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Educational software for analysis of diffraction experiences based on FDTD for acoustics and electromagnetics

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Abstract—Interference and diffraction of waves are elementary topics in physics. These phenomena can be identified in different disciplines such as Optics and Acoustics. Due to equivalence between the acoustics equations and the TE polarization in Electromagnetism a unified treatment can be defined for both problems. The aim of the work presented here is to develop a simulation software that provides a virtual laboratory for studying a set of canonical problems in both Optics and Acoustics. The application is based on a MATLAB GUI that invokes different C++ executables that implements the FDTD for both Electromagnetics and Acoustics. This application can enhance the experience of optics and physics undergraduates giving the opportunity of simulating a wider range of situations that hardly can be considered in laboratory. Specifically, the software simulates the diffraction produced by a user-configurable number of slits in Optics and a set of different obstacles and barriers in Acoustics.

I. INTRODUCCIÓN

In physics and technical education, laboratories are important environments for students learning. Setup and maintenance costs are limited. These constraints limit the range of activities that can be designed by the teacher. In response "virtual laboratories" have been developed to avoid this problem and improve student’s experience. Many examples of this can be found in the literature and virtual laboratories have become a hot topic for many researches [1]–[3]. Hands-on experiments in laboratory sessions play an important role in improving student’s knowledge. In both optics and acoustics education, this type of sessions implies to use different instruments such as lasers, analyzers, detectors, sound sources, microphones, and the like. All these instruments are expensive and the selection must be done according several academic criteria. In many cases it is difficult to cover a wide range of activities with specific equipment. Therefore, the laboratory experience can be completed by means of virtual laboratories, which can be used for comparing experimental values obtained in laboratory, and for studying different configurations that implies more instrumentation and different setups.

In this work a user-friendly educational software is presented. The aim of the application is to be used for students and teachers in the first years of Optics, Physics and Telecommunications degrees. The software is structured in a API based on MATLAB that invokes a C++ library that implements the FDTD for acoustics and electromagnetics. The experiments that can be performed by the application are focused on demonstrating the wave nature of light [4]. Regarding Optics, the light diffracted by slits that illuminates a screen at a large distance is computed. The diffraction pattern is function of the number of slits, their width and gap between them. Moreover, the theoretical irradiance curves are also computed in order to compare the range of validity of Fraunhofer and Fresnel approximations [4], [5]. As for Acoustics, the propagation of waves and diffraction produced by different objects and barriers is predicted. The size, shape and situation can be configured by user and also the wavelength of the audio source can be modified. In both topics, the near field wave field is shown in real time by the application giving to students the opportunity of see the analogies between light wave propagation and pressure waves.

II. THEORY

In this section, a brief review of the FDTD method and the theory related with the diffraction analysis in both Acoustics and Optics is given.

A. FDTD for Acoustics and Electromagnetics

Yee introduced the notation that has been established as the reference of the Finite-Difference Time-Domain method. The development of the FDTD has been carried out by the electromagnetics community to simulate EM wave propagation and scattering phenomena. This method also has been applied to the simulation of acoustic wave propagation and scattering. The FDTD has been applied with success due to the form of the solution in 2D is very similar to the TE waves in electromagnetics [6].
The first order acoustics equations in homogeneous, lossy fluid medium are:

\[ \nabla p(x, t) = -\rho \frac{\partial}{\partial t} u(x, t) - \alpha^* u(x, t), \quad (1) \]

\[ \nabla \cdot u(x, t) = -\kappa \frac{\partial}{\partial t} p(x, t) - \alpha p(x, t), \quad (2) \]

where \( \rho \) is the pressure field, \( u \) is the vector velocity field, \( \rho \) is the mass density of the medium, and \( \kappa \) is the compressibility of the medium \( (c = 1/\sqrt{\kappa \rho}) \). The attenuation coefficient, \( \alpha \), is the usual compressibility attenuation in acoustic media. The attenuation coefficient associated with the density, \( \alpha^* \), is generally zero for an acoustic medium, although it can be related to a “mass-proportional” damping used in solid mechanics. These two terms are mainly used for modeling the Absorbing Boundary Conditions developed by Berenger [7].

Equations (1) and (2) are valid for 2D and 3D problems. Specifically, in 2D they are equivalent to the 2D electromagnetic equations for the transverse electric (TE) polarization:

\[ \nabla \times \mathbf{E}(x, t) = -\mu \frac{\partial}{\partial t} \mathbf{H}(x, t) - \sigma^* \mathbf{H}(x, t), \quad (3) \]

\[ \nabla \times \mathbf{H}(x, t) = \frac{\partial}{\partial t} \mathbf{E}(x, t) + \sigma \mathbf{E}(x, t), \quad (4) \]

with \( \mathbf{E} \) and \( \mathbf{H} \) being the electric and magnetic field intensities respectively. The time-varying conductivity of the medium is \( \sigma \) and the magnetic conductivity is represented with \( \sigma^* \).

Taking into account the expressions (1)-(4) a symbol equivalence can be established:

\[ [E_x, -H_y, H_x, \epsilon, \mu, \sigma, \sigma^*] \leftrightarrow [p, u_x, u_y, \kappa, \rho, \alpha, \alpha^*]. \quad (5) \]

Thus, the FDTD method can be applied following the same schemes in both set of equations. Specifically, the Maxwell’s curl equations can be discretized and solved using the central-difference expressions, for both the time and space derivatives.

As mentioned below it must be included a formalism in order to avoid the interferences produced by outgoing waves reaching the grid simulation limits. The PMLs are a good technique for the absorption of electromagnetic waves by means of a nonphysical absorbing medium adjacent to the outer FDTD mesh boundary. Regarding the illumination method, it must be said that the source is introduced along the connecting boundary by using a Total Field/Scattered Field (TF/ SF) algorithm [8]. This method avoids the computation of the incident wave in the whole bidimensional grid and only two one-dimensional arrays are needed.

### B. Diffraction of thin slits in Optics

Considering a single aperture of small area illuminated by a plane wave, the interference pattern of the light emitted from the aperture is modified as the distance between the observation and the slit plane grows. It can be identified the Fresnel or near-field diffraction and the Fraunhofer or far-field region. The analysis of the diffraction in these regions differs due to the approximations considered in each case [4], [5]. The range of applicability of these two theories is determined by means of the Fraunhofer’s condition \( F = b^2/L \lambda << 1 \), with \( b \)

<table>
<thead>
<tr>
<th>( b/2 )</th>
<th>( \theta )</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta z )</td>
<td>( \theta )</td>
<td>( P )</td>
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</table>

Fig. 1. One-slit diagram [5].

<table>
<thead>
<tr>
<th>Normalized irradiance as a function of the number of apertures</th>
<th>1 aperture</th>
<th>2 apertures</th>
<th>( N ) apertures</th>
</tr>
</thead>
<tbody>
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<tr>
<td>( \sin \frac{\theta}{\delta} )</td>
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<td>( 4 \cos^2 \alpha )</td>
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Thus, from the expressions collected in Table I and the Eq. (6), the theoretical curves of irradiance in any point of the observation plane far away from the slits can be calculated. An interesting activity that can be performed by students is the analysis of the validity of these theories as a function of the distance \( L \) comparing the results with the numerical values obtained from FDTD.

### C. Diffraction of barriers and obstacles in Acoustics

The direct example of diffraction in acoustics is the problem of the noise barriers. A noise barrier is a popular method to mitigate the noise of transportation. They are commonly used in highways or roads to shield the receiver intercepting the line-of-sight of the noise source. In this case, the transmission noise is negligible and the main noise source becomes the diffracted sound. Although there are mathematically rigorous solutions to the diffraction of sound in barriers [9], [10] and a wave theory approximation [11], the efficiency of a noise barrier is usually predicted following the empirical charts of Maekawa [12]. Maekawa measured experimentally the attenuation, or insertion loss, produced by a barrier for different barrier configurations.

Maekawa measured experimentally the attenuation, or insertion loss, produced by a barrier for different barrier configurations.
relate the source and receiver positions and the frequency of the source using a unique parameter, the Fresnel Number (see Eq.7).

$$N = \frac{2(A + B - d)}{\lambda}$$  \hspace{1cm} (7)

Where, the distances $A$, $B$ and $d$ are shown in figure 2 and $\lambda$ is the wavelength of the noise source. Since the Fresnel number combines distances and wavelengths different arrangements of the source/receiver pair can have the same $N$ value for different wavelengths.

The work of Maekawa has been refined by some authors [13], [14] but, in the practice, the predicted insertion loss is estimated using some best-fit formula to the Maekawa charts [15]. In this work we have chosen the following predicting equation:

$$IL = \left\{ \begin{array}{ll}
10\log_{10} N + 13 & N > 1 \\
5 + 8N|N|^{-0.55-0.143|N|} & -0.3 \leq N \leq 1 \\
0 & N < -0.3
\end{array} \right.$$  \hspace{1cm} (8)

Using 8 as the expected insertion loss value provided by an acoustic barrier, it is possible to check the operation of the software. Although, we must take into account that the Maekawa charts have several scattering around the best-fit curve, and that the noise source used by Maekawa was spherical.

### III. SOFTWARE DESCRIPTION

The structure of the software implemented is illustrated in Fig. 3. The interface is based on a MATLAB GUI called from the desktop as DifSim. The user must choose at startup the Optics or Acoustics module to be loaded. Each module opens a window in which the different parameters related with the simulation can be modified by user. Once the configuration is completed, user can start the simulation that represents in real time the wavefields in the space grid simulation for each time-step. When the simulation is over, different post-process can be performed depending the module. As for Optics, the comparison of the numerical irradiance with the curves obtained with both Fresnel and Fraunhofer approximations can be compared. Regarding Acoustics, the comparison of the insertion loss computed by the FDTD method can be compared with those obtained by theoretical expressions.

### IV. RESULTS

The output obtained by the postprocess options of both modules are summarized in this section by means of several examples. Concerning the Optics module, the normalized irradiance curves are represented in Fig. 4a and 4b. Both graphs show the normalized irradiance patterns obtained by FDTD and the Fresnel and Fraunhofer approximations. The irradiance is represented as a function of the $y$-axes for one and two slits respectively. The FDTD simulation parameters are: $\lambda = 633$ nm, $\Delta = \lambda/20$ and $\Delta t = \Delta/2c_0)$. As can be seen, the Fresnel approximation behaves better in both cases instead of the Fraunhofer irradiance curve. Specifically, for the case of two apertures, the Fraunhofer approximation is quite different from the numerical irradiance because of the plane in which the irradiance is represented can be considered near field ($F \approx 2.4$).

Fig. 4 shows a sample of the irradiance as a function of the space for a specific time step. The application permits understand how the diffraction pattern changes as the wavefront travels along the grid simulation, since the transient behavior of the electromagnetic field is represented by the application as a movie. On the other hand, is interesting to note that the
detail of the electromagnetic field in near field region is quite difficult to appreciate in a real laboratory.

Regarding the acoustics module, a set of experiments is resumed in Fig. 5. In this figure, a barrier of 2m of height is measured in the range of 500-1000 Hz for different source and receiver positions, each triplet (frequency, source position, receiver position) is represented by a Fresnel number with the corresponding insertion loss (Fig. 5a). In this case, it is clear that the insertion loss computed by FDTD is slightly different than theory. This is due to the 2D implementation of the FDTD method. Performing a 2D simulation means that the 3rd dimension is always the same than the simulated plane. Therefore, FDTD is simulating a cylindrical source instead of a spherical source. The attenuation factor of a cylindrical source is 3dB each time the distance is doubled instead of the 6dB of the spherical source. Nevertheless, authors consider that results obtained by 2D simulation are closer to those that could be obtained with cars in movement in a traffic noise measurements.

![Graph](image)

**Fig. 5. Results of the Acoustics module of DifSim.**

V. CONCLUSIONS

In this work an educational application focused on the analysis of diffraction in Optics and Acoustics is presented. The software is based on a MATLAB GUI that invokes an implemented library in C++ that computes the FDTD method for both Optics and Acoustics scattering problems. More specifically, the Optics module of the application represents the diffraction of a set of user-configurable slits illuminated by a plane wave. The results are compared with the theoretical expressions in Fresnel and Fraunhofer regions. Thus, the student can analyze the accuracy of both approximations as a function of the distance between the observation and illumination planes. Regarding Acoustics module, students can analyze the diffraction produced by different barriers or objects that can be placed in the simulation region. The size and position of the objects in terms of the wavelength can be modified by user giving the opportunity of study how these parameters affects the wavefront of a pressure wave. The dimensions of a barrier and its shape can also be studied in terms of the insertion loss parameter, taking into account that the sources are always cylindrical instead of spherical. Both modules give the opportunity of visualize the transient behavior of the waves as a function of the space being an highly value-added content. The application can be easily handled by students and also by teachers for complement both lectures and laboratory sessions respectively.

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