi_N(k - 1, r)$. Obviously, as the laboratory maximizes profits,

²Our results do not change if we impose nonnegative royalties whenever c is not very low.

in order for k firms to accept the contract,³ it will choose F such that $F = \Pi(k, r) - \Pi_N(k-1, r)$. This implies that the problem of choosing the optimal contract (F, r) is equivalent to that of choosing (k, r) . Then, in the first stage, the external patentee solves the following problem subject to $1 \leq k \leq n$ and $r \leq \varepsilon$:

$$\underset{k,r}{Max} \ k(\Pi(k, r) - \Pi_N(k-1, r)) + krq(k, r). \quad (1)$$

We proceed as follows. First, we prove that the research laboratory finds it profitable to license the innovation to all firms in the industry then, we calculate the optimal royalty once k is replaced by n in (1). With regard to the first result, we know that, with a fixed fee contract, the input would be sold to only a subset of firms in order to protect industry profits from competition (Kamien and Tauman 1986). But with a two-part tariff contract, the laboratory can always license to one more firm, without affecting the level of competition, by choosing an appropriate royalty. The following lemma shows that it is always profitable for the laboratory to license the innovation to all firms regardless of how many there are in the industry. Assume that the laboratory licenses the innovation to k firms with a royalty r . We seek to show that the laboratory can always increase profits by licensing the innovation to all firms through a higher royalty ($r < r_E < \varepsilon$) such that *total industry output* remains constant. In the particular case of homogeneous goods this claim is intuitive because it implies a constant final price⁴. With differentiated goods, however, we can obtain the result via a technical condition.

Lemma 1 *Assume that the laboratory licenses to k firms with a royalty r . It can always increase profits by licensing to all firms with a royalty $r < r_E \leq \varepsilon$ such that $nq(n, r_E) = (n-k)q_N(k, r) + kq(k, r)$.*

³Since $\frac{\partial(\Pi(k, w) - \Pi_N(k-1, w))}{\partial k} < 0$, this is the only equilibrium in the acceptance stage.

⁴This argument is used in Sen and Tauman (2007) to prove that, with an auction plus royalty contract, the input would be sold to all firms. It is also used in Faulí-Oller and Sandonís (2009) in the context of an input market. In both papers, only the case of homogeneous goods is analyzed.

Proof. See the Appendix. ■

Lemma 1 shows that it is always feasible (and profitable) for the laboratory to license the innovation to all firms while increasing the royalty so that total output remains constant. Let us provide some intuition for this result. First, this strategy is always feasible (regardless of n) because of how the level of competition increases as the laboratory licenses the innovation to more firms. Licensing to one more firm increases the level of competition less when there are already many firms using the innovation than when only a few of them do. This implies that, as more firms buy a license, the laboratory needs progressively smaller increases in the royalty in order to keep the level of competition constant. Second, that this strategy is profitable for the laboratory can be explained as follows. Its problem is one of maximizing total industry profits less the outside option of firms (i.e the profit of firms that refuse the licensing contract and produce with the old technology). Licensing to all firms leads, on the one hand, to an increase of total industry profits due to an efficiency effect. On the other hand, the effect with respect to the outside option is ambiguous: licensing the innovation to more firms for a given royalty reduces the outside option, but increasing the royalty leads to an increase in the outside option. As we show in the proof, the overall effect is negative and hence it is in the interest of the patentee to license the innovation to all firms.

This result is central to the paper, so it would be interesting to know whether it holds for more general demand functions. In the Appendix we show that, for the case of homogeneous goods, it holds for concave demands satisfying a technical restriction concerning the third derivative of the inverse demand. We show that it also holds for the class of demands $P = A - X^b$, where $b \geq 1$.

Our next result uses Lemma 1 to derive the optimal two-part tariff contract when licensing to n firms.

Proposition 1 *The laboratory optimally licenses the innovation to all firms. The optimal royalty is: $r^*(n) = r_1$ if $\varepsilon < \varepsilon_1$ and $r^*(n) = r_2$ otherwise, where*

$$\begin{aligned} r_1 &= \frac{\gamma(n-1)((a-c)(-2+\gamma)\gamma + \varepsilon(4 + \gamma(-6 + \gamma + 2n)))}{2(4 + \gamma(4(-2+n) + \gamma(6 + \gamma(n-1) + (n-6)n)))}, \\ r_2 &= \frac{(a-c+\varepsilon)\gamma(n-1)}{2 + 2\gamma(n-1)}, \\ \varepsilon_1 &= \frac{(a-c)(4 + \gamma(-6 + \gamma(n-3)(n-1) + 4n))}{\gamma(n-1)(2 + \gamma(n-1))}. \end{aligned}$$

Proof. See the Appendix. ■

Observe that the constraint $r \leq \varepsilon$ is never binding in equilibrium. Recall that the objective function of the patentee can be expressed as total industry profits $n\Pi(n, r) + nrq(n, r)$ minus the outside option of the licensees $n\Pi_N(n-1, r)$. Market profits are increasing in r up to $r^*(n)$, whereas reducing the outside option calls for a lower r . The balance of the two effects leads to an optimal royalty which is always lower than ε . Notice also that the optimal royalty increases with ε . The reason is that the greater the innovation's effect, the less the value of the licensees' outside option for a given r . This gives the patentee more flexibility to pursue the first incentive, that is, it increases the royalty in order to increase industry profits.

2.2 The case of an internal patentee

Here, we consider the case where an innovation is owned by one of the firms in the industry (say, firm 1). We must distinguish two cases. If the innovation is drastic ($-2a + 2c + (a - c + \varepsilon)\gamma > 0$)⁵, then the patentee earns the monopoly profits in its market by *not* licensing the innovation; if the innovation is not drastic, monopolization never occurs. The timing of the game is the same as in Section 2.1. In this case there are three different cost levels in the market stage of the game: the owner of the innovation produces at $c - \varepsilon$, the licensees at cost $c - \varepsilon + r$, and the

⁵Observe that this holds when the innovation is large enough, in particular when $\varepsilon \geq \frac{(a-c)(2-\gamma)}{\gamma}$.

non-licensees at cost c . As a consequence, the respective equilibrium outputs are given by the following expressions if the innovation is non-drastic:

$$q_P^I(k, r) = \begin{cases} \frac{-2(c - \varepsilon) + a(2 - \gamma) + \gamma(c + \varepsilon(n - k - 2) + kr)}{(2 - \gamma)(2 + \gamma(n - 1))} & \text{if } r > \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k} \\ \frac{(a - c + \varepsilon)(2 - \gamma) + \gamma kr}{(2 - \gamma)(2 + \gamma k)} & \text{otherwise.} \end{cases}$$

$$q^I(k, r) = \begin{cases} \frac{a(2 - \gamma)(c - \varepsilon + r) + \gamma(c + \varepsilon(n - k - 2) - (n - k - 1)r)}{(2 - \gamma)(2 + \gamma(n - 1))} & \text{if } r > \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k} \\ \frac{(a - c - \varepsilon)(2 - \gamma) - 2r}{(2 - \gamma)(2 + \gamma k)} & \text{otherwise.} \end{cases}$$

$$q_N^I(k, r) = \begin{cases} \frac{(a - c)(2 - \gamma) + \gamma(-\varepsilon(k + 1) + kr)}{(2 - \gamma)(2 + \gamma(n - 1))} & \text{if } r > \varepsilon + \frac{-2a + 2c + (a - c + \varepsilon)\gamma}{\gamma k} \\ 0 & \text{otherwise.} \end{cases}$$

If the innovation is drastic, non-licensees do not produce ($q_N^I(k, r) = 0$). In this case, outputs in equilibrium for the patentee and licensees are given, respectively, by:

$$q_P^I(k, r) = \begin{cases} \frac{(a - c + \varepsilon)}{2} & \text{if } r > \frac{(a - c + \varepsilon)(2 - \gamma)}{2} \\ \frac{(a - c + \varepsilon)(2 - \gamma) + \gamma kr}{(2 - \gamma)(2 + \gamma k)} & \text{otherwise.} \end{cases}$$

$$q^I(k, r) = \begin{cases} 0 & \text{if } r > \frac{(a - c + \varepsilon)(2 - \gamma)}{2} \\ \frac{(a - c - \varepsilon)(2 - \gamma) - 2r}{(2 - \gamma)(2 + \gamma k)} & \text{otherwise.} \end{cases}$$

Here the subindex P stands for patentee, subindex N for non-licensees and superindex I for the internal case. Market profits of the patentee, the accepting firms, and the non-accepting firms are given, respectively, by $\Pi_P^I(k, r) = (q_P^I(k, r))^2$, $\Pi^I(k, r) = (q^I(k, r))^2$, and $\Pi_N^I(k, r) = (q_N^I(k, r))^2$.

In the second stage, given that $k - 1$ firms accept the contract, the k th firm accepts the contract whenever $F \leq \Pi^I(k, r) - \Pi_N^I(k - 1, r)$. Like the independent laboratory in Section 2.1,

the internal patentee also seeks to maximize licensing revenues⁶ and so will choose F such that $F = \Pi^I(k, r) - \Pi_N^I(k - 1, r)$. This implies that the problem of choosing the optimal contract (F, r) is equivalent to that of choosing (k, r) . Then, in the first stage, the patentee solves the following problem for $0 \leq k \leq n - 1$ and $r \leq \varepsilon$.

$$\underset{k, r}{Max} \Pi_P^I(k, r) + k (\Pi^I(k, r) - \Pi_N^I(k - 1, r) + r q^I(k, r))$$

Much as we did for the case of an external patentee, before solving this maximization problem we show that it is always profitable for the patentee to license the innovation to all firms regardless of the total number of firms in the industry.

Lemma 2 *Assume that the patentee licenses to k firms with a royalty r . The patentee can always increase its profits by licensing to all firms with a royalty $r < r_I \leq \varepsilon$ such that*

$$(n - 1)q^I(n - 1, r_I) + q_P^I(n - 1, r_I) = (n - k - 1)q_N^I(k, r) + kq^I(k, r) + q_P^I(k, r).$$

Proof. See the Appendix. ■

The intuition for this is similar to that for the case of an external patentee. In the Appendix we show that, for the case of homogeneous goods, Lemma 2 holds for concave demands satisfying the same technical restrictions as in the case of an external patentee.

Next, we derive the optimal royalty when licensing the innovation to $n - 1$ firms.

Proposition 2 *The laboratory optimally licenses the innovation to all firms.*

If $\gamma < \frac{2}{n - 1}$, the optimal royalty is: $r^{I*}(n) = \varepsilon$ if $\varepsilon \leq \varepsilon_1^I$, $r^{I*}(n) = r_1^I$ if $\varepsilon_1^I < \varepsilon < \varepsilon_2^I$ and $r^{I*}(n) = r_2^I$ otherwise.

If $\gamma \geq \frac{2}{n - 1}$, the optimal royalty is: $r^{I*}(n) = r_I$ if $\varepsilon < \varepsilon_2^I$ and $r^{I*}(n) = r_2^I$ otherwise; where

$$r_1^I = \frac{\gamma(a(-2 + \gamma)(-2 + \gamma(n - 1)) + c(-4 + \gamma^2 - (-2 + \gamma)\gamma n) + \varepsilon(n - 1)(4 + \gamma(-8 + \gamma + 2n)))}{2(4 + 4\gamma(-2 + n) + \gamma^2(7 + (-7 + n)n))}$$

⁶Since $\frac{\partial(\Pi^I(k, r) - \Pi_N^I(k - 1, r))}{\partial k} < 0$, this is the only equilibrium in the acceptance stage.

$$r_2^I = \frac{(a - c + \varepsilon)(-2 + \gamma)^2 \gamma (n - 1)}{8 - 2\gamma(8 + 3\gamma(n - 1) - 4n)}$$

$$\varepsilon_1^I = \frac{(a - c)(-2 + \gamma)\gamma(-2 + \gamma(n - 1))}{8 - \gamma(-\gamma(6 + \gamma) - 4(n - 3) + \gamma(4 + \gamma)n)}$$

$$\varepsilon_2^I = \frac{(a - c)(2 - \gamma)(8 + 8\gamma(n - 2) + 2\gamma^2(5 - 6n + n^2) - \gamma^3(2 - 3n + n^2))}{\gamma(-8 + 8n - 2\gamma^2(n^2 - 1) + 4\gamma(2 - 3n + n^2) - \gamma^3(2 - 3n + n^2))}$$

Proof. See the Appendix. ■

It is interesting to note that $r^{I*}(n) > r^*(n)$. This result is intuitive because an internal patentee obtains revenues not only from licensing but also from selling the good in the market. Therefore, it is more interested in controlling competition by charging a higher royalty. Observe that, for the particular case $n = 2$, the outside option does not depend on the royalty and, so the internal patentee maximizes industry profits. This implies that $\varepsilon_1^I = \varepsilon_2^I$.

3 Incentives to innovate

So far, we have analyzed licensing contracts while assuming that the innovation already existed. The next step in the analysis is to endogenize the size of an innovation by allowing firms to choose the amount of R&D. Our aim is to compare R&D investments of both an internal and an external patentee.

We assume that, previous to the licensing stage, the patentee chooses the level of R&D investment ε at the cost $C(\varepsilon) = d\varepsilon^2$, where the parameter d represents the expense of achieving a given cost reduction. In the licensing stage, the profits of the external patentee, $(B(\varepsilon))$, net of R&D costs, can be written as:

$$B(\varepsilon) = \begin{cases} \frac{n(\varepsilon^2(-2 + \gamma)^2(2 + \gamma(n - 1))^2 + a^2\gamma^4(n - 1)^2 + c^2\gamma^4(n - 1)^2 - 2c\varepsilon(2 + \gamma(n - 1))(8 + 8\gamma(n - 2) + \gamma^3(n - 1) + 2\gamma^2(5 + (n - 5)n)) + \varepsilon(2 + \gamma(n - 1))(8 + 8\gamma(n - 2) + 2a(-c\gamma^4(n - 1)^2 + \gamma^3(n - 1) + 2\gamma^2(5 + (n - 5)n))))}{4(2 + \gamma(n - 1))^2(4 + \gamma(4(n - 2) + \gamma(6 + \gamma(n - 1) + (n - 6)n)))} & \text{if } \varepsilon < \varepsilon_1 \\ \frac{(a - c + \varepsilon)^2 n}{4 + 4\gamma(n - 1)} & \text{otherwise.} \end{cases}$$

With an internal patentee, for simplicity we focus on the case $\gamma \geq \frac{2}{n-1}$, which allows us to avoid corner solutions.⁷ In this case, the profits of the internal patentee, net of R&D costs, are given by:

$$\text{If } B^I(\varepsilon) = \begin{cases} \Pi_P^I(k, r_I) + k(\Pi^I(k, r_I) - \Pi_N^I(k - 1, r_I) + rq^I(k, r_I)) & \text{if } \varepsilon \leq \varepsilon_2^I \\ \frac{(a - c + \varepsilon)^2(-\gamma^2 + (2 - \gamma)^2 n)}{4(4 - \gamma(8 + 3\gamma(n - 1) - 4n))} & \text{otherwise.} \end{cases}$$

Details on computing the actual value of $B^I(\varepsilon)$ when $\varepsilon \leq \varepsilon_2^I$ are given in the Appendix.

Therefore, an external patentee will choose:

$$\widehat{\varepsilon} = \arg \max_{\varepsilon} \{B(\varepsilon) - C(\varepsilon)\}$$

and an internal patentee will choose:

$$\widehat{\varepsilon}_I = \arg \max_{\varepsilon} \{B^I(\varepsilon) - C(\varepsilon)\}.$$

The specific values of $\widehat{\varepsilon}$ and $\widehat{\varepsilon}_I$ are calculated in the Appendix. The condition $d > \frac{n}{4}$ guarantees concavity and interior solutions.

Next, we compare both investments. This comparison, although interesting, has not received much attention in the literature. The seemingly lone exception is Sen and Tauman (2007) who also compare the incentives to innovate of both an external and an internal patentee. They show that the difference between the post-innovation and pre-innovation profits is always higher

⁷Observe that this implies $n \geq 3$.

Figure 1: $d = 10$

for an external patentee. The reason is that whereas an external patentee earns no profit with no innovation, an internal patentee still earns market profits. We adopt a different modelling strategy because, in our model, R&D is a continuous variable that determines the cost reduction and the post-innovation profits. In our setting, then an investment's incremental profits are less important than its marginal profitability. We show that in this case an internal patentee may have more incentive to innovate. Although the complexity of the equilibrium investments precludes an explicit comparison, we can plot in a three-dimensional space $\hat{\epsilon}_I - \hat{\epsilon}$, for different values of γ , n , and d . Figure 1 plots the difference for $d = 10$, $\gamma \in [\frac{2}{9}, 1]$, and $n \in [10, 30]$. Ranges for γ and n are chosen such that $\gamma \geq \frac{2}{n-1}$ and $d > \frac{n}{4}$ are satisfied. Figures 2 and 3 plot the difference for values $d = 30$ and $d = 50$, respectively.

If we look at the three figures, it is easy to see that the region where an internal patentee invests more in R&D increases in size as d increases. This result has a nice empirical implication regarding the internal organization of leading innovative firms. When the technological oppor-

Figure 2: $d = 30$

Figure 3: $d = 50$

tunity in an industry is high, which means that the cost of achieving a given cost reduction is relatively low, we can expect that innovation will be dominated by independent research laboratories. In contrast, when the technological opportunity is low, vertically integrated firms are expected to be the leaders in innovation activities.

A second finding is that an internal patentee invests more in R&D only when the goods are close enough substitutes. The reason could be that an internal patentee is in a better position to control for the level of competition because it is an active firm in the final good's industry.

In order to derive explicit results on the R&D comparison, it seems worthwhile to analyze the particular case where the goods are homogenous ($\gamma = 1$). This will facilitate the comparison of our result with that of Sen and Tauman (2007), who also consider homogenous goods.

3.1 The case of homogeneous goods

In the case $\gamma = 1$, the constraint $d > \frac{1}{2}$ guarantees that

$$\hat{\varepsilon} = \frac{(a - c)n(1 + n(2n - 1))}{-n(1 + n) + 4d(1 + n^3)} \text{ and}$$

$$\hat{\varepsilon}_I = \frac{(a - c)(3 + n(3 + n(-1 + n(-3 + 2n))))}{-3 - n(3 + (-3 + n)n) + 4d(1 + n)^2(3 + (-3 + n)n)}$$

are global maxima. As expected, the higher d is the lower are $\hat{\varepsilon}$ and $\hat{\varepsilon}_I$, but we also have that total expenditure in R&D ($d\hat{\varepsilon}^2$ and $d\hat{\varepsilon}_I^2$) is decreasing in d . Therefore, $\frac{1}{d}$ measures the degree of technological opportunity of the industry.

The literature has extensively studied the relationship between R&D investment and competition. It is possible to check that $\hat{\varepsilon}_I$ is always increasing in n and that $\hat{\varepsilon}$ increases with n whenever R&D investment is expensive enough (d high enough). When d is low, the trajectory of $\hat{\varepsilon}$ follows a U-shape with respect to the number of firms.

Our next proposition compares both investments for the particular case of homogeneous goods.

Proposition 3 *The internal patentee invests more in R&D than the external patentee when $n \geq 3$ and $d > \hat{d}(n)$, where $\hat{d}(n) = \frac{n^2(-3 + n(3n - 4))}{2(1 + n)(-3 + (n - 2)n^2)}$.*

This result confirms the intuitions gained from the figures.

3.2 Empirical evidence

In this section we present some empirical evidence that supports the main theoretical result of the previous section namely, that an external laboratory invests more (less) in R&D than an internal patentee when the technological opportunity of the industry is high (low). We use data from the "Encuesta sobre Innovación Tecnológica en las Empresas" (Survey on Technical Innovation in Companies) conducted by the Spanish Statistic Institute.⁸ The survey offers aggregate information on industries' annual R&D investment, considering separately the expenditures from the firms' own R&D laboratories (internal R&D) as well as the amount they have spent contracting R&D outside the firm (external R&D)⁹ as well as other R&D variables.

We use information about manufacturing firms, expenditures on *internal* R&D is a direct measure of incentive to innovate a vertically integrated laboratory's. Unfortunately, there is no comparable information about the R&D expenditures that *external* laboratories undertake in each industry. So, as a proxy of external laboratories' R&D, we use the external R&D expenditures of the manufacturing firms.¹⁰ We then use these two measures to define the ratio of internal to external R&D. According to the theoretical model, this ratio should be higher in those industries with lower technological opportunities and, conversely, lower in industries with

⁸The data from this survey are the official figures on R&D in Spain. We use information on 27 manufacturing industries from 2003 to 2007. The level of aggregation corresponds to the NACE 2- and 3-digit. The data is available from the INE website (www.ine.es).

⁹Organizing innovation in terms of internal versus external sourcing has been an issue in a number of empirical papers (see, e.g., Veugelers and Cassiman 1999; Piga and Vivarelli 2004). Firms can develop new products through their own R&D spending (internal R&D) or through contract with external laboratories (external R&D). In many cases, firms conduct both types of R&D.

¹⁰External R&D includes R&D acquisitions from outside the firm (usually through contracts); it does not include equipment acquisition or patent licenses. If one considers that external R&D is acquired mainly from external laboratories, then this measure, although incomplete, is a reasonable approximation of an external lab's incentive to innovate.

higher technological opportunities.

We use the standard classification of manufacturing firms to categorize industries according to technological opportunities, distinguishing among four types.¹¹ high-technology, medium-high-technology, medium-low-technology, and low-technology industries.¹² As is common in this literature, we assume that firms in high-tech industries (e.g., pharmaceutical, aircraft, spacecraft, communication equipment, computing) enjoy the highest level of technological opportunities. At the other extreme, low-tech manufacturing industries (e.g. wood, food, textiles) exhibit the lowest level of technological opportunities.

¹¹Technological opportunities are defined in the empirical literature as exogenous variations in the state of technology (see Jaffe 1986).

¹²The category *high-technology industries* comprises: Aircraft and spacecraft; Pharmaceuticals; Office, accounting, and computing machinery; Radio, TV, and communication equipment; and Medical, precision, and optical instruments. *Medium-high technology industries*: Electrical machinery; Motor vehicles; Chemicals (excluding pharmaceuticals); Railroad equipment and transport equipment; and Machinery and equipment. *Medium-low technology industries*: Rubber and plastic products; Other nonmetallic mineral products; Building ships; Basic metals; and Fabricated metal products (excluding machinery and equipment). *Low-technology industries*: Recycling; Wood, paper, and printing and publishing; Food products, beverages, and tobacco; and Textiles, cloths, leather products, and footwear.

Table 1 : R&D indicators for Spanish manufacturing firms, 2003-2007

	Ratio	R&D	Firms with
	Int/Ext	effort	R&D
High-technology industries	5.4	7.5	40.9%
Medium-high-technology industries	5.8	2.4	24.7%
Medium-low-technology industries	9.79	1.7	10.2%
Low-technology industries	10.2	1.9	8.2%
Total	8.0	3.2	19.5%

Table 1 shows the main R&D indicators by industry type. The first column presents the *average* value of the ratio of internal to external R&D. All industries exhibit a ratio that exceeds 1, but there are significative differences among industry types. Low-tech industries have the highest ratio, with internal R&D expenditure that are 10 times greater than external ones. Overall, the figures suggest that external laboratories are more active in industries with higher technological opportunities (i.e., high and medium-high technology), and this result agrees with the theoretical model's prediction. Moreover, high-tech industries have a larger proportion of firms with R&D activities, and the average R&D effort is significantly greater.¹³ In contrast, low-tech industries have a lower proportion of innovative firms and less R&D effort.

¹³Empirical studies have shown that technological opportunities induce a positive covariation with the total industry-level R&D expenditures and a higher proportion of firms undertaking R&D activities.

Table 2: OLS regression results, ratio of internal to external R&D

	(1)		(2)		(3)	
	Coef.	<i>T</i> -ratio	Coef.	<i>T</i> -ratio	Coef	<i>T</i> -ratio
Constant	13.57	(5.3)	13.73	(5.7)	14.93	(5.3)
High-tech	-3.80	(-3.3)				
R&D effort			-0.38	(-6.2)		
Proportion of R&D firms					-0.15	(-3.2)
Trend	-0.45	(-1.1)	-0.44	(-1.1)	-0.46	(-1.1)
Proportion of large firms	-0.07	(-3.4)	-0.09	(-3.5)	-0.07	(-3.2)
Observations	132		132		132	
Adjusted R^2	11.1%		9.5%		13.4%	

Table 2 presents the result of different OLS regressions using aggregate industry data, where the dependent variable is the ratio between internal and external R&D. The explanatory variables are three different measures of technological opportunities and two control variables: a temporal trend and the proportion of large firms (more than 250 employees) in each industry. The sources of technological opportunities are usually unobserved, so the empirical literature has typically used industry indicators. We use a dummy variable that takes the value 1 for those sectors included in high- and medium-high-technology classifications. Other variables include the average level of R&D effort (aggregate R&D expenditures over total revenues) in a given industry and the proportion of firms with R&D activities.

Column (1) of the table shows that the estimated parameter for the high-tech industry dummy is negative and significant. The parameter value indicates that the ratio between internal and external R&D is, on average, almost a 30% lower in high-tech industries. Column (2) shows

that the ratio decreases as the R&D effort of an industry increases, and this estimated parameter is also significant. Finally, column (3) shows a negative correlation between the proportion of firms with R&D in a given industry and the ratio between internal and external R&D. The parameter of the temporal trend is negative, but it is estimated with low precision because of the short period of time analyzed. Finally firms' size also has a negative effect on the ratio.

These results suggest that the main theoretical result of Section 2 explains what is actually observed in the Spanish economy.

4 Conclusions

In this paper we obtain the optimal two-part tariff licensing contract for the general case of an n -firm oligopolistic industry producing differentiated goods. We analyze the case where the patentee is an independent laboratory as well as the case where it is an incumbent patentee. We show in both cases that regardless of the size of the innovation, the number of firms in the industry, and the degree of product differentiation, the innovation is licensed to all firms in the industry.

This result has been previously obtained for the particular case of homogenous goods and an external patentee by Erutku and Richelle (2007). We show that it extends to the case of differentiated goods, and even when the patentee is an incumbent firm in the industry. This may seem counterintuitive, given that an internal patentee has market profits to protect. However, we have shown that the patentee can always protect profits i.e. control competition not by restricting the number of licensees but rather by increasing the royalty.

In the second part of the paper, we compare the incentive to innovate of an incumbent patentee with that of an independent laboratory. We find that this comparison depends on the level of technological opportunity. In particular, we find that when technological opportunity is low

(high), an internal (external) patentee optimally invests more in R&D. Our result is interesting compared it with that of Sen and Tauman (2007) who find that an external patentee has always more incentive to innovate. This difference arises because R&D, in our model, determines the cost reduction whereas in Sen and Tauman increases the probability of developing a given innovation. Therefore what matters, in our setting, is not the difference between post-innovation and pre-innovation profits but instead the marginal profitability of R&D investment. Finally, we test the theoretical prediction of our model concerning the comparison of the incentive to innovate of an internal versus an external patentee. We use data from the Spanish manufacturing industry that comes from the Survey on Technological Innovation in Companies conducted by the Spanish Statistic Institute (INE), which are the official figures on R&D in Spain. The results of the empirical analysis seem to support our theoretical prediction.

All the results in this paper have been obtained under the assumption of Cournot competition. It would be interesting to extend the results to the case of Bertrand competition. Although the formal analysis is cumbersome, we can provide some intuition at least for the case of an internal patentee. Our main result -that the innovation is always licensed to all firms- should remain true under Bertrand competition because an internal patentee may have less incentive to restrict the number of licensees under price competition than under quantity competition. The reason is that, under the former regime, the patentee has an additional means for moderating competition namely, using the royalty to establish a higher price. This effect is the main result in Fauli-Oller and Sandonis (2002).

5 Appendix

Proof of Lemma 1

Let $\pi(k, r)$ represent the laboratory's profit if it licenses to k firms and sets a royalty $r \leq \varepsilon$.

We have that

$$\pi(k, r) = B(k, r) - k(q_N(k-1, r))^2 - (n-k)(q_N(k, r))^2 - \varepsilon(n-k)q_N(k, r).$$

where

$$B(k, r) = k(q(k, r))^2 + (n-k)(q_N(k, r))^2 + krq(k, r) + \varepsilon(n-k)q_N(k, r).$$

Observe that we have expressed the profits of the incumbent patentee as the difference between total industry profits ($B(k, r)$) and the profits of the remaining firms. The efficiency term appears because we are computing industry profits *as if* total output were produced using the new technology.

Let r_E solve $nq(n, r_E) = (n-k)q_N(k, r) + kq(k, r)$.

We next prove that licensing to all firms with a royalty $r < r_E \leq \varepsilon$ is more profitable than licensing to k firms with a royalty r . This is equivalent to showing that the following expression is positive:

$$\begin{aligned} \pi(n, r_E) - \pi(k, r) &= B(n, r_E) - B(k, r) + k(q_N(k-1, r))^2 + (n-k)(q_N(k, r))^2 + \\ &\quad + \varepsilon(n-k)q_N(k, r) - n(q_N(n-1, r_E))^2. \end{aligned}$$

It is convenient to proceed in two steps. We first prove that $B(n, r_E) - B(k, r)$ is positive and then prove that the remaining terms are also positive.

If $\varepsilon - \frac{(a-c)(2-\gamma)}{\gamma k} \leq r \leq \varepsilon$, we have that:

$$B(n, r_E) - B(k, r) = \frac{k(1-\gamma)(n-k)(\varepsilon-r)^2}{n(2-\gamma)^2} \geq 0$$

$$k(q_N(k-1, r))^2 + (n-k)(q_N(k, r))^2 - n(q_N(n-1, r_E))^2 = \frac{k\gamma^2(n-k)(\varepsilon-r)^2}{n(2-\gamma)^2(2+\gamma(n-1))^2} \geq 0$$

If $\varepsilon - \frac{(a-c)(2-\gamma)}{\gamma(k-1)} < r < \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma k}$, then

$$r_E = \frac{(a-c+\varepsilon)(2-\gamma)(n-k) + k(2+\gamma(n-1))r}{(2+\gamma(k-1))n}$$

where $\varepsilon > r_E > r$ and $q_N(k, r) = 0$.

We must now distinguish two cases. First

$$\text{if } \frac{(\varepsilon\gamma k(2+\gamma(n-1))(n-1) + a(-2+\gamma)(2n+\gamma(k+(-2+n)n)) - c(-2+\gamma)(2n+\gamma(k+(-2+n)n)))}{(\gamma k(2+\gamma(n-1))(n-1))} < r \leq \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma k},$$

then $q_N(n-1, r_E) > 0$. It follows directly that

$$B(n, r_E) - B(k, r) = \frac{k(1-\gamma)(n-k)(a-c+\varepsilon-r)^2}{n(2+\gamma(k-1))^2} \geq 0 \quad (2)$$

Moreover, we have

$$kq_N(k-1, r) - nq_N(n-1, r_E) = \frac{(n-k)(2+\gamma(n+k-2))((a-c)(-2+\gamma) + \gamma k(\varepsilon-r))}{(2-\gamma)(2+\gamma(k-1))(2+\gamma(n-1))}. \quad (3)$$

It is clear that (3) is decreasing in r and that it is in the upper bound of the region, which implies

$$k(q_N(k-1, r))^2 - n(q_N(n-1, r_E))^2 > 0$$

Second,

$$\text{if } \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma(k-1)} < r \leq \frac{(\varepsilon\gamma k(2+\gamma(n-1))(n-1) + a(-2+\gamma)(2n+\gamma(k+(-2+n)n)) - c(-2+\gamma)(2n+\gamma(k+(-2+n)n)))}{(\gamma k(2+\gamma(n-1))(n-1))},$$

then $q_N(n-1, r_E) = 0$. The previous calculations applied to this case prove the result.

If $r \leq \varepsilon - \frac{(a-c)(2-\gamma)}{\gamma(k-1)}$, we have that $q_N(n-1, r_E) = 0$ and $q_N(k-1, r) = 0$. Then

$\pi(n, r_E) - \pi(k, r) = B(n, r_E) - B(k, r)$, which is positive by (2).

Proof of Proposition 1

$$r_1 = \arg \max_r \{n(\Pi(n, r) - \Pi_N(n-1, r)) + nrq(n, r)\}$$

$$r_2 = \arg \max_r \{n\Pi(n, r) + nrq(n, r)\}$$

If $\varepsilon = \varepsilon_1$, then $r_1 = r_2$. This implies that the optimal royalty is r_1 when $\varepsilon \leq \varepsilon_1$ and r_2 otherwise.

Proof of Lemma 2

Let $\pi^I(k, r)$ represent the profit of the incumbent patentee if it licenses to k firms and sets a royalty r . It is

useful to express this profit as

$$\pi^I(k, r) = B^I(k, r) - k \left(q_N^I(k-1, r) \right)^2 - (n-k-1) \left(q_N^I(k, r) \right)^2 - \varepsilon(n-k-1) q_N^I(k, r),$$

where

$$B^I(k, r) = k \left(q^I(k, r) \right)^2 + (n-k-1) \left(q_N^I(k, r) \right)^2 + \left(q_P^I(k, r) \right)^2 + k r q^I(k, r) + \varepsilon(n-k-1) q_N^I(k, r).$$

Observe that we have expressed the profits of the incumbent patentee as the difference between total industry profits ($B^I(k, r)$) and the profits of the remaining firms. The efficiency term appears because we are computing industry profits *as if* total output were produced using the new technology.

Let r_I solve $(n-1)q^I(n-1, r_I) + q_P^I(n-1, r_I) = (n-k-1)q_N^I(k, r) + kq^I(k, r) + q_P^I(k, r)$.

We next prove that licensing to all firms with a royalty $r < r_I \leq \varepsilon$ is more profitable than licensing to k firms with a royalty r . This is equivalent to showing that, next expression is positive:

$$\begin{aligned} \pi^I(n, r_I) - \pi^I(k, r) &= B^I(n-1, r_I) - B^I(k, r) + k \left(q_N^I(k-1, r) \right)^2 + (n-k-1) \left(q_N^I(k, r) \right)^2 \\ &\quad + \varepsilon(n-k-1) q_N^I(k, r) - (n-1) \left(q_N^I(n-2, r_I) \right)^2. \end{aligned}$$

It is convenient to proceed in two steps. We first prove that $B^I(n-1, r_I) - B^I(k, r)$ is positive and then prove that the remaining terms are also positive. We begin by analyzing the non-drastic case, where $(-2a + 2c + (a - c + \varepsilon)\gamma < 0)$.

If $\varepsilon \geq r > \varepsilon + \frac{-2a+2c+(a-c+\varepsilon)\gamma}{\gamma k}$, we have $r \leq r_I = \frac{\varepsilon(n-k-1)+kr}{n-1} \leq \varepsilon$. It is easy

to check that

$$B^I(n-1, r_I) - B^I(k, r) = \frac{k(1-\gamma)(n-k-1)(\varepsilon-r)^2}{(2-\gamma)^2(n-1)} \geq 0,$$

$$\begin{aligned} & k \left(q_N^I(k-1, r) \right)^2 + (n-k-1) \left(q_N^I(k, r) \right)^2 - (n-1) \left(q_N^I(n-2, r_I) \right)^2 \\ &= \frac{\gamma^2 k(n-k-1)(\varepsilon-r)^2}{(2-\gamma)^2(n-1)(2+\gamma(n-1))^2} \geq 0. \end{aligned}$$

If $\frac{(a-c)(-2+\gamma)+\varepsilon\gamma k}{\gamma(k-1)} < r \leq \varepsilon + \frac{-2a+2c+(a-c+\varepsilon)\gamma}{\gamma k}$, then

$$r_I = \frac{(a-c+\varepsilon)(1+k-n)(-2+\gamma)+k(2+\gamma(n-1))r}{(2+\gamma k)(n-1)},$$

where $\varepsilon > r_I > r$ and $q_N^I(k, r) = 0$.

We must now distinguish two cases. First

$$\text{if } \frac{(2+\gamma k)(n-1)(\varepsilon + \frac{-2a+2c+(a-c+\varepsilon)\gamma}{\gamma(n-2)} - \frac{(a-c+\varepsilon)(-2+\gamma)(1+k-n)}{(2+\gamma k)(n-1)})}{k(2+\gamma(n-1))} < r \leq \varepsilon + \frac{-2a+2c+(a-c+\varepsilon)\gamma}{\gamma k},$$

we have that $q_N^I(n-2, r_I) > 0$. It follows directly that:

$$B^I(n-1, r_I) - B^I(k, r) = \frac{(1-\gamma)k(n-k-1)((a-c+\varepsilon)(-2+\gamma)+2r)^2}{(2-\gamma)^2(2+\gamma k)^2(n-1)} \geq 0. \quad (4)$$

Moreover,

$$kq_N^I(k-1, r) - (n-1)q_N^I(n-2, r_I) \geq 0,$$

which implies that

$$k \left(q_N^I(k-1, r) \right)^2 - (n-1) \left(q_N^I(n-2, r_I) \right)^2 > 0$$

.

$$\text{Second, if } \frac{(a-c)(-2+\gamma)+\varepsilon\gamma k}{\gamma(k-1)} < r \leq \frac{(2+\gamma k)(n-1)(\varepsilon + \frac{-2a+2c+(a-c+\varepsilon)\gamma}{\gamma(n-2)} - \frac{(a-c+\varepsilon)(-2+\gamma)(1+k-n)}{(2+\gamma k)(n-1)})}{k(2+\gamma(n-1))},$$

then $q_N^I(n-2, r_I) = 0$. The previous calculations applied to this case prove the result.

If $r \leq \frac{(a-c)(-2+\gamma)+\varepsilon\gamma k}{\gamma(k-1)}$, we have that $q_N^I(n-2, r_I) = 0$ and $q_N^I(k-1, r) = 0$. Then

$\pi^I(n, r_I) - \pi^I(k, r) = B^I(n-1, r_I) - B^I(k, r)$, which is positive by (4).

Next we analyze the case of a drastic innovation, where $(-2a + 2c + (a - c + \varepsilon)\gamma > 0)$.

In this case, non-licensees do not produce and so we have $\pi^I(n, r_I) - \pi^I(k, r) = B^I(n - 1, r_I) - B^I(k, r)$.

If $\frac{(a - c + \varepsilon)(2 - \gamma)}{2} \leq r \leq \varepsilon$, then only the incumbent patentee produces, therefore, $r_I = r$ and $\pi^I(n, r_I) - \pi^I(k, r) = B^I(n - 1, r_I) - B^I(k, r) = 0$.

If $r < \frac{(a - c + \varepsilon)(2 - \gamma)}{2}$, the licensees and the patentee are active. As a result,

$$\pi^I(n - 1, r_I) - \pi^I(k, r) = B^I(n - 1, r_I) - B^I(k, r) = \frac{(1 - \gamma)k(n - k - 1)((a - c + \varepsilon)(-2 + \gamma) + 2r)^2}{(2 - \gamma)^2(2 + \gamma k)^2(n - 1)} \geq 0.$$

Proof of Proposition 2

$$r_1^I = \arg \max_r \{ \Pi_P^I(n - 1, r) + (n - 1) (\Pi^I(n - 1, r) - \Pi_N^I(n - 2, r)) + (n - 1)r q^I(n - 1, r) \}$$

$$r_2^I = \arg \max_r \{ \Pi_P^I(n - 1, r) + (n - 1)\Pi^I(n - 1, r) + (n - 1)r q^I(n - 1, r) \}$$

If $\varepsilon = \varepsilon_2^I$, then $r_1^I = r_2^I$. This implies that the optimal royalty is r_1^I when $\varepsilon \leq \varepsilon_2^I$ and r_2^I otherwise. On the other hand, $\varepsilon < \varepsilon_1^I$ and $\gamma < \frac{2}{n - 1}$, $r_1^I > \varepsilon$, hence the participation constraint is binding and the optimal royalty is ε .

The value of $B^I(\varepsilon)$ when $\varepsilon \leq \varepsilon_2^I$

$$\begin{aligned}
& \left(\frac{1}{(-2+\gamma)^2(2+\gamma(n-1))^2} \right) \\
& ((-2+\gamma)\gamma(2+\gamma(n-1))(n-1)(a(-2+\gamma)(-2+\gamma(n-1)) \\
& +c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1)(4+\gamma(-8+\gamma+2n)))((a-c+\varepsilon) \\
& \gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+ \\
& +\varepsilon(n-1)(4+\gamma(-8+\gamma+2n)))) \\
& (-2+\gamma)+\frac{4+\gamma(-8+\gamma+2n))}{4+4\gamma(n-2)+\gamma^2(7+(n-7)n)})) \\
& (2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n)))-(2+\gamma(n-2))(n-1) \\
& \gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1) \\
& (4+\gamma(-8+\gamma+2n))) \\
& (\varepsilon-\frac{4+\gamma(-8+\gamma+2n))}{2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n))}) \\
& (2a(-2+\gamma)-2c(-2+\gamma)+\varepsilon(-2+\gamma n+ \\
& \gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1) \\
& (4+\gamma(-8+\gamma+2n))) \\
& \frac{4+\gamma(-8+\gamma+2n))}{4+4\gamma(n-2)+\gamma^2(7+(n-7)n)}- \\
& \gamma^2(-2+n)\left(\frac{\gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1) \\
& (4+\gamma(-8+\gamma+2n)))}{2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n))}\right)+ \\
& (((a-c+\varepsilon)(-2+\gamma)\left(\frac{\gamma^2(-2+n)\left(\frac{\gamma(a(-2+\gamma)(-2+\gamma(n-1))+c(-4+\gamma^2-(-2+\gamma)\gamma n)+\varepsilon(n-1) \\
& (4+\gamma(-8+\gamma+2n)))}{2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n))}\right)}{\frac{\varepsilon(n-1)(4+\gamma(-8+\gamma+2n))}{2(4+4\gamma(n-2)+\gamma^2(7+(n-7)n))}}\right)\right)^2)
\end{aligned}$$

The values of the optimal R&D investment.

$$\widehat{\varepsilon} = \frac{(a-c)(n(8+8\gamma(n-2)+\gamma^3(n-1)+2\gamma^2(5+(n-5)n)))}{((2+\gamma(n-1))(-(-2+\gamma)^2n+4d(4+\gamma(4(n-2)+\gamma(6+\gamma(n-1)+(n-6)n)))));}$$

$$\begin{aligned}
& -(a-c)((-2+\gamma)(\gamma^5(n-1)^3-32n \\
& +2\gamma^4(n-1)^2(3+(n-5)n)-16\gamma(1+n(3n-7)) \\
& -4\gamma^3(n-2)(-2+n(10+(n-8)n))-8\gamma^2(-3+n(19+n(3n-16)))))) \\
\widehat{\varepsilon}_I = & \frac{(\gamma^6(n-1)^3+4\gamma^5(n-4)(n-1)^3+64n+64\gamma(n-3)n+16\gamma^2n(16+(n-11)n) \\
& -16\gamma^3(-2+(n-3)n(3n-5))-4d(-2+\gamma)^2(2+\gamma(n-1))^2 \\
& (4+4\gamma(n-2)+\gamma^2(7+(n-7)n))-4\gamma^4(11+n(-41+n(43+(n-15)n))))}{.}
\end{aligned}$$

Proof of Lemmas 1 and 2 for a general demand and homogenous goods

Assume we have n firms and market demand is given by $P(X)$, where $P'(X) < 0$ and $XP(X)$ is concave. Firms have constant marginal costs. Denote by C the sum of marginal costs. Then, in an interior equilibrium, we have:

$$nP(X) - C + P'(X)X = 0. \quad (5)$$

The profits of a firm i with marginal cost c are given by

$$\pi(c, \bar{C}) = \frac{(P(X(\bar{C})) - c)^2}{-P'(X(\bar{C}))},$$

where $\bar{C} = \sum_{j \neq i} c_j$ and $X(C)$ is implicitly defined in (5). We have that $\frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} > 0$.

We start with Lemma 1. In the case of homogenous goods, $B(n, r_E) - B(k, r) = 0$. Then, a sufficient condition for the laboratory to sell to all firms is

$$k(\pi(c, C^*) - \pi(c, \bar{C})) - (n-k)(\pi(c, \bar{C}) - \pi(c, \underline{C})) \geq 0, \quad (6)$$

where $C^* = (n-k)c + (k-1)(c-\varepsilon+r)$, $\bar{C} = (n-1)(c-\varepsilon+r_E)$, and $\underline{C} = (n-k-1)c + k(c-\varepsilon+r)$.

We also have that $C^* - \bar{C} = \frac{(n-k)(\varepsilon-r)}{n}$ and $\bar{C} - \underline{C} = \frac{k(\varepsilon-r)}{n}$. Hence, (6) can be rewritten as

$$\frac{\pi(c, C^*) - \pi(c, \bar{C})}{\pi(c, \bar{C}) - \pi(c, \underline{C})} = \frac{\int_{\bar{C}}^{C^*} \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} dC}{\int_{\underline{C}}^{\bar{C}} \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} dC} \geq \frac{n-k}{k}.$$

A sufficient condition for this to hold is that $\frac{\partial^2 \pi(c, \bar{C})}{\partial \bar{C}^2} > 0$. Then

$$\frac{\int_{\bar{C}}^{C^*} \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} dC}{\int_{\underline{C}}^{\bar{C}} \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} dC} > \frac{(C^* - \bar{C}) \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}}}{(\bar{C} - \underline{C}) \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}}} = \frac{n-k}{k}$$

For the case of an internal patentee (Lemma 2), a similar line of argument applies. Here, we have that $B^I(n-1, r_I) - B^I(k, r) = 0$. Then, a sufficient condition for the patentee to sell to all firms is:

$$k(\pi(c, C^*) - \pi(c, \bar{C})) - (n-k-1)(\pi(c, \bar{C}) - \pi(c, \underline{C})) \geq 0 \quad (7)$$

where $C^* = c - \varepsilon + (k-1)(c - \varepsilon + r) + (n-k-1)c$, $\bar{C} = c - \varepsilon + (n-2)(c - \varepsilon + r_I)$, and $\underline{C} = c - \varepsilon + k(c - \varepsilon + r) + (n-k-2)c$. We also have that $C^* - \bar{C} = \frac{(n-k-1)(\varepsilon-r)}{n-1}$ and $\bar{C} - \underline{C} = \frac{k(\varepsilon-r)}{n-1}$.

Now (6) can be rewritten as

$$\frac{\pi(c, C^*) - \pi(c, \bar{C})}{\pi(c, \bar{C}) - \pi(c, \underline{C})} = \frac{\int_{\bar{C}}^{C^*} \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} dC}{\int_{\underline{C}}^{\bar{C}} \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} dC} \geq \frac{n-k-1}{k}$$

A sufficient condition for this to hold is that $\frac{\partial^2 \pi(c, \bar{C})}{\partial \bar{C}^2} > 0$. Then,

$$\frac{\int_{\bar{C}}^{C^*} \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} dC}{\int_{\underline{C}}^{\bar{C}} \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}} dC} > \frac{(C^* - \bar{C}) \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}}}{(\bar{C} - \underline{C}) \frac{\partial \pi(c, \bar{C})}{\partial \bar{C}}} = \frac{n-k-1}{k}$$

It is tedious but direct to show that $\frac{\partial^2 \pi(c, \bar{C})}{\partial \bar{C}^2} > 0$ if $-P'(X)$ is log-concave and $P'''(X) \leq 0$.

We show that this also holds for the class of demands $P = A - X^b$, where $b \geq 1$.

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