Esta tesis doctoral contiene un índice que enlaza a cada uno de los capítulos de la misma.
Existen asimismo botones de retorno al índice al principio y final de cada uno de los capítulos.

Ir directamente al índice

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Aquesta tesi doctoral conté un índex que enllaça a cadascun dels capítols.
Existeixen així mateix botons de retorn a l'index al principi i final de cadascun deis capítols .

Anar directament a l'index

Per a una correcta visualització del text és necessària la versió d' Adobe Acrobat Reader 7.0 **o posteriors.**

PARTIAL CARTELS

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Als meus: el Santi, la Isabel, l 'àvia i l' Enric.

AGRAIMENTS

Estic agraït, molt agraït, a la gent que ha estat amb mi durant tots aquests anys. A tots aquells que sempre hi eren, quan els hi ho demanava o també quan no era capaç d'expressar l'ho molt que els necessitava, quan eren a prop o quan ben sovint, eren lluny però jo els sentia a prop perquè ells m'hi feien sentir. Ells ho han fet possible. És gràcies a tots ells i als meus que sempre he trobat les ganes de tirar endavant. Per sort la llista és llarga però no per això deixaré de anomenar-vos. Seria injust no començar amb en Ramon perquè la seva dedicació, paciència infinita i entusiasme amb aquest treball han estat imprescindibles i m'han permès continuar en els pitjors moments. Als clàssics de l´interrail, Alfred, Roger i Àlex, perquè estic segur que el viatge junts anirà molt més enllà. A l´Eloi perquè el teu bon humor és sempre una inspiració vital i imprescindible per mi. Al David, i l'Eloi Torcal pel carinyo sincer que sempre m'heu demostrat, i al David Puibert pel teu humor malvat amb el que tant hem rigut. A vosaltres, Christian i Irene, per una amistat llarga, profunda i sincera com n'hi ha molt poques, sempre sereu al meu cor. A Eli, por esas historias de Fabero. A l'inevitable Rafi. A vosotras, Ana, Teresa y Pepa, por la alegría indescriptible que suponían para mi mis escapadas a Madrid y vuestras visitas a Alicante, por todos los momentos juntos que tanto han significado, y por lo mucho que os quiero. A l'Anaís per tants anys de plàcida convivència i per la teva honestedat i sinceritat. A Antonio, porque tu tranquilidad y nuestras conversaciones mateando fueron el descanso ideal para poder acabar este trabajo. A Rebeca, por la complicidad que siempre ha habido entre nosotros. A Trino, el mejor coleguita. A Paula, Nico, Francis, Tate, Santi, Carlos, Elvira, Ana y toda la gente de Alicante con quien tantos momentos he compartido y que han sido importantes para mi. También a Jorge porque toda esta movida la empezamos juntos. Al Carles, Risto, David, Pablo i tots els altres, també vosaltres sou importants per mi malgrat no molt sovint tinc la oportunitat de dir-vos-ho. També a tu malgrat els nostres camins es separessin ja fa molt temps. Gràcies de tot cor a tothom.

Frankfurt, 28 d'Octubre de 2003

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1 INTRODUCTION

The literature about collusion, mainly deal with two different approaches. Firstly, there are the papers that investigate cartel stability in static models. By their very nature, in these models cartel members do not cheat on a cartel agreement as it is assumed that agreements are sustained through *binding* contracts. For many years it was widely held among economists that firms could not exercise market power collectively without some form of explicit coordination. However, nowadays these binding contracts restricting competition are prohibited in all European countries. Furthermore, the theory of repeated games has confirmed that stable arrangements may require little coordination between firms, and possibly none at all. There is another strand in the literature on cartel stability, which takes a quite different route. Seemingly independent, but parallel actions among competing firms in an industry are driven to achieve higher profits. It is termed tacit or *implicit* collusion. The distinguishing feature of implicit collusion is the lack of any explicit agreement. The key is that each firm seems to be acting independently, perhaps each responding to the same market conditions. This focuses on firms "incentive constraints" that are the incentives to cheat on the cooperative agreement. Then, what this approach leaves out, are firms' "participation constraints": it cannot explain why many real world cartels do not comprise all firms in the industry.

On the other hand, looking at the most recent measures of the European Commission against collusion, we see that several cartels have been fined hundreds millions euros by the European Commission for price fixing and setting sales quotas. At the same time it is also verified that in many of these industries, not all firms in the industry were fined. In practice, there are many cases where collusive agreements do not involve all firms in the industry. We have several examples. In the carbonless paper industry, the joint market share of the fined firms was between 85 and 90%. In the North Atlantic shipping industry, the market share of the cartel was calculated around 70-80%, or in the cartonboard industry where the market share of the cartel is assumed to be around 80%. Another significant example is the citric acid industry where three North-American and five European firms were convicted in United States, Canada and the European Union and fined more than one hundred million euros for fixing prices and allocating sales in the worldwide market, issuing coordinated price announcements and monitoring one another's prices and sales volumes during the period 1991-1995. The joint market share of these eight firms was only between 50% and 60%. Despite this empirical evidence, the Industrial Organization analysis of tacit collusion in quantity setting supergames, has generally focused on the symmetric subgame perfect equilibrium that maximizes industry profits.

On the contrary, this thesis is devoted to study the case where only a subset of firms are trying to tacitly collude while the rest maximize individual per period profits.

This thesis consists of three different chapters. In the first chapter, I study how does the size of a cartel affect the possibility that its members can sustain a collusive agreement in a supergame theoretical framework. I obtain that collusion is easier to sustain the larger the cartel is. Then, I explore the implications of this result on the incentives of firms to participate in a cartel. Firms will be more willing to participate because otherwise, they risk that collusion completely collapses, as remaining cartel members are unable to sustain collusion. The main point of this chapter is that the argument of selecting the most profitable symmetric equilibrium because it would be the one that firms will agree to play if they secretly meet to discuss their pricing plans, is compelling but it does not take into account that firms may prefer not to attend this meeting in order not to participate in the coordination to a collusive agreement. The previous result has implications on cartel formation, because it reduces the incentives to free-ride from a cartel by defecting from it. This idea can be illustrated with the following extreme example. I find that for some discount factors, the only sustainable cartel is the cartel that comprises all firms in the industry. Then all firms have incentives to participate in the cartel, because otherwise collusion completely collapses. This completely eliminates the gains from free-riding at the participation stage. The model predicts that the size of the cartels enforced can be larger in the *implicit* collusion model than in the *binding* collusion model. Then, the policy measures that induce firms to replace *binding* with *implicit* collusion to escape antitrust prosecution may have its costs. Forbidding *binding* collusion (and forcing firms to collude tacitly) has the positive effect of weaken the incentives to maintain a collusive agreement but the negative effect of making stronger the incentives to participate in a cartel.

In a second chapter, the partial cartel model is devoted to the issue of mergers. The equilibrium obtained is not like the standard equilibrium of the repeated game where only if the discount factor is large enough collusion is possible, but of the type; given the restraints that firms face according their discount factor, the equilibrium of the repeated game is the output that maximizes cartel firms' joint profits. Firms' ability to collude depends on the discount factor. Curiously, while there exists a substantial literature on the effect of mergers when pre-and post-merger behavior is noncooperative, little attention has been given by theorists to the consequences for firms of mergers when they take place in a collusive environment. In addition, the purpose of this chapter is also to analyze the competitive effects of horizontal mergers on profits and welfare in a Partially Cartelized market. It is shown that both mergers among fringe and cartel firms increase market price. Regarding merger profitability, the discount factor decreases cartel members' merger profitability. The intuition is that when mergers do not involve any cost saving, firms merge basically to restrict competition. However, when competition is already low because firms are sustaining collusive agreements, mergers lose attractiveness as an anticompetitive device. Thus, the more firms can collude, the less they are interested in merging. However, the higher cartel members' discount factor, the more fringe firms will be willing to merge. In this case, the negative effect on the profitability of mergers because outsiders increase their production is attenuated as an increase in the discount factor reduces the reaction of firms belonging to the cartel. An example of this could be the intense wave of mergers among oil firms that coincided with a large period of high oil prices caused by the OPEC production. In this chapter we observed that the higher cartel members' discount factor cartel firms increase less their output after a merger of fringe firms. In a short subchapter, a model where a group of firms (leaders) choose output before another group of firms (followers) is analyzed. Followers are also less efficient than leaders. It is obtained that leaders reduce their output when followers merge and this reduction renders the merger profitable. The model proves useful because it allows to obtain that mergers of two symmetric firms i.e. without efficiency gains maybe profitable in a setting where firms choose output. Now, the reaction of leaders after a followers' merger is more extreme than the reaction of cartel firms. Leaders may even cut production after a merger of two followers. This renders the merger profitable.

In the third chapter, the relationship between innovation and market structure and how initial production costs affect the incentives to innovate is addressed. It is a common feature in the real world that firms differ, and this asymmetry might refer to size,

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cost structure or R&D commitment. This chapter considers a theoretical model of n asymmetric firms which reduce their initial unit costs by spending on R&D activities. In accordance with Schumpeterian hypotheses it is obtained that more efficient (bigger) firms spend more in R&D and this leads to a more concentrated market structure. A positive relationship between innovation and market concentration is found. This calls for a corrective tax on R&D activities to curtail strategic incentives to over-invest in R&D trying to achieve a higher market share. The main point is therefore, that the relationship between market concentration and innovation is positive and should be corrected by an optimal industrial policy. Within the limited context of the model presented in this chapter, some implications on the design of a policy can be drawn, and this is that when R&D activities are used to reduce competition a corrective tax is needed. This corrective tax, together with a production subsidy reduces market concentration. A firm-specific industrial policy, which is different R&D taxes among firms is also introduced. The intuition behind it is that the support of R&D activities may be, firm-specific or even project-specific. The firm-specific policy prescribes that more efficient firms should be taxed at a lower rate, basically due to the fact that in the welfare maximization, the specific tax is used to divert production to the more efficient firms. Thus, by its nature, this firm-specific industrial policy causes an increase in market concentration.

2 INTRODUCCIÓN

La literatura sobre colusión se ha basado tradicionalmente en dos enfoques distintos. El primero analiza la estabilidad de los cárteles en modelos estáticos. Por sus características, estos modelos no explican porque los miembros del cártel no se desvían de los acuerdos tomados por el cártel ya que se supone que éste puede estructurarse mediante un acuerdo vinculante. Durante muchos años los economistas creyeron que las empresas no podían ejercer de forma conjunta su poder en el mercado sin algún tipo de coordinación explícita. Actualmente sin embargo, estos acuerdos están prohibidos en todos los países europeos. Además, la teoría de juegos repetidos ha confirmado que los acuerdos estables pueden requerir poca coordinación entre empresas o quizás incluso ninguna. Existe pues un segundo enfoque muy distinto en el estudio de la estabilidad de los cárteles. Así pues, a las acciones aparentemente independientes pero efectuadas efectivamente de forma paralela para aumentar los precios y los beneficios de una industria, frecuentemente se le llama colusión tácita o colusión implícita. La característica distintiva de la colusión implícita es la ausencia de un acuerdo explícito. La clave es que cada empresa parece actuar independientemente quizás respondiendo a las mismas condiciones de mercado. Este enfoque se centra en las "restricciones de incentivos" que son los incentivos que las empresas pueden tener en desviarse de los acuerdos del cártel. Así, lo que este enfoque no considera son las "restricciones de participación": no puede explicar porque en la vida real muchos cárteles no están formados por la totalidad de empresas de la industria.

Por otro lado, observando las últimas medidas tomadas por la Comisión Europea contra la colusión podemos ver que a diversos cárteles les han sido impuestas multas por valor de varios centenares de millones euros por acordar precios o distribuir cuotas de mercado conjuntamente. Al mismo tiempo, también podemos comprobar que en muchos de los sectores donde presuntamente las empresas coludían, las empresas multadas no constituían el total de empresas del sector. Tenemos muchos ejemplos. En la industria del papel las empresas multadas significaban entre el 85 y el 90 por ciento del total del mercado. En la industria del transporte marítimo en el Atlántico norte, la cuota de mercado del cártel era alrededor del 70-80 por ciento. Otro ejemplo significativo es la industria del ácido cítrico, donde tres empresas americanas y cinco europeas fueron multadas por diversas prácticas anticompetitivas durante el periodo de 1991 a 1995. La cuota de mercado de todas las empresas multadas conjuntamente era de entre el 50 y el 60 por ciento. A pesar de esta evidencia empírica, el análisis de la Teoría de la Organización Industrial sobre colusión implícita en juegos repetidos con las cantidades producidas como variable estratégica de las empresas, se ha basado generalmente en los equilibrios perfectos en subjuegos simétricos, es decir con todas las empresas del sector formando parte del acuerdo colusivo. Esta tesis pues, está dedicada al estudio del caso en que sólo una proporción de empresas intentan coludir tácitamente mientras que el resto maximiza beneficios individualmente.

Esta tesis está formada por tres capítulos distintos.

En el primer capítulo, titulado "Estabilidad y sostenibilidad de los cárteles", se estudia cómo el tamaño de un cártel afecta a la posibilidad que sus miembros puedan sostener un acuerdo colusivo en un modelo teórico de superjuegos. Se obtiene que la colusión es más fácilmente sostenible cuanto más grande es el cártel. Luego se exploran las implicaciones de este resultado en los incentivos que las empresas pueden tener en participar en un cártel. Las empresas desearán formar parte del acuerdo colusivo porque si no lo hacen se arriesgan a que ningún acuerdo colusivo sea sostenible puesto que las empresas restantes pueden no ser capaces de sostenerlo. La clave de este primer capítulo es que el argumento de seleccionar el equilibrio simétrico más provechoso para las empresas porque sería la asignación que las empresas llevarían a cabo si pudieran secretamente encontrarse y discutir sus planes, es incuestionable pero no tiene en cuenta que las empresas podrían preferir no asistir a ese encuentro para no tener que participar en la coordinación de un acuerdo colusivo. El resultado antes mencionado afecta a la formación de un cártel porque reduce los incentivos que las empresas pueden tener a secretamente desviarse y engañar al resto de empresas del cártel. Un ejemplo puede ilustrar esta idea; Para algunas tasas de descuento, el único cártel sostenible es aquel formado por todas las empresas de la industria. Luego todas las empresas tendrán incentivos a formar parte del acuerdo porque sino no hay colusión posible. El modelo predice que el tamaño del cártel será más grande si la colusión es implícita que si es vinculante. Luego, las políticas que inducen a sustituir colusión vinculante por colusión implícita pueden tener sus costes. Prohibir la colusión vinculante tiene el efecto positivo de disminuir los incentivos de mantener un acuerdo colusivo pero también el efecto negativo de aumentar los incentivos a participar en un cártel.

En el segundo capítulo, titulado "Fusiones en un mercado parcialmente cartelizado", el modelo de cárteles parciales es utilizado para analizar fusiones entre empresas. Ahora la perspectiva es de alguna forma distinta; ¿Para cada tasa de descuento, cuál es la producción de las empresas que maximiza sus beneficios conjuntos? Luego, el equilibrio que se obtiene no es el equilibrio usual de la teoría de juegos repetidos donde sólo si la tasa de descuento es suficientemente alta la colusión es posible. Es un equilibrio que nos dice, de acuerdo con las restricciones a las que las empresas se enfrentan en términos de como descuentan el futuro, cómo es el equilibrio del juego repetido que maximiza los beneficios conjuntos del cártel. Ahora la capacidad de las empresas para coludir se ve reflejada en la tasa de descuento. Curiosamente, aunque existe mucha literatura acerca de las consecuencias de las fusiones entre empresas cuando antes y después de las fusiones, las empresas compiten entre ellas, hay muy poca literatura teórica al respecto de cuando las fusiones tienen lugar en un mercado no competitivo. Así pues, el objetivo de este capítulo es analizar el efecto que en los beneficios y en el bienestar tienen las fusiones horizontales entre empresas en un mercado parcialmente cartelizado. Se demuestra que las fusiones, entre ya sea empresas del cártel o del margen, aumentan el precio en el mercado. Por lo que se refiere a la rentabilidad de las fusiones, cuanto más alta sea la tasa de descuento de las empresas menos incentivos tendrán las empresas del cártel en fusionarse puesto que su objetivo de restringir la competencia es más provechosamente obtenido con un acuerdo colusivo que con una fusión. Por otro lado, cuanto más alta sea la tasa de descuento de las empresas del cártel, más incentivos tendrán las empresas del margen a fusionarse. En este caso el efecto negativo sobre la rentabilidad de las fusiones que tiene el hecho de que las empresas que no se fusionan reaccionan aumentando su producción, se ve atenuado porque éstos reaccionan menos a la fusión a medida que su tasa de descuento aumenta. Un ejemplo práctico puede ser la gran oleada de fusiones entre empresas petroleras a finales de la década de los 90 que coincidió con un periodo de precios del petróleo altos debido a recortes en la producción por parte de la OPEP. En un apartado del segundo capítulo llamado "Fusiones en mercados de Stackelberg asimétricos", se analiza un modelo con un grupo de empresas (líderes) que eligen la producción antes que otras (seguidoras). Las seguidoras se suponen menos eficientes que las empresas líderes. Se obtiene que las líderes reducen su producción cuando las seguidoras se fusionan y eso hace que la fusión sea rentable. El modelo es útil porque permite ver fusiones provechosas entre dos empresas iguales cuando éstas eligen cantidades y no hay ganancias en eficiencia asociadas a la fusión. Ahora la reacción de las líderes a la fusión de las seguidoras es incluso más extrema que en el caso del cártel. Las líderes pueden incluso disminuir su producción en respuesta a una fusión entre empresas seguidoras. Esto puede hacer que la fusión sea rentable.

En el tercer capítulo, llamado "Innovación y concentración de mercado con empresas asimétricas", se analiza la relación entre la innovación que las empresas pueden llevar a cabo y la estructura del mercado y cómo los costes iniciales de producción afectan a los incentivos que las empresas pueden tener para innovar. Es muy frecuente observar empresas de un mismo sector económico diferir en muchos aspectos; tamaño, estructura de costes de producción o estrategia de inversión en I+D. Este capítulo considera un modelo teórico con un número arbitrario de empresas asimétricas que pueden reducir sus costes de producción si invierten en I+D. De acuerdo con las tesis Schumpeterianas, se obtiene que las empresas más eficientes (las más grandes en términos de cuota de mercado) gastan más en I+D y eso lleva al mercado a aumentar su grado de concentración. Luego, se encuentra una relación positiva entre innovación y concentración en el mercado. Esta relación requiere un impuesto correctivo para disminuir los incentivos que las empresas puedan tener en invertir más simplemente para conseguir una cuota de mercado más elevada. Así pues, lo más importante del capítulo es que la relación entre el grado de concentración del mercado e innovación es positiva y debería ser corregida con una política industrial óptima. En este contexto pues, algunas implicaciones en el diseño de la política industrial se pueden considerar; Cuando la inversión en I+D se utiliza para reducir la competencia, una política industrial adecuada correctiva debería llevarse a cabo. Un impuesto correctivo junto con un subsidio a la producción reduce la concentración del mercado. También se analiza una política industrial específica por cada empresa. La intuición es que el apoyo institucional a las actividades en I+D puede ser específico para cada empresa o incluso para cada proyecto de investigación. La política industrial específica aconseja que las empresas eficientes deberían tener un impuesto más pequeño básicamente debido a que la maximización del bienestar global prescribe que el impuesto se utilice para transferir cuota de mercado de las menos eficientes a las más eficientes. Así pues, por su naturaleza, este tipo de impuestos causa un aumento en el grado de concentración del mercado.

Chapter 1

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Cartel sustainability and cartel stability

March, 2003

The paper studies how does the size of a cartel affect the possibility that its members can sustain a collusive agreement. I obtain that collusion is easier to sustain the larger the cartel is. Then, I explore the implications of this result on the incentives of firms to participate in a cartel. Firms will be more willing to participate because otherwise, they risk that collusion completely collapses, as remaining cartel members are unable to sustain collusion.

JEL Classification:L11, L13,L41, D43.

Keywords: Collusion, Partial Cartels, Trigger strategies, Optimal Punishment.

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[2]I would like to thank my supervisor Ramon Faulí-Oller for his advice and encouragement, and Lluís Bru and George Symeonidis for his helpful comments. Financial support by the I.V.I.E, *(Instituto Valenciano de Investigaciones Económicas)*, Ministerio de Ciencia y Tecnologia through its project BEC2001-0535, the European Commission through its RTN programme (Research Training Network), the Center for Financial Studies (Frankfurt) and The Department of Economics of the University of Alicante is gratefully acknowledged. All errors are mine.

3 Introduction.

For many years it was widely held among economists that firms could not exercise market power collectively without some form of explicit coordination. However the theory of repeated games has cast some doubt on this approach. Stable arrangements may require little coordination between firms, and possibly none at all. This has raised a dilemma for the design of a policy towards collusion. If the legal standard focuses on explicit coordination, a large number of collusive outcomes can fall outside the prohibition, and if it tries to cover collusion without explicit coordination, it will prohibit non-cooperative practises.

Article 81(1) of the Rome Treaty stipulates that agreements or concerted practises between firms which distort competition are prohibited. What is meant by "agreements" and "concerted practises" is not further specified in the treaty. However, decisions recently taken by the Commission show that often firms behavior that do not involve a process of coordination are overlooked although they could mean an exercise of market power.

The literature about collusion, mainly deal with two different approaches. Firstly, there are the papers that investigate cartel stability in static models. They have mainly focused on the incentives of firms to participate in a cartel agreement. These papers focus on firms "participation constraints". Two different incentives play a role here. Firms face a trade off between participation and nonparticipation in the cartel: firms have an incentive to join the cartel so as to achieve a more collusive outcome, but on the other hand have an incentive to stay out of the cartel to free-ride on the cartel effort to restrict production. By their very nature, in these models cartel members do not cheat on a cartel agreement as it is assumed that agreements are sustained through binding contracts, they may, therefore, be viewed as models of *binding* collusion[3]. The seminal papers in this literature are Selten (1983) and d'Aspremont et al. (1983)

There is another strand in the literature on cartel stability, which takes a quite different route. The supergame-theoretic approach to collusion has focused on the problem

[3]A formal collusion agreement among competing firms (mostly oligopolistic firms) in an industry designed to control the market, raise the market price, and otherwise act like a monopoly is frequently also termed explicit collusion. Binding collusion refers, therefore, to an explicit collusive agreement enforceable at law.

of enforcement of collusive behavior (see for example Friedman (1971)). In these models, seemingly independent, but parallel actions among competing firms in an industry are driven to achieve higher profits. It is termed tacit or *implicit* collusion. This focuses on firms "incentive constraints"[4]. Then, what this approach leaves out, are firms' "participation constraints": it cannot explain why many real world cartels do not comprise all firms in the industry. Instead, they have studied under which circumstances collusion can be sustained as an equilibrium of the repeated game. Most research on the field has studied symmetric settings and have focused on the sustainability of the most profitable symmetric equilibrium. The reason to select this equilibrium is that it will be the one that firms will agree to play if they secretly meet to discuss their pricing plans (Mas-Colell et al (1996)).

The main point of the paper is that this argument is compelling but it does not take into account that firms may prefer not to attend this meeting in order not to participate in the coordination to a collusive agreement. This takes us back to the literature on the incentive to participate in a cartel, mentioned above. However, now the analysis is richer because one has to study how does the participation incentive interact with the incentive to maintain a collusive agreement. As a first step, I study how does the size of a cartel affect the possibility that its members can sustain a collusive agreement in a supergame theoretical framework. I obtain that collusion is easier to sustain the larger the cartel is. To obtain the result I study the sustainability of partial cartels i.e. cartels that do not include all the firms in a given industry. Partial cartels are often observed in reality, being the OPEC the most well known example.

The previous result has implications on cartel formation, because it reduces the incentives to free-ride from a cartel by defecting from it. I can illustrate the idea with the following extreme example. I find that for some discount factors, the only sustainable cartel is the cartel that comprises all firms in the industry. Then all firms have incentives to participate in the cartel, because otherwise collusion completely collapses. This completely eliminates the gains from free-riding at the participation stage.

Obviously, in practice it is easier to fight *binding* collusion than against implicit collusion. The model highlights that policy measures that induce firms to replace *binding*

[4]"Participation constraints" are firms incentives to join the cartel or the fringe; meanwhile "Incentive constraints" are the incentives to cheat on the cooperative agreement.

with implicit collusion[5] to escape antitrust prosecution may have its costs. Forbidding *binding* collusion (and forcing firms to collude tacitly) has the positive effect of weaken the incentives to maintain a collusive agreement but the negative effect of making stronger the incentives to participate in a cartel[6].

Therefore the total effect on price is uncertain. In the particular model I analyze price is higher with implicit than with *binding* collusion. The model predicts that the size of the cartels enforced can be larger in the *implicit* collusion model than in the *binding* collusion model.

We can think of several interesting cases where these results could be of interest. In several European countries, before governments moved to adapt its domestic competition policy to the European regime, agreements restricting competition among firms were not only permitted but also enforceable at law. Namely, Denmark (see OCDE (1993)) where before the Competition Act of 1990 was passed agreements were widespread in several sectors and often took the form of binding agreements and also West Germany where before 1987 hundreds of legalized cartels were enforced through a contract (see Audretsch (1989)). Switzerland and Sweden are other examples of countries where Cartels were sustained for decades by means of enforceable contracts. Therefore, the present model points out a possible consequence of banning binding collusion that perhaps has been unnoticed by antitrust authorities.

The structure of the paper is as follows. In the following section, the central model of the paper is set. The sustainability of the partial cartel is analyzed with the "trigger strategies"[7]. In the next section, the participation game is set. Firms decide first wether to

[5] "When the legal advisors of cartel members discovered that Article 85 had to be taken seriously, they had their clients throw their agreements in the waste basket. Simultaneously, the attention of DG\ IV shifted to the detection of tacit collusion, on the assumption that explicit collusion was being replaced by tacit collusion" (Phlips (1995)).

[6] "Although strict prohibition and strong sanctions probably reduce the incidence of explicit collusion, continuing cases are good evidence that firms find it profitable to engage in the practice. Firms are naturally careful not to create good evidence of such agreements. (...) It should be noted, however, that even if enforcement were 100 percent effective, this would not necessarily put an end to coordinated interaction. It could simply cause firms to opt for substitutes which are less likely to attract legal sanctions and offer the further advantages of greater flexibility and lower costs to arrange: It could be even argued that firms prefer less risky, more flexible alternatives to explicit collusion" (OECD (1999))

[7] The sustainability of partial cartels in a dynamic setting is considered by Martin (1990) in a ho-

join the cartel or not, afterwards the firms infinitely play a quantity game. The main aim of this section is to study the interaction between incentive and participation constraints. Afterwards, the sustainability of the partial cartel is analyzed using an optimal penal code to enforce collusion following Abreu (1986).

mogeneous firms context. However unlike our model, decision is sequential, that is cartel firms act as Stackelberg leaders.

4 Partial Cartels.

Assume that n firms, where $n > 2$, indexed i, $i = 1, 2, 3, ..., n$ compete in a market whose demand is given by $P(Q) = 1 - Q$. Cost functions of firms are given by: $c(q_i) = \frac{q_i^2}{2}$, where q_i denotes the production of firm i. Assume firms simultaneously choose quantities [8].

A (partial) cartel will be said to be active in this market if there is a group of firms (cartel members) that maximize joint profits and the remaining firms (nonmembers or fringe firms) maximize individual profits. When a cartel of k firms is active, members (m), and nonmembers (nm), simultaneously produce respectively:

$$q_m^k = \frac{2}{nk - k^2 + 3k + 2 + n} \tag{1}$$

$$q_{nm}^k = \frac{k+1}{nk - k^2 + 3k + 2 + n} \tag{2}$$

In this situation, Profits of members and nonmembers are given respectively by π_m^k and π_{nm}^k. Observe that if $k = 1$, we have standard Cournot competition and $q_m^1 = q_{nm}^1$.

We are going to study under which conditions playing (1) and (2) in each period can be sustained as an equilibrium of a game where the one stage game described above is repeated infinite times. Firms will be assumed to discount the future at a factor of δ. Member firms are denoted with a natural number from 1 to k.

Cartel members will sustain cooperation by using "trigger strategies", that is, when cheating, firms are punished with infinite reversion to the Nash Cournot equilibrium. Trigger strategies for a partial cartel can be formulated the following way, where $q_{t,i}$ denotes the strategy played by firm i in period t:

Firm i, $i = 1, ..k$ plays

$$\begin{cases} q_{1,i} = q_m^k \\ q_{t,i} = q_m^k \text{ if } q_{l,j} = q_m^k \text{ for any } l < t \text{ for } j = 1, ..., k \\ q_{t,i} = q_m^1 \text{ otherwise.} \end{cases}$$

Firm i, $i = k + 1, ...n$ plays

[8] Shaffer(1995) considers the cartel as a Stackelberg leader because of its power to impose its most preferred timing.

$$\begin{cases} q_{1,i} = q_{nm}^k \\ q_{t,i} = q_{nm}^k \text{ if } q_{l,j} = q_m^k \text{ for any } l < t \text{ for } j = 1, ..., k \\ q_{t,i} = q_m^1 \text{ otherwise.} \end{cases}$$

Nonmember firms play optimally, because the future play of rivals is independent of how they play today and they maximize current profits. Member firms will behave optimally if the discount factor is high enough. To obtain the conditions on the discount factor such that using "trigger strategies" is also optimal for member firms, we have to calculate the profits of a member firm that deviates from the cartel. They will choose:

$$q_d^k =_q^{\arg\max} P((k-1)q_m^k + (n-k)q_{nm}^k + q)q - \frac{q^2}{2}$$

and will obtain π_d^k like the profits obtained in the period of deviation.

Then trigger strategies are optimal for member firms if:

$$\frac{1}{1-\delta}\pi_m^k \geq \frac{\delta}{1-\delta}\pi_m^1 + \pi_d^k$$

If we let $\delta_k = \dfrac{\pi_d^k - \pi_m^k}{\pi_d^k - \pi_m^1}$, the previous condition can be written in the following way.

If $\delta_k \geq 1$ the cartel of size k can not be sustained for any δ. If $\delta_k < 1$, the cartel can be sustained for $\delta \geq \delta_k$.

Although it may be surprising at first sight that some cartel sizes can not be sustained in equilibrium, it comes from the well-known result in the literature that with Cournot competition, mergers (or any other collusive agreement) of a small number of firms reduces profits because non-participating firms react by increasing their production (see Salant et al.(1983)).

Next proposition shows that the previous intuition extends to any cartel size in the sense that whenever a cartel of size k is sustainable, cartels of larger size are also sustainable[9].

Proposition 1 *The cutoff discount factor (δ_k) that sustain the strategies described above, is strictly decreasing in the size of the cartel.*

[9]Remark the similarity with the result in Salant et al.(1983) that if a merger of k firms is profitable, a merger with more firms is also profitable.

We have that δ_k can be rewritten like :

$$\delta_k = \frac{1 - \dfrac{\pi_m^k}{\pi_d^k}}{1 - \dfrac{\pi_m^1}{\pi_d^k}}$$

Therefore variations of k have two different effects. First, $\dfrac{\pi_m^k}{\pi_d^k}$ decreases when k increases because deviation profits increase more than profits from being in the cartel of k firms. This would increase δ_k. Second, as k increases, $\dfrac{\pi_m^1}{\pi_d^k}$ also decreases because π_m^1 does not depend on k, and deviation profits increase with k. Thus punishment becomes proportionally more painful. This second effect would decrease δ_k.

The result from the Proposition 1 comes from the fact that the second effect dominates the first one.

5 The participation game.

In the previous Section we have obtained conditions on the discount factor under which cartels of different sizes are active. In this Section, we will allow firms to coordinate in the different outcomes by showing their willingness to participate in a collusive agreement. Those decisions will not affect the payoff of firms, but they will only be used as a coordination device: if k firms decide to participate in a cartel agreement, only cartels of size k can be observed in the repeated game.

This pre-communication play is modelled as a stage prior to market competition. The decision of each firm relates to selecting a zero-one variable w_i where:

$$w_i \; : \; \begin{array}{l} 1 \text{ iff firm } i \text{ joins the cartel} \\ 0 \text{ iff firm } i \text{ joins the fringe} \end{array}$$

If k firms announce joining the cartel , the future play is only modified if the discount factor allows a cartel of k firms to be active ($\delta \geq \delta_k$). Otherwise, all firms play the Cournot quantity in all periods. In short, once a cartel of k firms is formed, we will assume that discounted payoffs of member and nonmember firms are respectively given by the following expressions:

$$\Pi_m^k = \begin{cases} \frac{1}{1-\delta}\pi_m^k \text{ if } \delta \geq \delta_k \\ \frac{1}{1-\delta}\pi_m^1 \text{ otherwise} \end{cases} \tag{3}$$

$$\Pi_{nm}^k = \begin{cases} \frac{1}{1-\delta}\pi_{nm}^k \text{ if } \delta \geq \delta_k \\ \frac{1}{1-\delta}\pi_{nm}^1 \text{ otherwise} \end{cases} \tag{4}$$

We are going to look for the Nash equilibrium of the game.

In our model, a cartel of size k is an equilibrium configuration (stable cartel) if the following two conditions are satisfied:

-Internal stability: Either $k = 1$, or:

$$\Pi_m^k \geq \Pi_{nm}^{k-1} \tag{5}$$

Which means that no cartel firm wants to leave the cartel, as the profits that this firm would obtain by joining the fringe would be no larger than profits obtained by remaining in the cartel.

-External stability: Either $k = n$, or:

$$\Pi_m^{k+1} \leq \Pi_{nm}^k \tag{6}$$

If this condition holds no fringe firm has incentives to join the cartel, as doing so the profits obtained would be no larger than the profits obtained by staying in the fringe.

This participation game has been previously analyzed in the literature in cases where firms can sign binding contracts to sustain the outcome of the cartel[10]. In that case collusion is said to be binding, while in our model is called implicit. With binding collusion sustainability of cartels is not at issue. Then payoffs of players would be like (5) and (6) taking $\delta_k = 0$. Solving the participation game for the case of binding collusion will be both a helpful step to solve it in our case and will provide us a benchmark to compare the results.

[10]See Donsimoni (1985). The only difference is that it considers the Cartel behaves as a Stackelberg leader while in our case the cartel and nonmember firms compete à la Cournot.

The key point in the binding collusion case is that for any cartel size, internal stability is never satisfied. Firms know that the goal of the cartel is to reduce production. Then firms will have incentives to leave the cartel in order to free ride from the output reduction agreed by the remaining cartel members.

Proposition 2 *No cartel is stable when collusion is binding.*

We are ready now to determine the Nash equilibrium of the participation game. This game has many equilibria in which no cartel is active. For example all firms deciding not to join the cartel is always an equilibrium. For $\delta < \delta_n$ any choice by firms is an equilibrium because the participation decisions are irrelevant because no cartel can be sustained. To clarify the analysis I will focus on the equilibria where cartels are active whenever they exist. It turns out that when they exist, they are unique (except for a permutation of players). We state the results in the following Proposition:

Proposition 3 *No cartel is active in equilibrium if $\delta < \delta_n$. Whenever $\delta \in [\delta_k, \delta_{k-1})$ and $\delta_k < 1$, a cartel of k firms is active in equilibrium.*

The fact that for $\delta < \delta_n$ no cartel is active comes from Proposition 1. Therefore we have only to explain the second part of the Proposition. For $\delta_{k-1} > \delta \geq \delta_k$ only cartels of size greater or equal than k can be sustained. Cartels of sizes greater than k are not stable, because the result in Proposition 2 applies: internal stability does not hold.

The cartel of size k is internally stable, because firms know that quitting the cartel means that collusion fully collapses and they would obtain the Cournot profits. Therefore the cartel of size k is stable. That is, only the smallest cartel among those which can be sustained are stable in the Participation Game.

Once characterized the equilibrium of the participation game, there are two corollaries we can extract from Proposition 3.

Simply comparing Proposition 2 and Proposition 3 we get the following conclusion:

Corollary 1 *If $\delta \in [\delta_n, 1)$ the size of active cartels is bigger with* implicit *collusion than with* binding *collusion.*

Although with binding collusion cartels are always effective, because collusion consists of cartel members committing themselves to produce a certain agreement by signing binding contracts, we can not find stable cartels. However with implicit collusion firms do not dispose of any commitment power, but when $\delta > \delta_n$ (see Proposition 3) a cartel of certain size is stable. It is precisely the success of the cartels what reduces the incentive to participate in them in explicit collusion.

In the previous Section, we checked that cartels were only active if the discount factor was high enough. Therefore, prices were increasing in the discount factor. In the present Section, the size of the cartel is determined endogenously. Then, price may decrease with the discount factor, because it reduces the size of stable cartels. The failure of small cartels to be sustainable when δ is low, induces firms to create bigger cartels. This result is recollected in the following corollary:

Corollary 2 *When the size of the cartel is endogenously determined, if $\delta \in [\delta_n, 1)$ price decreases with the discount factor.*

The reason is basically that as long as the cutoff of the discount factor is decreasing with k, when $\delta \geq \delta_n$, the larger the discount factor, the lower the size of the cartel that is stable. Thus as δ increases, smaller cartels associated to lower prices are enforced. However, when δ is very low ($\delta < \delta_n$), as long as no agreement is possible, the price is the Nash equilibrium price.

6 Optimal punishment.

The literature about *implicit* collusion has treated repeated game models using basically two different types of strategies to enforce subgame perfect Nash equilibria (S.P.N.E.), the "trigger strategies" and the "stick and carrot" strategies defining an optimal punishment[11]. Trigger strategies have been used in the first three sections of the model. I obtained that the cutoff of the discount factor is decreasing in the size of the cartel, and this led us to the results of the third section. I will see in this section, if it is also true when cooperation is sustained by an optimal punishment.

Cooperation is sustained now with strategies where cheating firms are punished with the fastest and most severe possible punishment. Abreu (1986) outlines a symmetric, two-phase output path that sustains collusive outcomes for an oligopoly of quantity setting firms. The output path considered by Abreu has a "stick and carrot" pattern. The path begins with a period of low per-firm output for cartel members (q_m^k). The strategy calls for all cartel members to continue to produce q_m^k, unless an episode of defection occurs. If some firm cheats on the agreement, all cartel firms expand output for one period (q_m^p) (stick stage) and return to the most collusive sustainable output in the following periods, provided that every player of the cartel went along with the first phase of the strategies (carrot stage). As far as fringe firms are concerned, as the future play of the other firms is independent of how they play today, they optimally maximize per period profits. The "stick and carrot" strategies for a partial cartel can be formulated in the following way, where $q_{t,i}$ denotes the strategy played by firm i in period t:

Firm i, $i = 1, ..., k$ plays:

$$(\alpha)\begin{cases} q_{1,i} = q_m^k \\ q_{t,i} = q_m^k \text{ if } q_{t-1,j} = q_m^k \text{ for } j = 1, ..., k \ \forall t = 2, 3, ..., \\ q_{t,i} = q_m^k \text{ if } q_{t-1,j} = q_m^p \text{ for } j = 1, ..., k \ \forall t = 2, 3, ..., \\ q_{t,i} = q_m^p \text{ otherwise.} \end{cases}$$

[11] The latter were firstly set in a seminal paper from Abreu (1986). These strategies became popular in the literature given their optimality and their renegotiation-proofness quality.

Firm i, $i = k+1, ..., n$ plays:

$$(\beta)\begin{cases} q_{1,i} = q^k_{nm} \\ q_{t,i} = q^k_{nm} \text{ if } q_{t-1,j} = q^k_m, \text{ for } j = 1, ..., k \quad \forall t = 2, 3, ..., \\ q_{t,i} = q^k_{nm} \text{ if } q_{t-1,j} = q^p_m, \text{ for } j = 1, ..., k \quad \forall t = 2, 3, ..., \\ q_{t,i} = q^p_{nm} \text{ otherwise.} \end{cases}$$

Following Abreu (1986), the strategies described above are considered optimal, that is, they sustain the highest range of collusive outcomes among all possible punishment phases, if continuation profits after unilateral deviation in any period, equal the Minimax value of firms. In our model it is equal to 0, as firms can always decide not to be active and get 0 profits. Therefore, (α, β) are optimal when this condition holds:

$$\pi^s_m(q^p_m) + \frac{\delta}{1-\delta}\pi^k_m = 0 \tag{7}$$

π^k_m has been defined in section two like profits obtained by cartel firms when cartel and fringe firms produce (1) and (2) respectively, and $\pi^s_m(q^p_m)$ are cartel firms profits if firms are in a punishment phase (stick stage):

$\pi^s_m(q^p_m) = (1 - kq^p_m - (n-k)q^p_{nm})q^p_m - \frac{(q^p_m)^2}{2}$

We need the conditions for the strategies (α, β) to conform a S.P.N.E. On one hand, we need that if firms are in a collusive phase (carrot stage), profits that a firm would obtain if deviates from collusion should be no bigger (given that the rest of the firms adhere to the strategies described) than the profits obtained colluding. That is given by the next condition.

$$\pi^k_d - \pi^k_m \leq \delta(\pi^k_m - \pi^s_m(q^p_m)) \text{ no deviation in the carrot stage} \tag{8}$$

π^k_d has already been defined in section two like the profits that a cartel firm obtains when unilaterally deviates from the collusive agreement.

On the other hand, if we are in a punishment phase (stick stage), we need that firms obtain higher profits in the punishment phase than deviating from it. Therefore, firms do not unilaterally deviate in the stick stage if the following condition holds:

$$\pi_d^s(q_m^p) - \pi_m^s(q_m^p) \leq \delta(\pi_m^k - \pi_m^s(q_m^p)) \text{ no deviation in the stick stage} \tag{9}$$

where we define the profits that a cartel firm obtains by unilaterally deviating in the stick stage like:

$$\pi_d^s(q_m^p) = \max_{q_i} \, (1 - (k-1)q_m^p - q_i - (n-k)q_{nm}^p)q_i - \tfrac{(q_i)^2}{2}.$$

We have to see how to get q_m^p, and q_{nm}^p, such that if the discount factor is high enough, collusion will be sustained with the strategies (α, β), conforming at the same time, an optimal punishment.

Regarding q_m^p, it must be such that (7) holds. From (9) and (7) we obtain that no deviation in the stick stage is only possible if $\pi_d^s(q_m^p) \leq 0$, since otherwise a firm can deviate in the first period and keep doing so every time the punishment is reimposed. Hence, the total output produced by $(k-1)$ firms must be large enough that[12] $P((k-1)q_m^p) \leq 0$. We have that:

$$P((k-1)q_m^p) \leq 0 \Longleftrightarrow q_m^p \geq x, \tag{10}$$

which sets a lower bound on the quantity produced in the stick stage. This also implies that $q_{nm}^p = 0$. To obtain the lower bound of the discount factor such that (9) and (7) hold, that we will call δ_a, we compute (7) for the lowest value of q_m^p that satisfies (10):

$$\pi_m^s(x) + \frac{\delta_a}{1-\delta_a}\pi_m^k = 0$$

what leads us to:

$$\delta_a = \frac{\pi_m^s(x)}{\pi_m^s(x) - \pi_m^k} \tag{11}$$

[12]The price $P(q)$ is interpreted as price net of marginal cost at zero.

When $\delta > \delta_a$, you need a harsher punishment such that (7) is satisfied.

As far as deviation in the carrot stage is concerned, we have that from (8) and (7), we obtain that firms do not deviate if:

$$\frac{1}{1-\delta}\pi_m^k \geq \pi_d^k \tag{12}$$

This gives us the lower bound of the discount factor such that (8) and (7) are satisfied:

$$\delta \geq \frac{\pi_d^k - \pi_m^k}{\pi_d^k} = \delta_b$$

Finally, we have that (7), (8) and (9) are satisfied if the following condition on the discount factor holds:

$$\delta \geq \max\{\delta_a, \delta_b\} = \delta_k \tag{13}$$

On the one hand, δ_a is decreasing in k. When the number of firms in the cartel increases, it is possible to dissuade unilateral deviations without the need of expanding so much total output. Then profits in the stick stage ($\pi_m^s(x)$) are increasing in k what given (11) implies the result.

On the other hand, δ_b increases in k, because $\delta_b = 1 - \dfrac{\pi_m^k}{\pi_d^k}$ and deviation profits increase in k more than cartel profits. This make δ_b increase in k.

Analyzing the behavior of δ_k in (13), we obtain the following result:

Proposition 4 *: The cutoff discount factor that sustain the strategies (α, β) as a S.P.N.E. and define an optimal punishment, is strictly decreasing in the size of the cartel (k), if $k \leq \min\{n, f(n)\}$.*

where $f(n) = \frac{13+3n+\sqrt{(9n^2+138n+249)}}{10}$ and strictly increasing otherwise.

If k is small compared to n, the decreasing effect over δ_a dominates the increasing effect over δ_b(see fig. 1 for a graphic representation). The result is no so tight as in

Proposition 1, because the cutoff is always decreasing only if $n \leq 8$ $(n < f(n))$. However, the fact that is decreasing for low enough values of k (observe that $f(n) > \frac{n}{2}$) will allow us to obtain similar results as far as the participation game is concerned.

We proceed to solve the participation game, as we did in section three. Firms show their willingness to participate in a collusive agreement in a stage previous to play the "stick and carrot" strategies. The payoffs are given by (3) and (4) where now δ_k is the one in (13). For the result, we need to define $\overline{\delta} = \min\limits_{k} \ \delta_k$

Proposition 5 *: If $\delta < \overline{\delta}$ or $\delta \geq \delta_2$, no cartel is active in equilibrium. Otherwise, a cartel of k firms is active in equilibrium if $\delta_{k-1} > \delta \geq \delta_k$.*

This result is analogous to the result in Proposition 3. For $\delta_{k-1} > \delta \geq \delta_k$ only the smallest cartel among those which can be sustained is stable in the participation game. That is because in the smallest sustainable cartel, if firms do not remain in the cartel, it means that collusion fully collapses and they would obtain the Nash-Cournot profits which is worse for them if the cartel enforced has a size greater than two. If $\delta < \overline{\delta}$ collusion is not sustainable and if $\delta \geq \delta_2$ firms are better off with the Nash-Cournot profits than with the cartel of size two (see fig. 1).

Given that for $\delta \in [\overline{\delta}, \delta_2)$ cartels are active, similar results to the ones in Corollaries 1 and 2 can be derived from Proposition (7).

Corollary 3 *: If $\delta \in [\overline{\delta}, \delta_2)$ the size of active cartels is bigger with* implicit *collusion than with* binding *collusion.*

Corollary 4 *: When the size of the cartel is endogenously determined, if $\delta \in [\overline{\delta}, \delta_2)$ price decreases with the discount factor.*

This means that we have exactly the same result we obtained for the case of the "trigger strategies" when δ belongs to the interval $[\overline{\delta}, \delta_2)$.

Again, it is the success of the cartels what reduces the incentive to participate in them with binding collusion. Meanwhile, although firms do not dispose of any commitment power in implicit collusion, it is the threat to the collapse of collusion what provokes the existence of stable cartels.

Corollary 4 says that, if $\delta \in [\overline{\delta}, \delta_2)$, the larger the discount factor, the lower the size of the cartel that is stable. Thus, as δ increases, smaller cartels associated to lower prices are enforced.

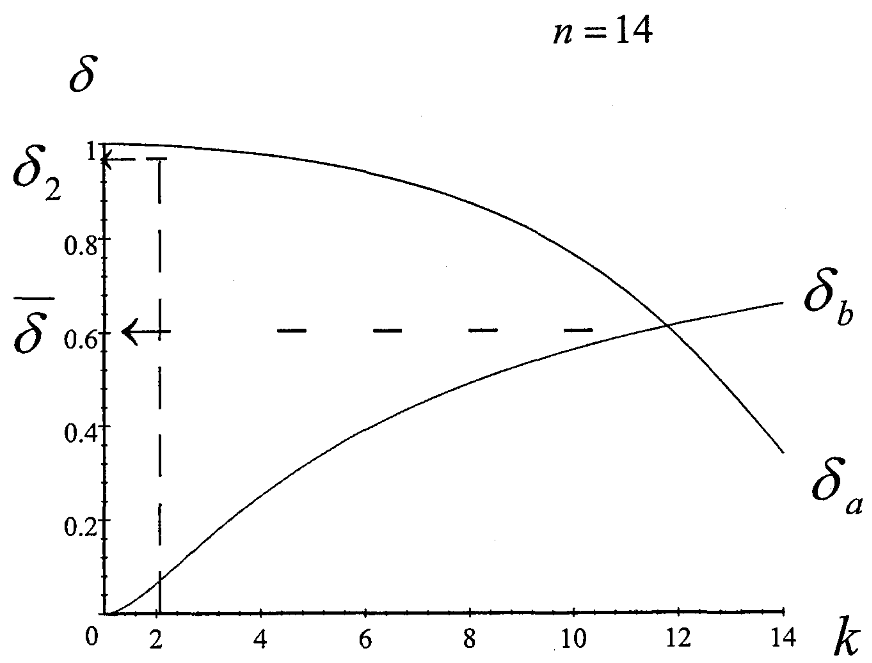

Figure1

7 Conclusions.

The main aim of the paper has been basically to analyze a model of partial collusion under two different approaches. The implicit collusion model approach with two different types of strategies to enforce collusion, showed that the larger the cartel, the easier is to sustain the cartel. When collusion is binding, that is, firms can somehow meet and sign a binding contract, it has been proved that the incentives to free ride the cartel play a central role, therefore only very small cartels can be enforced.

To be able to compare both models, a participation game has been set. In this model, an interaction between the incentive and the participation constraints, takes place. The main conclusion is that implicit collusion can enforce larger cartels than binding collusion, becoming therefore perhaps of greater concern for antitrust authorities, especially those countries, namely Holland, Denmark, Switzerland, etc., who moving to adapt its domestic competition policy to the European regime banned binding collusion.

8 Appendix.

Proof of Proposition 1: We have $\delta(k) = \frac{\pi^d - \pi^k}{\pi^d - \pi^1}$. If we calculate $\frac{\partial\delta(k)}{\partial k}$, we have that it is the following expressions in our model:

$$-24\frac{28n-60k-84nk+48k^2+24n^2k^2-12nk^3+60nk^2-39n^2k+30n^2-12k^3+2n^4+13n^3-3n^2k^3+3n^3k^2-6n^3k+8}{(9k^3-18nk^2-45k^2+5n^2k+2nk-16k+28+28n+7n^2)^2}$$

It is tedious but straightforward to show that, as long as $k \leq n$, we obtain that the derivative is negative. ■

Proof of Proposition 2: The conditions for stability are the following:

Internal stability:

$$2\frac{2k+1}{(nk-k^2+3k+2+n)^2} \geq \frac{3}{2}\frac{k^2}{\left(n(k-1)-(k-1)^2+3k-1+n\right)^2}$$

External stability:

$$(k+1)\frac{\frac{3}{2}k+\frac{3}{2}}{(nk-k^2+3k+2+n)^2} \geq 2\frac{2k+3}{\left(n(k+1)-(k+1)^2+3k+5+n\right)^2}$$

We can show that the expression of Internal stability is decreasing in k. Therefore showing that the condition does not hold at $k = 3$ also proves that coalitions of $k \geq 3$ are not stable. When $k = n = 2$, cooperation is sustainable. For $k = 2$, we can see in the internal stability that if $n \geq 3$ there are incentives to leave the cartel. ■

Proof of Proposition 3: If $\delta < \delta_n$, no collusive agreement is sustainable, therefore looking at (3) and (4) we can see that firms profits are the Nash equilibrium profits and no cartel can be active in equilibrium.

If $\delta \in [\delta_n, \delta_k]$ cartels of size k', for all $k' \in [k, n]$ are sustainable. Using Proposition 2 we can see that no cartel is stable in the explicit game. However, looking again at (3) and (4), we can see that $\Pi_m^{k'} \geq \Pi_{nm}^{k'-1}$ is hold if $k' = k$ with $k' \geq 3$. This is true as the profits of a cartel of size equal or bigger than 3 are larger than Cournot equilibrium profits. At the same time, $\Pi_m^{k'+1} \leq \Pi_{nm}^{k'}$ also holds. Therefore, for every discount factor, only the smallest cartel among those which are sustainable is stable and will be active in equilibrium. ■

Proof of Corollary 2: This is straightforward to show, only seeing that the price of the market is decreasing with k. Therefore, as the configuration enforced in the market involves smaller cartels, prices decrease. ■

Proof of Proposition 4: We obtain the cutoff δ for both stages of the punishment phase, where the envelope of both will be the significative cutoff that sustain the strategies.

It is easy to show that $\delta_a = 3\frac{\left(-nk+k^2-3k-2-n\right)^2}{16+36k+3k^2+12n+30nk+3n^2k^2-6nk^3+12nk^2+6n^2k+3n^2+3k^4-10k^3}$ and $\delta_b = \frac{k^2-2k+1}{(k+2)^2}$ are respectively strictly decreasing and strictly increasing with k. Therefore the minimum value of the decreasing δ will be at $k = n$. So we just have to calculate up to which value the decreasing part is above the increasing part.. Thus the envelope from above of both cutoffs is decreasing with k. If we construct the function $\delta_a - \delta_b$. We have that this is 0 whenever $k = \frac{13+3n+\sqrt{(9n^2+138n+249)}}{10}$. We can see that for smaller k, $\delta_a > \delta_b$ therefore the envelope is decreasing. ■

Proof of Proposition 5: If we see δ_a and δ_b we can check that for every (k, n) whenever $\delta_b = \delta_a = \overline{\delta}$, this $\in (0, 1)$. That is, the envelope $\max(\delta_b, \delta_a)$ is never increasing with k for all range of k. We know from Proposition 2 that no cartel is stable in the binding collusion model. Therefore if we look at which are the stable cartels in the implicit collusion model we see that if $\delta < \overline{\delta}$, no cartel is either stable with implicit collusion because $\overline{\delta} = \min(\max(\delta_b, \delta_a))$ and represents the minimum discount factor from which a cartel of any size can be sustainable. When $\delta > \delta_2$ all sizes of cartels are sustainable but as no cartel is stable in the participation game, if we apply to the argument of Proposition 3 this fails because Nash-Cournot profits are larger than cartel of size 2 profits. Therefore no cartel configuration is stable. Whenever $\delta \in (\overline{\delta}, \delta_2)$ if we check the stability of those which are sustainable we can apply exactly the same argument of proposition 3, and the Corollary 1, and the smallest sustainable cartel is stable. ■

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Chapter 2

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Mergers in a Partially Cartelized Market

July, 2003

The paper studies a Partial Cartel model where only a subset of firms colludes. In this model, firms' ability to collude depends on the discount factor. In addition, as hardly any attention has been given by the literature to the case where mergers take place in a collusive framework, the purpose of this paper is to analyze the competitive effects of horizontal mergers on profits and welfare in a Partially Cartelized market. We show that both mergers among fringe and cartel firms increase market price. Regarding merger profitability, the discount factor decreases cartel members' merger profitability. However, the higher cartel members' discount factor, the more fringe firms will be willing to merge. An example of this could be the intense wave of mergers among oil firms that coincided with a large period of high oil prices caused by the OPEC production cuts.

JEL Classification: L13,L40,L41

Keywords: Collusion, Partial Cartels, Trigger strategies, Mergers

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[14]I would like to thank my supervisor Ramon Faulí-Oller for his advice and encouragement, and Joel Sandonís for his helpful comments. Financial support by the I.V.I.E, *(Instituto Valenciano de Investigaciones Económicas)* and The Department of Economics of the University of Alicante is gratefully acknowledged. All errors are mine.

9 Introduction.

Faced with the multiplicity of equilibria arising in repeated games[15], most applied papers have focused on studying the equilibrium where industry profits are maximized assuming that all firms participate in the collusive agreement. However, at least to the best of our knowledge, little attention has been paid to the situation where not all the firms of the industry join the cartel (an exception is Martin (1993) to be commented later on)[16].

In practice, there are many cases where collusive agreements do not involve all firms in the industry. However given that it is difficult to determine which firms really belong to the cartel, we will refer to tested cases previously investigated by antitrust authorities.

Looking at the most recent measures of the European Commission against collusion, we see that several cartels have been fined hundreds millions euros by the European Commission for price fixing and setting sales quotas. At the same time it is also verified that in many of these industries, not all firms in the industry were fined. For example, in the carbonless paper industry, the joint market share of the fined firms was between 85 and 90%. In the North Atlantic shipping industry, the market share of the cartel was calculated around 70-80%, or in the cartonboard industry where the market share of the cartel is assumed to be around 80%. Another significant example is the citric acid industry where three North-American and five European firms were convicted in United States, Canada and the European Union and fined more than one hundred million euros for fixing prices and allocating sales in the worldwide market, issuing coordinated price announcements and monitoring one another's prices and sales volumes during the period 1991-1995[17]. The joint market share of these eight firms was only between 50% and 60%

[15] The "Folk Theorems" show that any individually rational payoff vector of a one-shot game of complete information can arise in a perfect equilibrium of the infinitely-repeated game if players are sufficiently patient (see for example Friedman (1971), Aumann, R, Shapley, L. (1976), Rubinstein, A. (1979) or Fudenberg et al. (1986)).

[16] There is a well-known literature where the industry structure is characterized by a small group of firms plus a competitive fringe. However this literature investigates cartel stability in static models where it is assumed that binding contracts could be signed. The seminal papers in this literature are Selten (1973) and d ´Aspremont et al. (1983), see also Shaffer (1995), Donsimoni (1986) or Thoron (1998) for example.

[17] U.S. Department of Justice, *Justice Department's Ongoing Probe Into Food and Feed Additives Yields Second Largest Fine Ever*, Press Release, January 29, 1997.

(see Levenstein, Suslow et al.(2002)).

Despite this empirical evidence, the Industrial Organization analysis of tacit collusion in quantity setting supergames has generally focused on the *symmetric* subgame perfect equilibrium that maximizes industry profits. However, the continuum of equilibria in supergames allows us to select equilibria such that only a subset of firms colludes. Indeed, there is a seminal paper on partial cartels (Martin 1993 p.111) from where we get the inspiration. He considers that only a subset of firms belongs to the cartel and the rest are called fringe firms. Then, he studies conditions on the discount factor such that the outcome where cartel firms behave as a Stackelberg leader and fringe firms as followers can be sustained as an equilibrium of the repeated game using "trigger" and "stick and carrot" strategies. We extend Martin (1993) model by identifying the best equilibrium for cartel firms using "trigger" strategies for each discount factor. (This means that for any value of the discount factor of cartel firms, some degree of collusion is always achieved). It coincides with the Stackelberg-follower model for high discount factors and converges to the Cournot outcome when the discount factor tends to zero. Thus, a contribution of this paper is that using the essence of the "trigger" strategies, we approach from a somewhat different and we believe more realistic perspective by asking: *for a given discount factor,* what is the output that cartel firms can produce to maximize their joint profits? Then, the equilibrium obtained is not like the standard equilibrium of the repeated game with "trigger" strategies where only if the discount factor is large enough collusion is possible, but of the type; given the restraints that firms face according their discount factor, the equilibrium of the repeated game is the output that maximizes cartel firms joint profits.

Although the framework with partial cartels we set can be used to study different subjects, we specialize the model to deal with the issue of mergers. Curiously, while there exists a substantial literature on the effect of mergers when pre-and post-merger behavior is noncooperative, little attention has been given by theorists to the consequences for firms of mergers when they take place in a collusive environment. In fact, despite its influence in matters of policy, there is almost no formal evidence from theory regarding the relationship between mergers and collusion[18]. Then, the purpose of this paper is to

[18]One contribution of interest is that of Davidson and Deneckere (1984) where they do not allow for a redistribution of output among firms after the merger, so it is assumed that each merged firm conserves their pre-merger cartel quota after the merger. This implies that horizontal mergers may make more

analyze the competitive effects of horizontal mergers on profits and welfare in a partial cartel model. In the sparse literature on the subject the effect of mergers is analyzed seeing which is the effect on the potential sustainability of the collusive agreements. This is reflected on the effect of mergers on the minimum discount factor required for collusion to be sustainable and it is commonly believed that mergers, by reducing the number of competitors, facilitate collusion. Due to the particular characteristics of our partial cartel model, the result of studying the effect of mergers on the threshold of the discount factor above which (full) collusion is sustained is ambiguous. In our case, when firms fail to sustain full collusion they can nevertheless achieve some degree of collusion. Thus, it would be more informative to study the effect of mergers on a variable that better reflects the degree of collusion achieved in the industry. It seems that price is the most indicated candidate.

Then, the first result obtained is that mergers of either cartel or fringe firms lead us to a more collusive market as long as they raise price. The second result obtained is that the capacity to achieve agreements, represented by the discount factor, crucially determines merger profitability. When mergers among cartel firms are considered, we obtain that the higher the discount factor, the lower merger profitability. The basic intuition is that, in the absence of cost savings derived from mergers, the main goal of a merger is to reduce competition. However, if firms can achieve this goal by colluding given that the discount factor is high enough, mergers are not profitable.

In contrast, as the discount factor increases, mergers among fringe firms become more profitable. In a linear oligopoly Cournot market, two firms never have an incentive to merge (see Salant et al., 1983), as non-merging firms react to the merger expanding its production. Our last result is explained by the fact that when the discount factor increases, cartel firms react expanding less their production in response to the merger of fringe firms.

Therefore, comparing our results with those in Salant et al. (1983) we obtain that merger profitability crucially depends on whether firms pre-merger behavior is cooperative or not. In our partial cartel model, merger profitability is increased with respect to the standard Cournot setting when we analyze mergers among non-colluding firms. The reverse is true if mergers among cartel firms are considered.

difficult for a cartel to maintain its integrity

We have an interesting example. Looking at what happened to the oil market in 1998-2000, we identify two different phenomena. On one hand after being elected in 1998, the Venezuelan leader Chávez worked with the OPEC president Mr. Alí Rodríguez to reinvigorate the cartel[19]. In June 1998, in its 105th meeting, the cartel agreed a cut of more than 2.5 million barrels a day. In March 1999, Mr. Rodriguez cut a deal with Saudi Arabia, the world's biggest producer, to reduce production by almost 2 million barrels a day and reverse the slide in prices. That move encouraged the rest of OPEC's 11 member nations to follow suit.

On the other hand, during this period, many oil firms merged. We have different examples: In 11/8/98, B.P. and Amoco. In 28/10/98, Japan Oil and Mitsubishi Oil. In 2/12/98, Exxon and Mobil. In 31/3/99, B.P. and Arco and in 14/9/99, TotalFina and Elf.

Therefore, more or less simultaneously, at the end of the 90's there was a wave of mergers among fringe firms together with a reinforcement of the collusive strategy of the cartel. Our model illustrates the basic economic intuition about the relationship between these two facts.

The structure of the paper is as follows. In the following section, the central model of the paper is set. In Section 3, the effect of mergers on the price is analyzed. In Section 4, merger profitability is considered. We conclude in Section 5. All proofs are relegated to the appendix.

10 Partial Cartels.

Consider n firms, which produce the same homogenous good in the same market for infinite periods. Firms discount future at common and known factor of $\delta \in [0, 1)$. Suppose they make output decisions simultaneously at the beginning of each period.

The Stage Game description is the following: We assume that the industry inverse demand is piecewise linear:

[19]"And now, OPEC has arisen again," President Chavez declared, adding that "its resurrection will rightly be celebrated in this land of the birth of OPEC" CNN, September 26, 2000.

$$p(Q) = \max(0, a - Q) \tag{14}$$

where Q is the industry output, p is the price for the output and $a > 0$. Every firm has a constant marginal cost of production d, where $a > d$.

Let (K, F) be a partition of the player set. We assume that the subset K comprises k $(\leq n)$ firms of the industry, while the remaining $(n-k)$ firms belong to the subset F. We will call hereafter firms that belong to the partition K, like cartel firms, and firms that belong to the partition F, like fringe firms.

We are going to consider "trigger strategies". The essence of these strategies is the following; firms join the cartel agreement given that all cartel members do so. In the event of deviation, the "loyal" members of the coalition revert forever to the static (Cournot) noncooperative equilibrium. This forces the deviant to do the same. The threat of such retaliation is a deterrent to deviation if the gain from cheating is no greater than the (discounted) per-period losses, which arise from the punishment.

Assume q and q^f as the output produced by cartel and fringe firms respectively. Let $\sigma_{i,t}$ denote the action of player $i \in \{1, ..., n\}$, at moment t, $t = 1, 2, ..., \infty$. The "trigger strategies" can be described in the following way. When the agreement is to produce q, the "trigger strategies" for cartel firms, $i \in K$, is given by:

$$\sigma_{i,1} = q \tag{15}$$

$$\sigma_{i,t} = \begin{cases} q \text{ if } \sigma_{j,h} = q \text{ for any } h < t \text{ for } j \in K,\ t \geq 2. \\ q_n \quad \text{Otherwise.} \end{cases} \tag{16}$$

For fringe firms, $i \in F$, we have:

$$\sigma_{i,1} = q^f \tag{17}$$

$$\sigma_{i,t} = \begin{cases} q^f \text{ if } \sigma_{j,h} = q \text{ for any } h < t \text{ for } j \in K,\ t \geq 2. \\ q_n \quad \text{Otherwise.} \end{cases} \tag{18}$$

We now look for the conditions on q and q^f that make these strategies conform a Subgame Perfect Nash Equilibrium of the repeated game. As far as fringe firms are concerned, we have that as the future play of their opponents is independent of how they play today, their optimal response is to maximize their current period's payoff.

$$q^f = \max\{0, (\frac{a-d-kq}{n-k+1})\} \tag{19}$$

It is the Cournot equilibrium among fringe firms when the output of each cartel firm is given by q.

We proceed to look for the value of q such that, given (19), the strategies (15) and (17) are a Subgame Perfect Nash Equilibrium.

We should define first the profit functions for cartel firms:

Cartel firms profits when they play q and fringe firms play q^f are:
$\Pi^c(q, q^f) = (a - d - kq - (n-k)q^f)q$

Deviation profits that a cartel firm obtains when it unilaterally deviates are:
$\Pi^d(q, q^f, q_i^d) = (a - d - (k-1)q - q_i - (n-k)q^f)q_i^d$
Where the quantity produced by the cheating cartel firm (q_i^d) is defined as follows:
$q_i^d = \arg\max_{q_i} \Pi^d(q, q^f, q_i)$

A trigger strategy supports noncooperative collusion if the present-discounted value of the income stream from adhering to the cartel is at least as great as the present-discounted value of the income stream from defection, which is deviating profits of one period, plus the profits of the punishment path, that is (discounted) static Nash equilibrium profits. For the case at hand, the condition for the cartel firms to be playing a Subgame Perfect Nash Equilibrium is the following:

$$\frac{\Pi^c(q, q^f)}{1-\delta} \geq \Pi^d(q, q^f, q_i^d) + \frac{\delta\, \Pi^c(q^n, q^n)}{1-\delta} \tag{20}$$

However, we have multiplicity of equilibria as for each δ, (20) holds for several cartel firms' production (q). To solve the multiplicity of equilibria we consider the allocation, which is the best for the cartel i.e. maximizes cartel firms' profits[20]. Therefore, it is the solution of the following program:

[20]This is also the widely extended way to solve the multiplicity of equilibria in infinitely repeated games with collusion at the industry level.

$$\max_{q} \ \Pi^c(q, q^f) \tag{21}$$

st.

$$\frac{\Pi^c(q, q^f)}{1-\delta} \geq \Pi^d(q, q^f, q_i^d) + \frac{\delta \ \Pi^c(q^n, q^n)}{1-\delta} \tag{22}$$

Solving (21) we denote the unique maximizer of $\Pi^c(q, q^f)$ by $\overline{q}$, where $\overline{q} = \frac{a-d}{2k}$. If $\delta \geq \overline{\delta}$, restriction (22) evaluated at $\overline{q}$ is satisfied and, in those cases, $\overline{q}$ is the solution to the whole program, where $\overline{\delta} = \dfrac{(n+1)^2}{(n+1)^2 + 4k(n+1-k)}$. For $\delta < \overline{\delta}$ the solution to the program is given by the output that satisfies (22) with equality.

Thus, for cartel firms, the solution to the program is the following:

$$q^c = \begin{cases} \dfrac{(a-d)((1+n)^2 + (1-2k+n)(3-2k+3n)\delta)}{(1+n)^3 - (1+n)(1-2k+n)^2\delta} & \text{if } \delta < \overline{\delta} \\ \\ \dfrac{a-d}{2k} & \text{if } \delta \geq \overline{\delta} \end{cases}$$

The equilibrium strategies we are going to consider are (15) and (17) when:

$$q = q^c$$

$$q^f = (\frac{a-d-kq^c}{n-k+1})$$

and thus,

$$q^f = \begin{cases} \dfrac{(a-d)(1-n(2+n)(-1+\delta) - \delta + 4k^2\delta)}{(1+n)^3 - (1+n)(1-2k+n)^2\delta} & \text{if } \delta < \overline{\delta} \\ \\ \dfrac{a-d}{2(n-k+1)} & \text{if } \delta \geq \overline{\delta} \end{cases}$$

Observe that as in (21) we chose the best equilibrium for the cartel, $k\overline{q}$ is the output of a unique Stackelberg leader. This is the production that cartel members want to achieve, because then their profits are maximized. It turns out that $k\overline{q} = \dfrac{a-d}{2}$ i.e. it does not depend on the number of firms in the fringe and it amounts to the monopoly output. Then, the best for cartel firms is to sustain the monopoly output in the repeated game. It is only possible when $\delta \geq \overline{\delta}$. In this case full collusion is sustained. Otherwise, full collusion can not be obtained because firms discount the future too much. The question is how the cartel quota should be adjusted in those cases in order to achieve the maximal level of collusion. It turns out that it crucially depends on the number of firms in the cartel.

When most firms belong to the cartel ($k > \frac{n+1}{2}$), quotas should be adjusted upwards in order to reduce the incentives to deviate from the cartel agreement. The reason is that output from all firms except the deviator move in the same direction as the quotas. Then the residual demand left to the deviator is lower, and therefore it gains less by deviating. This adjustment should be greater the lower the discount factor, reaching the standard Cournot output when $\delta = 0$.

However, when most firms belong to the fringe ($k < \frac{n+1}{2}$), quotas must be adjusted downwards in order to reduce the incentives to deviate. The reason is that the output from all firms except the deviator move in the opposite direction than the quotas because fringe firms increase their output when the quota is reduced. And this effect now dominates because fringe firms are the majority. This adjustment should be greater the lower the discount factor, reaching again the standard Cournot output when $\delta = 0$.

Figures 1 and 2 represent the value of the quotas and the output of the fringe firms as a function of the discount factor for the two cases discussed above.

The evolution of individual outputs has a direct consequence on the evolution of price. It decreases with the discount factor when $k < \frac{n+1}{2}$ and it increases when $k > \frac{n+1}{2}$. As δ increases, cartel firms are closer to achieve their objective. The question is that it differs depending on whether they are a majority or a minority. In the former case, the objective is to reduce output in order to increase price given that they control most of the market. In the latter case, their objective is to increase market share although this leads to a price cut. In any case, profits of the cartel firms are increasing with the discount factor because as δ increases, cartel firms are closer to the joint profit maximizing outcome. However, fringe firms' profits are only increasing in δ whenever $k > \frac{n+1}{2}$. In this case, as δ increases, fringe firms expand their output in order to take advantage of the high price motivated by the output reduction agreed by cartel members. This leads fringe firms' profits to increase with δ.

An interesting comparison that could clarify how this partially cartelized market works would be to compare cartel and fringe firms profits. Thus we define the following profit functions, which are cartel and fringe firms' profits respectively when the structure of the industry is formed by n firms with k of them in the cartel:

$$\Pi^c_{n,k}(q^c, q^f) = (a - kq^c - (n-k)q^f)q^c - dq^c \tag{23}$$

$$\Pi^f_{n,k}(q^c, q^f) = (a - kq^c - (n-k)q^f)q^f - dq^f \tag{24}$$

Comparing (23) and (24), we have that fringe firms obtain larger profits than cartel firms ($\Pi^f_{n,k}(q^c, q^f) > \Pi^c_{n,k}(q^c, q^f)$) whenever they are in a minority position ($k > \frac{n+1}{2}$). The reason can be obtained in Figure 1: fringe firms produce then more than cartel firms ($q^f > q^c$). If instead, fringe firms are in a majority position ($k < \frac{n+1}{2}$), fringe firms' profits are not only lower than cartel firms' profits but also lower than the standard Nash-Cournot equilibrium profits ($\Pi^f_{n,k}(q^c, q^f) < \frac{(a-d)^2}{(1+n)^2}$).

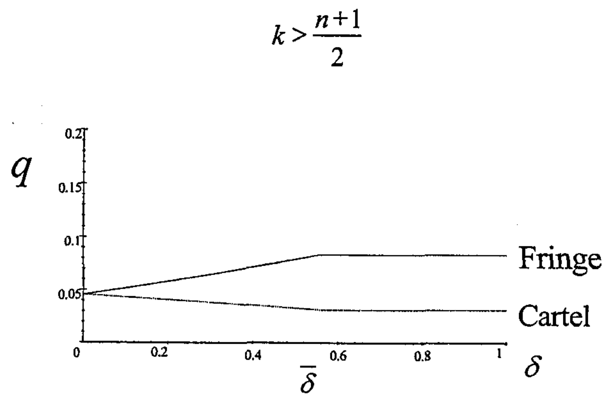


Figure 1

$$k < \frac{n+1}{2}$$

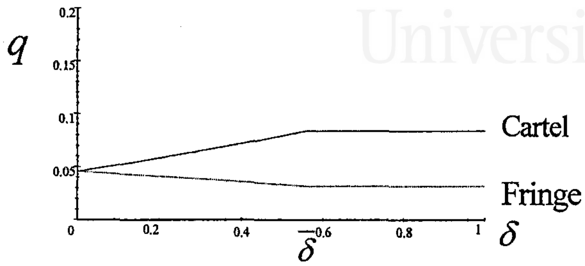


Figure 2

We can see the output of fringe and cartel firms as a function of δ for a given industry size n and partition K.

11 The impact of mergers on collusion.

This Section is motivated by the fact that little attention has been given by theorists to the consequences of mergers when postmerger behavior is collusive. In the sparse literature on the subject the effect of mergers is analyzed seeing which is the effect on the potential sustainability of the collusive agreements. This is reflected on the effect of mergers on the minimum discount factor required for collusion to be sustainable[21].

Therefore, once we understand how this partially cartelized market works, we proceed to study how mergers among firms change the picture. There are two different types of firms in the model, the ones that belong to the cartel and the ones that belong to the fringe. We only consider mergers among firms of the same type and the merged firm has the same type as the merging partners[22]. Furthermore, given the assumptions on costs, the merger does not bring any cost efficiency and then the merged entity is like any other independent firm of the same type. Therefore, starting from a situation with n firms and k cartel members, the merger of $m+1$ cartel firms leads to a market with $n-m$ independent firms and $k-m$ cartel members. Similarly, the merger of $m+1$ fringe firms turns the market into a situation with $n-k$ firms and k cartel members.

To focus the discussion I recall the result belonging to this literature most closely related to our setting. In the repeated symmetric Cournot game with linear demand and costs (Vives (1999)), the monopoly outcome can be sustained if $\delta \geq \frac{(n+1)^2}{(n+1)^2+4n} = \delta(n)$, where n is the number of firms. Then as $\frac{\partial \delta(n)}{\partial n} > 0$, we have that it is harder to collude with

[21] Compte, Jenny and Rey (2002) study collusion in a setting with firms with asymmetric capacities. They show that the main problem for collusion is to prevent the largest firm from deviating. Then, the effect of mergers is ambiguous whenever it involves the largest firm. On the one hand, it reduces the number of competitors what tends to hurt collusion. But, on the other hand, it increases the size of the largest firm, what increases its incentives to deviate. In their case, restricting attention to the sustainability of (full) collusion is validated by the fact that (full) collusion is sustainable whenever some collusion is sustainable.

[22] Mergers between cartel and fringe firms are not considered. If we did so, it seems reasonable to follow somehow Huck et al. (2001) reasoning: "If a leader merges with a follower in a market with quantity competition the new firm will stay a leader because the old firm can still use the old commitment technology of the former leader to commit itself on high output". However, in the present model of tacit collusion, this is not obvious as for example, if $k > \frac{n+1}{2}$, fringe firms profits are larger than cartel firms profits. Hence, it is not quite clear which should be the status of the new entity.

more firms. Then, mergers by reducing the number of competitors facilitate collusion[23].

My model differs from those models in two accounts. On the one hand, I do not study collusion at the industry level but the ability of a subset of firms to reach a collusive agreement (although the first case is obtained in the limit case when $k = n$). On the other hand, for any δ some degree of collusion is always achieved (Cournot competition is only obtained when $\delta = 0$).

When $\delta \geq \overline{\delta}$ cartel firms can sustain full collusion i.e. their best preferred market outcome. But for $\delta < \overline{\delta}$ profits of cartel firms are always greater than the ones obtained under Cournot competition as cartel firms achieve the maximum degree of collusion they are able given their discount factor.

Following the logic of the aforementioned models we could study the effect of mergers on $\overline{\delta}$. Surprisingly, the result is ambiguous. Mergers can make either easier or more difficult for cartel firms to sustain full collusion.

On the one hand, as

$$\overline{\delta} = \overline{\delta}(n,k) = \frac{(n+1)^2}{(n+1)^2 + 4k(n+1-k)}$$

the merger of $m+1$ cartel firms helps (full) collusion if

$$m < (n+1)\frac{2k-n-1}{k} \tag{25}$$

because then $\overline{\delta}(n-m, k-m) < \overline{\delta}(n,k)$. It hinders (full) collusion if the inequality in (25) is reversed. On the other hand, the merger of $m+1$ fringe firms helps (full) collusion if:

$$m < (n+1)\frac{n+1-2k}{n-k+1} \tag{26}$$

because then $\overline{\delta}(n-m, k) < \overline{\delta}(n,k)$. It hinders (full) collusion if the inequality in (26) is reversed.

These disappointing results may not be so important in the present case where firms, when they fail to sustain full collusion, can nevertheless achieve some degree of collusion. In our case, it would be more informative to study the effect of mergers on a variable that

[23] In the same setting, Davidson and Deneckere (1984) obtain the opposite result by assuming that firms are not allowed to redistribute output after a merger, which means that whenever the cartel is active, merged firms produce their pre-merger cartel quota. If $m+1$ firms merge, then full collusion is obtained if $\delta \geq \delta(n,m) = \frac{(n-1)^2(n-m+2)^2}{(n-1)^2(n-m+2)^2+4n((n-m+2)^2-4n)}$, where $\frac{\partial\delta(n,m)}{\partial m} > 0$.

better reflects the degree of collusion achieved in the industry. It seems that price is the most indicated candidate for that purpose. And then, a robust conclusion is obtained: mergers always increase price. This result is developed in the following Propositions that deal respectively with the case of mergers of cartel firms and mergers of fringe firms. This result seems to invalidate the approach taken so far to focus on the ability of firms to achieve (full) collusion.

Proposition 6 *Any merger of cartel members does not decrease price.*

The increase is always strict except for high values of the discount factor such that (full) collusion is sustained both pre-merger and post-merger. If $m+1$ cartel firms merge, price does not change with the merger if

$$\delta \geq \max\{\overline{\delta}(n-m,k-m),\overline{\delta}(n,k)\}$$

Then both pre-merger and post-merger full collusion is sustained what means that joint output of the firms in the cartel amounts to $\frac{a-d}{2}$. Then, fringe firms do not change their output and price is unaffected by the merger.

Observe that when $k=n$ the above Proposition encompasses the case previously analyzed in the literature where all firms participate in the collusive agreement. My model is richer though because, for any δ, collusion is exploited at its maximum level. Nevertheless this model confirms the results obtained in previous models that mergers help to sustain higher prices.

The most intriguing part of Proposition 1 is to understand why even when full collusion is more difficult to sustain after the merger, price increases. We are considering a situation where

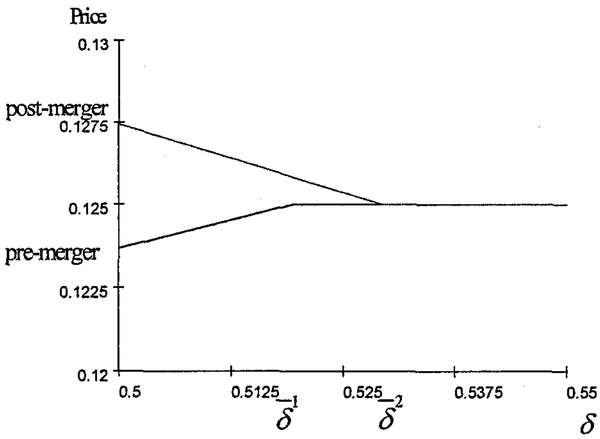
$$\overline{\delta}(n,k) < \overline{\delta}(n-m,k-m) \tag{27}$$

and then for $\delta \in (\overline{\delta}(n,k),\overline{\delta}(n-m,k-m))$ (full) collusion was sustainable before merger but it is not possible after the merger. (38) holds if and only if

$$m > (n+1)\frac{2k-n-1}{k} \tag{28}$$

The key point to solve the puzzle is to realize that (28) implies $k-m < \frac{n-m+1}{2}$ i.e. cartel firms are in a minority position after the merger. Then Figure 2 shows that when full collusion is not sustainable cartel firms adjust their quotas downwards what drives the price upwards. So the puzzle created because we may have simultaneously that full collusion becomes more difficult with the merger and price increases is explained by the anomalous feature of this model that price can decrease with the discount factor when cartels find themselves in a minority position.

The following picture illustrates the issue. It depicts the evolution of price pre-merger and post-merger for the specific case that $n = 10$, $k = 7$ and $m = 5$, with $a = 1$ and $d = 0$. Observe that in this case (28) holds.



Regarding mergers among fringe firms, the following proposition establishes its impact on price

Proposition 7 *Any merger of fringe firms strictly increases price.*

In this case, price strictly increases even when the discount factor is such that (full) collusion is sustained both pre-merger and post-merger. We have just seen that, in this case, the joint output of the cartel firms does not change with the merger. However, as the merger reduces the number of fringe firms their production is lower after the merger what explains that price strictly increases.

12 The Effect of Collusion on Mergers.

In the last Section, we studied the effect of mergers on price. In the present Section, we focus on the private incentives to merge. It is well-known that the price increase by itself does not guarantee that a merger is profitable. For example, in a Cournot setting (see the seminal paper Salant et al. (1983)), although mergers increase price, they are (generally) not profitable, because non merging firms react to the merger expanding their production[24]. To understand this, recall that the output sold in equilibrium in the standard linear Cournot model is given by: $q_i = \frac{a-c}{n+1}$. When n decreases (because of a merger), q_i increases, harming the newly merged firm, reducing incentives to merge.

If as in Salant et al. (1983) we refer to the subset of firms that do not participate in the proposed mergers as "outsiders", we have that the present model (that encompasses the Cournot case when $\delta = 0$) shares the same characteristic that outsiders increase their market share and therefore merger profitability can not be taken for granted. In our model, the outputs of cartel and fringe firms when $\delta < \overline{\delta}(n,k)$ are given respectively by:

$$q^c(n,k,\delta) = \frac{(a-d)((1+n)^2 + (1-2k+n)(3-2k+3n)\delta)}{(1+n)^3 - (1+n)(1-2k+n)^2\delta}$$

$$q^f(n,k,\delta) = \frac{(a-d)(1-n(2+n)(-1+\delta) - \delta + 4k^2\delta)}{(1+n)^3 - (1+n)(1-2k+n)^2\delta}.$$

Both the merger of cartel firms and the merger of fringe firms increase the individual output of nonmerging firms. For the merger of cartel firms the result follows from:

$$\frac{\partial q^c(n-m,k-m,\delta)}{\partial m} > 0, \text{ and } \frac{\partial q^f(n-m,k-m,\delta)}{\partial m} > 0$$

and for the mergers of fringe firms from:

[24] Salant et al. (1983) shows that at least 80% of the firms must merge in order to make the merger profitable. See also Faulí-Oller (1997) or Hennessy (2000) for an extension to a wider range of demand functions.

$$\frac{\partial q^c(n-m,k,\delta)}{\partial m} > 0, \text{ and } \frac{\partial q^f(n-m,k,\delta)}{\partial m} > 0.$$

The greater the reaction of outsiders to a merger the lower its profitability. We check below that it crucially depends on the discount factor and therefore the discount factor will be a key determinant of merger profitability.

With these preliminaries at hand we are going to present the results on how merger profitability depends on the discount factor. First of all, I am going to define what I understand by merger profitability. Using the profit functions defined in (23 and 24), the profitability of a merger of $m+1$ cartel firms is given by:

$$\Pi^c_{n-m,k-m}(q^c,q^f) - (m+1)\ \Pi^c_{n,k}(q^c,q^f) \tag{29}$$

and the profitability of a merger of $m+1$ fringe firms by:

$$\Pi^f_{n-m,k}(q^c,q^f) - (m+1)\ \Pi^f_{n,k}(q^c,q^f) \tag{30}$$

They are simply the difference between the profits obtained by merging firms after and before the merger. The following Propositions represent our main results. Proofs can be found in the appendix. We restrict attention to the case where full collusion is not sustained either before or after the merger.

Proposition 8 *Merger profitability among cartel firms decreases with the discount factor.*

Proposition 3 shows that the following condition holds:

$$\frac{\partial(\Pi^c_{n-m,k-m}(q^c,q^f) - (m+1)\Pi^c_{n,k}(q^c,q^f))}{\partial\delta} < 0. \tag{31}$$

The intuition for the result is as follows. In this model, firm's ability to collude depends on the discount factor: the greater the discount factor the greater the scope for collusion. When mergers do not involve any cost saving, firms merge basically to restrict competition. However, when competition is already low because firms are sustaining collusive agreements, mergers lose attractiveness as an anticompetitive device. Thus, the more firms can collude, the less they are interested in merging. Last result shows that, in our model, the more firms collude, mergers among colluding firms become less profitable than in the Salant et al. (1983) model where firms do not collude.

We turn now our attention to the private incentives of fringe firms to merge. The following Proposition makes clear that they are closely related with the value of the discount factor that parametrizes the ability of cartel firms to reach collusive agreements. The key point of Proposition 4 is that it establishes a connection between the level of collusion achieved by cartel firms and the profitability of mergers among firms not belonging to the cartel. This connection will be used in the next Section to present an example where we simultaneously have that cartel firms reduce their production and that fringe firms find profitable to merge. This example will be related with the evolution of the oil market at the end of 90's that is characterized by the same two features, namely, the success of the cartel and a wave of mergers of firms not belonging to the cartel.

Proposition 9 *The profitability of mergers among fringe firms increases with the discount factor.*

Proposition 4 shows that the following condition holds:

$$\frac{\partial(\Pi^f_{n-m,k}(q^c,q^f)-(m+1)\Pi^f_{n,k}(q^c,q^f))}{\partial\delta}>0. \tag{32}$$

The key point to understand the result is that as δ increases, cartel firms increase less their output after a merger of fringe firms. In algebraic terms, this means that:

$$\frac{\partial^2 q^c(n,k,\delta)}{\partial n\partial\delta}>0. \tag{33}$$

In this case, the negative effect on the profitability of mergers because outsiders increase their production is attenuated as an increase in the discount factor reduces the reaction of firms belonging to the cartel. Therefore, comparing our last result to those obtained by Salant et al. (1983), we obtain that as δ increases, mergers among not colluding firms become more profitable than in a market where firms do not collude.

12.1 Example.

We present below a numerical example that illustrates the implications of the last Proposition. We show that given a market structure the profitability of the merger of fringe firms is positive for high values of the discount factors whereas it is negative for low values of the discount factor. We show that the reason of this result is that when δ is high cartel firms increase their output after a merger much less than when δ is low.

The exact specification of the example is the following. Assume market demand is given by $P = 1 - Q$ and the marginal cost of firms is $d = \frac{1}{2}$. We have $n = 6$ firms and $k = 4$ firms belong to the cartel. We are going to consider the effect of the merger of fringe firms for the case $\delta = 0.1$ and $\delta = 0.5$.

If $\delta = 0.1$ we have that without the merger each cartel firm produces $q^c = 0.07$ and with the merger $q^c = 0.08$. The profitability of the merger is given by $\Pi^f_{6-1,4} - 2\Pi^f_{6,4} = -2.6338 \times 10^{-3} < 0$.

If $\delta = 0.5$ we have that without the merger each cartel firm produces $q^c = 0.0625$ and with the merger $q^c = 0.063$. The profitability of the merger is given by $\Pi^f_{6-1,4} - 2\Pi^f_{6,4} = 1.1703 \times 10^{-3} > 0$ (Results are summarized on the table).

$$\delta = 0.1 \Longrightarrow \begin{cases} q^c = 0.07 & \text{Without a merger.} \\ q^c = 0.08 & \text{With a merger.} \\ \Pi^f_{6-1,4} - 2\Pi^f_{6,4} = -2.6338 \times 10^{-3} < 0 \end{cases} \tag{34}$$

$$\delta = 0.5 \Longrightarrow \begin{cases} q^c = 0.0625 & \text{Without a merger.} \\ q^c = 0.063 & \text{With a merger.} \\ \Pi^f_{6-1,4} - 2\Pi^f_{6,4} = 1.1703 \times 10^{-3} > 0 \end{cases}$$

Table 1. Numerical Example.

The merger is only profitable when δ is high. Cartel firms increase their output with the merger by 0.01 if $\delta = 0.1$ and by 0.0005 if $\delta = 0.5$. This explains the different results on profitability.

The example shows that if for some reason unexpectedly the discount factor jumped from $\delta = 0.1$ to $\delta = 0.5$ we would observe the following adjustment in the strategies played by firms: on the one hand, fringe firms would decide to merge and, on the other hand, cartel firms would reduce their output from 0.07 to 0.063. This situation resembles what happened in the oil market at the end of the 90's: OPEC members agreed to cut their quotas and private firms not belonging to the cartel decided to merge.

13 Conclusion.

The main aim of this paper has been to develop a theoretical foundation of the impact of collusion on merger profitability and the effect of horizontal mergers on a previously collusive market.

We show that although the critical discount factor above which joint profit maximization could be sustained may increase (due to a merger), the effect of a merger on price is unambiguous, and price increases.

There exists a traditional view in Industrial Organization, following Salant al. (1983), according to which there is little scope for merger profitability when mergers do not involve any cost saving and firms are in a Cournot environment. Our partial cartel model predicts a positive (negative) correlation between the degree of collusion among cartel firms and merger profitability of fringe (cartel) firms. This can reinforce the tendency of some groups of firms to merge in a non-competitive environment.

In a simple example, we have shown that some mergers are only profitable whenever the degree of collusion in the market is large enough.

Hence, the model predicts that in a Partial Cartel model, fringe firms would have an incentive to merge whenever the cartel is successful enough. Therefore, it should be taken into account that collusion may not only directly increase price but also indirectly by rendering profitable mergers among fringe firms. The closest real example to our model comes from the oil market. In the oil market at the end of the 90´s, more or less simultaneously, mergers among, what we considered fringe firms, and production cuts by the cartel, the OPEC, took place. That could perhaps explain the wave of mergers among oil firms at the end of the 90´s.

Several issues have been left for future research. It could be fruitful to consider different types of mergers, like in Huck et al.(2001), considering mergers among firms of different types.(i.e. mergers within fringe and cartel firms). Second, as we let cartel firms pick up the best equilibrium for them, it could be also considered other punishment phases that could lead the cartel to better outcomes than using trigger strategies like for example using an Optimal punishment, following Abreu (1986) and Abreu (1988).

14 Appendix.

Proof of Proposition 6: The market price is given by the following expression:

$$p(n,k,\delta) = \begin{cases} f(n,k,\delta) = \frac{-(1+n)^2(a+dn)+(1-2k+n)(a(1+2k+n)+d(n+n^2-2k(2+n)))\delta}{-(1+n)^3+(1+n)(1-2k+n)^2\delta} & \text{if } \delta < \overline{\delta} \\ \frac{a+d(1+2(n-k))}{2(n-k+1)} & \text{if } \delta \geq \overline{\delta} \end{cases} \tag{35}$$

Observe that

$$p(n,k,\overline{\delta}) = f(n,k,\overline{\delta})$$

Define $\overline{\delta}^1$ and $\overline{\delta}^2$as the pre-merger and post merger cutoffs respectively.

First we will prove that the following is true:

$$f(n-m,k-m,\delta) > f(n,k,\delta) \text{ if } \delta < \min\{\overline{\delta}^1, \overline{\delta}^2\} \tag{36}$$

First, we see that $f(n-m,k-m,0) > f(n,k,0)$ On the other hand, we can see that there exists only one $\delta \in (0,1)$ that solves $f(n-m,k-m,\delta) = f(n,k,\delta)$. We will call it δ^0.

We can check that if $\overline{\delta}^1 < \overline{\delta}^2$, then $\delta^0 > \overline{\delta}^1$, but if $\overline{\delta}^1 > \overline{\delta}^2$, then $\delta^0 > \overline{\delta}^2$. If $\overline{\delta}^1 = \overline{\delta}^2$, then $\delta^0 = \overline{\delta}^2 = \overline{\delta}^1$. Therefore, (36) is true.

If $\delta < \min\{\overline{\delta}^1, \overline{\delta}^2\}$, (36) ensures that a merger (strictly) increases market price.

If $\delta \geq \max\{\overline{\delta}^1, \overline{\delta}^2\}$ the market price pre and post merger is the same, as $\frac{a+d(1+2(n-k))}{2(n-k+1)} = \frac{a+d(1+2(n-m-(k-m)))}{2(n-m-(k-m)+1)}$.

For the remaining values of δ, we have two different relevant cases to consider: the first one is when $m > (n+1)\frac{2k-n-1}{k}$ and $\overline{\delta}^1 < \overline{\delta}^2$. The second case is when $m < (n+1)\frac{2k-n-1}{k}$ and $\overline{\delta}^1 > \overline{\delta}^2$.

We know that $p(n,k,\overline{\delta}^1) = p(n-m,k-m,\overline{\delta}^2) = \frac{a+d(1+2(n-k))}{2(n-k+1)}$.

In the first case, it is enough to check that $f(n-m,k-m,\delta)$ is strictly decreasing with δ. This is true if $m > 2k-n-1$, and this holds as $m > (n+1)\frac{2k-n-1}{k}$.

In the second case it is enough to check that $f(n,k,\delta)$ is strictly increasing with δ, and this is true if $2k-n-1 > 0$, and this holds as otherwise $m < (n+1)\frac{2k-n-1}{k}$ could never hold. ■

Proof of Proposition 7: Market price is given by (35). We can easily see that the following holds:

$$\frac{\partial f(n,k,\delta)}{\partial n} < 0 \tag{37}$$

Define $\overline{\delta}^1$ and $\overline{\delta}^2$like pre-merger and post-merger cutoff respectively

If $\delta < \min\{\overline{\delta}^1, \overline{\delta}^2\}$, (37) is enough to prove that price strictly increases.

If $\delta \geq \max\{\overline{\delta}^1, \overline{\delta}^2\}$, we can see that as $p(n,k,\overline{\delta}^1) = \frac{a+d(1+2(n-k))}{2(n-k+1)} < p(n-m,k,\overline{\delta}^2) = \frac{a+d(1+2(n-m-k))}{2(n-m-k+1)}$, price strictly increases.

For the remaining cases, when, as we have seen, we have two relevant cases: the first is if $m > (n+1)\frac{n+1-2k}{n-k+1}$, then we have then that $\overline{\delta}^1 < \overline{\delta}^2$. The second case is if $m < (n+1)\frac{n+1-2k}{n-k+1}$, then we have then that $\overline{\delta}^1 > \overline{\delta}^2$.

In the first case, as $p(n-m,k,\overline{\delta}^1) > p(n,k,\overline{\delta}^1)$, it is enough to check that $f(n-m,k,\delta)$ is increasing with δ, which is true if $m > n-2k+1$, and this holds as $m > (n+1)\frac{n+1-2k}{n-k+1}$.

In the second case, as $p(n-m,k,\overline{\delta}^2) > p(n,k,\overline{\delta}^2)$ (remember 37), it is enough to check that $f(n,k,\delta)$ is decreasing with δ, which is true if $n+1 > 2k$, and this holds as otherwise $m < (n+1)\frac{n+1-2k}{n-k+1}$ could never be true. ■

Proof of Proposition 8: Basically what we have to do is proving that (31) holds. If we consider the following expression:

If $\delta < \overline{\delta}$, then $\Pi^c_{n,k}(q^c,q^f) = \frac{(a-d)^2(1-n(2+n)(-1+\delta)-\delta+4k^2\delta)((1+n)^2+(1-2k+n)(3-2k+3n)\delta)}{(1+n)^6-2(1+n)^4(1-2k+n)^2\delta+(1+n)^2(1-2k+n)^4\delta^2}$. Therefore, it is tedious but straightforward to show that (31) holds. ■

Proof of Proposition 9: As in the last Proof, we just have to take the following expression:

If $\delta < \overline{\delta}$, then $\Pi^f_{n,k}(q^c,q^f) = \frac{(a-d)^2(-1+n(2+n)(-1+\delta)+\delta-4k^2\delta)^2}{(1+n)^6-2(1+n)^4(1-2k+n)^2\delta+(1+n)^2(1-2k+n)^4\delta^2}$. We can also see that it is easy but tedious to see that (32) holds. ■

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Mergers in Asymmetric Stackelberg Markets

November 18^{th}, 2002

We analyze a Stackelberg model with a group of leaders and a group of followers. We obtain that leaders reduce their output when followers merge and this reduction renders the merger profitable..

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15 Introduction.

Salant et al (1983) showed that in a symmetric linear Cournot setting, two-firm mergers are never profitable. Unprofitability comes from the fact that nonmerging firms react to the merger by increasing their output. In the present paper, although firms choose output, two-firm mergers are profitable because some of the nonmerging firms reduce their output after a merger. We analyze a model where a group of firms (leaders) choose output before another group of firms (followers). Followers are also less efficient than leaders. We obtain that leaders reduce their output when followers merge and this reduction increases as followers become less efficient. This explains that mergers become profitable when the costs of followers are high enough. The present paper has extended the analysis in Huck et al. (2001) to the case where followers are less efficient than leaders. The extension proves useful because it allows to obtain that mergers of two symmetric firms i.e. without efficiency gains maybe profitable in a setting where firms choose output. In Huck et al. (2001) the merger of symmetric firms is not profitable. Furthermore, in an asymmetric Cournot model with linear demand and constant marginal costs the merger of symmetric firms is also not profitable.

16 The model and merger profitability.

Consider a market for a homogenous product with n firms. Inverse demand is given by $P(X) = 1 - X$. Competition occurs in two stages. In the first stage, m firms (leaders) choose the output they want to sell. In the second stage, the remaining $n - m$ firms (followers), knowing the outputs chosen by the leaders in the first stage, choose their level of production. Apart from their strategic advantage, leaders are assumed to be more efficient than followers[27]. The (constant) marginal cost of production of leaders is normalized to 0, whereas the unit cost of followers is given by $d \geq 0$. When $d = 0$ we are back to the standard (symmetric) Stackelberg model. We assume that

$$d < \frac{1}{m + 1 + m(n - m)} ,$$

[27] This is assumed following the reasoning of the folk theorem that relatively large firms are committed leaders and small firms are followers. Sadanand and Sadanand (1996) obtain a formal result for sufficiently small amounts of uncertainty.

so that followers are active in equilibrium.

The output sold in equilibrium by leaders and followers is given respectively by:

$$\begin{aligned} x_l &= \frac{d(n-m)+1}{m+1} \\ x_f &= \frac{d(-m(n-m)-1-m)+1}{(m+1)(n-m+1)} \end{aligned}$$

We can observe that x_l is decreasing with the number of leaders and increasing with the number of followers. On the other hand, x_f is decreasing with the number of followers but the effect of a change in the number of leaders is ambiguous as it depends on whether leaders are majority or a minority.

Last expressions lead to the following equilibrium profits obtained respectively by leaders and followers:

$$\begin{aligned} \Pi_l(n,m) &= \frac{(d(n-m)+1)^2}{(m+1)^2(n-m+1)} \\ \Pi_f(n,m) &= \frac{(d(m+1+m(n-m))-1)^2}{(m+1)^2(n-m+1)^2} \end{aligned}$$

As in Huck et al. (2001) we consider three different types of mergers. (a) a merger of two leaders, (b) a merger of two followers and (c) the merger between a leader and a follower. In case (c) the merged entity chooses output (only) in the first stage. A merger is considered to be profitable if the profits of merging firms increase after merger. In case (a) it implies that:

$$\Pi_l(n-1,m-1) - 2\Pi_l(n,m) > 0 \tag{38}$$

For a merger of case (b) it implies that:

$$\Pi_f(n-1,m) - 2\Pi_f(n,m) > 0$$

and a merger of type (c) is profitable if the following condition holds:

$$\Pi_l(n-1,m) - \Pi_f(n,m) - \Pi_l(n,m) > 0$$

As a benchmark we recall the result for the symmetric case analyzed in Huck et al. (2001): when $m > 2$ and $n - m > 2$ only type (c) mergers are profitable. Some of the results of the symmetric case extend to the asymmetric case. In particular, mergers of two leaders are not profitable if $m > 2$ and the merger between a leader and a follower is always profitable.

Condition (38) can be rewritten as $\dfrac{\Pi_l(n-1,m-1)}{\Pi_l(n,m)} > 2$. It is the same condition that in the symmetric case because the left hand side does not depend on d and, therefore, we obtain the same result. The merger of a follower and a leader was already profitable in the symmetric case. Profitability can only increase when followers become inefficient because the merger has the additional positive effect of allowing some cost savings by transferring output from a high cost firm to a low cost firm.

These two results that are common both to the asymmetric and the symmetric case are formally stated in the following two propositions.

Proposition 10 *If $m > 2$ the merger of two leaders is not profitable.*

Proof. Merger is profitable if:

$\Pi_l(n-1,m-1) - 2\Pi_l(n,m) = (-dn+dm-1)^2 \frac{m^2-2m-1}{m^2(-n-1+m)(m+1)^2} > 0$. We can see that its sign does not depend on d if $d > 0$ It is only positive if $m = 2$. ■

Proposition 11 *A merger between a leader and a follower is always profitable.*

Proof. Let be $\Pi^m(d) = \Pi_l(n-1,m) - \Pi_f(n,m) - \Pi_l(n,m) = \frac{(d(n-m-1)+1)^2}{(m+1)^2(n-m)} - \frac{(d(n-m)+1)^2}{(m+1)^2(n-m+1)} - \frac{(d(m+1-m^2+mn)-1)^2}{(m+1)^2(n-m+1)^2}$

$\Pi^m(d)$ is a concave function (as $\frac{\partial^2\Pi^m(d)}{\partial^2 d} < 0$). We also have that $\Pi^m(0) > 0$.

On the other hand,

$$\frac{\partial \Pi^m(d)}{\partial d} \begin{array}{l} >0 \text{ if } d < \bar{d} \\ <0 \text{ if } d > \bar{d} \end{array}$$

Thus, we can see that $\exists\ d_1, d_2$ such that $\Pi^m(d_1) = \Pi^m(d_2) = 0$ where $d_1 < 0 < \frac{1}{m+1-m^2+mn} < d_2$. and $\bar{d} \in (d_1, d_2)$. Then, for all $d \in (0, \frac{1}{m+1-m^2+mn})$, $\Pi^m(d) > 0$. ■

The main contribution of this paper is that mergers of two followers are profitable if d is high enough. In a Cournot setting, mergers are unprofitable because nonparticipants expand their output after the merger. The profitability of mergers in the asymmetric Stackelberg model is explained by the fact that leaders reduce their output after the merger of two followers. In particular, the marginal reduction in the output of leaders given a marginal decrease in the number of followers is given by:

$$-\frac{\partial x_l}{\partial n} = \frac{d}{k+1}$$

It is zero for the symmetric case and negative for the asymmetric case if $d > 0$. The reduction becomes more important as followers become more inefficient. This explains that mergers are only profitable when d is high enough.

The distinguishing result with respect to the symmetric case is stated in the last Proposition.

Proposition 12 *A merger between two followers is profitable if their cost is high enough.*

Proof. We call again:
$\Pi^m(d) = \Pi_f(n-1,m) - 2\Pi_f(n,m) = \frac{(d(m+1-m^2+m(n-1))-1)^2}{(m+1)^2((n-1)-m+1)^2} - 2\frac{(d(m+1-m^2+mn)-1)^2}{(m+1)^2(n-m+1)^2}$

If $2 < n-m$, we have that as $\Pi^m(d)$ is a concave function (as $\frac{\partial^2\Pi^m(d)}{\partial^2 d} < 0$). If $1 - \sqrt[2]{2} < n-m$,we have that $\Pi^m(0) < 0$.Then if $2 < n-m$, $\exists\ d_1, d_2$ such that $\Pi^m(d_1) = \Pi^m(d_2) = 0$ where $0 < d_1 < \frac{1}{m+1-m^2+mn} < d_2$. Then $\Pi^m(d) > 0$ for all $d \in (d_1, \frac{1}{m+1-m^2+mn})$.We also have that if $d > d_2$, $\frac{\partial\Pi^m(d)}{\partial d} < 0$

If $n-m = 2$ we have the same as before except that now $d_1 < 0$ and $\Pi^m(d) > 0$ if $d \in (0, \frac{1}{m+1-m^2+mn})$. Huck et al. (2001) already identified that, in this case, mergers are profitable in the symmetric case. ■

The intuition is that as d increases, leaders take less into account the rivalry of the followers. In this case, when the number of inefficient followers is reduced, leaders use less their "strategic power" to anticipate a large output. The consequence is that leaders reduce production.

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Chapter 3

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Innovation and market concentration with asymmetric firms.[29]

June, 2003

This paper considers a theoretical model of n asymmetric firms that reduce their initial unit costs by spending on R&D activities. In accordance with Schumpeterian hypotheses we obtain that more efficient (bigger) firms spend more in R&D and this leads to a more concentrated market structure. We also find a positive relationship between innovation and market concentration. This calls for a corrective tax on R&D activities to curtail strategic incentives to over-invest in R&D trying to achieve a higher market share.

JEL Classification: L11; L52; O31

Keywords: R&D; Asymmetries; Market Concentration; Optimal Industrial Policies.

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[29]I would like to thank my advisor Ramon Faulí Oller for his advice and encouragement. Financial support by The Department of Economics of the University of Alicante is gratefully acknowledged. All errors are mine.

18 Introduction

The advantages for innovation of a firm being large were firstly pointed out by Schumpeter in his *Capitalism, Socialism and Democracy (1942)*. He argued that there were innovation "capability advantages" of large firm size stemming from economies of scale in research and development (R&D) and management, greater capabilities for risk spreading, finance, etc. In summary, large firms have a level of production, productive capacity, marketing arrangements, and finance that enables them quickly to exploit a new technology at relatively large scale. However, the argument that large firms can be more efficient in R&D has been countered by arguments like that the bureaucratic control structure of large firms may partially or even fully offset these latent advantages, or even by the fact that weak competition may reduce the spur to innovation in large firms.

Despite substantial interest in the question little direct evidence on R&D and market structure has appeared in the literature. Simultaneous influences between R&D and concentration have been suggested and tested (see for example Connolly and Hirschey (1984) or Nelson and Winter (1982)). However, the literature on innovation and market structure has never reached a definitive conclusion on the relationship between firm size and investment in R&D activities. Neither empirical observations nor theoretical models come to any clear conclusions on this subject.

In this paper, I address the important question of the relationship between innovation[30] and market structure and how initial production costs affect the incentives to innovate.

On the one hand, it is also a common feature in the real world that firms differ, and this asymmetry might refer to size, cost structure or R&D commitment. Although in the context of R&D competition, it is important to understand how the outcome is influenced by the presence of asymmetries amongst firms, most of the literature on non-tournament models of innovation focuses on symmetric or identical firms. (see for example Spence (1984) or D'Aspremont and Jacquemin (1988)). In our model, asymmetry is presented by allowing firms to differ in their initial production costs.

[30]Innovations that reduce the cost of production of an existing good are called process innovations, while those that create new goods are called product innovations. We will focus on process innovations.

On the other hand, while there are some theoretical models that have tried to capture the advantages of firms large size in R&D in a duopoly (see for example Rosen (1991), Barros and Nilssen (1999), Poyago-Theotoky (1996) or Xiangkang and Zuscovitch (1998)), to the best of our knowledge, there has been no attempt to extend the model to the more general case of the oligopoly of n firms[31].

Therefore, the aim of the paper is the following: in an asymmetric model of n firms performing cost-reducing R&D activities, we analyze the incentives of the firms to innovate depending on their initial degree of efficiency. We also check how the implementation of R&D activities affects market concentration. R&D is assumed to be undertaken before the output is produced, with firms anticipating the effect of the R&D on the resolution of their market shares. I obtain that efficient firms spend more in R&D than inefficient firms, which means that larger firms, in terms of market share, invest more than smaller firms. Furthermore, they over-invest in R&D in order to increase their market share. This leads market concentration to increase confirming the Schumpeterian positive relationship between innovation and concentration.

As it was stressed, the strategic game played by firms leads to overuse R&D in absence of government policy. As national governments in a number of countries subsidize R&D of firms, in our model, industrial policy is also discussed[32]. Two different measures are analyzed: production and R&D taxes (or subsidies). The motivation for the government policy in the paper is to tax R&D efforts to curtail the strategic incentives of firms to over-invest in R&D to achieve a higher market share. The Optimal Industrial Policy also prescribes a production subsidy to compensate possible output decreases due to the R&D tax. What is obtained is that the Optimal Industrial Policy decreases market concentration, as it is corrective to the increase in the initial production cost gap among firms provoked by the implementation of R&D activities.

I also introduce a firm-specific industrial policy, which is different R&D taxes among firms. The intuition behind it is that the support of R&D activities may be, firm-specific or even project-specific. We obtain that firms are also generally taxed to reduce their in-

[31] Belleflame (2001) is an exception. However, his model differs from mine at least in two aspects. First, in his work firms are *ex − ante* identical and second, firms can also perform R&D to differentiate the product.

[32] It has also been discussed in a few papers of international R&D competition, see for example Spencer and Brander (1983) or Miyagiwa and Ohno (1997).

vestment in R&D. However, the firm-specific policy prescribes that more quantity-efficient firms should be taxed at a lower rate, basically due to the fact that in the welfare maximization, the specific tax is used to divert production to the more efficient firms. Thus, by its nature, this firm-specific industrial policy causes an increase in market concentration.

In section 2 we present the model. In section 3 we analyze the relationship between innovation and concentration. In section 4 the optimal industrial policy is characterized. Section 5 concludes. All proofs are relegated to the appendix.

19 The Model

We consider a market whose demand is given by a linear inverse demand function,

$$P = a - Q \tag{39}$$

Where P is the price and Q is total quantity supplied in the market. There are n firms competing in quantities and selling a homogeneous good[33], with $Q = \sum_{i=1}^{n} q_i$. The unit production cost depends on the R&D activity performed by the firm in such a way that the R&D outcome reduces the constant marginal cost of producing the final good. In particular, the unit production cost of firm i is given by:

$$d_i = c_i - x_i \tag{40}$$

Where $c_i < a$ is the initial level of unit production cost of firm i, and x_i is the level of firm i R&D investment, where $i = 1, ..., n$. That is, as indicated by the subscripts, we do not restrict firms to be equal.

The R&D costs are given by γx_i^2 with $\gamma > 0$. We assume γ to be equal across firms. This is done for convenience as my interest lies in how asymmetries in costs functions, and therefore in firms' size in terms of market share, affect industrial policy and market structure. Therefore, firm i profit function is:

$$\Pi_i(q_i, d_i) = (P - d_i)q_i - \gamma(c_i - d_i)^2 \tag{41}$$

We assume in our model that R&D is strategic and it involves a two-step game. The corresponding nonstrategic model would be one in which R&D would be used only to minimize costs, and the equilibrium would be the standard cost-minimization Cournot equilibrium that would naturally arise if R&D and output were simultaneously determined.

[33]The assumption of a homogeneous good leads naturally to Cournot competition. Under Bertrand competition, no asymmetry can survive with a homogeneous good.

In our model, firms simultaneously choose R&D levels, these R&D levels are made known to each other, and then output levels are also simultaneously determined. In the first stage, firms choose R&D levels, and in the second stage, output levels. I look for the subgame perfect Nash equilibrium of this two stage game.

The quantities produced by each firm as a function of the vector $d = (d_1, ..., d_n)$, total output and price respectively in the second-stage equilibrium are:

$$q_i(d) = \frac{a - (n+1)d_i + \sum_{i=1}^{n} d_i}{n+1} \qquad i = 1, ..., n; \tag{42}$$

$$\sum_{i=1}^{n} q_i \equiv Q = \frac{na - \sum_{i=1}^{n} d_i}{n+1} \tag{43}$$

$$P = \frac{a + \sum_{i=1}^{n} d_i}{n+1} \tag{44}$$

Then, from (42) and (44) respectively we obtain the effect of changes in marginal costs in the standard linear Cournot setting, that is:

$$\frac{\partial q_i}{\partial d_i} = -\frac{n}{n+1} \qquad \frac{\partial P}{\partial d_i} = \frac{1}{n+1} \tag{45}$$

To obtain the equilibrium in the first stage we make use of a technique developed in Saracho (2002) to deal with asymmetric situations. Although, in fact, firms choose the level of R&D (x_i), for computational reasons it will be more convenient to think that they choose the level of its marginal cost in the production stage (d_i). (40) relates directly both variables. We assume also $\gamma \geq 1$, and therefore the convexity property required with respect to x_i to ensure that second-order condition of firm i's maximization problem is satisfied. Firm i looks its final unit cost of production (d_i) that maximizes its profits. The first order conditions are:

$$\frac{\partial \Pi_i(q_i, d_i)}{\partial d_i} = (P - d_i)\frac{\partial q_i}{\partial d_i} + (\frac{\partial P}{\partial d_i} - 1)q_i + 2\gamma(c_i - d_i) = 0 \qquad i = 1, ..., n \tag{46}$$

We introduce (42), (43), (44) and (45) in (46) and simplifying we obtain that the first order condition for firm i becomes:

$$\frac{\partial \Pi_i(q_i, d_i)}{\partial d_i} = -2na - 2n\sum_{i=1}^{n} d_i + (2n(n+1) - 2\gamma(n+1)^2)d_i + 2\gamma(n+1)^2 c_i = 0 \quad (47)$$

We proceed now adding all n first order conditions, leading us to the following expression:

$$-2n^2 a - 2n^2\sum_{i=1}^{n} d_i + (2n(n+1) - 2\gamma(n+1)^2)\sum_{i=1}^{n} d_i + 2\gamma(n+1)^2\sum_{i=1}^{n} c_i = 0 \qquad (48)$$

Then,

$$\sum_{i=1}^{n} d_i = \frac{-n^2 a + \gamma(n+1)^2\sum_{i=1}^{n} c_i}{\gamma(n+1)^2 - n} \qquad (49)$$

Thus, defining $c = \sum_{i=1}^{n} c_i$, if we replace (49) in (47) we obtain d_i.

$$d_i = \frac{an^2 - (a + c + c_i)\gamma n(1+n) + c_i\gamma^2(1+n)^2}{(\gamma + (-1+\gamma)n)(-n + \gamma(1+n)^2)} \qquad (50)$$

Using (49) and (50) in (42) we can obtain the level of production[34]. Taking into account that $x_i = c_i - d_i$, from (50) we also obtain the optimal level of R&D for firm i[35]. They are respectively:

[34]It is implicitly assumed that firms are not too asymmetric in terms of their initial costs in such a way that in equilibrium we have $q_i > 0 \; \forall i = 1, ..., n$. This is implied by the following condition : $c_i < \dfrac{\gamma\sum_{j\neq i}^{n} c_j(1+n) + a(\gamma + (-1+\gamma)n))}{n(-1+\gamma(1+n))} \quad \forall i = 1, ..., n$

Observe that this implies that in equilibrium all firms obtain positive profits and perform a positive amount of R&D.

[35]High levels of R&D characterize this equilibrium for the output levels chosen. That is, the strategic behavior may induce firms to use more R&D than required to minimize the cost of the output produced (see Brander and Spencer (1983)).

$$q_i = \frac{\gamma(1+n)(c\gamma(1+n) + a(\gamma + (-1+\gamma)n) - c_i(-n+\gamma(1+n)^2))}{(\gamma + (-1+\gamma)n)(-n+\gamma(1+n)^2)} \qquad i = 1, ..., n, \quad (51)$$

$$x_i = \frac{n(nc_i - \gamma((1+n)^2 c_i - c) + a(\gamma + (-1+\gamma)n))}{(\gamma + (-1+\gamma)n)(-n+\gamma(1+n)^2)} \qquad i = 1, ..., n, \qquad (52)$$

Whereas total output produced by firms in equilibrium is[36]:

$$Q = \frac{(1+n)(\gamma(an-c))}{-n+\gamma(1+n)^2} \quad . \qquad (53)$$

20 The R&D Competition and market concentration

From the previous equations we obtain that since R&D reduces constant marginal cost of producing, an equal R&D outcome is proportionally more effective for a low-cost firm, as the cost reduction is applied to a greater amount of production. This leads us to the following result:

Proposition 13 *The effects on outputs and R&D investments of changes in parameters are:*

$$\frac{\partial x_i}{\partial \gamma} < 0; \qquad \frac{\partial x_i}{\partial ci} < 0; \qquad \frac{\partial q_i}{\partial \gamma} < 0; \qquad i = 1, ..., n \qquad (54)$$

Thus, the initial cost gap among firms is broadened by the performance of R&D activities. This shows that *ex-ante* asymmetries in production costs result in a higher incentive to invest in R&D by a lower cost firm, and also shows that R&D activities leads to an increase in production costs asymmetries. That means that low cost firms increase

[36]Observe that $Q > 0$ is always satisfied as $c_i < a \Longrightarrow c < an$.

their lead over high-cost firms. R&D activities can then be viewed as an instrument to leverage market power. This result has been already obtained for the duopoly case (see for example Barros and Nilssen (1999), Poyago-Theotoky (1996) or Rosen (1991)), however we have extended it for the n firms case.

On the one hand, we see that if R&D becomes more expensive (γ increases), the direct effect is that firms invest less, in such a way that when γ goes to infinite, firms do not invest at all ($x_i = 0$), and the model becomes the standard Nash-Cournot model without R&D. At the same time, as introduced by Proposition 1, and in line with its intuition, the level of R&D performed by a firm depends negatively on its initial per unit production cost (c_i).

On the other hand, there is an indirect effect, when R&D becomes more expensive firms reduce their production given that they do less R&D and therefore they are less efficient.

As I mentioned, an important issue would be to consider the effect of R&D investment on market performance. Together with firm size, the relationship between market structure and innovative behavior is of major concern for economists and policy makers. The interest derives from the Schumpeterian hypothesis that "large firms are more than proportionately more innovative than small firms" (see Kamien and Schwartz, (1991)). Schumpeter suggests a positive relationship between market concentration and innovative activity. The possibility available to the innovator to exert market power provides him with the incentives to undertake the required investment. However, the theoretical models that have been developed in order to analyze the different aspects of the relationship do not provide us with clear conclusions. In this sense, several arguments are exposed; On the one hand, a more concentrated market would allow firms to better capture consumer value than a less concentrated market providing incentives for early adoption (Saloner and Shepard, (1995)). On the other hand, the counterargument seems to centre on the fact that this higher concentration would however, undermine the pressures to adopt exerted by the existence of higher levels of competition. A key issue in the analysis of market structure is endogeneity: market structure may impact on R&D decisions, but R&D decisions will also influence subsequent market structure. In this sense, we will try to shed some light from a somewhat different perspective: in a scenario where asymmetric firms spend on R&D activities, to which structure does the market evolve?

In the United States, The Federal Trade Commission (FTC) uses the Hirschman-Herfindahl Index:

$$HHI = \sum_{i=1}^{n} (\frac{\text{firm sales}}{\text{total sales}} * 100)^2$$

as an indicator of whether or not an industry is subject to monopoly power. An HHI under 1000 is considered as an indicator of healthy competition. An HHI increase of 100 or more is likely to trigger an investigation, and a HHI above 1800 could be considered as evidence of a monopoly. We analyze the effect of the R&D cost on the Herfindahl Index (HI):

$$HI = \sum_{i=1}^{n} (\frac{\text{firm sales}}{\text{total sales}})^2$$

It is just in a different scale of that the HHI, then: $HHI = 10.000 * HI$. In our model we obtain the following result:

Proposition 14 *The HI is decreasing in* γ.

What Proposition 2 tells us is that as R&D becomes more expensive, the market becomes less concentrated. The intuition comes basically from the following:

$$\frac{\partial x_i}{\partial \gamma \partial c_i} = \frac{n^2(1+n)(2n+\gamma(1+n)^2(-2+\gamma(1+n))}{(\gamma+(-1+\gamma)n)^2(n-\gamma(1+n)^2)^2} > 0.$$

As we know from Remark 1, as γ increases firms spend less in R&D. However, from the last derivative we know that the rate at which firms decrease their R&D depends positively on their initial unit production cost (c_i). Therefore, as γ increases, the more inefficient is the firm, the smaller is its R&D reduction. We have then, that inefficient firms get their market share increased through a smaller R&D cut. In Proposition 2, we find a positive relationship between innovative and concentrated markets which is clearly Schumpeterian[37].

[37] "Creative firms prosper and in contrast, firms that do not innovate, or that innovate in ways consumers do not value, are destroyed by their more creative competitors" (Schumpeter (1942)). He calls this process of economic selection, the culling on non-innovative firms, *creative destruction.*

Remark 1 *The HI is higher when firms can perform cost reducing R&D activities.*

On balance, our model predicts that increased cost of R&D (γ), controlling for other factors, has a negative effect on market concentration consistent with the Schumpeterian hypothesis that less concentrated-less innovative and concentrated-innovative markets schemes can be observed.

We have seen which is the effect on market concentration of the strategic R&D. In the following section we will see which is the optimal government intervention and its consequences on firms behavior and on market structure.

21 The Optimal Industrial Policy

Market failures can provide a rationale for government intervention to either support or curtail incentives to perform private R&D. It is frequently found in the literature[38] that firms may use excessive strategic R&D to restrict competition and not to minimize costs. What we are interested to see in this subsection is what does this behavior call for regarding the industrial policy. In our model of domestic firms without international competition, the strategic motive of the government for intervention to diminish the rivalry of foreign firms is excluded[39]. To that extent, two different policies are considered. Basically we want to check wether the industrial policy should either subsidize or tax R&D activities and production. Therefore, we introduce a tax (or subsidy) on R&D by itself and a tax (or subsidy) on production[40].

The introduction of these policies affect both the levels of R&D committed by firms and the resolution of the output game given R&D levels. We allow for a tax on each firm per unit of its R&D investment. Denote this tax rate by σ. We also allow for a production tax on each firm of α per unit of output produced. Thus, we are now interested in the

[38] See for example Brander and Spencer (1983) and Spencer and Brander (1983).

[39] See for example Spencer and Brander (1983) or Barros and Nilssen (1999).

[40] An Issue not raised here which would have a bearing on the question of a tax vs. a subsidy on R&D is uncertainty. Bagwell and Staiger (1994) concluded that optimal R&D policy would require a precise assessment of the role that uncertainty plays in the R&D process. Moreover, R&D subsidy can play a positive strategic role.

following three-stage game: in stage 1 the government decides on the optimal vector (σ, α) of taxes, that is the simultaneous introduction of R&D and output taxes. In stage 2 firms decide their level of R&D activities, thus determining their production costs for the subsequent output decision. Finally in stage 3 firms decide their output levels.

Now firm i has a profit function given by:

$$\Pi_i(q_i, x_i, \sigma, \alpha) = (P - \alpha)q_i - (c_i - x_i)q_i - \gamma x_i^2 - \sigma x_i \tag{55}$$

We seek the subgame-perfect Nash equilibrium of this game. The government maximizes social welfare, taking into account consumers' surplus, firms' profits and revenues from the taxes. Thus, the government maximizes[41]:

$$W = \Sigma_{i=1}^{n}(\Pi_i(q_i, x_i, \sigma, \alpha) + \sigma x_i + \alpha q_i) + \frac{1}{2}Q^2 \tag{56}$$

Thus, now the equilibrium quantity produced by firm i that is the solution to the third-stage problem, and the level of R&D implemented by each firm that is the solution to the second-stage problem, both depend on the magnitude of the taxes (σ, α) chosen. Thus, we proceed like in the previous section and we obtain that maximizing firms profits (55) with respect to d_i and adding all first order conditions again, the equivalent for (49) is now:

$$\sum_{i=1}^{n} d_i = \frac{2\gamma c(1+n)^2 + n(1+n)^2\sigma - 2n^2(a-\alpha)}{2\gamma(n+1)^2 - 2n} \tag{57}$$

So it is straightforward to obtain the R&D and output levels in equilibrium that depend on the vector of taxes (σ, α).

At the same time, substituting in (56) the values for q_i and x_i obtained and maximizing with respect to σ and α, we get the explicit forms for the optimal R&D and output taxes that would be implemented by the government. This leads us to the following result[42]:

[41]The absence of a distortionary cost of public funds is assumed.

[42]See the Appendix for the Second Order Conditions.

Proposition 15 *The optimal industrial policy calls for a production subsidy* ($\alpha < 0$) *and for a tax on the level of R&D performed by firms* ($\sigma > 0$). *The equilibrium tax and the equilibrium subsidy are respectively:*

$$\sigma^* = \frac{2\gamma(1+n)(-1+3n)(an-c)}{n(-1+3n(-2-3n+2\gamma(1+n)^2))} \tag{58}$$

$$\alpha^* = \frac{2(c-an)(n+3(n^2+\gamma(1+n)^2))}{n(-1+3n(-2-3n+2\gamma(1+n)^2))} \tag{59}$$

We can observe that the policy prescribes taxing the level of R&D performed by the firms to curtail the strategic incentive to over-invest, while at the same time prescribes an output subsidy to stimulate production. Then, the effect (see Barros and Nilssen (1999)) of the profit-shifting motive that calls for a R&D subsidy in quantity competition is offset by the fact that firms spend too much in R&D. Thus, the tax corrects the incentives to do R&D beyond what cost minimization prescribes, meanwhile the output subsidy encourages firms' production.

Some comparative statics' about (58) and (59) can be obtained to clarify:

Remark 2

$$\frac{\partial \sigma^*}{\partial \gamma} < 0; \qquad \frac{\partial \alpha^*}{\partial \gamma} > 0$$

The first derivative tells us that when the cost of R&D increases, as the level implemented by firms is reduced (see Proposition 1), the optimal policy calls for a lower tax on the R&D activities. At the same time the second derivative, following the intuition described above, claims that when the cost of R&D increases, the optimal output subsidy is reduced. The reason is that the need to stimulate production to offset the effect of the R&D tax on firms' production is lower when the tax is lower because the cost of R&D increases.

Once the optimal industrial policy has been characterized, we are now ready to see the effect of both policy tools on market concentration. The consequences of the introduction of both policy instruments on firms' production and on firms' level of R&D clarify matters. Defining $q_i(\sigma^*, \alpha^*)$ and $x_i(\sigma^*, \alpha^*)$ like the quantity produced and the level of R&D implemented by each firm under the optimal industrial policy characterized by (58) and (59)[43]. Using q_i and x_i from (51) and (52), and comparing, we obtain that:

$$q_i(\sigma^*, \alpha^*) - q_i = \frac{\gamma(1+n)(an-c)(1+3n^2+6\gamma(1+n)^2)}{(n(-n+\gamma(1+n)^2)(-1+3n(-2-3n+2\gamma(1+n)^2))}$$
$$x_i(\sigma^*, \alpha^*) - x_i = \frac{(c-an)(-2n^2(1+3n)+\gamma(1+n)^2(-1+n(-4+3n)))}{(n(-n+\gamma(1+n)^2)(-1+3n(-2-3n+2\gamma(1+n)^2))},$$

thus, (as $\gamma \geq 1$ and $c < an$):

$$\begin{aligned} q_i &< q_i(\sigma^*, \alpha^*) \qquad \forall i \in 1, ..., n \\ x_i &> x_i(\sigma^*, \alpha^*) \qquad \forall i \in 1, ..., n \end{aligned} \tag{60}$$

That is, the simultaneous output subsidy and R&D tax expand firms' production. However it is important to remark that this is achieved reducing the level of R&D performed by firms in equilibrium. Therefore, the effect of strategic R&D to lower own marginal cost to get a higher market share is reduced by the tax but at the same time the potential reduction in output is compensated by the subsidy. The linearity of the model allows us to present an interesting lemma that introduces which is the effect of (58) and (59) on the market structure.

Lemma 1 *The following holds:*

$$q_i(\sigma^*, \alpha^*) - q_j(\sigma^*, \alpha^*) = q_i - q_j \qquad \forall i, j \in 1, ..., n$$

This means that variations on firms' production due to the introduction of the optimal industrial policy formed by (σ, α) do not depend on the initial production cost of the firm. This leads us directly to the following result:

[43] See the Appendix for a proof that the equilibrium with the Optimal Industrial Policy is also interior.

Proposition 16 *The introduction of the optimal industrial policy characterized by (58) and (59) reduces the HI.*

The market becomes less concentrated. Basically we have that with a subsidy on production and with a tax on the R&D, the effect described in Proposition 2 of efficient firms achieving a higher market share via an over-investment in strategic R&D vanishes. This proposition implies that one of the consequences of the strategic R&D performed by firms in an asymmetric market, that the industrial policy wants to mitigate, is the raise in market concentration.

A related question would be the relationship between both policy tools. In this sense, the following result shows how the prescribed production subsidy is influenced by the need to tax the over-investment in R&D. We get the following:

Lemma 2 *The optimal production subsidy that would be prescribed in absence of a R&D tax is smaller than (59).*

The intuition of the last result is clear. The presence of a R&D tax calls for a higher output subsidy given that the tax reduces the level of the R&D performed by firms, and also has the effect of cutting firms production.

Another interesting issue would be then to analyze which is solely the effect of the tax on R&D on the HI. In this sense, an interesting comparison would be to see Proposition 3 and 5 when there are no production subsidies. Assume $\alpha = 0$. It is easy to see that now we have that the optimal tax on the level of R&D is:

$$\sigma' = \frac{2(an-c)(-2n^2+\gamma(1+n)^2(3n-2))}{n(1+n)^2(-2-5n+6\gamma(1+n)^2)} \tag{61}$$

We obtain again that $\sigma' > 0$ [44]. The absence of production subsidies does not change the policy prescription of taxing firms to reduce firms' incentives to over-invest in strategic

[44] As in Barros and Nilssen (1999) we find that when production subsidies are not implemented, the optimal industrial policy calls for a tax on the R&D. Their tax however, is a firm-specific industrial policy in an open economy with foreign competition.

R&D. However, the homogeneity of the tax lead us to the following result, which is the equivalent to Proposition 6:

Proposition 17 *The introduction of the optimal industrial policy characterized by (61) increases the HI.*

From the last Proposition we see that with the introduction of an industrial policy where production is not subsidized, the market becomes more concentrated. Although the R&D tax reduces the level of R&D implemented by the firm, efficient firms achieve a higher market share because under this tax structure firms are equally penalized independently of their initial production costs. Therefore, with the introduction of (61), firms become more quantity asymmetric. This Proposition implies that it is precisely the introduction of the production subsidy (α) what reduces the concentration of the market.

21.1 A Firm-specific industrial policy

Our concern in this subsection is about one important aspect of R&D subsidies or taxes that distinguishes these policy instruments from other trade-policy instruments. While the former instruments tend to be industry-specific, the support (or taxation) of R&D activities can also be, by its nature, firm specific and even project specific.

Barros and Nilssen (1999) do comparative statics about the nature of a firm-specific industrial policy. Their model however, basically differs from ours as they do not specify a particular form for R&D costs and therefore they can not explicitly solve the model. Furthermore they do not consider consumers' surplus in national welfare.

However, when R&D activities are either taxed or subsidized differently among firms, the question that naturally arises is which firm should receive such support or be taxed. In our asymmetric model we face this question, asking whether firms should get their R&D output taxed or not and which firms should pay the lowest tax or get the highest subsidy.

To that extent we consider the introduction of a firm specific R&D tax (or subsidy) that we call σ_i. The timing of the game is the same three-stage situation described in the last subsection. Therefore the profits that firm i would obtain are:

$$\Pi_i(q_i, x_i, \sigma_i) = q_i^2 - \gamma x_i^2 - \sigma_i x_i \tag{62}$$

Output produced by firm i that depend on σ_i and total output produced by firms are easily obtained. We proceed again like in the previous subsection looking for the new production cost (d_i) that maximizes firms profits. They are the solution to the second stage problem:

$$q_i = \frac{(1+n)(2c\gamma^2(1+n)+2a\gamma(\gamma+(-1+\gamma)n)-(-n+\gamma(1+n)^2)(2c_i\gamma+\sigma_i)+\gamma(1+n)\Sigma_{i=1}^n\sigma_i))}{2(\gamma+(-1+\gamma)n)(-n+\gamma(1+n)^2)} \tag{63}$$

$$Q = \frac{(1+n)(-2c\gamma + 2an\gamma - \Sigma_{i=1}^n \sigma_i)}{2(-n+\gamma(1+n)^2)} \tag{64}$$

At the same time we know that each firm's first-order condition with respect to the level of its R&D investment is:

$$\frac{\partial \Pi_i(q_i, x_i, \sigma_i)}{\partial x_i} = 2q_i \frac{\partial q_i}{\partial x_i} - 2\gamma x_i - \sigma_i = 0 \tag{65}$$

Whereas the government maximizes social welfare that is:

$$W = \Sigma_{i=1}^n(\Pi_i(q_i, x_i, \sigma_i)) + \frac{1}{2}Q^2 \tag{66}$$

To have governments' preferred outcome, we proceed like in Barros and Nilssen (1999). To obtain the equilibrium, we proceed to the first stage, the government chooses an R&D tax for each firm. The tax is obtained in the following way. It is assumed that the government is able to choose R&D activities for each firm, x_i, directly. Therefore, the optimal government choices solve the following condition:

$$\frac{\partial W}{\partial x_i} = 2[\Sigma_{i=1}^n q_i \frac{\partial q_i}{\partial x_i}] - 2\gamma x_i + \Sigma_{i=1}^n q_i \Sigma_{i=1}^n \frac{\partial \Sigma_{i=1}^n q_i}{\partial x_i} = 0 \tag{67}$$

Therefore, the optimal firm-specific policy should make (67) and (65) hold. Since in our model $\dfrac{\partial q_i}{\partial x_i} = \frac{n}{n+1}$ and $\dfrac{\partial q_i}{\partial x_j} = -\frac{1}{n+1}$, we have that:

$$\sigma_i = \frac{Q - 2q_i}{n+1} \tag{68}$$

Where q_i and Q are "post-tax" quantities, respectively (63) and (64). Barros and Nilssen (1999) without considering consumers surplus in the social welfare obtain $\sigma_i = \frac{2(Q-q_i)}{n+1}$. That is, they obtain that the industrial firm-specific policy prescribes always a tax for each firm. However, we obtain the specific form for the optimal industrial firm-specific tax (or subsidy). To that extent, we replace (63) and (64) in (68) and adding up to n. We obtain:

$$\Sigma_{i=1}^{n} \sigma_i = \frac{2(a-c)\gamma(-2+n)n}{-2-n+2\gamma(1+n)^2} \tag{69}$$

Replacing (69) in (63) and (64), we are ready to obtain the explicit form for the firm-specific policy described in (68). It leads us to the following result:

Proposition 18 *The optimal industrial firm-specific policy prescribes a tax on the R&D when the firm does not have a market share larger than $\frac{1}{2}$. Otherwise, the policy prescribes a subsidy. The equilibrium tax (or subsidy) is:*

$$\sigma_i = \frac{\gamma(a(-2+n)(\gamma+(-1+\gamma)n)+c(n-3\gamma(1+n))+2c_i(-n+\gamma(1+n)^2)+\frac{(a-c)(-2+n)n(n-3\gamma(1+n))}{-2-n+2\gamma(1+n)^2})}{(-1+\gamma)(1+n)(-n+\gamma(1+n)^2)} \tag{70}$$

Last proposition specifies that a firm-specific policy also prescribes taxing firms to curtail the strategic incentive to over-invest in R&D unless the firm is so efficient that the market is almost monopolized. In this case, the optimal policy calls for a subsidy to the R&D of this firm.

A related question is how firms are penalized by the tax because they are better positioned or not from the start. Looking at (70) , we see that:

Remark 3 *Those firms that are more efficient are taxed less.*

This remark implies that more efficient firms are penalized less than inefficient firms by the government because they are somehow more cost-effective in conducting R&D activities. More quantity-efficient firms should be taxed at a lower rate. This means that the tax is used to divert production to the more efficient firms.

A central question is, of course, how this firm specific policy affects market structure. As we can deduce from the last proposition, we obtain the following result:

Proposition 19 *The introduction of the optimal firm-specific policy increases the HI.*

We can see from the last proposition that the firm-specific policy described in Proposition 8 increases market concentration. It is clear form Remark 4 that through the policy described by the vector (70) ex-ante asymmetries in production costs are increased and then the structure of the market becomes more concentrated.

22 Conclusions

In this paper we have presented a simple model of R&D competition of n firms placed asymmetrically at the start of the game. Firms compete to get a cost reduction. What it is obtained, in accordance with Schumpeterian hypotheses, is that if we consider large firms as those who have a larger initial market share, they spend more in R&D than small firms. Basically, this is true due to the fact that large firms use R&D activities to reduce competition achieving a higher market share. The more efficient is the firm from the start, as an equal R&D cost-reducing outcome is applied to a greater amount of production, the larger are the incentives to innovate. The conclusions are that the overinvestment in R&D beyond what cost minimization would prescribe leads to an increase in market concentration.

The main point is therefore, that the relationship between market concentration and innovation is positive and should be corrected by an optimal industrial policy. Within the limited context of the model presented in this paper, some implications on the design of a policy can be drawn, and this is that when R&D activities are used to reduce competition a corrective tax is needed. This corrective tax, together with a production subsidy reduces market concentration.

When the policy is firm-specific, the government taxes less the more efficient firms, basically because the policy is used as an instrument to divert production to the more efficient firms.

Several issues have been left for future research. First, it could be fruitful to apply to a more general framework, allowing firms to differ in their R&D efficiency or to a more general demand and cost functions. Second, as we pointed out in the introduction, the relationship between firm size and investment in R&D activities is not clear in the literature. The present model considers firms' size in terms of market share, however, other different means to measure firms' size can also be used, considering for example firms with different fixed costs or facing asymmetric costs to entry the market.

23 Appendix

Footnote 42: The Second Order Conditions for the government maximizing social welfare are hold as the Matrix:

$$\begin{pmatrix} \frac{\partial^2 W}{\partial^2 \sigma} & \frac{\partial^2 W}{\partial \sigma \partial \alpha} \\ \frac{\partial^2 W}{\partial \alpha \partial \sigma} & \frac{\partial^2 W}{\partial^2 \alpha} \end{pmatrix} = \begin{pmatrix} \frac{-n(1+n)^2(-2-5n+6\gamma(1+n)^2)}{4(n-\gamma(1+n)^2)^2} & \frac{n(n+3n^2-\gamma(1+n)^2(-1+4n))}{2(n-\gamma(1+n)^2)^2} \\ \frac{n(n+3n^2-\gamma(1+n)^2(-1+4n))}{2(n-\gamma(1+n)^2)^2} & \frac{-\gamma n^2(-2+\gamma(1+n)^2)}{(n-\gamma(1+n)^2)^2} \end{pmatrix}$$

is negative definite.

Footnote 43: The assumption taken in footnote 34 on production costs asymmetries is also enough to ensure that all firms produce under the optimal industrial policy ($q_i(\sigma^*, \alpha^*) > 0 \ \forall i$) and we are in an interior equilibrium. The condition required for $q_i(\sigma^*, \alpha^*) > 0 \ \forall i$ is $c_i \leq c_i^* = \frac{6an(1+n)(-1+\gamma)n+\sum_{j\neq i}^n c_j(-1-3n^2+6\gamma(-1+n)(1+n)^2)}{1-n-3n^2(1+3n)+6\gamma(1+n+n^3+n^4)}$. We assumed that $c_i < c_i^1 = \frac{\gamma \sum_{j\neq i}^n c_j(1+n)+a(\gamma+(-1+\gamma)n))}{n(-1+\gamma(1+n))}$ and we have that $c_i^1 < c_i^*$ if $a + \sum_{j\neq i}^n c_j < an$, which is true if $c_i < a \ \forall i$.

Proof. Proposition 13: The level of R&D implemented by firms in equilibrium is given by the expression:

$$x_i(c_i, \gamma) = \frac{n(nc_i - \gamma((1+n)^2 c_i - c) + a(\gamma + (-1+\gamma)n))}{(\gamma + (-1+\gamma)n)(-n+\gamma(1+n)^2)}$$

So we have:

$\frac{\partial x_i(c_i,\gamma)}{\partial \gamma} = \frac{n(1+n)(-a(1+n)(\gamma+(-1+\gamma)n)^2 - c(\gamma^2+3\gamma^2 n+(-1+3\gamma^2)n^2+\gamma^2 n^3+c_i(n-\gamma(1+n)^2)^2)}{(\gamma+(-1+\gamma)n)^2(n-\gamma(1+n)^2)^2}$ which is negative whenever the initial condition on c_i is hold.

$\frac{\partial x_i(c_i,\gamma)}{\partial ci} = -\frac{n}{\gamma+(-1+\gamma)n} < 0$ (as $\gamma \geq 1$)

$\frac{\partial q_i}{\partial \gamma} < 0$ is implied by (40), (42) and $\frac{\partial x_i(c_i,\gamma)}{\partial \gamma} < 0$. ∎

Proof. Proposition 14: The HI is given by the following expression:

$$HI = \frac{-2ac(\gamma+(-1+\gamma)n)^2+a^2 n(\gamma+(-1+\gamma)n)^2+d(n-\gamma(1+n)^2)^2-c^2\gamma(1+n)(-2n+\gamma(1+n)(2+n))}{(c-an)^2(\gamma+(-1+\gamma)n)^2}$$

where $d = \sum_{i=1}^n c_i^2$. Thus, $\frac{\partial HI}{\partial \gamma} = \frac{2n(1+n)(c^2-dn)(-n+\gamma(1+n)^2)}{(c-an)^2(\gamma+(-1+\gamma)n)^3}$. The sign of this derivative depends on $(c^2 - dn)$. We have that this is negative when $(\sum_{i=1}^n c_i)^2 < n\sum_{i=1}^n c_i^2$ which is true if $0 < n\sum_{i=1}^n c_i^2 - (n\frac{\sum_{i=1}^n c_i}{n})^2$ that is when $0 < n\sum_{i=1}^n c_i^2 - n^2\bar{c}^2$. (Where $\bar{c} = \frac{\sum_{i=1}^n c_i}{n}$ is the average cost). So, $0 < \frac{n}{n}(n\sum_{i=1}^n c_i^2 - n^2\bar{c}^2)$holds if $0 < n^2(\frac{\sum_{i=1}^n c_i^2 - n^2\bar{c}^2}{n})$. On the other hand, we can develop $\sum_{i=1}^{n} \frac{(c_i-\bar{c})^2}{n}$ which is the sample variance and therefore always positive, that is $\frac{\sum_{i=1}^{n}(c_i^2-2\bar{c}c_i+\bar{c}^2)}{n} = \frac{\sum_{i=1}^{n} c_i^2-2\bar{c}\sum_{i=1}^{n} c_i+n\bar{c}^2}{n} = \frac{\sum_{i=1}^{n} c_i^2-2n\bar{c}^2+n\bar{c}^2}{n} = \frac{\sum_{i=1}^{n} c_i^2-n^2\bar{c}^2}{n} > 0$ ■

Proof. Proposition 15: We have that the taxes in equilibrium are respectively:

$$\sigma^* = \frac{2\gamma(1+n)(-1+3n)(an-c)}{n(-1+3n(-2-3n+2\gamma(1+n)^2))}$$
$$\alpha^* = \frac{2(c-an)(n+3(n^2+\gamma(1+n)^2))}{n(-1+3n(-2-3n+2\gamma(1+n)^2))}$$

The denominator of (σ^*, α^*) is positive if $\gamma > \frac{(1+3n)^2}{6n(1+n)^2}$, which is true as $\gamma \geq 1$. Then the sign of the taxes depends only on its numerator. We have that $c_i < a$, therefore $\sum_{i=1}^{n} c_i = c < an$. So, $\sigma^* > 0$ and $\alpha^* < 0$ ■

Proof. Lemma 1: We have that $q_i(\sigma, \alpha) = \frac{a-\alpha-(n+1)d_i(\sigma,\alpha)+\sum_{i=1}^{n} d_i(\sigma,\alpha)}{n+1}$ where:

$$d_i(\sigma, \alpha) = -\frac{2c_i\gamma(1+n)^2 + (1+n)^2\sigma + 2n(\alpha - a) - \frac{n(2c\gamma(1+n)^2+n(-2an+(1+n)^2t+2n\alpha)}{-n+\gamma(1+n)^2}}{2(1+n)(n-\gamma(1+n))}$$

$$\sum_{i=1}^{n} d_i(\sigma, \alpha) = \frac{2c\gamma(1+n)^2 + n(1+n)\sigma - 2n^2(a-\alpha)}{-2n + 2\gamma(1+n)^2}$$

and

$$q_i = \frac{\gamma(1+n)(c\gamma(1+n) + a(\gamma + (-1+\gamma)n) - c_i(-n+\gamma(1+n)^2))}{(\gamma + (-1+\gamma)n)(-n+\gamma(1+n)^2)}$$

it is tedious but straightforward to check that $q_i(\sigma, \alpha) - q_j(\sigma, \alpha) = q_i - q_j$ holds. ■

Proof. Proposition 16: We have that $HI \equiv \sum_{i=1}^{n}(\frac{q_i}{Q})^2$. Thus, using Lemma 4 we have that the introduction of the optimal industrial policy characterized by σ^* and α^* means that the concentration index turns to $HI' = \sum_{i=1}^{n}(\frac{q_i+\epsilon}{Q+n\epsilon})^2$ where ϵ is the variation on firms production due to $(\sigma^*\alpha^*)$ and it is the same across all firms. Therefore, as $\sum_{i=1}^{n}(\frac{q_i+\epsilon}{Q+n\epsilon})^2 = \frac{\sum_{i=1}^{n} q_i^2+n\epsilon^2+2\epsilon\sum_{i=1}^{n} q_i}{(Q+n\epsilon)^2}$, if we compare both indexes we have: $HI - HI' = \sum_{i=1}^{n}(\frac{q_i}{Q})^2 - \frac{\sum_{i=1}^{n} q_i^2+n\epsilon^2+2\epsilon\sum_{i=1}^{n} q_i}{(Q+n\epsilon)^2} > 0 \iff \frac{n^2\epsilon^2\sum_{i=1}^{n} q_i^2+2Qn\epsilon\sum_{i=1}^{n} q_i^2}{Q^2(Q+n\epsilon)^2} > \frac{n\epsilon^2+2\epsilon\sum_{i=1}^{n} q_i}{(Q+n\epsilon)^2}$. So $n^2\epsilon^2\sum_{i=1}^{n} q_i^2 + 2Qn\epsilon\sum_{i=1}^{n} q_i^2 > Q^2n\epsilon^2 + 2\epsilon Q^2\sum_{i=1}^{n} q_i \iff \epsilon(n^2\epsilon\sum_{i=1}^{n} q_i^2 + 2Qn\sum_{i=1}^{n} q_i^2) > \epsilon Q^2(n\epsilon + 2Q)$ given that $\epsilon > 0$ (it is easy to see that $q_i(\sigma^*, \alpha^*) > q_i$) The last conditions turns to

$n\sum_{i=1}^{n} q_i^2(n\epsilon + 2Q) > Q^2(n\epsilon + 2Q)$, which is always true whenever $HI \equiv \sum_{i=1}^{n}(\frac{q_i}{Q})^2 > \frac{1}{n}$ as it is assumed in our model where firms are not restricted to be equal. ■

Proof. Lemma 2: The optimal production subsidy it is obtained assuming $\sigma = 0$. Then we obtain that:

$$\alpha = \frac{(c - an)(\gamma + n + 2\gamma n + (-1 + \gamma)n^2)}{n^2(-2 + \gamma(1 + n)^2)} < 0$$

then we can check that the proposition holds by simply seeing $\alpha^* - \alpha < 0$. ■

Proof. Proposition 17: We can see that when the industrial policy consists of assuming $\alpha = 0$ and introducing $\sigma' > 0$. We can see that the equivalent for Lemma 4 also holds: $q_i(\sigma', 0) - q_j(\sigma', 0) = q_i - q_j$. Therefore, we can apply the reasoning of the Proof of Proposition 5 in the following way: now $\epsilon < 0$, so HI decreases whenever $HI \equiv \sum_{i=1}^{n}(\frac{q_i}{Q})^2 > \frac{1}{n}$, which is always true in our asymmetric model. ■

Proof. Proposition 18: It is immediate to see that as we obtained:

$$\sigma_i = \frac{Q - 2q_i}{n + 1}$$

so, $\sigma_i > 0$ whenever $\frac{q_i}{Q} < \frac{1}{2}$ ■

Proof. Proposition 19: The introduction of the firm-specific industrial policy is characterized by:

$$\sigma_i = \frac{\gamma(a(-2+n)(\gamma+(-1+\gamma)n)+c(n-3\gamma(1+n))+2c_i(-n+\gamma(1+n)^2)+\frac{(a-c)(-2+n)n(n-3\gamma(1+n))}{-2-n+2\gamma(1+n)^2})}{(-1+\gamma)(1+n)(-n+\gamma(1+n)^2)}$$

So if we compare the HI before and after the introduction of δ_i we can see that they are respectively $HI \equiv \sum_{i=1}^{n}(\frac{q_i}{Q})^2$ and $HI' = \sum_{i=1}^{n}(\frac{q_i + \sum_{i=1}^{n}\epsilon_i}{Q + \sum_{i=1}^{n}\epsilon_i})^2$, where ϵ_i is firms production variation due to the introduction of the tax δ_i . It is also easily verified that $q_i(\sigma_i) < q_i \ \ \forall i$, therefore $\epsilon_i < 0$. Simplifying, we have that $HI' > HI$ whenever $\sum_{i=1}^{n} q_i\epsilon_i < \sum_{i=1}^{n} q_i \sum_{i=1}^{n} \epsilon_i$, which is true. ■

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UNIVERSIDAD DE ALICANTE
Comisión de Doctorado

Reunido el Tribunal que suscribe en el día de la fecha acordó otorgar, por UNANIMIDAD a la Tesis Doctoral de Don/Dña. MARC ESCRIHUELA VILLAR la calificación de SOBRESALIENTE CUM LAUDE

Alicante 5 de DICIEMBRE de 2003

El Secretario

El Presidente,

LUIS CORCHÓN. JOEL SANDONÍS

POR MEDIO DE LA PRESENTE SE HACE CONSTAR QUE EL PROCEDIMIENTO DE LECTURA DE TESIS DOCTORAL REALIZADA POR MARC ESCRIHUELA VILLAR SE HA REALIZADO EN LENGUA INGLESA, OBTENIENDO DE ESTA MANERA LA MENCIÓN DE DOCTOR EUROPEO

ALICANTE, 5 DICIEMBRE, 2003

PRESIDENTE SECRETARIO