Room: Clinton Auditorium

12:45-13:00 STUDENT PRESENTATION

Look-up table of quadrics applied to corneal topography

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Summary
We present a new look-up table method to convert acquired corneal data from reflected Placido rings to curvature radii. The technique avoids extrapolating corneal data and overcomes the difficulty with the lack of symmetry of projected rings and provides a better description of the corneal morphology.

Introduction
The information in obtaining a corneal topography using a commercial device usually is hidden, in spite of being of interest for researchers. In previous works [1,2], the authors detailed the algorithms and procedures to obtain the curvature data of cornea. Radial distances from Placido rings were related to the correspondent local curvature radii through a set of calibration curves (look-up table method). If the method was able to distinguish and classify l rings, then lookup tables contained L calibration equations on the form $R = m_l + n_l$, where R are the mean radial distances measured for calibration balls of curvature radius C in the $l = 1, \ldots, L$ different labeled rings. Mean radial distance was computed as an arithmetic mean of distances obtained after transforming from Cartesian to polar coordinates and this averaging introduces quite uncertainty.

Espinosa et al. [1] proposed obtaining a calibration equation, alternatively to that set of calibration curves in [2]. They considered three variables, the independent variables L, corresponding to the ring's label, C, the curvature radius, and R, the radial distance of each ring, which was set as the radius obtained from least square fitting them to a circle. The radial distance of each reflected ring on a surface depends of both the curvature radius of the surface and the ring's label as $R=(a\,l+b)C$. Coefficients a and b could be obtained through least square fitting of computed radial distances. In order to compute the curvature radius $C(x,y)$ at each pixel at a distance $R(x,y)$ from center, and being part of a reflected ring L, we have

$$C(x,y)=\frac{R(x,y)}{(a\,l+b)} \qquad (x,y)\in Ring_l \tag{1}$$

Discussion
Equation (1) introduces a dependence of radial distance with the ring's label. Moreover, it is a unique calibration equation, alternatively to the set of curves in [2]. However, when we compute both a and b parameters in the calibration process, we do not take into account the indetermination in the radial distance R. This radial distance is obtained from a fitting of Cartesian sampled data to a circle. Therefore, the assignment of a unique radial distance to each ring interpolates data sampled in a Cartesian grid.
The indetermination in the radii of fitted circles depends both on the ring's label and the curvature radius of the theoretical eye used in the calibration. This is an intrinsic inconvenience to the method which comes from the sampling of an image with polar symmetry, the reflected rings, with a Cartesian grid, the CCD array of the camera. Therefore, we reformulate the problem from the beginning. From equation (1), it is clear that the curvature radius is proportional to the radial distance. In order to avoid the above exposed interpolation of data, we look for a calibration equation directly considering all measured data.
If we perform the expansion of

$R = m_l + n_l$, we get $x^2+y^2=m_l^2C^2+2m_ln_lC+n_l^2$,

an expression with terms in a similar way to a quadric equation. Therefore, as an initial general case, we consider that curvature radii and spatial coordinates of captured reflected rings are related following a quadric surface in the Euclidean space which may be compactly written in vector and matrix notation as

where $v_l = \{x_l, y_l, C\}$ are row vectors containing the coordinates of data of each l ring or edge for each calibration surface of curvature radius is the transpose of v_l, D_l are 3x3 matrices, E_l are 3-dimensional row vectors and F_l are scalar constants. Expression (2) provides a calibration equation for each ring as
Curvature radii and (x_l, y_l) coordinates of data for each l ring are least-squares fitting to (3), thus providing a look-up table of quadrics. The method overcomes the interpolation of Cartesian data so it improves the morphological description of corneal surface.

Results
Figure 1 and Table 1 show results for the fitting of ring l=16.

a	40.78 (38.99, 42.57)	***f***	3.01 (2.67, 3.34)
b	40.88 (39.09, 42.68)	***g***	1.10 (0.70, 1.41)
c	-0.08 (-0.13, -0.04)	***h***	-26.16 (-28.04, -24.29)
d	-0.35 (-0.39, -0.31)	***k***	92.43 (85.47, 99.38)
e	-0.11 (-0.14, -0.07)		

Table 1. Parameters and confidence bounds in brackets of the fitting to (5)

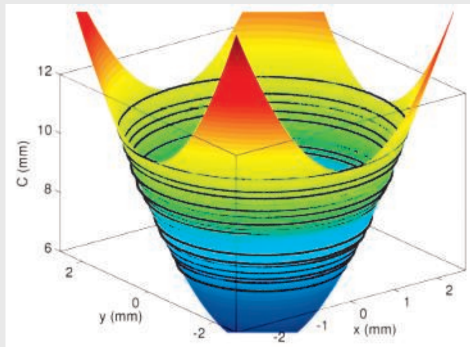

Fig. 1. Data and surface resulting from the fitting

References
[1] J. Espinosa, A.B. Roig, D. Mas, C. Hernández and C. Illueca, *Proc. SPIE Photonics Europe*, 2012.
[2] D. Mas, M.A. Kowalska, J. Espinosa and H. Kasprzak, *J. Mod. Opt.*, **57**, 94-102 2010.

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