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Do first-year University students understand the language of Mathematics?

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Abstract

The aim of the project is to determine if the understanding of the language of Mathematics of students starting university is propitious to the development of an appropriate cognitive structure. The objective of this current work was to analyse the ability of first-year university students to translate the registers of verbal or written expressions and their representations to the registers of algebraic language. Results indicate that students do not understand the basic elements of the language of Mathematics and this causes them to make numerous errors of construction and interpretation. The students were not able to associate concepts with definitions and were unable to offer examples.

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Keywords: The language of mathematics, register, difficulties, symbol.

1. Introduction

This paper is the result of a research project, jointly undertaken by the Faculty of Agronomy of the National University of Central Buenos Aires (Azul) and the Faculty of Chemical Engineering (Rosario) of the Catholic University of Argentina, entitled "An analysis of the Language of Mathematics and its influence on the Validation Processes of University Students of Engineering**".**

In order to formulate its concepts and developments, Mathematics uses its own system of codes and symbols which facilitates the transmission of ideas with clarity and precision. The comprehension of the concepts and developments of this science requires an understanding of the syntax and semantics of that system of codes and symbols. The language of Mathematics is an essential instrument for the formation of concepts and procedures; it not only fulfils a communicative function whose only objective is the establishment of an unobstructed dialogue between the student and the teacher, but it should also be considered as an environment for analysis and the optimisation of mathematical activity. From the point of view of communication, the most important characteristic

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of Mathematics is its rigorous language which is linked to the fact that its concepts are abstract notions whose representations are determined as much by semiotics as by noetics (Duval, 1998). The relationships between the symbols and signs therefore, depends on the conceptual domain in which they are found. However simple they may be, mathematical expressions are semiotic registers that determine meaning (semantics), irrespective of the manner in which they are represented (syntax). These meanings are mediated, fundamental concepts that are the foundations for the construction of mathematical knowledge.

The solution of mathematical problems depends firstly on understanding the problem formulation and subsequently on converting the information presented: from a discursive description of the objects to a symbolic transcript (numerical or literal) of their relationships, that is to say, a symbolic model of the situation. This should not be considered as an automatic and direct process. Moreover, the student should not be expected to successfully convert between registers, even if he/she can work effectively in the initial and final registers and carry out operations involving the representations on an individual basis (Sastre Vázquez, Boubée, Rey and Delorenzi, 2008).

An important factor that contributes to the difficulties experienced by students solving a mathematical problem is a failure to 'take the first step': the exercise of reading and comprehending the formulation of the problem. Full understanding is the foundation to construct the subsequent solution, which can also cause problems, albeit of a different kind (Sastre Vázquez et al, 2008). Among the many shortcomings evidenced by students, one of the most striking is an inability to express their knowledge and ideas in colloquial language. Students are not able to write coherently, have little awareness of synonyms and use a vocabulary that lacks clarity and precision.

The results presented in this study are derived from the first phase of a research project undertaken by the Faculty of Agronomy of the National University of Central Buenos Aires (UNCPBA) and the Faculty of Chemical Engineering "Fray Rogelio Bacon", UCA, in the Province of Santa Fé, Rosario. The aim of the project is to determine if a good command of the language of Mathematics when starting University helps students to develop an appropriate cognitive structure. Does this understanding affect students' conceptualisations of mathematical objects? Does it influence validation processes? The ability of the student to translate the registers of verbal or written expressions (propositional language) and their representations to the registers of Algebraic language (use of mathematical symbols) are of particular interest.

This paper analyses the results obtained from a pilot survey which was designed as an initial stage in statistical data collection and handling for the joint research project. The results were used to determine the difficulties and obstacles faced by first-year university students in understanding symbolic and natural language. When presented with the questions, it was hoped that students would become aware of any gaps in their knowledge and, thereby, recognise the need to overcome their shortcomings to be able to undertake Mathematics courses at University level. The specific objectives were to determine: the level of understanding of mathematical symbols and signs in first-year university students, and the difficulties faced by the students when reading a mathematical expression, understanding the representations of a concept and the meanings of mathematical symbols and signs.

2. Methodology

The data was collected by means of a diagnostic survey, based on the proposal of Ortega (2002) with the modifications and adaptations that were considered necessary for the purposes of this paper. The first section requested information on: the personal details of the student; the quality of the Mathematics teaching that they had received; their opinion on the usefulness of mathematics; whether they liked or enjoyed studying mathematics. The second section asked questions regarding the language of Mathematics: knowledge of the most common symbols and mathematical formulations that appear in textbooks. More specifically, the questions of the second section referred to:

1)Knowledge of mathematical symbols: Students were asked to give the meaning of ten mathematical symbols or logograms (Pimm, 1990): $\forall; \exists; \in; \subset; \notin; \Rightarrow; \Leftrightarrow; \sum; \cup; \cap$. They were also asked if the symbols were familiar to them and if they had ever used them. 2) Knowledge of mathematical definitions: Matching ten concepts with the most suitable description. 3)The interpretation of symbolic expressions and explanation of its meaning. 4) Writing an expression in symbolic language, relating mathematical concepts and colloquial language.

Items in the first section were set out in five categories (Very Poor, Poor, Average, Good, Very Good) and results for the second section were divided into three categories (Poor, Average, Good).

3. Results.

Student survey results on the quality of teaching received and attitudes towards mathematics.

Table 1. Quality of teaching received

Quality of teaching received					
Cat.	Very poor	Poor	Average	Good	Very good
%	2 %	1 %	37 %	53 %	7 %
Enjoyment					
Cat.	None	Not much	Average	Quite a lot	Very much
%	1 %	7 %	46 %	38 %	8 %
Difficulty					
Cat.	Very difficult	Difficult	Average	Easy	Very easy
%	0 %	12 %	71 %	17 %	0 %
Interest					
Cat.	Not at all	Not very	More or less	Quite	Very
%	0 %	4 %	22 %	59 %	15 %
Usefulness					
Cat.	Not at all	Not very	More or less	Quite	Very
%	0 %	1 %	6 %	31 %	62 %

Results concerning knowledge of different symbols and formulations (taken as independent elements) of the language of mathematics:

Table 2. Percents of affirmative answers regarding the knowledge of the knowledge of the different types of symbols and propositions.

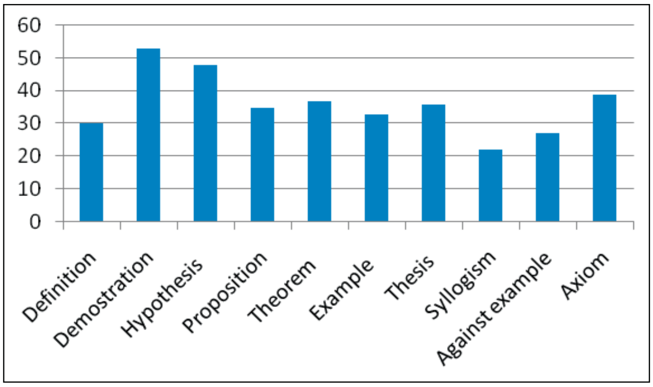
Symbol	Is it Family for you?	Have you used it?	Do you know the meaning?	Does the student describe the meaning well?
$\forall$	57	51	54	4
$\exists$	43	40	50	8
$\in$	30	31	44	16
$\subset$	54	49	49	9
$\notin$	30	30	41	18
$\Rightarrow$	34	31	37	12
$\Leftrightarrow$	35	34	43	13
Σ	31	29	42	12
$\cup$	40	35	40	11
$\cap$	41	36	43	12

Most students gave negative responses or failed to answer the questions on the familiarity and use of the symbols. There were no symbols for which the number of positive responses to familiarity and use was greater than the number of negative replies and unanswered questions. The most familiar and most used symbols were $\notin$, $\in$ and Σ; no other symbol received more than 40% of positive replies. Most students were unable to give the meaning of the symbols or did not respond to the question. The best understood symbols were $\notin$ and $\in$ (29% and 26%). These symbols were also those which received the highest percentages of correct interpretations (13% and 15%,). Between 69% and 93% of respondents could not describe the meaning of the symbols.

Table 3. Affirmative answers (%) regarding the knowledge of different types of symbols and propositions.

Concept	Category			Concept	Category		
	Yes	No	W/A.		Yes	No	W/A.
Definition	39 %	30 %	31 %	Example	9 %	33 %	58 %
Demostration	9 %	53 %	38 %	Thesis	1 %	36 %	63 %
Hypothesis	11 %	48 %	41 %	Syllogism	9 %	22 %	69 %
Proposition	8 %	35 %	57 %	Against example	17 %	27 %	56 %
Theorem	6 %	37 %	57 %	Axiom	2 %	39 %	59 %

Figure 1. Affirmative answers (%) regarding the knowledge of different types of symbols and propositions.



Most of respondents were not able to recognise the definitions of the concepts. The concept 'definition' received the highest percentage of correct answers (39%); 'thesis' and 'axiom' were the least recognised concepts (only 1% and 2%). The least recognised concept was 'demonstration' which 53% of fails. There were high percentages of unanswered questions – between 31% and 69%. Given that the term "example" is common and used daily by students, the low percentage of correct answers (9%) is surprising; even the explicitly linked concept 'counterexample' was better recognised (17%).

Table 4: Interpreting symbolic expressions.

$\sum_{j=1}^{5} j^2 = 12$				
Cat.	Wrong	Regular	Right	No Answer
%	5 %	1 %	9 %	85 %
$\exists x \in R / (x-1)(x-2)(x-3)(x^5-9) = 0$				
Cat.	Wrong	Regular	Right	No Answer
%	8 %	1 %	4 %	87 %
$R = Q \cup Z$				
Cat.	Wrong	Regular	Right	No Answer
%	13 %	0 %	3 %	84 %
$R^+ \subset R$				

Cat.	Wrong	Regular	Right	No Answer
%	7 %	0 %	6 %	87 %
$\forall x \in R : \forall y \in R : x^2 = y^2 \Rightarrow x = y$				
Cat.	Wrong	Regular	Right	No Answer
%	9 %	0 %	3 %	88 %

In Table 4, most students were unable to interprete the mathematical expressions. More than 84% failed to respond to any of the expressions. The first expression had most responses, although only 9% recognised that it was erroneous.

4. Conclusions

Generally speaking, most students expressed their satisfaction with the Mathematics teaching they had received. They recognized the usefulness of the discipline and demonstrated an interest in it. Mathematics is considered to be of intermediate difficulty: neither very easy nor very difficult. The results of this pilot study indicate that on entering University, students have a very limited understanding of mathematical symbols.

The response rate was very low in the exercises which asked students to identify mathematical concepts, interpret symbolic expressions and translate and articulate the different languages (natural and symbolic). Consequently, it is difficult to draw any conclusions to establish and characterize the shortcomings of the students as regards these skills. From the partial results obtained in this study, it can, however, be concluded that students need to overcome their descriptive difficulties if they are to successfully complete their University studies. In short, the results appear to show that students do not understand the basic elements of the language of Mathematics which leads to numerous construction and interpretation errors. The students were unable to associate concepts with their definitions and much less of providing examples.

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