Essays on Macroeconomic Fluctuations

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To my mother and Iryna.
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0.1 Introducción

La última recesión económica global y la actual crisis de endeudamiento en la Unión Europea han incrementado notablemente el interés por entender los orígenes y los mecanismos subyacentes liderando la propagación de recesiones. Dado que esta información puede ser útil para mitigar los efectos adversos de los episodios contractivos a través de apropiadas políticas de estabilización, investigadores académicos y responsables de la política económica han empezado a observar atentamente la desagregación del ciclo económico con el objetivo de explorar los tipos de interconexiones predominantes entre sub-economías, a nivel regional.

Esta tesis doctoral se enfoca en analizar algunos aspectos de las fluctuaciones macroeconómicas, tales como los orígenes, la propagación y el pronóstico de las recesiones económicas, junto con la interacción entre ciclos económicos reales y nominales, además del pronóstico en tiempo real de la producción en una economía en términos nominales. Los enfoques metodológicos usados para evaluar dichas cuestiones se basan fundamentalmente en la unificación entre dos conceptos fundamentales. Primero, las dos características que definen a los ciclos económicos, comovimientos y no-linealidades. Segundo, el grado y tipo de interconexión entre un conjunto de sub-economías que pertenecen a un ciclo económico desagregado por medio del análisis de redes.

El primer capítulo, desarrolla una metodología útil para monitorear el grado de sincronización entre muchos procesos estocásticos que están sujetos a cambios de regímenes utilizando una red que varía a través del tiempo con dinámicas Markovianas. Una aplicación empírica dedicada al estudio de la sincronización de las fases de los ciclos económicos en los estados de Estados Unidos sugiere que las recesiones nacionales podrían ser anticipadas por un índice que mide la gran heterogeneidad existente en las recesiones regionales. Además, la manera en la cual una recesió
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cional venidera podría afectar a cada una de las sub-economías, a nivel de los estados, puede ser dinámicamente evaluada. De los resultados también se intuye que uno de los determinantes fundamentales gobernando la sincronización de los ciclos económicos, a parte de la similitud entre la composición industrial de las sub-economías, es su correspondiente ubicación geográfica.

El segundo capítulo, toma en cuenta que dada la heterogeneidad de recesiones, determinar que parte de la economía es el principal origen de cada episodio contractivo es crucial para los responsables de la política económica. Por lo tanto, en este capítulo se desarrolla un modelo probabilístico basado en comovimientos y no linealidades usado para obtener inferencias simultaneas sobre tres asuntos de las fluctuaciones macroeconómicas en Estados Unidos. ¿Cuando ocurren simultáneamente las fases de los ciclos de actividad real y ciclos de inflación?, ¿Cuál es el grado de sincronización entre ambas fases? Y ¿Qué parte de la economía es la más afectada durante cada una de dichas fases? Las respuestas a estas preguntas, permitirán fechar y categorizar episodios contractivos entre recesiones de demanda, recesiones de oferta y recesiones mixtas, basándose en la variación dinámica de la contribución de choques al ciclo económico.

Finalmente, el tercer capítulo se enfoca en la situación restrictiva que han estado enfrentando durante los últimos años los responsables de la política económica en Estados Unidos y otras economías desarrolladas, el haber alcanzado el nivel del límite inferior en la tasa de interés. Dada dicha situación algunos distinguidos expertos en política económica han sugerido que una alternativa no convencional como establecer al PIB nominal como objetivo, podría ser la solución para disminuir las altas tasas de desempleo en dichas economías. Por lo tanto, este capítulo se enfoca en proveer pronósticos tempranos de la tasa de crecimiento del PIB nominal actual para la economía de Estados Unidos. Dichos pronósticos son computadas usando exactamente el conjunto de información que poseen los responsables de la política
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económica en el momento que los pronósticos son realizados. El análisis explora la habilidad predictiva de varios modelos univariados y multivariados, enfocándose además en encontrar los indicadores más adecuados en la realización de dicha tarea. Los resultados muestran que, entre los candidatos propuestos, un modelo de factores dinámicos de pequeña escala que contiene información sobre la actividad real económica, la dinámica de inflación y agregados monetarios de Divisia, produce los pronósticos del PIB nominal más precisos.

Capítulo I. Monitoreo de Sincronización de Recesiones Regionales

El interés in identificar los cambios en la sincronización de los ciclos económicos empezó a incrementarse marcadamente desde la implementación de la Unión Monetaria Europea, dado que era de esperarse que países experimentando una mayor sincronización entre ellos, mostraran costos menores al incorporarse a la unión, que aquellos países experimentando ciclos menos sincronizados, Camacho et al. (2006). En otras palabras, analizar cambios de sincronización es crucial para los responsables de la política económica para poder determinar los países, regiones o inclusive sector de una misma economía, que podría ser más influyenciados por políticas económicas globales o choques económicos agregados.

Dada la naturaleza asimétrica de los ciclos económicos, desde el trabajo seminal de Hamilton (1989), en el cual las fases de la economía de Estados Unidos son caracterizadas utilizando un modelo de cambios de regímenes con dinámicas Markovianas (MS), un gran número de extensiones de este enfoque han sido propuestas por diversos autores debido a su gran éxito. En particular, los modelos multivariados MS se han convertido en una herramienta muy útil para analizar sincronización entre las fases del ciclo económico en diferentes países, como es el caso de Smith y Summers (2005) y Camacho y Perez-Quirós (2006), o en diferentes regiones del mismo país,
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Owyang et al. (2005) y Hamilton y Owyang (2012). A pesar de que estos estudios proveen una visión global acerca de cuan sincronizados están los ciclos económicos de varias economías, no son capaces de capturar endógenamente potenciales cambios de sincronización. Estos es debido a que, con el objetivo de preservar la parsimonia en los modelos, una pregunta clave que serviría para revelar esta característica ha pasado inadvertida: ¿Cuál es la relación dinámica de dependencia entre las variables de estado no observadas que gobiernan un modelo multivariado MS?

Los enfoques seguidos en la literatura tradicionalmente asumen una relación de dependencia entre las variables de estado que es fija a través del tiempo. Las mismas pueden ser divididas en dos categorías. La primera, se refiere a estudios en los cuales dicha relación es simplemente asumida a priori en base al juicio del econometrista, es decir totalmente independiente, totalmente dependiente o que posean una relación rezagada. La segunda categoría se enfoca en realizar evaluaciones a posteriori de la sincronización entre los procesos Markovianos durante un periodo de tiempo específico, proveyendo estimadores de la relación de dependencia promedio. Ambas categorías no permiten determinar cómo puede haber variado la relación de dependencia entre las variables de estado a través del tiempo.

En este capítulo se propone una metodología útil para monitorear los cambios en el grado de sincronización global entre muchos procesos estocásticos, los cuales están sujetos a cambios de regímenes. Específicamente, el método consiste en computar las inferencias sobre los regímenes de un modelo multivariado MS y simultáneamente obtener una medida de la sincronización entre las variables de estado gobernando los procesos estocásticos para cada periodo del tiempo. Dicha medida es estimada endógenamente como un promedio ponderado entre los casos extremos de total independencia y total dependencia haciendo inferencia sobre los regímenes de alta y baja sincronización. Aparte de capturar no linealidades, otra de las principales ventajas de esta metodología es que las sincronizaciones en pares de economías, las cuales son
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estimadas a través de métodos Bayesianos, pueden ser fácilmente transformadas en medidas de desincronización para luego ser combinadas con análisis de escalamiento multidimensional dinámico y análisis de redes, con el objetivo de evaluar los posibles cambios en el patrón de agrupamiento que podrían experimentar un sistemas de muchas series de tiempo y los componentes claves gobernando el sistema.

La metodología propuesta es utilizada en una aplicación empírica en la cual se analizan los cambios mensuales en el grado de sincronización en pares y global entre las fases económicas de los estados de Estados Unidos y el ciclo económico nacional. Los datos utilizados son indicadores coincidentes de actividad económica de cada uno de los 48 estados, los cuales son calculados y publicados, con un rezago de no más de 2 semanas, por el Banco de la Reserva Federal de Filadelfia. El periodo analizado corresponde desde Agosto de 1979 hasta Marzo del 2012.

La primera parte del análisis se enfoca en obtener la sincronización dinámica en pares de economías. Dado que se tienen 48 economías será necesario analizar todas las posibles combinaciones de pares entre ellas, lo que conlleva estimar 1128 diferentes modelos. El método de estimación es por medios Bayesianos, específicamente siguiendo el procedimiento de Gibbs sampling. Adicionalmente, se estimarán 48 modelos que corresponden a la sincronización dinámica entre el indicador coincidente de estado y ciclo económico nacional, para poder determinar cambios en la concordancia entre los ciclos económicos regionales y el nacional. Los resultados muestran que existen economías tales como Louisiana, Oklahoma y Wyoming que experimentan una sincronización baja y relativamente constante a través del tiempo. Mientras que otras economías tales como Alabama, Minnesota, Nebraska, South Carolina, Tennessee y Vermont muestran una sincronización alta y relativamente constante con respecto al ciclo económico nacional. Por otra parte, el resto de economías usualmente experimentan abruptos cambios de sincronización que toman lugar antes o después de las recesiones nacionales. Dichas características concuerdan con los resultados en
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Hamilton y Owyang (2012) quienes claman que las diferencias en sincronización a través de estados parece ser una cuestión de tiempo. Por medio de este análisis, se puede evaluar cuan afectado podría ser el estado “A” por un choque económico que está ocurriendo en el estado “B”, o que estados sería más afectados por una recesión económica venidera, tomando en cuenta que además esta información puede ser actualizada de manera mensual.

Sin embargo, para analizar cómo afectaría una recesión nacional a todas sus sub-economías simultáneamente, necesitaríamos un enfoque unificado. Para obtener una visión más global y unificada de los resultados obtenidos hasta ahora, se utilizará la técnica conocida como escalamiento multidimensional dinámico, la cual es una metodología que permite representar elementos de grandes dimensiones, como matrices, en sistemas de coordenadas de menores dimensiones, como por ejemplo un plano, en el cual las distancias entre dos puntos de dicho plano o mapa representarán el grado de desincronización entre dos economías. De esta manera se pueden obtener mapas de sincronizaciones para cada año con el objetivo de analizar su evolución. Por ejemplo, durante la recesión de 1990, los 48 estados estaban divididos en tres grupos, específicamente, un grupo se encontraba altamente sincronizado con el ciclo económico nacional y por tanto siendo afectado de gran manera por dicha recesión, el segundo grupo estaba medianamente sincronizado con el ciclo económico nacional, mientras que las economías correspondientes al tercer grupo se encontraban experimentando dinámicas completamente independientes con respecto a los otros estados y más aun con respecto al ciclo económico nacional.

El objetivo final de esta aplicación se centra en utilizar las distancias en los mapas de sincronización para combinarlo con el análisis de redes y poder inferir el grado de interconexión global entre todas las sub-economías de una manera unificada. Esto permitirá averiguar hasta qué punto la información contenida en la sincronización de recesiones regionales podría ser útil para predecir recesiones nacional. Para deter-
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Minar las sub-economías, en este caso interpretadas como los nodos de la red, más prominentes, esta metodología se basa en la medida de centralidad de cercanía. Dicha medida provee información sobre cuán cercano estaría cada nodo con respecto a los otros nodos restantes en la red, lo que para este contexto tomaría la interpretación de la sincronización simultánea de una sub-economía con respecto al resto. Luego de haber obtenido las centralidades de cercanía de cada nodo para cada periodo de tiempo, se puede obtener la centralidad de toda red, la cual se puede interpretar como un índice de sincronización global de las economías regionales de Estados Unidos. Una vez computado dicho índice, se puede observar que posee una marcada tendencia a incrementarse algunos meses antes de que las recesiones nacionales ocurran, manteniéndose en valores altos durante todo el periodo de recesión e inclusive algunos meses después de que dicha recesión haya finalizado. Con el objetivo de evaluar la capacidad predictiva de este índice para anticipar recesiones nacionales, el índice fue reestimado dos veces, cada una utilizando solo datos hasta un mes antes de que ocurrieran las dos últimas recesiones nacionales, para simular las condiciones en tiempo real. Los resultados corroborando su confiabilidad, dado que los incrementos en el índice, efectivamente se realizaban algunos meses antes de las dos últimas recesiones.

Con esto podemos concluir que este capítulo provee una metodología útil para monitorear los cambios en sincronización de muchos procesos estocásticos que están sujetos a cambios de régimen utilizando una red ponderada con dinámicas Markovianas. Esta metodología es aplicada al análisis de la sincronización de los ciclos económicos regionales en los Estados Unidos, al nivel de estados. Los resultados ofrecen una herramienta útil para evaluar el grado en el cual una recesión nacional venidera podría afectar simultáneamente a cada una de sus sub-economías regionales. Además, se obtiene un índice que mide la sincronización regional global de este país, el cual es útil para anticipar recesiones nacionales.
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Capítulo II. Ciclos Reales versus Ciclos nominales: Un enfoque de dos factores dinámicos con cambios Markovianos de regímenes.

El Buró de Investigación Económica de Estados Unidos (NBER) define el concepto de ciclo económico como periódicos pero irregulares incrementos y decrementos de actividad económica, típicamente observados en indicadores macroeconómicos tales como PIB real y Producción Industrial. Sin embargo, el Comité encargado de fechar el ciclo económico, no indaga en las causas de dichas recesiones, las cuales tradicionalmente se asume que provienen de dos orígenes diferentes. Por una parte, las recesiones que empiezan en el lado de la oferta agregada de una economía son causadas por choques contractivos de oferta los cuales típicamente afectan a los costos de producción. Por otra parte, las recesiones que empiezan en el lado de la demanda agregada de la economía son causadas por choques contractivos de demanda los cuales afectan los niveles de gasto de una economía. Para discriminar entre estos dos tipos de orígenes de recesiones económicas, es importante enfatizar que a pesar de que ambos choques causan decrementos en la actividad económica, el efecto que ambos tienen sobre el nivel de precios, es diferente.

En un trabajo seminal, Blanchard (1989) investiga si el comportamiento conjunto de las variables reales y nominales en Estados Unidos es consistente con la interpretación tradicional de las fluctuaciones macroeconómicas, es decir, que choques en la demanda (oferta) agregada ocasionarán movimientos en la misma (opuesta) dirección, encontrando un claro “si” como respuesta. En otras palabras, mientras choques contractivos de demanda están acompañados por decrementos en los precios, choques contractivos de oferta están acompañados por incrementos en el nivel de precios.

Recientemente, Aruoba y Diebold (2010) examinaron la interacción dinámica entre actividad económica real y precios a través del ciclo económico con el objetivo de extraer información acerca de los choques contractivos. Para esto, dichos autores
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propusieron dos modelos de factores dinámicos lineales por separado y usaron el filtro de Kalman para obtener extracciones óptimas de la evolución de la actividad real y nominal. De acuerdo con estos autores, la coherencia de los movimientos respectivos y la cronología del ciclo económico fechada por el NBER son la clave para determinar si los choques contractivos provienen de la demanda o de la oferta agregada de una economía.

Basándose en la ampliamente aceptada visión de que las recesiones son causadas por choques contractivos de diferente naturaleza, con la correspondiente mezcla variando substancialmente a través de recesiones, Galí (1992), Ireland (2010) y Forni y Gambetti (2010), este capítulo propone un modelo de dos factores dinámicos con cambios de regímenes que supera dos principales desventajas del análisis en Aruoba y Diebold (2010) y que permite hacer inferencia sobre el tipo de choques agregados influyendo el ciclo económico con el objetivo de descubrir los orígenes de recesiones en Estados Unidos.

Primero, a pesar de que Aruoba y Diebold (2010) examinan la interacción entre actividad real y precios, lo hacen usando modelos de factores dinámicos separados para computar los índices reales y nominales, sin tomar en cuenta la interrelación potencial entre estos dos conceptos. El modelo propuesto en este capítulo extiende el enfoque anterior y considera una estructura unificada, en el cual dos factores son extraídos del mismo conjunto de indicadores de actividad real y nominal. Por lo tanto, ambos índices, de actividad real y nominal, son endógenamente computados y la interacción entre los indicadores y los factores es estimada sin imponer restricciones considerables.

Segundo, sin tomar en cuenta la definición de Burns y Mitchel (1946), en la cual se define a los ciclos económicos como asimétricos, Aruoba y Diebold (2010) extraen factores utilizando modelos lineales. El modelo propuesto en este capítulo cuenta incluye dinámicas no lineales permitiendo a los factores ser gobernados por
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dos procesos Markovianos de cambios de regímenes potencialmente dependientes. Por lo tanto, esta propuesta es una extensión natural del modelo de factor dinámico con un único índice sujeto a cambios de regímenes propuesto por Kim and Yoo (1995), Chauvet (1998) y Kim y Nelson (1998), dado que la misma no restringe a todos los indicadores incluidos en el modelo a seguir una única dinámica no lineal común. Por consiguiente, el algoritmo usado para estimar el modelo propuesto a través de máxima verosimilitud se extiende para considerar dos factores que dependan de dos variables de estado, tratando además con asuntos relacionados a la relación de interdependencia e identificación de los factores.

El conjunto de datos a utilizarse son obtenidos del Banco de la Reserva Federal de St. Louis y del Buró de Análisis Económico de Estados Unidos. Siguiendo la línea de Aruoba y Diebold (2010) la base de datos consiste en cinco indicadores de actividad económica real, los cuales corresponden al PIB real, índice de producción industrial, ingreso personal real menos transferencias, ventas reales de manufacturas y empleo total no agrícola. Además se incluyen seis indicadores de precios, los cuales son, deflactor del PIB, índice de precios al consumidor, índice de precios al productor, índice de precios de bienes no energéticos de Standard and Poor, precio del petróleo y compensación por hora en el sector de negocios no agrícolas. El análisis empírico usa las tasas de crecimiento de los indicadores económicos mencionados.

Una vez que el modelo ha sido estimado, las cargas se asignan endógenamente a cada factor, específicamente los indicadores de actividad real poseen mayor carga en un factor, mientras que los indicadores de precios poseen mayor carga en otro factor. Estas estimaciones sugieren que un factor puede ser interpretado como un índice de actividad económica, mientras que el otro factor como un índice de inflación. Además, el grado de interdependencia entre fases de los dos índices es significativo e igual a un tercio, sugiriendo que los ciclos de actividad económica real y los ciclos de inflación coinciden aproximadamente el treinta por ciento de las veces.
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Una vez obtenidos los estimadores de los parámetros se puede proceder a la obtención de los factores por medio del filtro de Kalman. Por una parte el primer factor, el cual fluctúa a través de su media condicionada, muestra cambios en su dirección que claramente reproducen de manera bastante precisa las cronologías del ciclo económico definida por el NBER. Durante periodos de expansión, el valor del factor se incrementa alcanzando valores cercanos a su media condicionada a un estado de expansión. Durante recesiones, el factor cae drásticamente alcanzando valores cercanos a su media condicionada a un estado de recesión. Adicionalmente, el modelo proporciona las probabilidades de que el factor se encuentre en cualquiera de los dos estados de la economía., las cuales nuevamente coinciden con los periodos de recesión definidos por el NBER.

Por otra parte, la evolución del segundo factor revela dinámicas diferentes a las del primer factor. Este índice toma valores negativos en los sesentas, se incrementa rápidamente durante los setenta y mediados de los ochentas, y vuelve a ser negativo desde entonces. De acuerdo a las estimaciones de las medias condicionales de las variables de estado que gobiernan este segundo factor, la primera y la segunda parte de la muestra está gobernada por un estado, mientras que la parte de media de la muestra está gobernada por el otro estado. Adicionalmente, el modelo obtiene las probabilidades de estar en cada uno de los dos estados, las cuales son comparadas con los periodos de alta inflación documentados por el Banco de la Reserva Federal de Chicago, mostrando una alta concordancia. Esto finalmente aporta evidencia adicional para concluir que el primer factor es un índice de actividad económica real, mientras que el segundo factor es un índice de inflación.

La ventaja principal de la estructura unificada propuesta es que se pueden computar las probabilidades de recesión causadas por choques de demanda, es decir, la probabilidad de los eventos conjuntos caracterizados por periodos de baja actividad económica real y bajos precios. Específicamente, esta probabilidad se incrementa du-
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tividad económica real y altos precios. Dicha probabilidad se incrementa durante


oferta, pero terminan mostrando una alta probabilidad de choques contractivos de

demanda, lo cual es consistente con la visión de que dichas recesiones fueron causadas

por choques contractivos mixtos de demanda y de oferta.

En conclusión, el modelo propuesto en este capítulo permite categorizar las re-

cesiones fechadas por el NBER entre recesiones de demanda, recesiones de oferta y

recesiones mixtas en base a la contribución de los choques contractivos durante cada

episodio recesivo. Los resultados muestran que las recesiones son heterogéneas con

una mezcla de choques que substancialmente varía a través de los períodos de rece-


particular, la última recesión del 2007-2009, la cual es de gran interés, entra en la
categoría de las recesiones mixtas, dado que dos tercios del total del periodo contrac-
tivo, específicamente el principio y el final, estuvieron influenciados por choques de

oferta, mientras que durante el periodo medio de dicha recesión, fueron los choques
de demanda los que prevalecieron.
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Capítulo III. Pronóstico en Tiempo Real del PIB Nominal

Este trabajo está realizado de manera conjunta con Marcelle Chauvet, de la Universidad de California Riverside, y William A. Barnett, del Centro de Estabilidad Financiera, New York.

Durante los últimos años la Reserva Federal de los Estados Unidos ha alcanzado el nivel del límite inferior en la tasa de interés dado los continuos intentos para reducir la tasa de desempleo, la cual se mantienen altos niveles a pesar de que la economía está experimentando una lenta recuperación. En vista de esta situación, el Comité Federal de Mercado Abierto (FOMC) adicionalmente está usando herramientas complementarias para llevar a cabo la política monetaria, una de ellas corresponde a forward guidance (orientación hacia adelante). Como ha señalado el presidente de la Reserva Federal, Ben Bernanke, y Michael Woodford (2012) en el simposio económico anual Jackson Hole, esta herramienta consiste en declaraciones explícitas que realiza un banco central acerca de sus acciones futuras para especificar la evolución de la economía, adicionalmente a los anuncios acerca de las acciones de política inmediata a las cuales dicho banco está comprometido.

La estrategia forward guidance podría llevar a cambios en las expectativas futuras acerca de la evolución económica que podrían mejorar la situación presente, dependiendo del objetivo y la regla que los bancos centrales estén comprometidos a seguir. Para el caso de Estados Unidos, como ha sido sugerido por varios expertos en el tema, tales como Woodford (2012), Romer (2011) y Hall y Mankiw (1994), entre otros, la Reserva Federal debería comenzar a considerar como objetivo prioritario el control de la senda del PIB nominal, dado que ellos consideran que esta estrategia constituiría una herramienta poderosa de comunicación. Durante la última recesión, la trayectoria del PIB nominal sufrió una drástica contracción causada por varias significativas tasas de crecimiento negativas. Dado que el PIB nominal es la producción real de una economía multiplicada por el nivel de precios, establecer el objetivo de


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hacer retornar el PIB nominal a su trayectoria previa a la crisis podría mejorar las expectativas acerca de las condiciones económicas futuras. Dichas expectativas aumentarían los incentivos a incrementar el consumo presente por parte de los hogares y también las empresas serían más optimistas acerca de las decisiones de inversión presente.

Bajo un escenario de PIB nominal objetivo, los pronósticos de la tasa de crecimiento actual de dicha variable juegan un papel fundamental en monitorear sus continuas variaciones con el fin de evaluar la efectividad de dicha política. El trabajo seminal de Croushore y Stark (2001) enfatiza el uso de datos colectados en tiempo real para obtener resultados robustos al momento de realizar análisis de política económica y hacer pronósticos. Este parece ser el punto de partida de una literatura creciente acerca de pronóstico de variables macroeconómicas utilizando el conjunto de información exacto que se encuentra disponible al momento en el cual los análisis son llevados a cabo.

Sin embargo, el uso de datos en tiempo real podría traer algunas complicaciones cuando se requiere el uso de modelos multivariados, tales como la incorporación de series de tiempo en diferentes frecuencias y bordes en la base de datos. Recientemente, nuevos modelos econométricos de pronóstico han sido propuestos justamente para tratar con este tipo de problemas. La mayoría de los trabajos más relevantes utilizan modelos multivariados para los cuales son utilizadas representaciones de espacio de estados con el objetivo de tratar con observaciones faltantes para luego poder utilizar el filtro de Kalman y obtener inferencias óptimas sobre el comovimiento entre las variables usadas. La información contenida en dicho comovimiento será usada en predecir la variable objetivo. Es importante mencionar que este tipo de modelos ha sido mayoritariamente usado con el fin de obtener pronósticos precisos del PIB real, mostrando resultados satisfactorios, y que hasta ahora no han sido utilizados para obtener pronósticos del PIB nominal.
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Dado que el objetivo en este capítulo es hacer disponible información que pueda ser usada para llevar a cabo la política monetaria, nuestro enfoque será exclusivamente la dinámica del PIB nominal. Debido a la importancia de pronósticos tempranos de la tasa de crecimiento trimestral actual del PIB nominal, exploraremos modelos univariados y multivariados con el objetivo de determinar la especificación que provea los pronósticos del PIB nominal más precisos, usando exactamente el mismo conjunto de información que el responsable de política posee en el momento del análisis y además tomando en cuenta las potenciales revisiones periódicas asociadas a publicaciones pasadas que algunas variables podrían experimentar.

Primero se lleva a cabo un análisis basado en modelos univariados, el cual consiste en estimar con data en tiempo real modelos autorregresivos con diferentes rezagos. Los resultados muestran un desempeño poco preciso de dichos modelo, reportando errores cuadráticos medios elevados. Debido a esto procedemos a explorar especificaciones multivariadas.

El primer modelo multivariado toma como fundamento el hecho de que la tasa de crecimiento del PIB nominal es igual a la suma de las tasas de crecimiento del Deflactor de PIB y del PIB real. Dado esto, una estrategia sencilla para realizar inferencias tempranas del PIB nominal sería utilizar medidas aproximadas de Deflactor y el PIB real que posean una frecuencia más alta. Por una parte, una de las variables más utilizada como aproximación de la producción real de una economía es el índice de producción industrial en frecuencia mensual. Por otra parte, dado que el deflactor del PIB nos indica la evolución del nivel de precios, una de las medidas de aproximación más usadas para el deflactor es el índice de precios al consumidor, el cual se lo calcula con una frecuencia mensual. En base a estas dos variables mensuales se puede construir un índice sencillo sumándolas y estandarizándolas con respecto a la variable objetivo, en este caso el PIB nominal. Dicho índice mensual es utilizado para obtener pronósticos tempranos. Los resultados muestran que a pesar
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del alto comovimiento, computado dentro de la muestra, entre este índice sencillo y el PIB nominal es bastante alto, los pronósticos fuera de muestra y utilizando datos en tiempo real, no son tan satisfactorios, dado que están caracterizados por una alta volatilidad, implicando por lo tanto una alta incertidumbre con respecto a los pronósticos.

Desde el trabajo seminal de Stock y Watson (1991) el uso de modelos de factores dinámicos ha sido visto como una atractiva alternativa en obtener pronósticos. Debido a esto, nuestro siguiente análisis se basa en el uso de factores dinámicos con el fin de obtener pronósticos del PIB nominal más precisos que las obtenidas en los modelos univariados y el modelo multivariado sencillo. El modelo consiste en extraer el comovimiento entre un conjunto de variables, para luego ser utilizado en la construcción de pronósticos. Además, una característica fundamental del modelo propuesto es que permite la inclusión de observaciones faltantes y la mezcla entre variables en frecuencia trimestral y mensual.

El primer paso de nuestra estrategia consiste en seleccionar los indicadores que provean pronósticos con los errores cuadráticos medios más bajos, es decir los que tengan más habilidad predictiva. El conjunto de posibles indicadores está compuesto por variables de actividad económica real, tales como, índice de producción industrial, ingreso personal real menos pagos por transferencias, empleo en el sector no agrícola y ventas reales en el sector de manufactura. En este conjunto también se incluyen indicadores de inflación, tales como, el índice de precios al consumidor, el índice de precios al productor, el índice de precios de los gastos personales de consumo y el índice de precios de los gastos personales de consumo excluyendo los sectores de alimento y energía.

Finalmente, a este conjunto de variables se incluyen indicadores de naturaleza nominal o monetaria, tales como, ingreso personal, gasto personal de consumo, sueldo promedio por hora de empleados de producción y no supervisores y los agregados...
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monetarios de Divisia, M3, M4 y M4-, los cuales son obtenidos del Centro de Estabilidad Financiera en New York. Después de haber probado diferentes combinaciones de variables incluidas en el modelo de factor dinámico se obtiene que un conjunto selecto de trece especificaciones, las cuales proveen los pronósticos fuera de muestra más preciso, superando en todos los casos a los modelos univariados y al modelo multivariado sencillo.

Dados estos resultados podemos concluir que el modelo multivariado de factor dinámico que combina información de la variable objetivo, PIB nominal, junto con datos mensuales que corresponden al índice de producción industrial, el índice de precios al consumidor y el agregado monetario de Divisia M3, es la especificación que produce los pronósticos tempranos más exactos del PIB nominal.
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The last global economic recession and the current debt crises, in the European Union, have remarkably increased the interest in understanding the sources and the underlying mechanisms leading to the propagation of recessions. Since this information can be useful to mitigate the adverse effect of contractionary episodes through appropriate stabilization policies, academic researchers and policy makers have started to closely look at the disaggregation of the business cycle in order to explore the type of economic interconnections prevailing between smaller economies, at the regional level, along with its dynamic interaction with different types of markets, e.g. financial, housing, etc.

This doctoral dissertation focuses on studying the sources, propagation and prediction of recessions. The methodological approaches I use to assess these issues heavily rely on the unification between two fundamental concepts which have generally been considered in isolation from one another in the literature. First, they take into account the two defining characteristics of the business cycle, which are comovements and nonlinearities, as documented in Burns and Mitchell (1946), and second, they also account for the degree and type of interconnectedness between a set of sub-economies of a disaggregated business cycle by using network analysis.

The first chapter develops a framework for monitoring the degree of synchronization between many stochastic processes that are subject to regime changes by relying on a time-varying weighted network with Markov-switching dynamics. An empirical application to the study of business cycle phases synchronization in U.S. states suggests that national recessions can be anticipated by an index that measures the substantial heterogeneity across regional recessions. Moreover, the way in which an upcoming national recession could simultaneously affect each of its smaller economies, at the state level, can be dynamically evaluated. The results also give
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insights about some essential determinants driving business cycles synchronization as the similarities between the industrial composition of the economies and their geographic location.

Due to the heterogeneity of recessions, assessing which side of the economy is the main source of each contractionary period is crucial for policy makers in order to mitigate its adverse effects through appropriate stabilization policies. The second chapter develops a probabilistic model based on comovements and nonlinearities used to provide simultaneous inferences on three issues of U.S. macroeconomic fluctuations under a unified setup: When do the phases of business and inflation cycles simultaneously occur? What is their synchronicity degree? And, which side of the economy is the most affected during each of those phases? By answering those questions the proposed framework is able to date and categorize contractionary episodes into demand recessions, supply recessions and mix recessions based on the time-varying shock contributions to the business cycle.

Given that some economies have reached the lower-bound level of the interest rate, non-conventional economic policies are starting to emerge, one of the most suggested by the experts relies on a Nominal GDP targeting strategy. The third chapter focuses on providing early assessments of current quarterly Nominal GDP growth for the US economy. These nowcasts are computed by using the exact amount of information that policy makers have at hand at the time that predictions are done. We explore the predictive ability of several univariate and multivariate specifications, by also looking for the most helpful indicators in performing this task. The results show that, among the proposed candidates, a small scale dynamic factor model that contains information of real economic activity, inflation dynamics and Divisia monetary aggregates, produces the most accurate nowcasts of Nominal GDP.
References


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Chapter 1

Monitoring Synchronization of Regional Recessions

1.1 Introduction

The interest in identifying changes in business cycles synchronization started to markedly increase since the implementation of the European Monetary Union, due to more synchronized countries were expected to face smaller costs of joining the Union than those countries with relatively less synchronized cycles, Camacho et al. (2006). In other words, analyzing synchronization changes is crucial for policy makers in order to determine the countries, regions or even sectors of an economy that could be more sensitive to global policies or aggregate economic shocks and the others that could remain less affected by them.

Given the asymmetric nature of business cycles, since the seminal work by Hamilton (1989) in which the phases of the U.S. economy are characterized by using a Markov-switching (MS) model, a broad range of extensions related to this approach have been developed due to its great success. In particular, multivariate MS models
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have become a useful tool in analyzing synchronization between the business cycle phases of different countries, as is the case of Smith and Summers (2005) and Camacho and Perez-Quiros (2006), or different regions, as in Owyang et al. (2005) and Hamilton and Owyang (2012). Although all these studies provide a picture about how much some business cycles are in sync during a given sample period, they are not able to endogenously capture potential changes in the degree of synchronization. This is because in order to preserve parsimony in the model, a still non explored question that could help to unveil this feature has remained unnoticed: What is the dynamic relationship between the unobserved state variables governing a multivariate MS setting?

The approaches followed in the literature traditionally assume a fixed over time dependence relation between such state variables, they can be divided into two categories. The first category refers to studies in which such relation is just a priori assumed based on the econometrician’s judgment. Apart from the general Markovian specification, that involves the estimation of the full transition probability matrix, multivariate MS models are usually analyzed under three different types of relationships between the unobserved state variables governing each time series, see Hamilton and Lin (1996) and Anas et al. (2007). The first one refers to the case in which all series follow a common regime dynamics, Krolzig (1997). Second, the use of totally independent Markov chains, which is the most followed approach, Smith and Summers (2005) and Chauvet and Senyuz (2008). Third, the dynamics in one state variable precede those of other state variables, Hamilton and Perez-Quiros (1996) and Cakmakli et al. (2011).

The second category focuses on making a posteriori assessments of the synchronization between MS processes during a given sample period, providing average dependence relationship estimates, as in the case of Artis et al. (2004) who compute

\footnote{This approach presents computational difficulties as the model increases in the number of series, states or lags.}
cross-correlations between the smoothed state probabilities after estimating several univariate models. Also, some works focus on testing synchronization by relying on the extreme cases of independence and perfect synchronization.\textsuperscript{2} One intuitive approach followed by Camacho and Perez-Quiros (2006), Bengoechea et al. (2006), and Leiva-Leon (2011), is based on modeling the data generating process as a linear combination between the two polar cases, perfect dependence and complete independence, claiming that in most of real situations, the true MS multivariate dynamics should be somewhere in between them. Although this approach can be used to study how much synchronized are a set of MS processes during a given sample period, neither they are able to identify possible synchronization changes.

This paper proposes a novel framework for monitoring changes in the degree of global synchronization between many stochastic processes that are subject to regime changes. Specifically, it computes regime inferences from a multivariate Markov-switching model and simultaneously obtains a measure of the time-varying synchronization between the unobserved state variables governing each process. Such measure is endogenously estimated as a weighted average between the dependent and independent polar cases by making inference on the regimes of high and low synchronization without requiring \textit{a posteriori} computations as in the case of the concordance in Harding and Pagan (2006). Apart from capturing nonlinearities, One advantage of this approach in comparison to the conventional dynamic correlation measure, is that pairwise synchronizations, which are estimated through Bayesian methods, can be easily converted into desynchronization measures to be combined with dynamic multidimensional scaling and network analysis in order to make assessments regarding to possible changes in the clustering patterns that could experiment a system of time series, the key components leading the system, and even to make

\textsuperscript{2}Some examples are Harding and Pagan (2006), who propose tests of the hypotheses that cycles are either unsynchronized or perfectly synchronized under complications caused by serial correlation and heteroscedasticity in cycle states, and Pesaran and Timmermann (2009) who test independence between discrete multicategory variables based on canonical correlations.
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inference about its future behavior.

The proposed framework is used as a tool for monitoring the time-varying synchronization among U.S. states business cycle phases in order to assess its twofold ability, i.e. predictive and descriptive. Specifically, it is assessed, first, up to which extent the interconnectedness between smaller economies can be helpful to anticipate national recessions. And second, how an upcoming national recession could simultaneously affect each of its smaller economies at the state level.

In a network where U.S. states are interpreted as the nodes and the strength of the links between nodes is given by the degree of business cycle interdependence between two economies, the substantial heterogeneity across U.S. states recessions claimed in Hamilton and Owyang (2012) can be measured with an index of centrality that relies on the global synchronization. In sample results show that this index tends to rapidly increase some months before national recessions, dated by the NBER, occur. The reliability of this index to anticipate recessions is then confirmed with real-time exercises by reestimating it with the available information up to some periods before national recessions start, indicating that decreases in regional recessions heterogeneity are helpful in order to predict upcoming national recessions.

The clustering coefficient and closeness centrality measures are used to make time-varying assessments of the comovement among economic phases, reporting high values when economies are following the same pattern (recessions or expansions), and dropping when their phases are independent from each other by following idiosyncratic behaviors. Since the degree of interdependence between states can be monitored month-to-month, it is possible to assess how much state "i" could be affected by shocks hitting state "j" or the nation as a whole.

Additionally, the framework also provides two noteworthy features. First, centrality measures of the network reflect that there is a set of economies, given by states
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of North Carolina, Missouri, Wisconsin, Tennessee, and Alabama, that present the highest levels of interdependence with the rest of economies and that interestingly are also geographically linked. On the one hand, the industrial composition of the economies in this region, which can be interpreted as a business cycle core, is significantly similar to the one of the U.S. economy. On the other hand, the industrial composition of the states with the lowest levels of centrality, as Wyoming, Louisiana, North Dakota, Oklahoma and Texas, is markedly different to the U.S., providing evidence that industrial composition seems to be a determinant factor leading business cycles synchronization.

Second, apart from dynamic estimates of the interdependence degree, this framework also provides the possibility of computing stationary estimates. They are obtained with the ergodic or time-independent probabilities associated to a latent variable measuring synchronization. This stationary results report that states in the core roughly coincide with the ones showing high concordance with the national business cycle found in Owyang et al. (2005). Moreover, U.S. states can be grouped into three clusters, a highly, discreetly and lowly in sync with the national business cycle, and the states in each cluster roughly coincide with the ones found in Hamilton and Owyang (2012).

The paper is structured as follows. Section 2 provides the Markov-switching synchronization modeling approach, the filtering algorithm and the procedure of the Bayesian parameter estimation. Section 3 analyzes the business cycle phases synchronization in U.S. states relying on network analysis, providing multidimensional scaling, clustering coefficient, and closeness centrality estimates. Section 4 concludes.

3Note that unlike spatial econometric models, the proposed framework do not take into account any previous geographical consideration.
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1.2 Modeling Markov-Switching Synchronization

In this section, it is proposed an algorithm to monitor changes in the degree of pairwise synchronization between stochastic processes that are subject to regime changes. Let $y_{i,t}$ be a time series modeled as a function of a latent variable, $S_{i,t}$, that indicates the regime at which it is, an idiosyncratic component $\epsilon_{i,t}$, and a set of parameters, $\theta_i$, to be estimated. Accordingly, for $i = a, b$,

$$ y_{a,t} = f(S_{a,t}, \epsilon_{a,t}, \theta_a) $$ (1.1)

$$ y_{b,t} = f(S_{b,t}, \epsilon_{b,t}, \theta_b), $$ (1.2)

the goal will be to make an assessment on their synchronization for each period of time, that is,

$$ \delta_t^{ab} = \text{sync}(S_{a,t}, S_{b,t}) = \Pr(S_{a,t} = S_{b,t}). $$ (1.3)

Specifically, this paper will focus on the time-varying sync between the two unobserved state variables governing a bivariate Markov-switching model. In order to mainly focus on modeling this dynamic dependence relation and to avoid complex notation, it is considered the following parsimonious and very tractable bivariate two-state Markov-switching specification:

$$ \begin{bmatrix} y_{a,t} \\ y_{b,t} \end{bmatrix} = \begin{bmatrix} \mu_{a,0} + \mu_{a,1} S_{a,t} \\ \mu_{b,0} + \mu_{b,1} S_{b,t} \end{bmatrix} + \begin{bmatrix} \epsilon_{a,t} \\ \epsilon_{b,t} \end{bmatrix}, $$ (1.4)

where

$$ \begin{bmatrix} \epsilon_{a,t} \\ \epsilon_{b,t} \end{bmatrix} \sim N\left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2 \end{bmatrix} \right). $$ (1.5)

The results obtained in this section can be straightforwardly applied to an extended specification that could include more lags in the dynamics or even Markov-switching variance-covariance matrix. The state variable $S_{k,t}$ indicates if $y_{kt}$ is in regime 0 with a mean equal to $\mu_{k,0}$, where $S_{k,t} = 0$, or if $y_{kt}$ is in regime 1 with a mean equal to $\mu_{k,1}$, where $S_{k,t} = 1$. The transition probability matrix is given by

$$ \mathbf{P} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} $$

with $p_{ij}$ representing the probability of transitioning from regime $i$ to regime $j$. The model specifies how the regimes change over time, which is crucial for monitoring synchronization.


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\[ \mu_{k,0} + \mu_{k,1}, \text{ where } S_{k,t} = 1, \text{ for } k = a, b. \] Moreover \( S_{a,t} \) and \( S_{b,t} \) evolve according to irreducible two-state Markov chains, whose transition probabilities are given by

\[
\Pr(S_{k,t} = j | S_{k,t-1} = i) = p_{k,ij}, \text{ for } i, j = 0, 1 \text{ and } k = a, b. \tag{1.6}
\]

In order to characterize the dynamics of \( y_t = [y_{a,t}, y_{b,t}]' \), the information contained in \( S_{a,t} \) and \( S_{b,t} \) can be summarized in the state variable, \( S_{ab,t} \), it will account for the possible combinations that the vector, \( \mu_{S_{ab,t}} = [\mu_{a,0} + \mu_{a,1}S_{a,t}, \mu_{b,0} + \mu_{b,1}S_{b,t}]' \), could take through the different regimes. It is defined as:

\[
S_{ab,t} = \begin{cases} 
1, & \text{If } S_{a,t} = 0, S_{b,t} = 0 \\
2, & \text{If } S_{a,t} = 0, S_{b,t} = 1 \\
3, & \text{If } S_{a,t} = 1, S_{b,t} = 0 \\
4, & \text{If } S_{a,t} = 1, S_{b,t} = 1
\end{cases}. \tag{1.7}
\]

In contrast to the previous ways of modeling \( \Pr(S_{ab,t} = j_{ab}) \) in the literature, where some exogenous prior specific relationship between \( S_{a,t} \) and \( S_{b,t} \) is assumed, this paper proposes to model \( \Pr(S_{ab,t} = j_{ab}) \) based on the individual dynamics of \( S_{a,t} \) and \( S_{b,t} \), and simultaneously accounting for the dynamic degree of dependence between each other, which is endogenously estimated.

Although the degree of dependence between \( S_{a,t} \) and \( S_{b,t} \) is unknown, the two opposite extreme cases of dependence relationships are known. That is, on the one hand, if \( S_{a,t} \) and \( S_{b,t} \) are fully independent, then \( \Pr(S_{a,t} = j_a, S_{b,t} = j_b) = \Pr(S_{a,t} = j_a) \Pr(S_{b,t} = j_b) \). On the other hand, if \( S_{a,t} \) and \( S_{b,t} \) are totally dependent, in the sense that they are fully synchronized, then \( y_{a,t} \) and \( y_{b,t} \) are driven by the same state variable, \( S_t \), i.e. \( S_{a,t} = S_{b,t} = S_t \), remaining in this case \( \Pr(S_{a,t} = j_a, S_{b,t} = j_b) = \Pr(S_t = j) \). In empirical applications, the true dependence degree should be located between these two extreme possibilities.

In order to make inference on the type of dependence between \( S_{a,t} \) and \( S_{b,t} \), a
new unobserved state variable will be defined as:

\[
V_t = \begin{cases} 
0 & \text{If } S_{a,t} \text{ and } S_{b,t} \text{ are fully independent} \\
1 & \text{If } S_{a,t} \text{ and } S_{b,t} \text{ are totally dependent}
\end{cases}, \quad (1.8)
\]

In order to maintain the nonlinear nature of the framework, it will also evolve according to an irreducible two-state Markov chain whose transition probabilities are given by

\[
\Pr(V_t = j_v | V_{t-1} = i_v) = p_{v,kl}, \quad \text{for } i_v, j_v = 0, 1 \quad (1.9)
\]

Model in Equation (1.4) hence remains fully characterized by the state variable \(S_{ab,t}\), which collects information regarding to joint dynamics, individual dynamics and their dependence relationship over time simultaneously. In this way \(S_{ab,t}\) is defined as:

\[
S_{ab,t} = \begin{cases} 
1, & \text{If } V_t = 0, S_{a,t} = 0, S_{b,t} = 0 \\
2, & \text{If } V_t = 0, S_{a,t} = 0, S_{b,t} = 1 \\
3, & \text{If } V_t = 0, S_{a,t} = 1, S_{b,t} = 0 \\
4, & \text{If } V_t = 0, S_{a,t} = 1, S_{b,t} = 1 \\
5, & \text{If } V_t = 1, S_{a,t} = 0, S_{b,t} = 0 \\
6, & \text{If } V_t = 1, S_{a,t} = 0, S_{b,t} = 1 \\
7, & \text{If } V_t = 1, S_{a,t} = 1, S_{b,t} = 0 \\
8, & \text{If } V_t = 1, S_{a,t} = 1, S_{b,t} = 1
\end{cases}, \quad (1.10)
\]

Inference on the possible states of \(S_{ab,t}\^*\), i.e. \(\Pr(S_{ab,t}^* = j_{ab}^*)\) for \(j_{ab}^* = 1, \ldots, 8\), can be done by computing

\[
\Pr(S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = j_v) \Pr(V_t = j_v) \quad (1.11)
\]

where \(\Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = j_v)\) indicates the inference on the dynamics of \(S_{ab,t}\) conditional on total independence, \(V_t = 0\), or conditional on full dependence, \(V_t = 1\).

\footnote{States 6 and 7 in Equation (1.10) are truncated to zero by construction, since the two state variables cannot be in different states if they are perfectly synchronized, i.e. \(\Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = 1) = 0\) for any \(j_a \neq j_b\).}
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In the former case the joint probability of $S_{ab,t}$ will be

\[
\Pr(S_{a,t} = j_a, S_{b,t} = j_b, V_t = 0) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = 0) \Pr(V_t = 0)
\]

\[
= \Pr(S_{a,t} = j_a) \Pr(S_{b,t} = j_b) \Pr(V_t = 0),
\]

(1.12)

and in the latter case, it will be

\[
\Pr(S_{a,t} = j_a, S_{b,t} = j_b, V_t = 1) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = 1) \Pr(V_t = 1)
\]

\[
= \Pr(S_t = j) \Pr(V_t = 1),
\]

(1.13)

therefore probabilities of the state variable $S_{ab,t}$ in Equation (1.7) after accounting for synchronization, can be easily computed as

\[
\Pr(S_{a,t} = j_a, S_{b,t} = j_b) = \Pr(V_t = 1) \Pr(S_t = j) +
\]

\[
(1 - \Pr(V_t = 1)) \Pr(S_{a,t} = j_a) \Pr(S_{b,t} = j_b),
\]

(1.14)

which indicates that joint dynamics of $S_{a,t}$ and $S_{b,t}$ are characterized by a linear combination between the extreme dependent case and the extreme independent case, where the weights assigned to each of them are endogenously determined by their sync degree

\[
\delta_{t}^{ab} = \Pr(V_t = 1).
\]

(1.15)

1.2.1 Filtering Algorithm

Following the line of Hamilton’s (1994) algorithm, we propose an extension to estimate the model described in Equations (1.4) and (1.14). The algorithm is composed by two unified steps, in the first one the goal is the computation of the likelihoods, while in the second one the goal is to compute prediction and updating probabilities.

STEP 1: For the moment it is assumed that model’s parameters are known and collected in the vector

\[
\theta = (\mu_{a,0}, \mu_{a,1}, \mu_{b,0}, \mu_{b,1}, \sigma_a^2, \sigma_b^2, \sigma_{ab}, \sigma_{a,00}, \sigma_{a,11}, \sigma_{b,00}, \sigma_{b,11}, \sigma_{p,00}, \sigma_{p,11}, \sigma_{v,00}, \sigma_{v,11}),
\]
in the next section the Bayesian procedure to estimate $\theta$ will be clarified. By using the prediction probabilities\(^5\) $\Pr(S_{k,t} = j_k | \psi_{t-1}; \theta)$ for $k = a, b$, $\Pr(V_t = j_v | \psi_{t-1}; \theta)$ and $\Pr(S_t = j | \psi_{t-1}; \theta)$, the joint probability corresponding to the state variable that fully characterizes the model dynamics, $S_{ab,t}^*$, can be obtained relying on Equations (1.12) and (1.13), that is

$$
\Pr(S_{ab,t}^* = j_{ab}^* | \psi_{t-1}; \theta) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v | \psi_{t-1}; \theta) = \Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = j_v, \psi_{t-1}; \theta) \Pr(V_t = j_v | \psi_{t-1}; \theta), \quad (1.16)
$$

thereafter the density of $y_t$ given that it is on regime $S_{t}^*$, and the prediction probabilities of the realizations of $S_{t}^*$, are used to compute the joint likelihood

$$
f(y_t, S_{ab,t}^* = j_{ab}^* | \psi_{t-1}; \theta) = f(y_t | S_{ab,t}^* = j_{ab}^*, \psi_{t-1}; \theta) \Pr(S_{ab,t}^* = j_{ab}^* | \psi_{t-1}; \theta) = f(y_t, S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v | \psi_{t-1}; \theta), \quad (1.17)
$$

then summing across the respective terms, specific likelihoods corresponding individual processes are computed

$$
\sum_{j_a=0}^1 \sum_{j_b=0}^1 f(y_t, S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v | \psi_{t-1}; \theta) = j_a | \psi_{t-1}; \theta) = \sum_{j_a=0}^1 \sum_{j_b=0}^1 f(y_t, S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v | \psi_{t-1}; \theta) = j_a | \psi_{t-1}; \theta) = \sum_{j_a \neq j_b} f(y_t, S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v | \psi_{t-1}; \theta) = f(y_t, V_t = j_v | \psi_{t-1}; \theta) = \sum_{j_a=0}^1 \sum_{j_b=0}^1 f(y_t, S_{a,t} = j_a, S_{b,t} = j_b, V_t = j_v | \psi_{t-1}; \theta).
$$

---

\(^5\)The steady state or ergodic probabilities can be used as starting values of the filter.
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STEP 2: Once $y_t$ is observed at the end of time $t$, the prediction probabilities $\Pr(S_{k,t} = j_k|\psi_{t-1}; \theta)$ for $k = a, b$, $\Pr(V_t = j_v|\psi_{t-1}; \theta)$ and $\Pr(S_t = j|\psi_{t-1}; \theta)$ can be updated as follows

\begin{align*}
\Pr(S_{a,t} = j_a|\psi_t; \theta) &= \frac{f_a(y_{a,t}, S_{a,t} = j_a|\psi_{t-1}; \theta)}{f(y_t|\psi_{t-1}; \theta)} \\
\Pr(S_{b,t} = j_b|\psi_t; \theta) &= \frac{f_b(y_{b,t}, S_{b,t} = j_b|\psi_{t-1}; \theta)}{f(y_t|\psi_{t-1}; \theta)} \\
\Pr(S_t = j|\psi_t; \theta) &= \frac{f(y_t, S_t = j|\psi_{t-1}; \theta)}{f(y_t|\psi_{t-1}; \theta)} \\
\Pr(V_t = l|\psi_t; \theta) &= \frac{f(y_t, V_t = l|\psi_{t-1}; \theta)}{f(y_t|\psi_{t-1}; \theta)}
\end{align*}

(1.22)-(1.25)

Where $\psi_t = \{\psi_{t-1}, y_t\}$, and the unconditional likelihood function is given by

\[ f(y_t|\psi_{t-1}; \theta) = \sum_{j_t=1}^8 f(y_t, S_{ab,t}^* = j_{ab}^*|\psi_{t-1}; \theta). \]

(1.26)

Forecasts of the updated probabilities in Equations (1.22)-(1.25) are done by using the corresponding transition probabilities in the vector $\theta$, that is $p_{a,ij}, p_{b,ij}, p_{ij}, p_{v,ij}$ for $S_{a,t}, S_{b,t}, S_t, V_t$ respectively,

\begin{align*}
\Pr(S_{k,t+1} = j_k|\psi_t; \theta) &= \sum_{i_k=0}^1 \Pr(S_{k,t+1} = j_k, S_{k,t} = i_k|\psi_t; \theta) \\
&= \sum_{i_k=0}^1 \Pr(S_{k,t+1} = j_k|S_{k,t} = i_k) \Pr(S_{k,t} = i_k|\psi_t; \theta),
\end{align*}

(1.27)

for $k = a, b$.

\begin{align*}
\Pr(V_{t+1} = j_v|\psi_t; \theta) &= \sum_{i_v=0}^1 \Pr(V_{t+1} = j_v, V_t = i_v|\psi_t; \theta) \\
&= \sum_{i_v=0}^1 \Pr(V_{t+1} = j_v|V_t = i_v) \Pr(V_t = i_v|\psi_t; \theta)
\end{align*}

(1.28)

\begin{align*}
\Pr(S_{t+1} = j|\psi_t; \theta) &= \sum_{i=0}^1 \Pr(S_{t+1} = j, S_t = i|\psi_t; \theta) \\
&= \sum_{i=0}^1 \Pr(S_{t+1} = j|S_t = i) \Pr(S_t = i|\psi_t; \theta).
\end{align*}

(1.29)
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Finally the above forecasted probabilities are used to predict inferences on the realizations of $S_{ab,t+1}^*$, again relying on Equations (1.12) and (1.13)

$$\Pr(S_{ab,t+1}^* = j_{ab}|\psi_t;\theta) = \Pr(S_{a,t+1} = j_a, S_{b,t+1} = j_b, V_{t+1} = j_v|\psi_t;\theta)$$

$$= \Pr(S_{a,t} = j_a, S_{b,t} = j_b | V_t = j_v, \psi_t;\theta) \Pr(V_t = j_v|\psi_t;\theta), \quad (1.30)$$

By iterating these two steps for $t = 1, 2, \ldots, T$, the algorithm simultaneously provides inferences on the joint dynamics and individual dynamics along with their time-varying degree of dependence of the model in Equations (1.4) and (1.14).

1.2.2 Bayesian Parameter Estimation

The approach to estimate $\theta$ will be relied on a bivariate extended version of the multi-move Gibbs-sampling procedure implemented by Kim and Nelson (1998) for Bayesian estimation of univariate Markov-switching models. In this setting both the parameters of the model $\theta$ and the Markov-switching variables $\tilde{S}_k,T = \{S_{k,t}\}^T_1$ for $k = a, b$, $\tilde{S}_T = \{S_t\}^T_1$ and $\tilde{V}_T = \{V_t\}^T_1$ are treated as random variables given the data in $\tilde{y}_T = \{y_t\}^T_1$. The purpose of this Markov chain Monte Carlo simulation method is to approximate the joint and marginal distributions of these random variables by sampling from conditional distributions.

Priors

For the mean and variance parameters in vector $\theta$, the Independent Normal-Wishart prior distribution is used

$$p(\mu, \Sigma^{-1}) = p(\mu)p(\Sigma^{-1}), \quad (1.31)$$

---

6 The motivation for the use of Bayesian methods relies on the fact that as the number of possible states increase, the likelihood function could be characterized by many local maxima and there could be strong convergence problems in performing maximum likelihood estimation, Boldin (1996).
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where

\[ \mu \sim N(\mu, V_\mu) \]
\[ \Sigma^{-1} \sim W(S_\Sigma^{-1}, \Omega) \]

for the transition probabilities \( p_{a,00}, p_{a,11} \) from \( S_{a,t} \), \( p_{b,00}, p_{b,11} \) from \( S_{b,t} \), \( p_{00}, p_{11} \) from \( S_t \) and \( p_{v,00}, p_{v,11} \) from \( V_t \). Beta distributions will be used as conjugate priors

\[ p_{k,00} \sim Be(u_{k,11}, u_{k,10}), p_{k,11} \sim Be(u_{k,00}, u_{k,01}), \text{ for } k = a, b \] (1.32)
\[ p_{00} \sim Be(u_{11}, u_{10}), p_{11} \sim Be(u_{00}, u_{01}) \] (1.33)
\[ p_{v,00} \sim Be(u_{v,11}, u_{v,10}), p_{v,11} \sim Be(u_{v,00}, u_{v,01}) \] (1.34)

**Drawing \( \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T \) and \( \tilde{V}_T \) given \( \theta \) and \( \tilde{y}_T \)**

Following the result in Equation (1.14), in order to make inference on the bivariate dynamics of the model (1.4) driven by \( \tilde{S}_{ab,T} = \{ S_{ab,t} \}_1^T \) and described in (1.7), it is just needed to make inference on the dynamics of the single state variables \( \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T \) and \( \tilde{V}_T \), this can be done following the results in Kim and Nelson (1998) by first computing draws from the conditional distributions

\[ g(\tilde{S}_{k,T}|\theta, \tilde{y}_T) = g(S_{k,T}|\tilde{y}_T) \prod_{t=1}^T g(S_{k,t}|S_{k,t+1}, \tilde{y}_t), \text{ for } k = a, b \] (1.35)
\[ g(\tilde{S}_T|\theta, \tilde{y}_T) = g(S_T|\tilde{y}_T) \prod_{t=1}^T g(S_t|S_{t+1}, \tilde{y}_t) \] (1.36)
\[ g(\tilde{V}_T|\theta, \tilde{y}_T) = g(V_T|\tilde{y}_T) \prod_{t=1}^T g(V_t|V_{t+1}, \tilde{y}_t). \] (1.37)

In order to obtain the two terms in the right hand side of Equation (1.35)-(1.36) the following two steps can be employed:

**Step 1**: The first term can be obtained by running the filtering algorithm developed in Section 2.1, to compute \( g(\tilde{S}_{k,t}|\tilde{y}_t) \) for \( k = a, b \), \( g(\tilde{S}_t|\tilde{y}_t) \) and \( g(\tilde{V}_{k,t}|\tilde{y}_t) \) for \( t = 1, 2, \ldots, T \), saving them and taking the elements for which \( t = T \).
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**Step 2:** The product in the second term can be obtained for \( t = T−1, T−2, \ldots, 1, \) by following the result:

\[
g(S_t|\tilde{y}_t, S_{t+1}) = \frac{g(S_t, S_{t+1} | \tilde{y}_t)}{g(S_{t+1} | \tilde{y}_t)} \times g(S_{t+1} | S_t) g(S_t | \tilde{y}_t), \tag{1.38}
\]

where \( g(S_{t+1} | S_t) \) corresponds to the transition probabilities of \( S_t \) and \( g(S_{t} | \tilde{y}_t) \) were saved in Step 1.

Then, it is possible to compute

\[
Pr[S_t = 1 | S_{t+1}, \tilde{y}_t] = \frac{\sum_{j=0}^{1} g(S_{t+1} | S_t = j) g(S_t = j | \tilde{y}_t)}{g(S_{t+1} | S_t = 1) g(S_t = 1 | \tilde{y}_t)}, \tag{1.39}
\]

and generate a random number from a \( U[0, 1] \). If that number is less than or equal to \( Pr[S_t = 1 | S_{t+1}, \tilde{y}_t] \), then \( S_t = 1 \), otherwise \( S_t = 0 \). The same procedure applies for \( S_{a,t}, S_{b,t} \) and \( V_t \), and by using Equation (1.14) inference of \( \tilde{S}_{ab,T} \) can be done.

**Drawing** \( p_{a,00}, p_{a,11}, p_{b,00}, p_{b,11}, p_{00}, p_{11}, p_{v,00}, p_{v,11} \) given \( \tilde{S}_{a,T}, \tilde{S}_{a,T}, \tilde{S}_T \) and \( \tilde{V}_T \)

Conditional on \( \tilde{S}_{k,T} \) for \( k = a, b, \tilde{S}_T \) and \( \tilde{V}_T \), the transition probabilities are independent on the data set and the model’s parameters, hence the likelihood function of \( p_{00}, p_{11} \) is given by:

\[
L(p_{00}, p_{11} | \tilde{S}_T) = p_{00}^{n_{00}} (1 - p_{00})^{n_{01}} p_{11}^{n_{11}} (1 - p_{11})^{n_{10}}, \tag{1.40}
\]

where \( n_{ij} \) refers to the transitions from state \( i \) to \( j \), accounted for in \( \tilde{S}_T \).

Combining the prior distribution in Equation (1.33) with the likelihood, the posterior distribution is given by

\[
p(p_{00}, p_{11} | \tilde{S}_T) \propto p_{00}^{u_{00} + n_{00} - 1} (1 - p_{00})^{u_{01} + n_{01} - 1} p_{11}^{u_{11} + n_{11} - 1} (1 - p_{11})^{u_{10} + n_{10} - 1} \tag{1.41}
\]

which indicates that draws of the transition probabilities will be taken from

\[
p_{00} | \tilde{S}_T \sim Be(u_{00} + n_{00}, u_{01} + n_{01}), \quad p_{11} | \tilde{S}_T \sim Be(u_{11} + n_{11}, u_{10} + n_{10}) \tag{1.42}
\]
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Drawing \( \mu_{0,a}, \mu_{1,a}, \mu_{0,b}, \mu_{1,b} \) given \( \sigma_a^2, \sigma_b^2, \sigma_{ab}, \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T \) and \( \tilde{y}_T \)

The model in Equation (1.4) can be compactly expressed as

\[
\begin{bmatrix}
  y_{a,t} \\
  y_{b,t}
\end{bmatrix} =
\begin{bmatrix}
  1 & S_{a,t} & 0 & 0 \\
  0 & 0 & 1 & S_{b,t}
\end{bmatrix}
\begin{bmatrix}
  \mu_{a,0} \\
  \mu_{a,1} \\
  \mu_{b,0} \\
  \mu_{b,1}
\end{bmatrix} +
\begin{bmatrix}
  \varepsilon_{a,t} \\
  \varepsilon_{b,t}
\end{bmatrix},
\]

with

\[
\begin{bmatrix}
  \varepsilon_{a,t} \\
  \varepsilon_{b,t}
\end{bmatrix} \sim N\left(\begin{bmatrix}
  0 \\
  0
\end{bmatrix},
\begin{bmatrix}
  \sigma_a^2 & \sigma_{ab} \\
  \sigma_{ab} & \sigma_b^2
\end{bmatrix}\right)
\]

or alternatively as

\[
y_t = \tilde{S}_t \mu + \xi_t, \quad \xi_t \sim N(0, \Sigma),
\]

(1.43)

Stacking, it remains

\[
y =
\begin{bmatrix}
  y_1 \\
  y_2 \\
  \vdots \\
  y_T
\end{bmatrix}, \quad \tilde{S} =
\begin{bmatrix}
  \tilde{S}_1 \\
  \tilde{S}_2 \\
  \vdots \\
  \tilde{S}_T
\end{bmatrix}, \quad \text{and } \xi =
\begin{bmatrix}
  \xi_1 \\
  \xi_2 \\
  \vdots \\
  \xi_T
\end{bmatrix},
\]

the model in Equation (1.43) remains written as a normal linear regression model with an error covariance matrix of a particular form:

\[
y = S \mu + \xi, \quad \xi \sim N(0, I \otimes \Sigma)
\]

(1.44)

Conditional on the covariance matrix parameters, state variables and the data, by using the corresponding likelihood function, the conditional posterior distribution

\[
p(\mu | \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T, \Sigma^{-1}, \tilde{y}_T) \text{ takes the form}
\]

\[
\mu | \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T, \Sigma^{-1}, \tilde{y}_T \sim N(\overline{\mu}, \overline{V}_\mu),
\]

(1.45)
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where

\[ V = V_1 + T X_t = 1 S_0 t + 1 S_1 y_t : \]

After drawing \( \mu = [\mu_{a,0}, \mu_{a,1}, \mu_{b,0}, \mu_{b,1}]' \) from the above multivariate distribution, if the generated value of \( \mu_{a,1} \) or \( \mu_{b,1} \) is less than or equal to 0, that draw is discarded, otherwise it is saved, this is in order to ensure that \( \mu_{a,1} > 0 \) and \( \mu_{b,1} > 0 \).

Drawing \( \sigma_a^2, \sigma_b^2, \sigma_{ab} \) given \( \mu_{0,a}, \mu_{1,a}, \mu_{0,b}, \mu_{1,b}, \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T \) and \( \tilde{y}_T \)

Conditional on the mean parameters, state variables and the data, by using the corresponding likelihood function, the conditional posterior distribution

\[ p(\Sigma^{-1}|\tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T, \mu, \tilde{y}_T), \]

takes the form

\[ \Sigma^{-1}|\tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{V}_T, \mu, \tilde{y}_T \sim W(\tilde{S}^{-1}, \overline{\nu}), \tag{1.46} \]

where

\[ \overline{\nu} = T + \nu \]
\[ \overline{S} = S + \sum_{t=1}^{T} (y_t - \tilde{S}_t \mu) (y_t - \tilde{S}_t \mu)', \]

after \( \Sigma^{-1} \) is generated the elements is \( \Sigma \) are recovered.

1.3 Business Cycle Synchronization in U.S. States

The global financial crisis has stimulated the interest on the sources and propagation of contractionary episodes, calling to take a more careful look at the disaggregation of
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...the business cycle in order to study the mechanisms underlying economic fluctuations. Two recent works have shown interesting features about the business cycle phases in U.S. states, Owyang et al. (2005), and the propagation of regional recessions in the same country, Hamilton and Owyang (2012). On the one hand, the former study that follows a univariate approach, finds that U.S. states differ significantly in the timing of switches between regimes of expansions and recessions, indicating large differences in the extent to which state business cycle phases are in concord with those of the aggregate economy. On the other hand, the later work which follows a multivariate approach, focuses on clustering the states sharing similar business cycle characteristics finding that differences across states appear to be a matter of timing and they can be grouped into three clusters with some of them entering in recession or recovering before others.

Although these previous studies provide insights about the synchronization between different business cycles during a given period, they are not able to capture changes in the degree of synchronization between the phases of economic fluctuations, which can be potentially caused by economic unions, policy changes, aggregate recessionary shocks, etc. Analyzing synchronization changes is crucial for policy makers in order to determine at every period of time, which states could be more sensitive to specific policy or recessionary shocks and which others would remain less affected.

The framework developed in Section 2 is applied for monitoring monthly changes in the degree of sync among economic phases of U.S. states, and also between states and the national business cycle. The state coincident indexes constructed in Crone (2002) and provided by the Federal Reserve Bank of Philadelphia, are used as indicators of economic activity at state level. Alaska and Hawaii are excluded, as in Hamilton and Owyang (2012), and The Chicago Fed National Activity Index (CFNAI) is used as monthly measure of the aggregate U.S. business cycle, the sample period is 1979:8 to 2012:3.
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1.3.1 Bivariate Synchronization

The analysis that was performed for 48 U.S. states required to model each of the \( C_2^{48} = 1,128 \) pairwise comparisons, where in each of them 12,000 draws were generated from the posterior with an initial burn of 2,000 draws that were discarded.\(^7\) In order to assess the performance of the proposed MS synchronization model, two selected examples are analyzed.\(^8\)

The first one focuses on the case of two states which presents high and similar percentage of national GDP, that is, New York with 7.68% and Texas with 7.95%. Table 1.A shows the posterior means and medians for the model parameters along with their corresponding standard deviation. All means and medians of parameters are similar and also statistically significant. It is worth to highlight the estimates of the transition probabilities associated to the state variable that measures synchronization, \( V_t \). For the New York vs. Texas case, the probability of going from a regime of high sync to another regime of high sync is almost equal to the probability of going from a regime of low sync to another regime of low sync, since both are about 0.95. These estimates are corroborated in Chart A of Figure 1.A, where the probabilities of recession for New York and Texas along with their synchronization are plotted. As can be seen during the first half of the sample, both states were lowly synchronized, however since 1994 they started to experiment high sync levels.

The second example focuses on analyzing two states of different GDP share by selecting the polar case, that is, the state with the highest share, California with 13.34%, along with the state with the lowest share, Vermont with 0.18%. Table 1.A presents the Bayesian estimation results of the model parameters. Unlike to the previous case, in the California vs. Vermont model, the probability of remaining

\(^7\)Due to the high technology nowadays available, such high number of computations is not a problem anymore.

\(^8\)The results for the other cases are available upon request to the author.
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in a high sync regime (0.97) is higher than the probability of remaining in a low sync regime (0.93). This can be observed in Chart B of Figure 1.A, since with the exception of the 1987-1994 period, both states have experimented high levels of synchronization among their business cycle phases.

These two examples, apart from documenting the existence of abrupt changes in business cycle sync phases between U.S. states, show that their synchronization is independent on their GDP share, since the proposed approach only focuses on making a time-varying assessment of the comovement among economic phases, reporting high values when both are following the same pattern (recessions or expansions), and dropping when their phases are independent from each other by following idiosyncratic behaviors.

The analysis regarding to the concordance between states and national recessions is performed in Owyang et al. (2005) by following the line in Harding and Pagan (2006), where the degree to which two business cycles are in sync is calculated as the percentage of time the two economies were in the same regime. However, such approach provides an average synchronization measure without the possibility of making inference on the potential changes that it could experiment trough time. The approach in this paper is used to analyze this issue. Figures 1.A - 1.A show the dynamic sync degree between states and national business cycle, showing a high heterogeneity across states. On the one hand, some of them show an almost constant and low sync degree, e.g. Louisiana, Oklahoma and Wyoming, while others an almost constant and high sync degree, e.g. Alabama, Minnesota, Nebraska, South Carolina, Tennessee and Vermont. On the other hand, the rest of states in general experiment abrupt sync changes, that usually take place before or after national recessions, as in the case of Massachusetts and North Dakota respectively for the 1990’s recession, and North Carolina for 2007’s recession, these features agree with Hamilton and Owyang (2012) who claim that differences across states appear to be a matter of timing.
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By means of the performed analysis, it can be evaluated how much state "A" could be affected by an economic shock that is hitting state "B", or which states could be more sensitive when a period of national recession is coming, since the degree of interdependence between states and national economic activity can be monitored month-to-month. However, an aggregate dynamic representation of the results presented so far seems needed in order to deal with the question: How an upcoming national recession could simultaneously affect each of its smaller economies at the state level?

As suggested by Timm (2002) and Camacho et al. (2006), multidimensional scaling methods are a helpful tool to identify cyclical affiliations between economies. Although, traditionally studies focus in providing a map of just one general picture of the business cycle similarities for a given sample period, for the present case a dynamic approach is needed.

1.3.2 Dynamic Multidimensional Scaling

This methodology seeks to find a low dimensional coordinate system to represent $n$-dimensional objects and create a map of lower dimension ($k$) which gives approximate distances among them. The dimensional coordinates of the projection of any two objects, $a$ and $b$, are computed by minimizing a stress function which measure the squared sum of divergences between the true distances or the measure of desynchronization, $\gamma_{t}^{ab} = 1 - \delta_{t}^{ab}$, and the approximate distances, $\tilde{\gamma}_{t}^{ab}$, among these objects. Moreover given that this representation can be obtained for each $t = 1, \ldots, T$, it is applied a dynamic version, in which the evolution of states synchronization is presented by a discrete-time sequence of graph snapshots. In order to preserve the "mental map" between snapshots so that the transition between frames in the animation can be easily interpreted, the movements between time steps are constrained
by adding a temporal penalty to the stress function of a static graph layout method, Xu et al. (2011), so that the minimization problem is:

$$\min_{\tilde{n}^a} = \frac{1}{n} \sum_{a=1}^{n} \left( \sum_{b=1}^{n} (\gamma_{ab}^t - \tilde{\gamma}_{ab}^t)^2 \right) + \beta \sum_{a=1}^{n} \tilde{\gamma}_{t|t-1}^a,$$

where $\beta$ can be chosen as the average standard deviations of the series in order to adjust the movements according to the variability of the data and with

$$\tilde{\gamma}_{t|t-1}^a = \left( \left\| z_{a,t} - z_{b,t} \right\| \right)^{1/2} = \left[ \sum_{i=1}^{k} (z_{ai,t} - z_{bi,t})^2 \right]^{1/2}$$

$$\tilde{\gamma}_{t|t-1}^a = \left( \left\| z_{a,t} - z_{a,t-1} \right\| \right)^{1/2} = \left[ \sum_{i=1}^{k} (z_{ai,t} - z_{ai,t-1})^2 \right]^{1/2}$$

where $z_{a,t}$ and $z_{b,t}$ are the $k$-dimensional projection of the objects $a$ and $b$, and $z_{ai,t}$ and $z_{bi,t}$ each of their corresponding elements at time $t$.

Figure 1.A plots the U.S. states synchronization map for the first month of the 1990’s recession, showing that for this period of time U.S. states could be grouped into three clusters, coinciding with the number of clusters found in Hamilton and Owyang (2012). Specifically, there is a bunch of states that were more affected by that recession, being highly in sync with the national business cycle, some examples are Nevada, Ohio, Michigan, Florida, etc., and two separate groups that experimented low synchronization with respect to the national activity and moreover with respect to each other, in one of them are located states like Texas, Oklahoma, Wyoming, etc., while in the other asynchronized group are found states like Rhode Island, New Jersey, Maine, etc. The same exercise performed is applied for the 2001’s and 2007’s recessions respectively, showing that for both episodes U.S. states can be grouped only into two clusters, which can be interpreted as the "core" and the "periphery". The "core" that is highly in sync with the national cycle contains the great part of

9The full animated representation of the synchronization between U.S. states can be found at: https://sites.google.com/site/danileivaleon/media
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the states, while the "periphery" just few of them, some examples are North Dakota, Wyoming, Oklahoma, Montana, etc.

1.3.3 Network Analysis

The intuition behind the proposed framework for monitoring synchronization in Section 2, relies on the fact that if \( \delta_{t}^{ab} \approx 1 \), then it is likely that states \( a \) and \( b \) are sharing the same business cycle dynamics, or in other words, economic shocks affecting to \( a \) could also be affecting to \( b \) creating a link of dependence between them. By means of this, U.S. states, denoted by \( h_i \) for \( i = 1, \ldots, n \), and collected in \( H = \{h_i\}_{1}^{n} \), can be interpreted as nodes, interacting in the weighted network \( g_t \), with the relationship between each pair of nodes \( h_a \) and \( h_b \) at \( t \), given by the strength of their link \( \delta_{t}^{ab} \in [0, 1] \) and collected in \( \Theta_t \).

The previous graphs mapping states business cycle similarities for the beginning of the last three recessions showed two different scenarios. In the first one, 1990’s recession, states tend to be less synchronized between them and also with respect to the National business cycle than the other two, 2001’s and 2007’s recessions. Now this paper will try to explore: up to which extent the interconnectedness between smaller economies can be helpful to anticipate national recessions? The Markov-Switching Synchronization Network (MSYN) will be used to determine the connectedness between nodes (states) and therefore the "key" nodes (states) composing the U.S. business cycle core or leading economies, that will help to anticipate global recessions. In order to have a glimpse about the form that the MSYN would take during contractionary episodes, Figure 1.A plots the corresponding graph for the first month of the last three recessions. Given that the MSYN is a weighted network, in order to make possible the graphical representation, the link between \( a \) and \( b \) is plotted if \( \delta_{t}^{ab} > 0.5 \), otherwise there is no link between them. It is important
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to note that although the U.S. business cycle is not included in the network analysis, just the states, all the three plots show close relation with the ones in Figure 1.A, where the national cycle is included, corroborating the grouping pattern.

By looking at a disaggregated perspective, this paper has shown that U.S. states present a high dynamic heterogeneity among their business cycle phases, due to while some states are entering in recession, the others may or may not follow the same pattern. This study takes advantage of this fact departing from the hypothesis that when this heterogeneity starts to decrease, meaning that all states start to converge to the same phase, something bad is going to happen, agreeing with the traditionally view that economies tend to show higher converge during periods of recessions than during periods of expansion.

In order to find the core dynamics leading the national economy, it is required to look at the prominence of each node (state) in the present synchronization network, for this purpose the concept used to answer this kind of questions is the one of centrality. There are several measures regarding to the centrality of a node in a network, but given that desynchronization measures, $\gamma_{it}^{ab}$, under the present framework are interpreted as distances, providing information about the farness among U.S. states business cycles, the most appropriate measure for this context is the one of closeness centrality.

The farness of a given node is defined as the sum of its distances to all other nodes, where the distance between two nodes $i$ and $j$ is given by the length of the shortest path between them, denoted by $d(i, j)$. In order to compute $d(i, j)$, the Dijkstra’s (1959) algorithm is applied since it solves the shortest path problem on a weighted graph $G_t = (H, \Theta_t)$ for the case in which all edges weights are nonnegative.$^{10}$

$^{10}$For example, in a set $H' = \{a, b, c\}$ where the distances $\gamma = 1 - \delta$, are given by $\gamma_{ab} = 0.5$, $\gamma_{ac} = 0.9$ and $\gamma_{bc} = 0.2$, the shortest path between $a$ and $c$ will be $0.7$, since $\gamma_{ab} + \gamma_{bc} < \gamma_{ac}$, hence notice that $d(a, c)$ does not necessarily have to be equal to $\gamma_{ac}$. 

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The closeness between two nodes is defined as the inverse of the farness. Thus, the more central is a node, the lower is its total distance to all other nodes. Closeness can be regarded as a measure of how fast it will take to spread information, risk, economic shocks, etc., from node $i$ to all other nodes sequentially.\footnote{For an overview regarding to definitions in network analysis, see Goyal (2007).} The total distance from node $i$ to all other nodes in the network $g$, i.e. the farness of $i$ at time $t$, is given by $\sum_{j \neq i|t} d_t(i, j)$, being its reciprocal the closeness centrality of node $i$:

$$C_t(i) = \frac{1}{\sum_{j \neq i|t} d_t(i, j)}.$$  \hspace{1cm} (1.50)

This measure of centrality has a natural analogue at the aggregate network level. Let $i^*$ be the node that attains the highest closeness centrality across all nodes and let $C_t(i^*)$ be this centrality at time $t$, then the closeness centrality of the network is given by

$$C_t = \sum_{i=1|t}^k [C_t(i^*) - C_t(i)].$$ \hspace{1cm} (1.51)

This measure is calculated for every period of time with the information available in $\Theta_t$ for the whole sample, i.e. since 1979:8 until 2012:3, plotting in Chart A of Figure 1.A the dynamics of the closeness centrality for the MSYN involving U.S. states, showing a clear and easy to interpret pattern. The centrality shows a markedly high tendency to increase some months before national recessions take place, keeping high values during the whole contractionary episode and even some months after it ends, then returning to an stable level that seems to prevail in general during the rest of the expansion period. High centrality levels reported after recessions have ended could be in part attributed to the so-called "third phase" in the business cycle of some U.S. states, since the pattern associates stable growth with low centrality, while recessions and recoveries with high centrality.

The most useful feature of the centrality index, $C_{c,t}$, is the capacity to anticipate national recessions, since some periods before a contractionary episode begin,
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it starts to increase. In order evaluate this predictive power the index was reestimated two times with data up to one month before the beginning of the last two recessions.\textsuperscript{12} The Chart B of Figure 1.A plots the centrality index for the 2007’s and 2001’s recessions respectively, corroborating its reliability to anticipate the last two recessions U.S. recessions, since the index starts to increase some months before these national recessions took place.

Additionally, it is possible to explore the average pattern of the states’ centrality in the economy. By accounting for the centrality of each node (state) using Equation (1.50), and averaging them trough time, the Chart A of Figure 1.A plots the state’s centrality pattern for the period 1979:08 - 2012:03. The figure shows the regions with the highest and the lowest centrality. The regions with the highest centrality indicate the geographical zones that experiment the highest business cycle interdependence with the rest of the U.S. states’ economies simultaneously. Although in the analysis none geographic location information has been previously considered, there is a clear geographical association in Figure 1.A, showing that spatial proximities among regional economies seem to be one of the fundamental determinants in business cycle synchronization for the U.S. economy.

Some of the most central states are North Carolina, Wisconsin, Tennessee, Missouri and Georgia, while some states showing the lowest centrality are Montana, Oklahoma, Louisiana, North Dakota and Texas. Regarding the economies with high centrality, notice that some of them, as North Carolina and Georgia, represent a significant share of the U.S. GDP. However, Wisconsin, Tennessee and Missouri, are modest economies since each of them represent about 1.7% of the total GDP. Then, how is possible that modest economies tend to be highly representative of the national business cycle? One of the potential answers rely on the industrial composition of the economies. The top pies in Chart B of Figure 1.A show the industrial

\textsuperscript{12}The same analysis was not performed for the 1990’s, an the previous recessions due to the small size of the sample.
composition of the U.S. economy and the economy of Missouri, showing a similar composition, while the bottom pies of the same Chart, makes the same exercise, but comparing the industrial composition of the U.S. economy and the one of Texas, which is the second largest state in terms of GDP share, showing that these two pies are totally different. The intuition behind these comparisons rely on the fact that if there are two economies with similar (different) structure, it is very likely (unlikely) that macroeconomic shock affecting to one of them, will also affect to the other. This implies that regional industrial composition can be another fundamental determinant of business cycle synchronization in the U.S. economy. However, a deeper analysis of the factors driving synchronization remain as future research.

Apart from dynamic estimates of the interdependence degree, this framework also provides the possibility of computing stationary estimates, obtained with the ergodic or time-independent probabilities of $V_t$, as in Hamilton (1994), that can be interpreted as an *average sync* during the period under study. The same procedure can be followed to compute stationary *states vs. national* synchronizations, which are reported in Table 1.A. Not surprisingly the states in the core show high values, that is, Missouri (0.84), Wisconsin (0.87), Alabama (0.92), Tennessee (0.93), and North Carolina (0.94) with the highest value. Interestingly, these states also show high concordance with the National business cycle according to the results in Owyang et al. (2005), obtained under a univariate approach. Moreover, a whole stationary sync picture is obtained and the resulting map is reported in Chart A of Figure 1.A, showing that for 1979:8-2012:3, U.S. states can be grouped into three clusters, a highly, discreetly and lowly in sync with the national business cycle. This results also coincides with the one in Hamilton and Owyang (2012), where it is found three clusters with individual states in a given cluster sharing certain business cycle characteristics, as can be seen in Chart B of Figure 1.A, not coincidentally the states in each cluster roughly coincide in both studies giving robustness to the results found in this paper.
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1.4 Conclusions

This paper proposes a novel framework for monitoring changes in the degree of synchronization between many stochastic processes that are subject to regime changes. Specifically, it computes regime inferences from a multivariate Markov-switching model and simultaneously estimates a measure of the time-varying synchronization between the unobserved state variables governing each process involved in the system.

The proposed framework is used as a tool for monitoring the time-varying synchronization among U.S. states business cycles in order to assess, first, up to which extent the interconnectedness between smaller economies can be helpful to anticipate national recessions? and second, how a national recession that is coming soon is going to affect each of its smaller economies at the state level?

The results suggest that national recessions can be anticipated by an index that accounts for the global synchronization of states under an interdependent environment, confirming its reliability with real time exercises and indicating that decreases in regional recessions heterogeneity are helpful in order to predict upcoming national recessions. By characterizing the dynamic grouping pattern, it is possible to assess on a timely basis how an upcoming national recession could simultaneously affect each of its smaller economies at the state level. Finally, the results indicate that factors as the industrial composition and the geographic location of regional economies seems to be fundamental determinants of business cycle phases synchronization in the U.S. economy.
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1.A Tables and Figures

Table 1.1: Bivariate MS Synchronization Model for New York and Texas

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{ny,0}$</td>
<td>-0.0934</td>
<td>0.0131</td>
<td>-0.0944</td>
</tr>
<tr>
<td>$\mu_{ny,1}$</td>
<td>0.2480</td>
<td>0.0126</td>
<td>0.2485</td>
</tr>
<tr>
<td>$\sigma^2_{ny}$</td>
<td>0.0073</td>
<td>0.0005</td>
<td>0.0072</td>
</tr>
<tr>
<td>$p_{ny,11}$</td>
<td>0.9837</td>
<td>0.0073</td>
<td>0.9847</td>
</tr>
<tr>
<td>$p_{ny,00}$</td>
<td>0.9312</td>
<td>0.0263</td>
<td>0.9346</td>
</tr>
<tr>
<td>$\mu_{tx,0}$</td>
<td>-0.0601</td>
<td>0.0102</td>
<td>-0.0601</td>
</tr>
<tr>
<td>$\mu_{tx,1}$</td>
<td>0.1691</td>
<td>0.0107</td>
<td>0.1692</td>
</tr>
<tr>
<td>$\sigma^2_{tx}$</td>
<td>0.0064</td>
<td>0.0004</td>
<td>0.0064</td>
</tr>
<tr>
<td>$p_{tx,11}$</td>
<td>0.9839</td>
<td>0.0070</td>
<td>0.9852</td>
</tr>
<tr>
<td>$p_{tx,00}$</td>
<td>0.9358</td>
<td>0.0251</td>
<td>0.9390</td>
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<tr>
<td>$\sigma_{ny,tx}$</td>
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<td>0.0013</td>
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<tr>
<td>$p_{11}$</td>
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<td>0.9819</td>
</tr>
<tr>
<td>$p_{00}$</td>
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<td>0.0264</td>
<td>0.9323</td>
</tr>
<tr>
<td>$p_{V,11}$</td>
<td>0.9501</td>
<td>0.0481</td>
<td>0.9647</td>
</tr>
<tr>
<td>$p_{V,00}$</td>
<td>0.9480</td>
<td>0.0360</td>
<td>0.9565</td>
</tr>
</tbody>
</table>

Note: The selected example presents the case of two states with high and similar U.S. GDP share, New York with 7.68%, and Texas with 7.95%.
Chapter 1. Monitoring Synchronization of Regional Recessions

Table 1.2: Bivariate MS Synchronization Model for California and Vermont

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{ny,0}$</td>
<td>-0.0211</td>
<td>0.0100</td>
<td>-0.0215</td>
</tr>
<tr>
<td>$\mu_{ny,1}$</td>
<td>0.1674</td>
<td>0.0103</td>
<td>0.1673</td>
</tr>
<tr>
<td>$\sigma_{ny}$</td>
<td>0.0068</td>
<td>0.0005</td>
<td>0.0068</td>
</tr>
<tr>
<td>$p_{ny,11}$</td>
<td>0.9773</td>
<td>0.0090</td>
<td>0.9785</td>
</tr>
<tr>
<td>$p_{ny,00}$</td>
<td>0.9450</td>
<td>0.0205</td>
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<td>$\mu_{tx,0}$</td>
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<td>0.0136</td>
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</tr>
<tr>
<td>$\mu_{tx,1}$</td>
<td>0.2012</td>
<td>0.0141</td>
<td>0.2010</td>
</tr>
<tr>
<td>$\sigma_{tx}$</td>
<td>0.0127</td>
<td>0.0010</td>
<td>0.0127</td>
</tr>
<tr>
<td>$p_{tx,11}$</td>
<td>0.9764</td>
<td>0.0093</td>
<td>0.9774</td>
</tr>
<tr>
<td>$p_{tx,00}$</td>
<td>0.9413</td>
<td>0.0217</td>
<td>0.9432</td>
</tr>
<tr>
<td>$\sigma_{ny,tx}$</td>
<td>0.0028</td>
<td>0.0005</td>
<td>0.0027</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.9772</td>
<td>0.0092</td>
<td>0.9748</td>
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<tr>
<td>$p_{00}$</td>
<td>0.9432</td>
<td>0.0210</td>
<td>0.9458</td>
</tr>
<tr>
<td>$p_{V,11}$</td>
<td>0.9742</td>
<td>0.0242</td>
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</tr>
<tr>
<td>$p_{V,00}$</td>
<td>0.9362</td>
<td>0.0417</td>
<td>0.9461</td>
</tr>
</tbody>
</table>

Note: The selected example presents the case of the states with the highest and the lowest U.S. GDP share, i.e. California with 13.34% and Vermont with 0.18%.
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Table 1.3: Stationary MS Synchronization between State and U.S. business cycle phases

<table>
<thead>
<tr>
<th>State</th>
<th>Sync</th>
<th>State</th>
<th>Sync</th>
<th>State</th>
<th>Sync</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alabama</td>
<td>0.92</td>
<td>Maine</td>
<td>0.80</td>
<td>Ohio</td>
<td>0.92</td>
</tr>
<tr>
<td>Arizona</td>
<td>0.79</td>
<td>Maryland</td>
<td>0.48</td>
<td>Oklahoma</td>
<td>0.22</td>
</tr>
<tr>
<td>Arkansas</td>
<td>0.71</td>
<td>Massachusetts</td>
<td>0.44</td>
<td>Oregon</td>
<td>0.74</td>
</tr>
<tr>
<td>California</td>
<td>0.87</td>
<td>Michigan</td>
<td>0.90</td>
<td>Pennsylvania</td>
<td>0.88</td>
</tr>
<tr>
<td>Colorado</td>
<td>0.41</td>
<td>Minnesota</td>
<td>0.90</td>
<td>Rhode Island</td>
<td>0.58</td>
</tr>
<tr>
<td>Connecticut</td>
<td>0.66</td>
<td>Mississippi</td>
<td>0.85</td>
<td>S. Carolina</td>
<td>0.92</td>
</tr>
<tr>
<td>Delaware</td>
<td>0.80</td>
<td>Missouri</td>
<td>0.84</td>
<td>S. Dakota</td>
<td>0.71</td>
</tr>
<tr>
<td>Florida</td>
<td>0.68</td>
<td>Montana</td>
<td>0.39</td>
<td>Tennessee</td>
<td>0.93</td>
</tr>
<tr>
<td>Georgia</td>
<td>0.80</td>
<td>Nebraska</td>
<td>0.61</td>
<td>Texas</td>
<td>0.38</td>
</tr>
<tr>
<td>Idaho</td>
<td>0.62</td>
<td>Nevada</td>
<td>0.75</td>
<td>Utah</td>
<td>0.72</td>
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<tr>
<td>Illinois</td>
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<td>N. Hampshire</td>
<td>0.60</td>
<td>Vermont</td>
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<td>Indiana</td>
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<td>New Jersey</td>
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<td>Iowa</td>
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<td>New Mexico</td>
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<td>Washington</td>
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</tr>
<tr>
<td>Kansas</td>
<td>0.85</td>
<td>New York</td>
<td>0.77</td>
<td>W. Virginia</td>
<td>0.55</td>
</tr>
<tr>
<td>Kentucky</td>
<td>0.84</td>
<td>N. Carolina</td>
<td>0.94</td>
<td>Wisconsin</td>
<td>0.87</td>
</tr>
<tr>
<td>Louisiana</td>
<td>0.24</td>
<td>N. Dakota</td>
<td>0.32</td>
<td>Wyoming</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Note: The table reports the stationary degree of synchronization for 1979:8 to 2012:3. Those estimates correspond to the ergodic probability that the business cycle phases of the corresponding state and the one of the U.S. economic activity are totally dependent, i.e. \( \Pr(V_t = 1) \).
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Figure 1.1: Markov-Switching Synchronization between Selected States

Chart A

New York vs. Texas Synchronization

Note. The Chart A of the figure plots the probabilities of recession for New York and Texas and its time-varying synchronization. The Chart B of the figure plots the probabilities of recession for California and Vermont and its time-varying synchronization. Shaded areas correspond to NBER recessions. The full set of pairwise cases can be available upon request to the author.
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Figure 1.2: Markov-Switching Synchronization between States and U.S.

Note. Each chart plots the time-varying degree of synchronization of business cycle phases between each U.S. state and the whole U.S. economy. Shaded areas correspond to NBER-referenced recessions.
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Figure 1.3: Markov-Switching Synchronization between States and U.S. (cont.)

Note. Each chart plots the time-varying degree of synchronization of business cycle phases between each U.S. state and the whole U.S. economy. Shaded areas correspond to NBER-referenced recessions.
Figure 1.4: Markov-Switching Synchronization between States and U.S.
(cont.)

Note. Each chart plots the time-varying degree of synchronization of business cycle phases between each U.S. state and the whole U.S. economy. Shaded areas correspond to NBER-referenced recessions.
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Figure 1.5: Markov-Switching Synchronization between States and U.S. (cont.)

Note. Each chart plots the time-varying degree of synchronization of business cycle phases between each U.S. state and the whole U.S. economy. Shaded areas correspond to NBER-referenced recessions.
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Figure 1.6: U.S. States Synchronization Map for the start of Recessions

Note. The figure plots the multidimensional scaling map based on the Euclidean distance of the U.S. States business cycle characteristics for different periods. The distances are normalized with respect to the U.S. National Economic Activity, the red point in the center. The size of the points makes reference to the GDP share of the corresponding state. The full animated version is available at:

https://sites.google.com/site/daniloleivaleon/media
Figure 1.7: U.S. States Synchronization Network for the start of Reces-
sions

Note. The figure plots the interconnectedness in terms of synchronization between the business cycle
phases of U.S. States. Each node represent a State, and the green each green line represent the link
between two states, which take place only if Pr(V_t=1)>0.5. The full animated version is at:

https://sites.google.com/site/daniloleivaleon/media
Chapter 1. Monitoring Synchronization of Regional Recessions

Figure 1.8: Centrality of the Synchronization Network

Note. The Chart A plots the closeness centrality of the synchronization network composed by the U.S. state for each period of time. Shaded areas correspond to recessions as documented by the NBER. The top figure of Chart B plots the dynamic closeness centrality with the data until 12/2007. The bottom figure plots the dynamic closeness centrality with the data until 03/2001. Shaded areas correspond to recessions as documented by the NBER.
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Figure 1.9: Average Centrality Pattern through U.S. States

Chart A. U.S. States Average Centrality Pattern: 1979:8 – 2012:3

Chart B. Average Industrial Composition: 1997 – 2012

Texas
Missouri
US

Note. Chart A plots the centralities of all states averaged through time, the intensity of the colour makes reference to the levels of centrality. The more central is a state the more synchronized it is with the rest of states simultaneously. Chart B plots the industrial composition of the regional economies, Missouri and Texas, and the industrial composition of the U.S. economy.
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Figure 1.10: Ergodic Synchronization

Chart A. Stationary Synchronization Map in U.S. States

Note. Chart A plots the multidimensional scaling map based on the stationary Euclidean distance of the U.S. States business cycle characteristics for the sample 1979:08-2010:03. The distances are normalized with respect to the U.S. National Economic Activity, the red point in the center. The full animated version can be found at the author’s webpage. In Chart B, the shading for each state indicates the probability that such state belongs to any given cluster. Source: Hamilton and Owyang (2012)
Examples

References


References


References


Chapter 2

Real vs. Nominal Cycles: A Markov-Switching Bi-Factor Approach

2.1 Introduction

The National Bureau of Economic Research defines the notion of business cycles as periodic but irregular up-and-down movements in economic activity, typically observed in macroeconomic indicators such as real GDP and Industrial Production. However, its Business Cycle Dating Committee does not enter into the causes of these recessions, which are traditionally assumed to come from two different sources. On the one hand, recessions that start on the supply side of the economy are known to be caused by supply shocks which typically affect production costs. On the other hand, recessions that start on the demand side of the economy are known as caused by demand shocks which affect economy-wide expenditure levels. To discriminate between these two sources of business cycle downturns, it is worth to emphasize
that although both types of shocks cause decreases in actual economic activity, their effects on the price level is different.

In a seminal work, Blanchard (1989) investigates if the joint behavior of U.S. real and nominal variables is consistent with the traditional interpretation of macroeconomic fluctuations, i.e. that aggregate demand (supply) shocks move output and prices in the same (opposite) direction, finding a qualified yes as an answer. In other words, while recessionary demand shocks tend to produce price declines, negative supply shocks tend to increase the price level.

Recently, Aruoba and Diebold (2010) examine the dynamic interactions between real activity and prices over the business cycle to extract information about the sources of the contractionary shocks. For this purpose, they propose two separate state-space linear dynamic-factor models and use the Kalman filter to produce optimal extractions of real and nominal activities. According to these authors the coherence of their respective movements and the business cycle chronology determined by the NBER are the key to determine whether the recessionary shocks are demand- or supply-driven.

Relying on the widely accepted view that recessions are caused by adverse shocks of different nature, with the corresponding mix varying substantially across recessions, Galí (1992), Ireland (2010) and Forni and Gambetti (2010), this paper proposes a multistate Markov-switching dynamic bi-factor approach that overcomes two main drawbacks of the analysis in Aruoba and Diebold (2010) and that allows to make inference on the type of aggregate shocks hitting the business cycle in order to uncover the sources of the last U.S. recessions.

First, although they examined the interactions between real activity and prices, they use separate dynamic factor models to compute the real and nominal indexes, without taking into account the potential interrelation between these two concepts.
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The model in this paper extends the previous approach to consider a unified framework two separate factors extracted from the same set of real and nominal indicators. Hence, both real and nominal indexes are endogenously determined and the interactions between the indicators and the factors are estimated without strong restrictions.

Second, although since the early definition of Burn and Mitchel (1946) that business cycle dynamics is considered asymmetric, Arouba and Diebold (2010) extract the factors from linear models. The proposed model in this paper accounts for nonlinearities by allowing the factors to be governed by two potentially dependent Markov-switching processes. Then, this proposal is a natural extension of the single-index Markov-switching dynamic factor model proposed in the late nineties by Kim and Yoo (1995), Chauvet (1998) and Kim and Nelson (1998), since it relaxes the restriction that all the indicators depend on a unique common nonlinear dynamics. Accordingly, the algorithm used to estimate the model in this paper via maximum likelihood is extended to consider two factors that depend on two separate state variables, dealing with issues related to their dependence relation the identification of the factors.

The main findings suggest the following. First, coinciding with Stock and Watson (1999), real activity indicators, such as Industrial production, play a fundamental role on the dynamic behavior of the inflation index. As well as the real activity index is significantly influenced by nominal variables, such as consumer prices. Moreover, the synchronicity degree between the indexes shows that U.S. business and inflation cycle phases have coincided a 30% of the time during the last half century, supporting the need of a unified framework to analyze real and nominal interactions.


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1Kholodilin and Yao (2005) proposes a similar approach applied to the case of leading and coincident indicators, by setting one state variable to be the lag of the other.

The paper is structured as follows. Section 2, develops the algorithm to estimate the multistate Markov-switching dynamic bi-factor model, which can be straightforwardly generalized to the case of $K$ factors. Section 3 examines the empirical results, identification issues, analyses the dynamic interaction between real economic activity and inflation, and quantify the contribution of shocks to business cycle phases. Section 4 concludes.

### 2.2 The Model

#### 2.2.1 Model’s Dynamics

In this section, I combine the dynamic-factor and Markov-switching frameworks to create a statistical model capturing both regime shifts and comovements. Specifically, the log-level of each of the $N$ economic indicators, $y_{it}$, is modelled as composed of three stochastic autoregressive processes. The first component corresponds to the common factor among the real activity indicators, $f_{t}^{b}$, and refers to the business cycle conditions. The second component corresponds to the common factor among the nominal indicators, $f_{t}^{p}$, and refers to the underlying price evolution. Finally, the third component corresponds to the idiosyncratic dynamics, $e_{it}$, and refers to the particular evolution of the time series. According to previous studies (see Aruoba
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and Diebold, 2010, and references therein), a stochastic trend is not included in the dynamic factor based on assumption that each of the series studied are integrated but not cointegrated. Therefore, the empirical analysis is undertaken using the log of the first difference of the observable indicators

\[ y_{it} = \gamma^b_i f^b_t + \gamma^p_i f^p_t + e_{it}, \quad (2.1) \]

where \( \gamma^b_i \) and \( \gamma^p_i \) refer to the factor loadings.

To complete the specification of the data generating process, the factors, \( f^b_t \) and \( f^p_t \), are assumed to be governed by two unobserved regime-switching variables, \( S^b_t \) and \( S^p_t \), respectively. Hence, the dynamics of these factors can be specified as

\[ f^r_t = \mu^r_t + \sum_{h=1}^k b_r (f^r_{t-h} - \mu^r_{t-h}) + \omega^r_t, \quad (2.2) \]

where the errors, \( \omega^r_t \), are distributed as \( N(0, \sigma^2_r) \), and \( r = b, p \).\(^2\) Within this framework, one can label \( S^b_t = 1 \) as expansions and \( S^b_t = 2 \) as recessions at time \( t \) if \( \mu^b_1 > \mu^b_2 \). In addition, one can also label \( S^p_t = 1 \) as highly inflationary regimes and \( S^p_t = 2 \) as regimes of low inflation at time \( t \) if \( \mu^p_1 > \mu^p_2 \). In these cases, the first coincident indicator is expected to exhibit high (usually positive) growth rates in expansions and low (usually negative) growth rates in recessions, while the second coincident indicator is expected to exhibit higher growth rates in inflationary regimes and low growth rates periods of price stability. In addition, each state variables is assumed to evolve according to a irreducible 2-state Markov chains whose transition probabilities are defined by

\[ p(S^r_t = j | S^r_{t-1} = i, S^r_{t-2} = h, \ldots, \psi_{t-1}) = p(S^r_t = j | S^r_{t-1} = i) = p^r_{ij}, \quad (2.3) \]

where \( r = b, p \), \( \psi_t \) is the information set up to period \( t \), and \( i, j, h = 0, 1 \).\(^3\)

\(^2\)According to Albert and Chib (1993), an AR(0) Markov-switching model, provides a useful model of the U.S. quarterly output series, hence following Camacho and Perez-Quiros (2007), I use \( k = 0 \) in the empirical application. In this case, the economic indicators are modelled as a recurrent sequences of shifts between two fixed equilibria of high and low growth means.

\(^3\)The variances \( \sigma^2_b \) and \( \sigma^2_p \) are taken to be unity for identification of the model.
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The dynamics of the idiosyncratic components are stated as

\[ e_{it} = \sum_{h=1}^{m} \phi_{ih} e_{i,t-h} + \varepsilon_{it}, \]  
\[ (2.4) \]

where \( \varepsilon_{it} \) are distributed as \( N(0, \sigma^2_i) \), with \( i = 1, \ldots, N \). Finally, all the shocks, \( \varepsilon_{it} \) and \( \omega_{rt} \), are assumed mutually uncorrelated in cross-section and time-series dimensions.

2.2.2 Estimation Procedure

For estimation purpose, it is convenient to cast model into state space form. More compactly, the measurement equation is defined as

\[ y_t = H \beta_t + \xi_t, \]  
\[ (2.5) \]

where \( y_t \) is an \( N \)-vector that collects the observed indicators, and \( \xi_t \sim i.i.d. N(0, R) \). In addition, the expression for the transition equation is defined as

\[ \beta_t = \tilde{\mu}_{st} S_t + F \beta_{t-1} + v_t, \]  
\[ (2.6) \]

with \( v_t \sim i.i.d. N(0, Q) \). An extensive description of what these equations look like for the empirical model and a the detailed form of \( H, F, \tilde{\mu}, \xi_t, v_t \), and the state vector \( \beta_t \) have been set out in the Appendix 2.1.

If the regimes that determine the evolution of the two factors were observable, then the system would be a linear Gaussian dynamic factor model and the standard Kalman filter combined with procedures based on the likelihood functions could be applied to obtain parameter estimates and the paths of the unobservable components. However, since the regimes are not directly observed, rather it must be inferred from the data, the usual Kalman filter cannot be employed. Instead, each iteration of the Kalman filter produces a fourfold increase in the number of cases to consider and approximations to the Kalman filter are unavoidable.
Based on the approximate maximum-likelihood estimation method of Kim (1994), I propose an algorithm to estimate the nonlinear dynamic bi-factor model. Basically, the method contains three unified stages which are run in each iteration of the Kalman and Hamilton filters. In the first stage, the algorithm computes one-step-ahead predictions of the state vector and its associated mean squared error matrices by using as inputs the joint probabilities of the Markov-switching processes and the state vector. Adding a new set of observations, the Kalman filter updates the state vector and its mean squared errors and evaluates the likelihood function conditional on the bi-variate Markov processes. In the second stage, the algorithm applies the Hamilton’s (1989) filter which involves an evaluation of the likelihood function and updates the filtered probabilities. Accordingly, the likelihood function can be maximized with respect to the model parameters. In the third stage, the algorithm collapses the posteriors using the probability terms according to the Kim’s (1994) approximations.\footnote{The algorithm can straightforwardly be generalized to a Markov-switching dynamic $K$-factor model where each factor is governed by $M$-state variables. For the empirical purposes of this paper, we focus just on the case of a bi-factor model which largely facilitates notation.}

**Stage 1:** The goal is to form a forecast of the state vector, $\beta_t$, and its associated mean squared error matrices, $P_t$, conditional on the information set $\psi_{t-1}$, and on present and past states of each unobservable variables $S^b_t$ and $S^p_t$. Assuming that the state variables take on the values $j^b$ and $j^p$ at $t$, and take on the values $i^b$ and $i^p$ at $t - 1$, the forecasts are computed from the prediction equations

\begin{align}
\hat{\beta}_t^{(i^b,j^p,j^b,j^p)} &= \tilde{\mu}_{j^b,j^p} + F_t^{(i^b,i^p)} \hat{\beta}_{t-1|t-1}^{(i^b,i^p)}, \\
\hat{P}_t^{(i^b,j^p,j^b,j^p)} &= FP_{t-1|t-1} F' + Q,
\end{align}

where $i^b, i^p, j^b, j^p = 1, 2$.

Once a new set of observations is included, the algorithm computes the forecast
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eerror and its variance matrix that can be obtained as

\[
\eta^{(i^b,i^p,j^b,j^p)}_{i|t-1} = \gamma_{t} - H_{i|t-1}^{(i^b,i^p,j^b,j^p)},
\]

(2.9)

\[
f^{(i^b,i^p,j^b,j^p)}_{i|t-1} = H_\eta^{(i^b,i^p,j^b,j^p)}H' + R.
\]

(2.10)

In addition, the conditional likelihood of the observable variables can be evaluated
as a by-product of the algorithm at each \( t \), which allows estimation of the unknown
model parameters. The likelihood function at each \( t \) is:

\[
f(y_t|\psi_{t-1}) = \sum_{i^b,j^b,i^p,j^p} \ f(y_t|S^b_{t-1} = i^b, S^p_{t-1} = i^p, S^b_{t} = j^b, S^p_{t} = j^p, \psi_{t-1})
\times p(S^b_{t-1} = i^b, S^p_{t-1} = i^p, S^b_{t} = j^b, S^p_{t} = j^p|\psi_{t-1})
\]

(2.11)

where the first terms of these products are the conditional Gaussian

\[
(2\pi)^{-\frac{N}{2}} |f^{(i^b,i^p,j^b,j^p)}_{i|t-1}|^{-\frac{1}{2}} \exp \left[ -\frac{1}{2} \eta^{(i^b,i^p,j^b,j^p)}_{i|t-1} \left( f^{(i^b,i^p,j^b,j^p)}_{i|t-1} \right)^{-1} \eta^{(i^b,i^p,j^b,j^p)}_{i|t-1} \right]
\]

(2.12)

and the second probability terms are computed in the next stage.

**Stage 2:** The goal is to compute inferences about the different states by using
Hamilton’s nonlinear filter. Since the dependence relationship between the two
Markov-switching variables is unknown, in order to model the joint probability events
associated to the possible realizations of each unobserved state variable, I rely on the
two polar cases of dependence. First, the completely independent case, in which the
joint probability event is just the product of the individual probabilities.

\[
p(S^b_{t-1} = i^b, S^p_{t-1} = i^p, S^b_{t} = j^b, S^p_{t} = j^p|\psi_{t-1}) = p(S^b_{t-1} = i^b, S^b_{t} = j^b|\psi_{t-1}) \times
p(S^p_{t-1} = i^p, S^p_{t} = j^p|\psi_{t-1}).
\]

(2.13)

Second, the completely dependent or perfect synchronization case, in which both
Markov-switching variables follow exactly the same pattern, implying that there is
just one state variable governing the whole model’s dynamics, i.e. \( S^b_t = S^p_t = S_t \).

\[
p(S^b_{t-1} = i^b, S^p_{t-1} = i^p, S^b_{t} = j^b, S^p_{t} = j^p|\psi_{t-1}) = p(S_{t-1} = i, S_t = j|\psi_{t-1}).
\]

(2.14)
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Then, I follow the line of Bengoechea et al. (2005), who suggest that in empirical applications such degree of dependence should be located somewhere in between these two extreme possibilities. Hence, it can be seen as a linear combination between them, given by a parameter \( \delta \) which provides insights about the degree of synchronization between the state variables and that satisfies \( 0 \leq \delta \leq 1 \).

\[
p(S_{t-1}^p = i^p, S_t^p = j^p | \psi_{t-1}) = \delta \times p(S_{t-1} = i, S_t = j | \psi_{t-1}) + (1 - \delta) \times p(S_{t-1}^b = i^b, S_t^b = j^b | \psi_{t-1})
\]

The terms on the right hand side of equations (2.13) and (2.14) can easily be obtained by using the transition probabilities

\[
p(S_t = i, S_{t-1} = j | \psi_{t-1}) = p(S_t = j | S_{t-1} = i)p(S_{t-1} = i | \psi_{t-1}), \tag{2.17}
\]

where \( r = d, b \).

**Stage 3:** Using the new set of observations at the end of time \( t, y_t \), the probability terms can be updated using Bayes rule

\[
p(S_t^r = i^r, S_{t-1}^r = j^r | \psi_t) = \frac{f(y_t, S_{t-1}^r = i^r, S_t^r = j^r | \psi_{t-1})}{f(y_t | \psi_{t-1})}, \tag{2.18}
\]

\[
p(S_{t-1} = i, S_t = j | \psi_t) = \frac{f(y_t, S_{t-1} = i, S_t = j | \psi_{t-1})}{f(y_t | \psi_{t-1})}, \tag{2.19}
\]

where

\[
f(y_t, S_{t-1}^r = i^r, S_t^r = j^r | \psi_{t-1}) = \sum_{i', j'} f(y_t | S_{t-1}^r = i', S_t^r = j', S_{t-1}^r = i^r, S_t^r = j^r, \psi_{t-1}) \times p(S_{t-1}^r = i^r, S_{t-1}^r = i', S_t^r = j', S_t^r = j^r | \psi_{t-1}),
\]

\[
f(y_t, S_{t-1} = i, S_t = j | \psi_{t-1}) = \sum_{i', j'} f(y_t | S_{t-1}^r = i', S_t^r = j', S_{t-1}^r = i', S_t^r = j^r, \psi_{t-1}) \times p(S_{t-1}^r = i', S_{t-1}^r = i', S_t^r = j', S_t^r = j^r | \psi_{t-1}),
\]
with \( r, r' = b, p \). By the law of total probability, the state probabilities become

\[
p(S_t^r = j^r | \psi_t) = \sum_{i^r=1}^{2} p(S_{t-1}^{i^r} = i^r, S_t^r = j^r | \psi_t)
\]

(2.20)

\[
p(S_t = j | \psi_t) = \sum_{i^r=1}^{2} p(S_{t-1} = i, S_t = j | \psi_t)
\]

(2.21)

with \( r = b, p \).

The last step of the Kalman filter updates the inferences of the state vector and its variance matrix by using the updating equations

\[
\beta_{t|t}^{(i^b, i^p, j^b, j^p)} = \beta_{t|t-1}^{(i^b, i^p, j^b, j^p)} + P_{t|t-1}^{(i^b, i^p, j^b, j^p)} H' \left[ f_{t|t-1}^{(i^b, i^p, j^b, j^p)} \right]^{-1} \eta_{t|t-1}^{(i^b, i^p, j^b, j^p)},
\]

(2.22)

\[
P_{t|t}^{(i^b, i^p, j^b, j^p)} = \left( I - P_{t|t-1}^{(i^b, i^p, j^b, j^p)} H' \left[ f_{t|t-1}^{(i^b, i^p, j^b, j^p)} \right]^{-1} H \right) P_{t|t-1}^{(i^b, i^p, j^b, j^p)}.
\]

(2.23)

It is worth noting that the algorithm calculates a battery of \((2^2)^2\) different inferences of the state vector and its mean square error matrix, corresponding to every possible value of the vector \((i^b, i^p, j^b, j^p)^t\). This implies that after a few iterations the number of cases increases dramatically and the system become untractable.

To overcome this drawback, I extend the approximation to the filter suggested by Kim (1994) that reduces the number of different terms at each time \( t \) by collapsing the \((2^2)^2\) posterior \( \beta_{t|t}^{(i^b, i^p, j^b, j^p)} \) and \( P_{t|t}^{(i^b, i^p, j^b, j^p)} \), into \((2^2)\) posterior \( \beta_{t|t}^{(j^b, j^p)} \) and \( P_{t|t}^{(j^b, j^p)} \). In particular, I use

\[
\beta_{t|t}^{(j^b, j^p)} = \frac{\sum_{i^b=1}^{2} \sum_{i^p=1}^{2} p(S_{t-1}^{i^b} = i^b, S_{t-1}^{i^p} = i^p, S_t^{j^b} = j^b, S_t^{j^p} = j^p | \psi_t) \beta_{t|t}^{(i^b, i^p, j^b, j^p)}}{p(S_t^{j^b} = j^b, S_t^{j^p} = j^p | \psi_t)},
\]

(2.24)

and

\[
P_{t|t}^{(j^b, j^p)} = \frac{1}{p(S_t^{j^b} = j^b, S_t^{j^p} = j^p | \psi_t)} \sum_{i^b=1}^{2} \sum_{i^p=1}^{2} p(S_{t-1}^{i^b} = i^b, S_{t-1}^{i^p} = i^p, S_t^{j^b} = j^b, S_t^{j^p} = j^p | \psi_t)
\]

\[
\times \left[ P_{t|t}^{(i^b, i^p, j^b, j^p)} + \left( \beta_{t|t}^{(j^b, j^p)} - \beta_{t|t}^{(i^b, i^p, j^b, j^p)} \right) \left( \beta_{t|t}^{(j^b, j^p)} - \beta_{t|t}^{(i^b, i^p, j^b, j^p)} \right)^\top \right].
\]

(2.25)
2.2.3 Weights

The weights implicitly used by the Kalman Filter to perform factor estimates from the coincident variables can be calculated by measuring the effects of units changes in the lags of individual observations on the inference of the state vector $\beta_{t|t}$. They are useful for identification purposes since they give insights regarding to which are the key variables governing the evolution of each factor. The weights can be obtained directly from the Kalman filter matrices $\beta_t, P_t$ and $f_t$. However, contrary to the standard linear frameworks, these matrices are in this case state dependent. Since the Kalman filter is linear when the unobservable states are known, the expected value of the Kalman matrices conditional on the state variables is computed following the line used in Markov-switching impulse responses, that is

$$\Theta_{t|t-1} = \sum_{j^b=1}^{2} \sum_{j^p=1}^{2} \sum_{i^b=1}^{2} \sum_{i^p=1}^{2} \Theta_{t|t-1}^{(i^b,i^p,j^b,j^p)} p(S_t^b = j^b, S_{t-1}^b = i^b, S_t^p = j^p, S_{t-1}^p = i^p),$$

(2.26)

for $\Theta = \beta, P$ and $f$. According to Stock and Watson (1991) and Banbura and Rustler (2007), the weights are now easy to compute. Plugging the expression of the forecast errors into the forecasting equation leads:

$$\beta_{t|t} = \beta_{t|t-1} + P_{t|t-1} H' [f_{t|t-1}]^{-1} [y_t - H \beta_{t|t-1}].$$

(2.27)

Then, replacing $\beta_{t|t-1}$ in the right hand side of the above equation by the prediction equation and denoting the Kalman gain by $G_{t|t-1} = P_{t|t-1} H' [f_{t|t-1}]^{-1}$, it can be obtained

$$\beta_{t|t} = [I - G_{t|t-1} H] F \beta_{t-1|t-1} + G_{t|t-1} y_t + [I - G_{t|t-1} H] \tilde{\mu},$$

(2.28)

where

$$\tilde{\mu} = \sum_{j^b=1}^{2} \sum_{j^p=1}^{2} \tilde{\mu}_{j^b,j^p} p[S_t^b = j^b, S_t^p = j^p|\psi_t].$$

(2.29)

Since the matrix $F$ in the transition equation of the state-space representation is time invariant, $G_{t|t-1}$ converges to the steady-state Kalman gain, $G$. Under these
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conditions and with some algebra Equation (2.28) can also be expressed as:

$$\beta_{t|t} = M(L)y_t + J\tilde{\mu},$$

(2.30)

where $L$ denotes the lag-operator, $J = (I - (I - GH)FL)^{-1}(I - GH)$, and the elements of the matrix of lag polynomial $M(L) = (I - (I - GH)FL)^{-1}G$ measure the effect of changes in $y_t$ on the inference of $\beta_{t|t}$, which can be decomposed into the weighted sum of observations by letting $M_j$ be each of these matrices

$$\beta_{t|t} = \sum_{j=0}^{\infty} M_jy_{t-j} + J\tilde{\mu}. \quad (2.31)$$

Accordingly, $M(1) = (I - (I - GH)F)^{-1}G$, is the matrix that contains the cumulative impacts of the individual observations in the inference of the state vector, Camacho and Perez Quirós (2010).

2.2.4 Ragged Edges

The framework can also be extended to allow for missing observations in the data by following the approach in Mariano and Murasawa (2003). It consists in replacing missing observations with random draws $\epsilon_t$ from a $N(0, \sigma^2)$ that are independent from the model parameters that do not impact on the model estimation. As a consequence, some of the system matrices would be time-varying, remaining the elements in the measurement equation being replaced by the following expressions:

$$y_{it}^* = \begin{cases} y_{it} & \text{if } y_{it} \text{ observable} \\ \epsilon_t & \text{otherwise} \end{cases}, \quad (2.32)$$

$$H_{it}^* = \begin{cases} H_{it} & \text{if } y_{it} \text{ observable} \\ 0 & \text{otherwise} \end{cases}. \quad (2.33)$$
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\[ \xi_{it}^* = \begin{cases} 0 & \text{if } y_{it} \text{ observable} \\ \epsilon_t & \text{otherwise} \end{cases}, \]  
\[ (2.34) \]

\[ R_{iit}^* = \begin{cases} 0 & \text{if } y_{it} \text{ observable} \\ \sigma_i^2 & \text{otherwise} \end{cases}, \]  
\[ (2.35) \]

where \( y_{it} \) is the \( i \)-th element in the vector \( y_t \), \( R_{iit} \) its variance, \( H_{it} \) is the \( i \)-th row of the matrix \( H_t \) that contains \( \kappa \) columns, and \( \mathbf{0}_\kappa \) a row vector of \( \kappa \) zeroes. Accordingly, Equation (2.5) would be replaced by

\[ y_t^* = H^* \beta_t + \xi_t^*, \]  
\[ (2.36) \]

where \( \xi_t^* \sim i.i.d. N(0, R^*) \).

2.3 Empirical Results

2.3.1 Data

the indicators in logs, the standard tests for a unit root was unable to reject at standard significance levels. Accordingly, the empirical analysis uses the growth rates of the observable indicators.\footnote{Recall that we assume that the series are not cointegrated.} Finally, the variables are standardized to have a zero mean and a variance equal to one before estimating the model.

### 2.3.2 Maximum Likelihood Estimates

Several features of the maximum likelihood estimates of parameters in the three versions of the Markov-switching dynamic bi-factor model, reported in Table 2.B, deserve attention. First, the loading factors of $f_{it}^b$ that are associated to real activity and inflation indicators are positive and statistically significant, with the exception of GDP deflator and hourly compensation. However, the loading factors of $f_{it}^b$ that are related to real activity indicators are much higher than those related to price indicators. This result indicates that the first factor can be interpreted as a coincident index of the US real economic activity. Second, the loading factors of $f_{it}^p$ that are related to price indicators in the measurement equation are positive and statistically significant.\footnote{Again with the only exception of hourly compensation that has a negative loading factor related to $f_{it}^p$.} But, the loading factors of $f_{it}^p$ that are related to real activity indicators are negative. These estimates suggest that the second factor can be interpreted as an inflation index. Note that although the indicators has not been a priory classified as real and nominal, the model assigns endogenously the indicators loads on each factor. Third, the degree of dependence between the phases of the two indexes given by $\delta$, is equal to 0.32 and statistically significant at all levels. Since its interpretation refers to perfect synchronization when it is equal to one and total independence when it is equal to zero, it is suggesting that U.S. business cycles and inflation cycles coincide approximately 30\% of the time.
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Moreover, as it was pointed out in Section 2, the weights that variables have on each factor helps us to analyze further the extent to which indicators loads on each factor. These weights were computed, indicating that real activity variables have the 68% of the weight on the first factor dynamics, while price indicators have the 67% of the weight on the second factor. This result reinforces the interpretation of the first factor as an index of economic activity and the second factor as an index of inflation dynamics.

2.3.3 Factor Analysis

According to the results of the previous section, each of the eleven economic indicators is decomposed on two unobserved common dynamic factors plus an idiosyncratic component. The first factor is mainly driven by the five real activity indicators while the second factor is governed by the evolution of the six price indicators.

The Chart 1 of Figure 2.B depicts the first factor. While it fluctuates around its unconditional mean, the broad changes of direction in the factor seem to mark quite well the NBER-referenced business cycles. During expansions, the value of the factor rises up to about its estimated first-state mean of 0.46. During recessions, the factor drastically falls to its second-state mean of about -3.05. In addition, the figure also reveals the strong coherence of the second factor and the Chicago Fed National Activity Index (CFNAI) which is a leading index designed to gauge overall US economic activity.\footnote{To convert the monthly CFNAI into quarterly observations, the index is expressed as weighted averages}

$$w_t = \frac{1}{3}z_t + \frac{2}{3}z_{t-1} + z_{t-2} + \frac{2}{3}z_{t-3} + \frac{1}{3}z_{t-4},$$

where $w_t$ refers to quarterly and $z_t$ to monthly.
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with the NBER recessions. According this figure, it is easy to interpret state 2 as recessions and the series plotted in this chart as probabilities of being in recession. Therefore, one can interpret this factor an index of the broad economic activity which is much less noisy than the individual economic indicators.

The Chart 1 of Figure 2.B plots the second factor and reveals that the evolution of the factor does not follow as closely as the first factor the business cycle dynamics. This index takes negative values in the sixties, it sharply increases to during the seventies and mid-eighties and come back to negative variables since then. According to the estimates of the conditional means of the state variable that governs the evolution of the second factor reported in Table 2.B, the first and last part of the sample is governed by state 1 (estimated mean of state 1 is 3.02) while the middle part of the sample is governed by state 2 (estimated mean of state 1 is -0.35). The figure also points out that the evolution of the second factor and PCEPI (Personal Consumption Expenditure Price Index) growth strongly cohere. Finally, the Chart 2 of Figure 2.B displays the filtered probabilities of being in state 1 that come from the state variable that governs the evolution of the second factor, $p(S_t^p = 2|\psi_t)$, along with the high inflation periods referenced by the Chicago Fed. According this figure, one can interpret state 1 as periods of high inflationary pressures. Therefore, the second factor can be interpreted as a price index.

2.3.4 Inference of Shocks

It is now widely accepted that fluctuations in economic activity are caused by a mix of several types of shocks, e.g. demand, supply, monetary or fiscal, as shown in Forni and Gambetti (2010) or technology and nontechnology shocks, Galí (1999), which can have simultaneous or lagged, soft or strong, short or long, positive or negative impact on it. Some seminal attempts to study the effects of some of these shocks using
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structural VARs are presented in Blanchard and Quah (1989) in which disturbances that have a temporary effect and the ones that have a permanent effect on output fluctuations are interpreted as demand and supply disturbances respectively. This work was extended by Galí (1992) to allow the inclusion of monetary components, finding that the four main sources of fluctuations are money supply, money demand, investment, and aggregate supply shocks.

However, there is some criticism about the identification strategy of shocks when structural VARs are used, as can be seen in Lippi and Reichlin (1993). They show that a very simple modification of the underlying model can lead to significant changes in results. The main point of their criticism is based on the fact that economic theory does not in general provide sufficient structure to choose the most appropriate moving-average representation to estimate the structural VAR model, carrying to the dilemma of fundamentalness of the representation to be issued, Blanchard and Quah (1993). Hence another way to identify shocks without imposing strong restrictions on the structure of the model and without loss of economic intuition seems needed.

Two of the most relevant types of shocks are aggregate demand and aggregate supply shocks since their features are of great importance to the study of business cycles. As suggested by Aruoba and Diebold (2010), prices and quantities are related over the business cycle, and the nature of this relationship contains information about the sources of shocks. While adverse demand shocks lead to periods of business cycle downturns and low inflation, adverse supply shocks lead to reductions in economic activity along with inflationary pressures. In an analogous way, expansionary demand shocks lead to increases of economic activity along with prices, but expansionary supply shocks lead to periods of business cycle upturns and low inflation.

The proposed Markov-switching dynamic bi-factor model allows to perform inference on the four types of shocks since the probabilities of recession, $p(S_t^b = 2|\psi_t)$,
can be additively decomposed into the probability of a recession consistent with an 
adverse demand shock, \( p(S^b_t = 2, S^p_t = 2|\psi_t) \), and the probability of a recession 
consistent with a contractionary supply shock, \( p(S^b_t = 2, S^p_t = 1|\psi_t) \). The same criterion 
 applies for periods of expansions, that is 

\[
p(S^b_t = 1|\psi_t) = p(S^b_t = 1, S^p_t = 2|\psi_t) + p(S^b_t = 1, S^p_t = 1|\psi_t) \tag{2.37}
\]

\[
p(S^b_t = 2|\psi_t) = p(S^b_t = 2, S^p_t = 2|\psi_t) + p(S^b_t = 2, S^p_t = 1|\psi_t). \tag{2.38}
\]

The Chart 1 of Figure 2.B plots the probabilities of recessions that are caused by 
demand shocks, i.e., probabilities of the joint event that characterizes periods of low 
activity and low prices. The figure shows that this probability tends to raise during 
the view that these recessions are caused by adverse demand shocks. The Chart 2 of 
Figure 2.B shows the joint filtered probabilities of stagflation, i.e., low activity and 
and 1980.I-1980.III show high probabilities of decreased real activity and increased 
inflation, consistent with adverse supply shocks as the source of these recessions. 
Moreover, the recessions occurred during the periods 1981.III-1982.IV and 1990.III- 
1991.I, start showing high probability of contractionary supply shocks, but they end 
showing high probability of contractionary demand shocks. This is consistent with 
the view that they were caused by a mix of aggregate a supply and demand shocks.

The analysis of the 2008 Great Recession is of special interest. According to 
Aruoba and Diebold (2010), in this recession inflation falls later than real activity, 
plunging only in summer 2008, whereas real activity begins its descent in 2007. This 
agrees with the high values of \( p(S^b_t = 2, S^p_t = 1|\psi_t) \) observed at the beginning of 
this recession in the bottom panel of Figure 2.B. However, inflation follows the 
falls occurred in real activity within approximately six months, leading to the sharp 
increases on the joint probability of low real activity and prices, \( p(S^b_t = 2, S^p_t = 2|\psi_t) \), 
during the third quarter of 2008 plotted in the top panel of Figure 2.B. This positive
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cow-movement of real activity and inflation during the recent recession is consistent with the adverse demand shock documented by these authors. Finally, since the end of 2008 prices start to increase while real activity was still falling, as can be seen with the high values of \( p(S_t^b = 2, S_t^p = 1 | \psi_t) \) during the last part of the great recession. These results lead to interpret a mix of contractionary supply and demand shocks as the cause of the great recession.

Regarding to phases of expansions, the Chart 1 of Figure 2.B plots the probability of high real activity and high prices, \( p(S_t^b = 1, S_t^p = 1 | \psi_t) \), showing that some expansionary periods occurred in the 1970s and early 80s were caused by expansionary demand shocks. However, during the rest of expansionary periods in the sample, the probability of high real activity and low prices, \( p(S_t^b = 1, S_t^p = 2 | \psi_t) \), plotted in the bottom panel of Figure 2.B, remains high, indicating that the main source of these expansions are positive supply shocks. This result agrees with Galí (1992), who attributes a large estimate of the contribution of supply factors to short-run GNP fluctuations.

Finally, the proposed model allows quantifying the contribution of each type of shock on the phases of the U.S. business cycle by averaging the filtered probabilities of the joint events through each NBER-referenced recession periods. In order to obtain a better picture of the results described so far, Table 2.B reports such contributions, calculated as

\[
\alpha_{\tau}^{\text{supply}} = \sum_\tau \frac{\Pr(S_t^p = 1, S_t^b = 2)}{\Pr(S_t^b = 2)}
\]

(2.39)

\[
\alpha_{\tau}^{\text{demand}} = \sum_\tau \frac{\Pr(S_t^p = 2, S_t^b = 2)}{\Pr(S_t^b = 2)}
\]

(2.40)

where \( \alpha_{\tau}^{\text{supply}} \) and \( \alpha_{\tau}^{\text{demand}} \) denote the contribution of aggregate supply and demand shocks, respectively, to the recession occurred during the period \( \tau \).

The results reported in Table 2.B support the heterogeneity of recessions showing
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2.4 Conclusions

The most appropriate way to deal with the analysis of business cycle and its relationship with other aspects of an economy is taking into account its two defining characteristics, which are nonlinearities and comovement of macro variables through the cycle and modelling their dynamic interaction using a unified setup in order to avoid misleading comparisons. This paper basically presents two contributions. The first contribution is methodological, since it presents an extension to account for two nonlinear common factors of the joint Kalman and Hamilton filter early proposed by Kim and Yoo (1995), Chauvet (1998) and Kim and Nelson (1998) in the context of a unique nonlinear common factor. In the proposed model, each factor is governed by its own Markov switching process and the dependence degree between these processes can be simultaneously assessed.

The second contribution is empirical. Using U.S. macroeconomic indicators, the

\(^8\)This 70\% has been chosen based on the criterion of the author just for the purpose of defining an intermediate category.
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The proposed methodology computes an index of economic activity and an index of price developments taking into account their interdependence, obtaining that real activity indicators, such as Industrial production, play a fundamental role in the dynamic behavior of the inflation index, as well as the real activity index is influenced by nominal variables, such as consumer prices. Inferences on the interaction between real and nominal phases through economic cycles are computed, providing a tool to date periods of economic recessions and periods of high inflation simultaneously.

Relying on these inferences, the framework is able to categorize NBER contractionary episodes into demand recessions, supply recessions and mix recessions by quantifying the type of shock contribution to each period of time. The results show that recessions are heterogeneous with a substantial mix of shocks varying across recessions, agreeing with Galí (1992), Ireland (2010) and Forni and Gambetti (2010). Regarding to the 2007-2009 recession which deserves special attention, the inferences indicate that it should enter in the mix category, since two thirds of the contraction period, specifically the beginning and the end, were influenced by negative supply shocks, while during the middle of the recession, negative demand shocks were prevailing.
2.A Appendix

Let us assume that \( m = 2 \), and \( k = 0 \). According to the eleven-variable model used in the empirical application, the measurement equation, \( y_t = H\beta_t + \xi_t \), with \( \xi_t \sim N(0,R) \), is

\[
\begin{bmatrix}
\Delta GDP_t \\
\Delta IND_t \\
\Delta PIN_t \\
\Delta SAL_t \\
\Delta PAY_t \\
\Delta DEF_t \\
\Delta CPI_t \\
\Delta PPI_t \\
\Delta GSC_t \\
\Delta OIL_t \\
\Delta HCN_t \\
\end{bmatrix} = \begin{bmatrix}
\gamma_1^b \\
\gamma_2^b \\
\gamma_3^b \\
\gamma_4^b \\
\gamma_5^b \\
\gamma_6^b \\
\gamma_7^b \\
\gamma_8^b \\
\gamma_9^b \\
\gamma_{10}^b \\
\gamma_{11}^b \\
\end{bmatrix} \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
f_t^b \\
f_t^p \\
e_{1t} \\
e_{1,t-1} \\
e_{2t} \\
e_{2,t-1} \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix} \right)
\]

(A.1)

where \( R \) is a matrix of zeroes. The notation of the variables is defined as: \( GDP \) is real GDP, \( IND \) is industrial production, \( PIN \) is real personal income less transfers, \( SAL \) is real manufacturing trading sales, \( PAY \) is total non-farm labor, \( DEF \) is deflator of GDP, \( CPI \) is consumer price index, \( PPI \) is producer price index, \( GSC \) is Standard and Poor’s GSCI non-energy commodities price index, \( OIL \) is spot oil price, and \( HCN \) hourly compensation in the non-farm business sector.
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The transition equation, \( \beta_t = \tilde{\mu}_{S_t}^b + F \beta_{t-1} + \nu_t \), with \( \nu_t \sim N(0, Q) \), is

\[
\begin{bmatrix}
    f_t^b \\
    f_t^p \\
    \varepsilon_{1t} \\
    \varepsilon_{1,t-1} \\
    \varepsilon_{2t} \\
    \varepsilon_{2,t-1} \\
    \vdots \\
    \vdots \\
    \varepsilon_{11,t-1}
\end{bmatrix} = \begin{bmatrix}
    \mu_{S_t}^b \\
    \mu_{S_t}^p \\
    0 \\
    0 \\
    0 \\
    \phi_{11} \\
    \vdots \\
    \vdots \\
    1
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    0 \\
    \phi_{11,1} \\
    \phi_{11,2}
\end{bmatrix} \begin{bmatrix}
    \varepsilon_{1,t-1} \\
    \varepsilon_{1,t-1} \\
    \varepsilon_{2,t-1} \\
    \varepsilon_{2,t-1} \\
    \varepsilon_{2,t-2} \\
    0 \\
    0 \\
    0
\end{bmatrix}
\]  

(A.2)

where \( Q \) is a diagonal matrix in which the entries inside the main diagonal are collected in the vector

\[
(\sigma_b^2, \sigma_p^2, \sigma_1^2, 0, \sigma_2^2, 0, \sigma_3^2, 0, \sigma_4^2, 0, \sigma_5^2, 0, \sigma_6^2, 0, \sigma_7^2, 0, \sigma_8^2, 0, \sigma_9^2, 0, \sigma_{10}^2, 0, \sigma_{11}^2, 0)'
\]  

(A.3)
### Table 2.1: Maximum likelihood estimates

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
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<td>( \gamma_1^b )</td>
<td>0.4414</td>
<td>0.0415</td>
<td>( \gamma_5^p )</td>
<td>0.5078</td>
<td>0.0580</td>
<td>( \sigma_5 )</td>
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<td>0.0407</td>
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<td>0.1897</td>
<td>0.0498</td>
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<td>0.0431</td>
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<td>0.0465</td>
<td>( \phi_{62} )</td>
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<td>0.0669</td>
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<tr>
<td>( \gamma_4^b )</td>
<td>0.2745</td>
<td>0.0496</td>
<td>( \gamma_{11}^p )</td>
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<td>0.0493</td>
<td>( \phi_{71} )</td>
<td>1.7643</td>
<td>0.0607</td>
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<td>( \gamma_5^b )</td>
<td>0.3309</td>
<td>0.0276</td>
<td>( \mu_1^p )</td>
<td>3.0261</td>
<td>0.3699</td>
<td>( \phi_{72} )</td>
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<td>0.0535</td>
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<td>( \gamma_6^b )</td>
<td>0.0438</td>
<td>0.0225</td>
<td>( \mu_2^p )</td>
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<td>0.1052</td>
<td>( \phi_{81} )</td>
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<td>( \gamma_7^b )</td>
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<td>0.0394</td>
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<td>0.1962</td>
<td>0.1057</td>
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<td>( \gamma_8^b )</td>
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<td>0.0468</td>
<td>( p_{22}^b )</td>
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<td>0.0217</td>
<td>( \sigma_7 )</td>
<td>0.0364</td>
<td>0.0134</td>
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<tr>
<td>( \gamma_9^b )</td>
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<td>0.0493</td>
<td>( \phi_{11} )</td>
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<td>0.0802</td>
<td>( \sigma_8 )</td>
<td>0.5879</td>
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<td>( \gamma_{10}^b )</td>
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<td>0.0447</td>
<td>( \phi_{12} )</td>
<td>0.0393</td>
<td>0.0747</td>
<td>( \phi_{91} )</td>
<td>0.0181</td>
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<tr>
<td>( \gamma_{11}^b )</td>
<td>-0.0955</td>
<td>0.0470</td>
<td>( \sigma_1 )</td>
<td>0.5856</td>
<td>0.0326</td>
<td>( \phi_{92} )</td>
<td>-0.0001</td>
<td>0.0009</td>
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<td>( \mu_1^b )</td>
<td>0.4662</td>
<td>0.1036</td>
<td>( \phi_{21} )</td>
<td>-0.1122</td>
<td>0.1487</td>
<td>( \sigma_9 )</td>
<td>0.8747</td>
<td>0.0485</td>
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<td>( \mu_2^b )</td>
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<td>0.3805</td>
<td>( \phi_{22} )</td>
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<td>0.0083</td>
<td>( \sigma_{10} )</td>
<td>0.0957</td>
<td>0.0716</td>
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<td>( p_{11}^b )</td>
<td>0.9627</td>
<td>0.0221</td>
<td>( \sigma_2 )</td>
<td>0.3335</td>
<td>0.0388</td>
<td>( \phi_{10.1} )</td>
<td>-0.0023</td>
<td>0.0003</td>
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<td>( p_{22}^b )</td>
<td>0.9581</td>
<td>0.0792</td>
<td>( \phi_{31} )</td>
<td>0.0026</td>
<td>0.0747</td>
<td>( \phi_{10.2} )</td>
<td>0.6622</td>
<td>0.1977</td>
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<td>( \gamma_1^p )</td>
<td>-0.2286</td>
<td>0.0480</td>
<td>( \phi_{32} )</td>
<td>0.1715</td>
<td>0.0722</td>
<td>( \sigma_{11} )</td>
<td>0.0153</td>
<td>0.0707</td>
</tr>
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<td>( \gamma_2^p )</td>
<td>-0.2273</td>
<td>0.0452</td>
<td>( \sigma_3 )</td>
<td>0.6745</td>
<td>0.0357</td>
<td>( \phi_{11.1} )</td>
<td>0.0136</td>
<td>0.0698</td>
</tr>
<tr>
<td>( \gamma_3^p )</td>
<td>-0.2735</td>
<td>0.0468</td>
<td>( \phi_{41} )</td>
<td>0.4695</td>
<td>0.0611</td>
<td>( \phi_{11.2} )</td>
<td>0.0187</td>
<td>0.0914</td>
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<td>( \gamma_4^p )</td>
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<td>0.0456</td>
<td>( \phi_{42} )</td>
<td>0.0001</td>
<td>0.0713</td>
<td>( \sigma_{11} )</td>
<td>0.8857</td>
<td>0.0438</td>
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<td>( \gamma_5^p )</td>
<td>-0.1150</td>
<td>0.0302</td>
<td>( \sigma_4 )</td>
<td>0.7288</td>
<td>0.0396</td>
<td>( p_{11}^p )</td>
<td>0.9666</td>
<td>0.0337</td>
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<td>( \gamma_6^p )</td>
<td>0.0838</td>
<td>0.0228</td>
<td>( \phi_{51} )</td>
<td>1.1629</td>
<td>0.0751</td>
<td>( p_{22}^p )</td>
<td>0.6622</td>
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</tr>
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<td>( \gamma_7^p )</td>
<td>0.5008</td>
<td>0.0399</td>
<td>( \phi_{52} )</td>
<td>-0.3381</td>
<td>0.0437</td>
<td>( \delta )</td>
<td>0.3187</td>
<td>0.0914</td>
</tr>
</tbody>
</table>

Note. Superindexes \( p \) and \( b \) refer to the first (or business cycle) and the second (or price index) factors. Subindexes in the loadings, \( \gamma \), from 1 to 11 refer to real GDP (1), Industrial Production (2), Personal Income less Net Transfers (3), Real Manufacturing Trading Sales (4), Total Nonfarm Labor (5), GDP Deflator (6), Consumer Price Index (7), Producer Price Index (8), Spot Oil Price (9), Standard and Poor’s GSCI Non-energy Commodities Price Index (10), Hourly Compensation in the Non-farm Business Sector (11).
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<table>
<thead>
<tr>
<th>Recession Periods</th>
<th>Cont. Demand</th>
<th>Cont. Supply</th>
<th>Rec. type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960Q2 - 1961Q1</td>
<td>0.89</td>
<td>0.11</td>
<td>Demand</td>
</tr>
<tr>
<td>1969Q4 - 1970Q4</td>
<td>0.15</td>
<td>0.85</td>
<td>Supply</td>
</tr>
<tr>
<td>1973Q4 - 1975Q1</td>
<td>0.08</td>
<td>0.92</td>
<td>Supply</td>
</tr>
<tr>
<td>1980Q1 - 1980Q3</td>
<td>0.17</td>
<td>0.83</td>
<td>Supply</td>
</tr>
<tr>
<td>1981Q3 - 1982Q4</td>
<td>0.32</td>
<td>0.68</td>
<td>Mix</td>
</tr>
<tr>
<td>1990Q3 - 1991Q1</td>
<td>0.30</td>
<td>0.70</td>
<td>Mix</td>
</tr>
<tr>
<td>2001Q1 - 2001Q4</td>
<td>0.87</td>
<td>0.13</td>
<td>Demand</td>
</tr>
<tr>
<td>2007Q4 - 2009Q2</td>
<td>0.33</td>
<td>0.67</td>
<td>Mix</td>
</tr>
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</table>

Note: Average contribution of Contractionary Demand and Contractionary Supply shocks through periods of recession. If the contribution of one of the average shocks type, supply or demand, is less than or equal to 70% they are categorized as mix recessions.
Figure 2.1: Analysis of the first factor

Chart 1. First factor and the CFNAI index

Notes. In Chart 1, the solid line refers to the first factor (1959.2-2011.3) and the dashed line refers to the CFNAI (1967.2-2011.3). To facilitate graphing, both series are standardized. The probabilities plotted in Chart 2 come from the state variable that governs the dynamics of the first factor. Shaded areas correspond to recessions as documented by the NBER.
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Figure 2.2: Analysis of the second factor

Chart 1. Second factor and the PCEP index

Chart 2. Filtered probabilities of high mean state

Notes. In Chart 1, the solid line refers to the second factor (1959.2-2011.3) and the dashed line refers to the PCEPI (1959.2-2011.3). To facilitate graphing, both series are standardized. The probabilities plotted in Chart 2 come from the state variable that governs the dynamics of the second factor. Shaded areas correspond to high inflation periods documented by the Chicago Fed.
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Figure 2.3: Inference on Contractionary Shocks

Chart 1. Probability of contractionary demand shock

Chart 2. Probability of contractionary supply shock

Notes. Chart 1 plots the joint filtered probabilities of low economic activity and low prices. Chart 2 plots the joint filtered probabilities of low economic activity and high prices. Shaded areas correspond to recessions as documented by the NBER.
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Figure 2.4: Inference on Expansionary Schocks

Chart 1. Probability of expansionary demand shock

Chart 2. Probability of expansionary supply shock

Notes. Chart 1 plots the joint filtered probabilities of high economic activity and high prices. Chart 2 plots the joint filtered probabilities of high economic activity and low prices. Shaded areas correspond to recessions as documented by the NBER.
References


References


References


Chapter 3

Real-Time Nowcasting of Nominal GDP

Joint work with William A. Barnett and Marcelle Chauvet

3.1 Introduction

During recent years the Federal Reserve has reached the lower bound level of the interest rate due to its continuous attempts to reduce unemployment rate, which still remains in high levels although the economy is experiencing a slow recovery. In view of this situation, the Federal Open Market Committee (FOMC) additionally is using complementary tools to carry up monetary policy, one of them regards to forward guidance. As appointed by Ben Bernanke, the chairman of the Federal Reserve, and Michael Woodford (2012) at the Annual Jackson Hole Economic Symposium, this tool consist of explicit statements of a central bank about its future actions to specific

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developments in the economy, in addition to its announcements about the immediate policy actions that it is undertaking.

The forward guidance strategy could lead to changes in expectations of future economic developments that could improve the present situation, depending on the target and rule that central banks are committed to follow. For the U.S. case, as suggested by many economists as Woodford (2012), Romer (2011) and Hall and Makiw (1994), among others, the Fed should start targeting the path of nominal GDP, since they consider this would constitute a powerful communication tool. During the last recession, the path of nominal GDP suffered a drastic contraction, as can be seen in Chart A of Figure 3.B, caused by several significantly negative growth rates, see Chart B of Figure 3.B. Since nominal GDP is the output of the economy times the price level, setting the objective of returning nominal GDP to its pre-crisis trajectory could improve expectations about the future economic conditions. Such expectations would increase the incentives of households to consume more today and also firms would be more optimistic regarding their present investment decisions. For an extended discussion about forward guidance and targeting nominal GDP, see Belongia and Ireland (2012) and Del Negro et al. (2012).

Under the nominal GDP targeting scenario, nowcasting nominal output growth plays a fundamental role in monitoring its continuous development in order to assess the effectiveness of the policy. The seminal work of Croushore and Stark (2001) emphasizes the use of real-time data in order to obtain robust results at the time of making policy analysis and forecasting. This seems to be the starting point of an increasing literature regarding forecasting variables by using the exact amount of data available at the time in which the analysis was conducted, for example Chauvet and Hamilton (2006) focus on real-time assessments of the state of the U.S. economy in order to date turning points.

However, the use of real time data could bring some complications, such as mixed
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frequency and ragged edges, if a multivariate framework is desired. Recently, new econometric forecasting modes have been proposed in order to deal with these problems. Some relevant works in this line are Mariano and Murasawa (2003), Camacho and Perez-Quiros (2010), Banbura et al. (2012), among others, who rely on a state space representation of the multivariate system in order to deal with missing data, then the Kalman filter is applied to obtain optimal inferences on the comovement between the variables used. The information contained in this comovement will be helpful in forecasting a target variable. It is worth to mention that although this type of frameworks so far have been mainly used to obtain accurate inferences of real GDP, showing satisfactory results, they have not been used yet in nowcasting Nominal GDP.

Since the objective in this paper is to make available information that can be useful to conduct monetary policy, our focus will exclusively be the path of Nominal GDP (NGDP). Due to the importance of early assessments of current quarterly NGDP, we will explore univariate and multivariate approaches in order to determine the one providing the most accurate nowcasts of NGDP growth, by using the exact amount of data that the econometrician would have at the time the analysis is being done, and moreover, taking into account the potential periodic updates or "revisions" of past releases that some variables could experience.

The results show that univariate models perform poorly regarding real-time nowcasts of NGDP, while multivariate models provide substantial improvements. We rely on the use of dynamic factor models in order to combine information about real economic activity, inflation dynamics, and monetary aggregates to nowcast NGDP. By focusing on small scale dynamic factor models, we try with several combinations of variables in order to get the one providing the highest accuracy in nowcasting performance, finding that the best specification includes information of past releases of NGDP itself, Industrial Production, Consumer Price Index, and the Divisia M3.
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The paper proceeds as follows. Section 2 provides univariate analysis of nowcasting NGDP by relying on autoregressive models computed with real-time data. Section 3 proposes a multivariate approach based on the natural relationship between NGDP, real economic activity and inflation dynamics, discuss the variable selection strategy and proposes a dynamic factor model to compute NGDP nowcasts. Section concludes.

3.2 Univariate Nowcasts of Nominal GDP

This section is intended to provide real-time assessments of the quarterly nominal GDP growth on a monthly basis by relying on univariate models. At every quarter that new information of NGDP is published in the national accounts, its previous quarterly releases are also revised in order to obtain more accurate information of the past developments of such variable. Therefore, a real-time analysis at time $t$ requires the use of all available information up to time $v$, analogously the analysis at time $t+1$ will be done by using the new set of information collected in "vintage" $v+1$, which not necessarily will be equal to $v$ due to revisions of the past releases. Notice that $t$ represents quarters, while $v$ could represent months, since data is in general revised on a monthly basis. This allows us to compute inferences every three months about the same quarterly figure of NGDP. The real-time data of NGDP has been obtained from the Philadelphia Fed data base.

This section starts by exploring the performance of a univariate approach in doing real-time nowcasting. For this purpose we compute the following autoregression model:

$$y_{t|v} = \phi_{0|v} + \sum_{j=1}^{k} \phi_{j|v}y_{t-j|v} + e_{t|v}$$  \hspace{1cm} (3.1)

Were $y_{t|v}$ denotes the NGDP growth of quarter $t$ that is observed at monthly
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vintage $v$, and $\phi_{j|v}$ are the autoregression parameters computed with all the available information up to $v$. In this way, at the end of the sample $T$ a forecast of the next period will be computed as

$$
\hat{y}_{T+1|V} = \hat{\phi}_0|V + \sum_{j=1}^{k} \hat{\phi}_{j|V} y_{T|V}
$$

(3.2)

where $V$ denotes the last available vintage. In order to assess the accuracy of the model in Equation (3.1), these forecasts will be compared against the first published figure of the corresponding quarter, which is released in general one month after the end of the corresponding quarter, for example, the last quarter of 2000, $y_{00Q4}$, was released on 31/01/2001, and the first quarter of 2001, $y_{01Q1}$, was released on 27/04/2001. However, since 01/02/2001 we can nowcast $y_{01Q1}$ with information up to the fourth quarter of 2000, that corresponds to the vintage of February 2001, $y_{00Q4|01Feb}$. Then on 01/03/2001 we can update our nowcast by replacing $y_{00Q4|01Feb}$ for $y_{00Q4|01Mar}$, and moreover by using the previous data already revised in March. Finally, on 01/04/2001 we can perform another nowcast by replacing the last observation in the sample $y_{00Q4|01Mar}$ for $y_{00Q4|01Apr}$ and the previous revised data up to April. These three nowcast will be collected, and in May 2001, when the first release of $y_{01Q1}$ will be published, we will compute the Root Mean Square Error, RMSE, in order to assess the performance of the model in Equation (3.1).

The autoregressive model is estimated with data from 1967Q2 until 2000Q4, then real-time simulation nowcasts are performed every month during the period 2001Q1 - 2012Q3. The nowcast are shown in Figure 3.B for the cases of $k = 1, 2, 3$ in Equation (3.1), from the left to right chart respectively. As can be seen the performance of the autoregressive models in general are not accurate, since most of the time such inferences overestimate the target values, particularly during the period after the "great recession" (2007Q12 - 2009Q2). In the first three columns of Table 3.B are reported the RMSE corresponding to the three models, showing that the AR(2) is the one doing the best performance among them. Since new regressions are run every
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month, new autoregressive coefficients are compute at the same frequency, the path of these coefficient for each model are collected and shown in Figure 3.B. The figure shows evidence of parameter instability occurred after the last recession, given the increase in the autocorrelation of the process, which could be the origin of the notable overestimation in the real-time nowcasts in this period. This problem frequently seen in univariate specifications motivated us to take a look into the multivariate analysis to perform more accurate inferences of our target variable.

3.3 Multivariate Nowcasts of Nominal GDP

Nominal GDP is the market value at current prices of all final goods and services produced within a country in a given period of time. It also can be viewed as the real GDP times the price level of the economy. Therefore, letting $Z_t$ be Nominal GDP, $X_t$ real GDP and $P_t$ the price level, there is a natural link between these three concepts:

$$Z_t = X_t P_t$$

$$\ln(Z_t) - \ln(Z_{t-1}) = \ln(X_t) - \ln(X_{t-1}) + \ln(P_t) - \ln(P_{t-1})$$

$$z_t = x_t + p_t$$

The charts A and B of Figure 3.B show the real GDP growth and GDP deflator growth. As can be seen in the figure the former tracks the business cycle while the latter is represents the path of inflation dynamics. In order to provide more timely assessment of NGDP, we can take advantage of the fact that the our target variable contains a real activity component and an inflation component, as shown in Equation (3.3), and proxy $x_t$ and $p_t$, which are on quarterly frequency, by a couple of indicators with a higher frequency, as Industrial Production (IP) and Consumer Price Index (CPI), respectively, which can be found at a monthly basis. The charts
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C and D plots the developments of IP and CPI growth rates, respectively, showing how IP follows the business cycle in more timely manner than real GDP and that CPI follows the inflation path more timely than the GDP deflator growth. By adding IP and CPI growth rates and standardizing them with respect to NGDP we could have a "naive" monthly index of our target variable. Given that the naive index is in monthly frequency but NGDP in quarterly frequency, we use the transformation in Mariano and Murasawa (2003) in order to compare both variables in quarterly terms.

Quarterly time series $Z_t$ can be expressed into monthly time series $W_t$ as:

$$Z_t = 3 \left( \frac{W_t + W_{t-1} + W_{t-2}}{3} \right)$$

which can be approximated by using the geometric mean instead of the arithmetic mean, since when variations are small the difference between the two types of means tend to be negligible.

$$Z_t = 3 \left( W_t W_{t-1} W_{t-2} \right)^{1/3}$$

by taking logs to both sides of the equation above, taking three periods differences and after some algebra, it is obtained that

$$z_t = \frac{1}{3} w_t + \frac{2}{3} w_{t-1} + w_{t-2} + \frac{2}{3} w_{t-3} + \frac{1}{3} w_{t-4} \tag{3.4}$$

where the quarter-on-quarter growth rates, $z_t$, are expressed in month-on-month growth rates, $w_t$. In Chart A of Figure 3.B are plotted the common component and NGDP, providing evidence of a high comovement between them, since the naive index yields a relatively good in sample fit. Notice that the naive index also overestimates the true values after the last recession. Despite the relatively good in sample performance, when nowcasts with real-time data are performed by using this index, the results are not satisfactory. The Chart B of Figure 3.B provides the out of sample nowcasts, which although following the path of NGDP, are characterized by a high volatility and therefore a significant uncertainty. In the third column of Table 3.B is
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reported the associated RMSE that is higher than any of the autoregressive models described in Section 2. Notice that particularly during the period after the great recession the predictions are more unstable, decreasing the reliability of the naive model in providing early accurate assessments of the target variable. In order provide more stable forecasts we rely on notion of the comovement between real activity and inflation indicators rather than a simple sum.

3.3.1 The Model

Since the seminal work of Stock and Watson (1991) the use of factor models has been viewed as an attractive alternative, in comparison to the classical vector autoregression approach, for several reasons, such as, the fact that factor models allow to impose a considerable amount of structure on the data, being less general than the VAR models, and therefore much more parsimonious in terms of parameters, providing a useful tool to handle a large amount of information, as claimed in Forni and Gambetti (2010). Due to these reasons, and others to be discussed later, this paper relies on the use of dynamic factor models to produce more accurate nowcast of nominal GDP than the ones obtained with simple autoregression models or the naive index.

In this section we specify the model to nowcast nominal GDP, which allows for the inclusion of mixed frequency data and also missing observations. By using the approach proposed by Mariano and Murasawa (2003) in Equation (3.4) to express quarterly data in terms of monthly data, the framework adopted in this paper starts from computing the comovement, $f_t$, between the target variable, i.e. NGDP, denoted by $y_{1,t}$, an indicator of real economic activity, $y_{2,t}$, and an indicator of inflation dynamics, $y_{3,t}$.
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\[
\begin{bmatrix}
  y_{1,t} \\
  y_{2,t} \\
  y_{3,t}
\end{bmatrix}
= \begin{bmatrix}
  \gamma_1 \left( \frac{1}{3} f_t + \frac{2}{3} f_{t-1} + f_{t-2} + \frac{2}{3} f_{t-3} + \frac{3}{3} f_{t-4} \right) \\
  \gamma_2 f_t \\
  \gamma_3 f_t \\
  \frac{1}{3} v_{1,t} + \frac{2}{3} v_{1,t-1} + v_{t-2} + \frac{2}{3} v_{1,t-3} + \frac{3}{3} v_{1,t-4} \\
  v_{1,t} \\
  v_{3,t}
\end{bmatrix} + \text{ (3.5)}
\]

where \( \gamma_i \) are the factor loadings and \( v_{i,t} \) are the associated error terms for \( i = 1, 2, 3 \). The dynamics of the unobserved components in the system are modeled with autoregressive dynamics,

\[
f_t = \phi_1 f_{t-1} + \phi_2 f_{t-2} + \ldots + \phi_6 f_{t-6} + \epsilon_t, \quad \epsilon_t \sim i.i.d.N(0, 1) \quad \text{(3.7)}
\]

\[
v_{1,t} = \varphi_{11} v_{1,t-1} + \varphi_{12} v_{1,t-2} + \ldots + \varphi_{16} v_{1,t-6} + \epsilon_{1,t}, \quad \epsilon_{1,t} \sim i.i.d.N(0, \sigma_{\epsilon_1}^2) \quad \text{(3.8)}
\]

\[
v_{i,t} = \varphi_{i1} v_{i,t-1} + \varphi_{i2} v_{i,t-2} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim i.i.d.N(0, \sigma_{\epsilon_i}^2), \quad \text{for } i = 2, 3 \quad \text{(3.9)}
\]

In order to extract optimal inferences of the unobserved variable \( f_t \) and \( v_{i,t} \) by using the Kalman Filter, the system involving Equations (3.5) - (3.9) can be cast into a state space model

\[
y_t = H \beta_t + \xi_t, \quad \xi_t \sim i.i.d.N(0, R) \quad \text{(3.10)}
\]

\[
\beta_t = F \beta_{t-1} + \zeta_t, \quad \zeta_t \sim i.i.d.N(0, Q) \quad \text{(3.11)}
\]

where Equation (3.10) corresponds to the Measurement Equation that relates observed variables with their common component and idiosyncratic terms in Equation (3.5). Equation (3.11) refers to the Transition Equation that explicitly specify the dynamic of the unobserved variables in Equations (3.7) - (3.9). A complete representation of how these equations look like can be seen at the Appendix 3.1.

Following the line of Camacho and Perez-Quiros (2010) we adapt the model (3.10) - (3.11) to incorporate potential missing observations into the system. The strategy
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consist on substituting each missing observation with a random draw \( u_t \) from a \( N(0, \sigma_u^2) \). As shown in Mariano and Murasawa (2003) this substitution helps to make the matrices conformable and does not have any effect in the estimation of the model’s parameters. In order to do so, it is necessary to update the components of model (3.10) - (3.11) depending on if \( y_{i,t} \) is observed or not, in the following way:

\[
y_{i,t} = \begin{cases} y_{i,t} \text{ if } y_{i,t} \text{ observed} \\ u_t \text{ otherwise} \end{cases}, \quad H^*_{i,t} = \begin{cases} H_i \text{ if } y_{i,t} \text{ observed} \\ 0_{1,\kappa} \text{ otherwise} \end{cases}
\]

\[
\xi^*_{i,t} = \begin{cases} 0 \text{ if } y_{i,t} \text{ observed} \\ u_t \text{ otherwise} \end{cases}, \quad R^*_{i,t} = \begin{cases} 0 \text{ if } y_{i,t} \text{ observed} \\ \sigma_v^2 \text{ otherwise} \end{cases}
\]

where \( H^*_{i,t} \) is the \( i \)-th row of the matrix \( H \) which has \( \kappa \) columns and \( 0_{1,\kappa} \) is a row vector of \( \kappa \) zeros. Therefore, in the model robust to missing observations, the Measurement Equation (3.10) will be replaced by

\[
y_t = H^*_t \beta^*_t + \xi^*_t, \quad \xi^*_t \sim i.i.d. N(0, R^*_t) \quad (3.12)
\]

Finally the Kalman filter is applied to the time-varying state space model (3.12) - (3.11) to obtain optimal inferences on the vector \( \beta_t \), that contains the information on the comovement among the economic indicators, \( y_{i,t} \) for \( i = 1, 2, 3 \), collected in \( f_t \).

The Kalman filter is applied in two steps, the predicting step:

\[
\beta_{t|t-1} = F \beta_{t-1|t-1}
\]

\[
P_{t|t-1} = FP_{t|t-1}F' + Q
\]

\[
\eta^*_{t|t-1} = y_t - y_{t|t-1} = y_t - H^*_t \beta_{t|t-1}
\]

\[
\rho^*_{t|t-1} = H^*_t P_{t|t-1} H^{*'}_t + R^*_t
\]

and the updating step:

\[
\beta_t = \beta_{t|t-1} + K_t^* \eta^*_{t|t-1}
\]

\[
P_t = P_{t|t-1} - K_t^* H^*_t P_{t|t-1}
\]
where $K_t^* = P_{t|t-1} H_t^* \rho_{t|t-1}^*$ corresponds to the weight assigned to new information contained in the prediction error $\eta_{t|t-1}^*$, about the state vector $\beta_t$, also known as the Kalman gain. For further details, see Kim and Nelson (1999).

### 3.3.2 Variable Selection

The recent developments in econometric forecasting models regarding mixing frequency dynamic factor models have constantly been applied to real GDP in order to track the business cycle, some examples are Angelini et al. (2011) and Camacho and Perez-Quiros (2010) for the Euro area, or Aruoba and Diebold (2010) and Camacho and Martinez-Martin (2012) for US. However, few have been done in finding potential economic indicators that can be helpful in providing early assessments about the development of nominal GDP. For this reason, the approach we adopt in this paper starts by the construction of a "benchmark" model that incorporates information of our target variable, NGDP, one real activity indicator and one inflation indicator. Once this benchmark is obtained, it will be enlarged with additional variables based on the contribution of such variables in terms of predictive ability. Now the obvious question is: which real and inflation indicators should we use as benchmark?

On the one hand we consider as representative indicators of real economic activity to industrial production (IP), real personal income less transfer payments (PILT), nonfarm labor (NFL) and real manufacturing trading sales (MTS). These variables are the ones used Stock and Watson (1991) to build an coincident indicator of the business cycle. On the other hand we use the most representative indicators of inflation dynamics in the U.S. economy, which are Consumer Price Index (CPI), Producer Price Index (PPI), Personal Consumption Expenditures Price Index (PCEP) and Personal Consumption Expenditure Price Index excluding Food and Energy, known as the core inflation, (PCEF). Given our four real activity indicator and four inflation
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indicators, there are sixteen possible pairwise (one real, one inflation) combinations, that will constitute our set of potential benchmark models to nowcast nominal GDP.

We estimate sixteen models of the type in Equations (3.12) - (3.9) by using always the NGDP and one of the pairwise in the benchmark set and obtain an index based on the common component among the variables, then we compute the RMSE with respect to the first figure published of NGDP for the corresponding quarter, they are reported in Table 3.B. The information in the table helps us to clearly identify the combinations between real and inflation indicators that provide a better in sample fit to the target variable. The combination showing the best performance is \{MTS, CPI\}. However, there are others such as \{IP, CPI\}, and \{MTS, PPI\} showing similar performance. Hence, we save the three specifications to later assess their out of sample performance. The comparison between the best benchmark and the target variable can be seen in Figure 3.B, showing a high improvement with respect to the naive index case.

In order to assess the need for additional inflation and real activity indicators, we enlarge the basic model by including an additional variable in our benchmark data set. In Table 3.B are reported the RMSE of the all possible enlarged models, which are higher than the corresponding to any the benchmarks previously estimated. The results indicate that once one real and one inflation indicator has been already incorporated into the model, any additional indicator of the same nature (real or inflation) yields significant decreases in the fitting performance of the enlarged model.

The next step in this section is to assess the ability of additional indicators, others than the ones in the benchmark set, which could improve the fitting between our index and NGDP. We start by considering as additional indicators to Personal Income (PI), Personal Consumption Expenditures (PCE), Average Hourly Earnings of Production and Nonsupervisory Employees (AHETPI). Also, in order to explore the particular ability that the CFS Divisia monetary aggregates could have, we will
use M3, M4, and M4-. Notice that the difference between M4 and M4- is that the later removes Treasury bills to remove the overlap between monetary and fiscal policy. Additional, we treat the 3-Month Treasury Bill as another potentially helpful indicator and finally we also incorporate the S&P500 into our set of new indicators.

We repeat the same procedure as before in order to assess the ability of any additional indicator in the factor model. In Table 3.B are reported the RMSE for all the possible models enlarged with the new set of indicators. Although in most of the cases the performance substantially decreases, in six of them the performance is in general as good as the one of the best benchmark models. These combinations correspond to: {IP, CPI, M3}, {IP, CPI, M4}, {IP, CPI, TBILL}, {MTS, CPI, M3}, {MTS, CPI, M4}, {MTS, CPI, TBILL}. We also save these six specifications to later evaluate their out of sample performance.

From these results we are already able to identify the set of variables that have shown good performance across all the models estimated so far. This set of variables corresponds to IP, MTS, CPI, M3, M4 and TBILL. Given this set, we use the just mentioned six specifications and make enlargements by using any additional variable in the set, that has not been already included. The RMSE of these models are shown in Table 3.B, from the eight estimated models, four of them show a good performance. They correspond to the combinations {IP, CPI, M3, TBILL}, {IP, CPI, M4, TBILL}, {MTS, CPI, M3, TBILL}, {MTS, CPI, M4, TBILL}, we also save these four models to later explore their out of sample performance.

This section has provided 13 specifications that include different combinations of real activity, inflation, and monetary indicators and that have shown the best in sample performance in fitting the target variable, i.e. nominal GDP. In the next section, we will assess the nowcasting ability of these specifications by using real-time data. Before doing so, it is worth to analyze the contribution of each variable to the common factor for the 13 selected specifications. We focus on the factor loadings that
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are reported in Table 3.B. The table shows that, regarding real activity variables, across all the specifications, NGDP loads around 0.2, while IP loads 0.3, and MTS shows a slightly lower load of about 0.18. Regarding inflation indicators, CPI shows a higher load that PPI. Finally regarding monetary aggregates, the load of M3 is higher than the one of M4 and that the one of TBILL, which is the lowest among all these variables. Moreover, notice that the estimates indicate that TBILL is negatively related to the common factor.

3.3.3 Real-Time Nowcasting

In this section we use the 13 selected models that yielded the best in sample performance to assess their ability to compute out of sample predictions of the current quarterly growth of nominal GDP, by using the exact amount of data that the forecaster has available at the time the prediction is done, i.e. by also taking into account all the possible revision that some variables can experiment in their previous releases.

Each model will be estimated with data since 1967Q1 until 2000Q12, then the first figure to be predicted will be the nominal GDP growth for the first quarter of 2001, $y_{01Q1}$, this prediction will be done at the beginning of February, that is, once the forecaster observes the development of the economy during January 2001, 2001M01, by relying on the information of the monthly indicators. This first prediction will be done by using the available data up to the beginning of February, which consists on the set of monthly indicators up to 2001M01 and quarterly growth of nominal GDP up to 2000Q4, i.e. $y_{00Q4}$. The second prediction of $y_{01Q1}$, will be done at the beginning of March 2001, by using information of monthly indicators up to 2001M02 and nominal GDP growth up to $y_{00Q4}$. The third and last prediction of $y_{01Q1}$, will be done at the beginning of April 2001 by using information up to 2001M03 and also $y_{00Q4}$. Notice that this is the last prediction of $y_{01Q1}$ since at the beginning of
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the next month, May 1992, its first figure will be published in the national accounts. This published figure will be the one we use to assess the nowcasting performance of the model by calculating the RMSE associated to all the predictions during the period 2001Q1 - 2012Q3. The parameters of the model are reestimated once per year in order to account for parameter instability.

The real-time nowcasts of the 13 selected models are shown in Figure 3.B. In all the charts of this figure it is shown significant improvements with respect to the nowcasts based on the autoregressive models and also with respect to the ones based on the "naive" index, displayed in Figure 3.B and Chart B of Figure 3.B, respectively. However, there is some heterogeneity across the performance of the 13 models, since models 1, 4, 5, 6, 10 and 11 show better predictions than the rest of the models during 2001 - 2003. In general all models show a somehow similar performance during 2004 - 2007. Not surprisingly, the same models that performed good at the beginning of the sample did the same during the "great recession", since they provided a much more accurate prediction than the rest.

In Table 3.B we report the RMSE associated to the 13 selected models, showing that the one with the best real-time nowcasting ability is Model 4, which uses as input data of NGDP, IP, CPI and M3, followed by models 5 and 11, which contain the same real activity and inflation indicators, i.e. IP and CPI, but with other monetary indicators, such as M4 instead of M3 for Model 5, and M4 and TBILL instead of M3 for Model 11.

3.4 Conclusions

Given the non-conventional present situation that the Fed is facing regarding the lower bound level of the interest rate, many economists have suggested that non-conventional strategies should be adopted to decrease unemployment rate, one of
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the proposals is targeting nominal GDP. This paper focuses on the evaluation of univariate and multivariate econometric frameworks that can be useful in order to given earlier assessments of the current nominal GDP quarterly growth, under the real conditions that policy makers face at the time the predictions are done.

The univariate analysis shows that classical autoregressive models provide poor performance regarding real-time nowcasts of the target variable. Hence a multivariate framework is proposed by relaying on the use of dynamic factor models. Several specifications were used in order to identify the set of variables providing the best nowcasting performance. The variables used belong to the category of real economic activity, inflation, and monetary indicators, among others. The model showing the highest accuracy is constructed by using information of the previous releases of Nominal GDP, Industrial Production, Consumer Price Index and Divisia M3.
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3.A Appendix

The state space representation described in this appendix corresponds to a specification that contains information on quarterly growth of nominal GDP, \( y_{1,t} \), the growth rate of one monthly indicator of real economic activity, \( y_{2,t} \), and the growth rate of one indicator of inflation dynamics, \( y_{3,t} \). It is composed by a Measurement Equation:

\[
\begin{bmatrix}
y_{1,t} \\
y_{2,t} \\
y_{3,t}
\end{bmatrix} = \begin{bmatrix}
\gamma_1 & 2\gamma_1 & 2\gamma_1 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & 0 & 0 & \cdots & 0 \\
\gamma_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\
\gamma_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
f_t \\
f_{t-1} \\
f_{t-2} \\
f_{t-3} \\
f_{t-4} \\
f_{t-5} \\
v_t \\
v_{1,t} \\
v_{1,t-1} \\
v_{1,t-2} \\
v_{1,t-3} \\
v_{1,t-4} \\
v_{1,t-5} \\
v_{2,t} \\
v_{2,t-1} \\
v_{3,t} \\
v_{3,t-1}
\end{bmatrix}
\]

that relates the observed variables with the state vector \( \beta_t \). And the Transition Equation
which gives explicit dynamics to the state vector.
Chapter 3. Real-Time Nowcasting of Nominal GDP

3.B Tables and Figures

Table 3.1: Root Mean Square Errors for AR and Naive Models

<table>
<thead>
<tr>
<th></th>
<th>AR(1)</th>
<th>AR(2)</th>
<th>AR(3)</th>
<th>Naive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>0.7066</td>
<td>0.6748</td>
<td>0.7223</td>
<td>0.8350</td>
</tr>
</tbody>
</table>

Note: The first three columns report the in sample root mean square error associated to autoregressive models with one, two and three lags, respectively. The fourth column reports the root mean square associated to a Naive models, which consists on the sum of the growth rates of industrial production and consumption price index standardized with respect to Nominal GDP growth.

Table 3.2: Root Mean Square Errors for Potential Benchmarks

<table>
<thead>
<tr>
<th>Inflation Indicators</th>
<th>Real Activity Indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IP</td>
</tr>
<tr>
<td>CPI</td>
<td>0.2972</td>
</tr>
<tr>
<td>PPI</td>
<td>0.3699</td>
</tr>
<tr>
<td>PCEPI</td>
<td>0.3412</td>
</tr>
<tr>
<td>PCEPILFE</td>
<td>0.9700</td>
</tr>
</tbody>
</table>

Note: Each entry in the table reports the in sample root mean square error associated to a dynamic factor model with three variables. Specifically, with the corresponding inflation indicator, in the rows, and real activity indicator, in the columns. Moreover, Nominal GDP is always included in each model.
## Table 3.3: Root Mean Square Errors for Enlarged Models

<table>
<thead>
<tr>
<th></th>
<th>IP</th>
<th>NFL</th>
<th>MTS</th>
<th>PILT</th>
<th>CPI</th>
<th>PPI</th>
<th>PCEP</th>
<th>PCEF</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP, CPI</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NFL, CPI</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MTS, CPI</td>
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<td></td>
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<tr>
<td>IP, PPI</td>
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</tr>
<tr>
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</tr>
<tr>
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<td>MTS, PCEP</td>
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<tr>
<td>PILT, PCEP</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>IP, PCEF</td>
<td></td>
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<td>MTS, PCEF</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PILT, PCEF</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Each entry in the table reports the in sample root mean square error associated to a dynamic factor model with four variables. Specifically, with the corresponding pair of benchmark variables, in the rows, and an additional variable that has not been already included, in the columns. Moreover, Nominal GDP is always included in each model.
Table 3.4: Root Mean Square Errors for Enlarged Models (Cont.)

<table>
<thead>
<tr>
<th></th>
<th>PI</th>
<th>PCE</th>
<th>M3</th>
<th>M4</th>
<th>M4-SP500</th>
<th>TBILL</th>
<th>AHETPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP, CPI</td>
<td>0.435</td>
<td>0.327</td>
<td>0.298</td>
<td>0.295</td>
<td>0.312</td>
<td>0.303</td>
<td>0.296</td>
</tr>
<tr>
<td>NFL, CPI</td>
<td>0.534</td>
<td>0.423</td>
<td>0.808</td>
<td>0.809</td>
<td>0.807</td>
<td>0.809</td>
<td>0.809</td>
</tr>
<tr>
<td>MTS, CPI</td>
<td>0.457</td>
<td>0.320</td>
<td>0.290</td>
<td>0.291</td>
<td>0.325</td>
<td>0.827</td>
<td>0.278</td>
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<tr>
<td>PILT, CPI</td>
<td>0.762</td>
<td>0.420</td>
<td>0.804</td>
<td>0.805</td>
<td>0.759</td>
<td>0.823</td>
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</tr>
<tr>
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<td>0.421</td>
<td>0.318</td>
<td>0.374</td>
<td>0.364</td>
<td>0.386</td>
<td>0.380</td>
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<td>NFL, PPI</td>
<td>0.784</td>
<td>0.417</td>
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<td>0.808</td>
<td>0.809</td>
<td>0.810</td>
</tr>
<tr>
<td>MTS, PPI</td>
<td>0.417</td>
<td>0.321</td>
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<td>0.369</td>
<td>0.324</td>
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<td>0.477</td>
<td>0.541</td>
<td>0.442</td>
<td>0.442</td>
</tr>
<tr>
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<td>0.864</td>
<td>0.858</td>
<td>0.338</td>
<td>0.908</td>
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<tr>
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<td>0.529</td>
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<td>0.808</td>
<td>0.809</td>
<td>0.809</td>
</tr>
<tr>
<td>MTS, PCEP</td>
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<td>0.352</td>
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<td>0.870</td>
<td>0.801</td>
<td>0.912</td>
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<tr>
<td>PILT, PCEP</td>
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<td>0.452</td>
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<td>0.854</td>
<td>0.846</td>
<td>0.864</td>
<td>0.903</td>
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<tr>
<td>IP, PCEF</td>
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<td>0.969</td>
<td>0.969</td>
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</tr>
<tr>
<td>NFL, PCEF</td>
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<td>0.812</td>
<td>0.811</td>
<td>0.811</td>
<td>0.812</td>
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<td>0.965</td>
<td>0.966</td>
<td>0.988</td>
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</tbody>
</table>

Note: Each entry in the table reports the in sample root mean square error associated to a dynamic factor model with four variables. Specifically, with the corresponding pair of benchmark variables, in the rows, and an additional variable that has not been already included, in the columns. Moreover, Nominal GDP is always included in each model.
Table 3.5: Root Mean Square Errors for Enlarged Models (Cont.)

<table>
<thead>
<tr>
<th>Variables</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>IP, CPI, M3, M4</td>
<td>1.2609</td>
</tr>
<tr>
<td>IP, CPI, M3, TBILL</td>
<td>0.2977</td>
</tr>
<tr>
<td>IP, CPI, M4, TBILL</td>
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</tr>
<tr>
<td>IP, CPI, M3, M4, TBILL</td>
<td>1.4936</td>
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<tr>
<td>MTS, CPI, M3, TBILL</td>
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<tr>
<td>MTS, CPI, M4, TBILL</td>
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</tr>
<tr>
<td>MTS, CPI, M3, M4, TBILL</td>
<td>1.2623</td>
</tr>
</tbody>
</table>

Note: Each entry in the table reports the in sample root mean square error associated to a dynamic factor model with five or six variables. Specifically, with the corresponding set of selected variables, in the rows, and Nominal GDP is always included in each model.
### Table 3.6: Factor Loadings for the Selected Models

<table>
<thead>
<tr>
<th>Models</th>
<th>NGDP</th>
<th>IP</th>
<th>MTS</th>
<th>CPI</th>
<th>PPI</th>
<th>M3</th>
<th>M4</th>
<th>TBILL</th>
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<tbody>
<tr>
<td>1</td>
<td>0.2187</td>
<td>0.3097</td>
<td>0.2488</td>
<td>0.2488</td>
<td>0.2488</td>
<td>0.2488</td>
<td>0.2488</td>
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<tr>
<td>2</td>
<td>0.2098</td>
<td></td>
<td>0.1825</td>
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<td>0.2643</td>
<td>0.2643</td>
<td>0.2643</td>
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<tr>
<td>3</td>
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<td>0.2551</td>
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<td>0.2551</td>
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<td>0.2551</td>
<td>0.2551</td>
<td>0.2551</td>
<td>0.2551</td>
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<td>0.2548</td>
<td>0.2548</td>
<td>0.2548</td>
<td>0.2548</td>
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<td>0.2663</td>
<td>0.2663</td>
<td>0.2663</td>
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<td>0.2567</td>
<td>0.2567</td>
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<td>0.2573</td>
<td>0.2573</td>
<td>0.2573</td>
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<tr>
<td>12</td>
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<td>0.2394</td>
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<tr>
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<td>0.2582</td>
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</tbody>
</table>

Note: The first column of the table labels the different dynamic factor models which include the variables associated to the corresponding rows. Each entry in the table makes reference to the loading factor of each variable in the models.
Table 3.7: Root Mean Square Errors for Real-Time Nowcasts

<table>
<thead>
<tr>
<th>Model</th>
<th>Variables</th>
<th>RMSE</th>
<th>Model</th>
<th>Variables</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.5219</td>
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<td>MTS, CPI, M4</td>
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</tr>
<tr>
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<td>MTS, CPI</td>
<td>0.5638</td>
<td>9</td>
<td>MTS, CPI, TBILL</td>
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</tr>
<tr>
<td>3</td>
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<td>IP, CPI, M3, TBILL</td>
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</tr>
<tr>
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<td>IP, CPI, M4, TBILL</td>
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</tr>
<tr>
<td>5</td>
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<td>12</td>
<td>MTS, CPI, M3, TBILL</td>
<td>0.5134</td>
</tr>
<tr>
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<td>MTS, CPI, M4, TBILL</td>
<td>0.5974</td>
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<tr>
<td>7</td>
<td>MTS, CPI, M3</td>
<td>0.5667</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Each entry in the table reports the out of sample root mean square error associated to a dynamic factor model with five or six variables. Specifically, with the corresponding set of selected variables, in the rows, and Nominal GDP is always included in each model.
Chapter 3. Real-Time Nowcasting of Nominal GDP

Figure 3.1: Path of Nominal GDP

Chart A. Nominal GDP level
Chart B. Nominal GDP growth

Note: Chart A and Chart B plot Nominal GDP in levels and in quarterly growth rates for the period 1967Q1 - 2012Q4. The vertical bar makes reference to the great recession period, 2007Q4 - 2009Q2.

Figure 3.2: Real-Time Nowcasts of Nominal GDP based on Univariate Models

Chart A. AR(1)
Chart B. AR(2)
Chart C. AR(3)

Note: The Charts A, B and C plot the monthly real-time nowcasts of Nominal GDP from autoregressive models with one, two and three lags, respectively, for the period 2010Q4 - 2012Q3. The red points refer to the NGDP quarterly growth.
Chapter 3. Real-Time Nowcasting of Nominal GDP

Figure 3.3: Updated Autorregresive Parameters Model for Nominal GDP

Note: Charts A plots the autoregressive parameters from the AR(1) model, which are updated each period as new information is included in the model, for the period 2010Q4 - 2012Q3. The same applies for Chart B, which plots the two autoregressive parameters from the AR(2) model, and for the Chart C, which plots the three autoregressive parameters from the AR(3) model.
Chapter 3. Real-Time Nowcasting of Nominal GDP

Figure 3.4: Real Activity vs. Inflation

Chart A. Real GDP growth

Chart B. GDP Deflator growth

Chart C. IP growth

Chart D. CPI growth

Note: Chart A and B plot the quarterly growth of Real GDP and GDP Deflator, respectively, for the period 1967Q1 - 2012Q4. Charts C and D plot the monthly quarterly growth of the Industrial Production Index (IP) and the Consumer Price Index (CPI) for the period 1967M1 - 2012M12.
Chapter 3. Real-Time Nowcasting of Nominal GDP

Figure 3.5: NGDP vs. IP+CPI

Chart A. In Sample 1967.I-2012.III

Chart B. Out of Sample 2000.I-2012.III

Note: Chart A plots the in sample predictions of the Naive model, dashed line, and the NGDP growth, solid line. Chart B plots the monthly out of sample nowcast of the Naive model, solid line, and the NGDP quarterly growth, represented by the red points.

Figure 3.6: NGDP vs. Best Benchmark

Note: The figure plots the in sample prediction from the best benchmark, based on the mean square error criterion, which uses information of Manufacturing Trade Sales (MTS), Consumer Price Index (CPI) and NGDP, it is refers to the dashed line. The solid line refers to the NGDP quarterly growth.
Chapter 3. Real-Time Nowcasting of Nominal GDP

Figure 3.7: Real-Time Nowcasts based on a Dynamic Factor

Model 1

Model 2

Model 3

Model 4

Model 5

Model 6

Model 7

Model 8

Model 9

Note: Each chart in the figure plots the monthly real-time nowcasts associated to the different dynamic factor models, they are represented by the solid line. The red points refer to NGDP quarterly growth.
Chapter 3. Real-Time Nowcasting of Nominal GDP

Figure 3.8: Real-Time Nowcasts based on a Dynamic Factor (Cont.)

Note: Each chart in the figure plots the monthly real-time nowcasts associated to the different dynamic factor models, they are represented by the solid line. The red points refer to NGDP quarterly growth.
References


References


