Public Economics
Public Goods

Iñigo Iturbe-Ormaetxe

U. of Alicante

2011-12
A public good differs from a private good because it can provide benefits to several users simultaneously, whereas a private good benefits a single user.

A public good that can accommodate any number of users is called a pure public good.

It is impure when there is congestion.

Think of a public radio station versus a road.
Characteristics of Public Goods

- **Non-rivalry** (consumption by one individual does not reduce the quantity available for consumption by others)
- Pure public goods are non-rival
- **Non-excludability** (no individual can be excluded from consuming it except, possibly, at infinite cost)
- Radio vs. teaching
- With PGs, equilibrium outcome is not efficient
Economy with $H$ households ($h = 1, \ldots, H$)

Two goods $X$ and $G$

$X$ is always private, individual $h$ consumes quantity $X^h$

The total quantity of good $X$ in the economy is $X = \sum_h X^h$

Consumption of good $G$ by $h$ is $G^h$, with $G = \sum_h G^h$

Utility of $h$ is $U^h = U^h(X^h, G^h)$
Social welfare function is a weighted sum of utilities

Weight of $h$ is $\beta^h \geq 0$, at least one $\beta^h > 0$

Production possibility frontier is $F(X, G) = 0$

$U^h$ increasing in both $X$ and $G$
Solve the problem:

$$\max \sum_h \beta^h U^h(X^h, G^h)$$

s.t. $F\left(\sum_h X^h, \sum_h G^h\right) \leq 0$ [\lambda]

This is equivalent to maximize $U^1$ subject to $U^h \geq U^0_0$ for all $h \geq 0$ and $F \leq 0$. The Lagrangian is:

$$L = \sum \beta^h U^h - \lambda F$$

First order conditions are:

$$[X^h] : \beta^h U^h_X = \lambda F_X$$

$$[G^h] : \beta^h U^h_G = \lambda F_G$$
Can be simplified into:

\[
MRS_{GX}^h \equiv \frac{U_G^h}{U_X^h} = \frac{F_G}{F_X} \equiv MRT_{GX},
\]

one for each individual \( h \)

All Pareto efficient allocations must satisfy:

\[
MRS_{GX}^h = MRT_{GX},
\]

for each individual \( h \)

From the 1st Welfare Theorem we know that the market equilibrium allocation will be efficient
Now $G$ is the level of PG, the same for everyone

Utility of $h$ is $U^h = U^h(X^h, G)$

Production possibilities frontier $F(X, G) = 0$ same as before
The problem to solve is:

\[
\max \sum_h \beta^h U^h(\chi^h, G)
\]

s.t. \( F(\sum_h \chi^h, G) \leq 0 [\lambda] \)

The FOCs are:

\[
[\chi^h] : \beta^h U^h_{\chi} = \lambda F_{\chi}
\]
\[
[G] : \sum_h \beta^h U^h_G = \lambda F_G
\]

Using \( \beta^h = \lambda F_{\chi} / U^h_{\chi} \) we get:

\[
\sum_h \left[ \frac{U^h_G}{U^h_{\chi}} \right] = \frac{F_G}{F_{\chi}}
\]
Condition for Pareto efficiency: \textbf{sum of} MRS is equal to MRT:

\[ \sum_{h} MRS_{GX}^{h} = MRT_{GX} \]

The intuition is that one additional unit of the public good increases the utility of all individuals. When the good is private, one additional unit increases only the utility of one individual.

Consider that \( \sum_{h} MRS_{GX}^{h} > MRT_{GX} \)
Samuelson Rule

- Samuelson rule simple, but difficult to implement in practice
  - Government needs to know preferences of individuals
  - Moreover, it is the issue of how to finance the PG (cannot use LST)
- Samuelson analysis is first-best

- How can be implemented the optimal level of a PG with the available policy tools?
Private good $X$ and pure public good $G$

Both prices normalized to one

Individual $h$ has initial endowment $Y^h$ of good $X$

**Contribution** of individual $h$ to public good funding is $G^h$

Consumption of PG is $G = \sum_h G^h$ (same for everyone)

Private good consumption is $X^h = Y^h - G^h$ for individual $h$
Individual $h$ solves:

$$\max U^h(X^h, G^h + \sum_{j \neq h} G^j)$$

s.t. $X^h + G^h = Y^h$

Here $h$ takes $\sum_{j \neq h} G^j$ as given (Nash behavior)

In the Nash equilibrium: $U^h_X = U^h_G$

The Samuelson Rule is not satisfied

There is a Pareto improvement if each person invests $1/H$ more euros in the PG:

$$\Delta W = -U^h_X(1/H) + U^h_G = U^h_G(1 - 1/H) > 0$$

The market outcome is inefficient. It entails underprovision of $G$
Lindahl equilibrium

- How to achieve Pareto efficiency through a decentralized mechanism?
- A classical solution is the Lindahl mechanism: Individuals pay shares $\tau^h$ of the PG and can choose a level of $G$
- Individual $h$ chooses $G$ to maximize $U^h(Y^h - \tau^h G, G)$. The FOC is $\tau^h U^h_X = U^h_G$, and we get a demand function $G^h = G^h(\tau^h, Y^h)$
- A Lindahl equilibrium satisfies:
  1. $\sum_h \tau^h = 1$ (the public good is fully financed)
  2. All individuals choose the same $G$
- Since $\frac{U^h_G}{U^h_X} = \tau^h$, $\sum_h \left(\frac{U^h_G}{U^h_X}\right) = \sum_h \tau^h = 1$ ($MRT_{GX}$)
Lindahl: problems

A Lindahl equilibrium is Pareto efficient, but has severe implementation problems

First, we require information on individual preferences to set the personalized prices. It is very likely that individuals will have no incentives to reveal their preferences

Second, we have to be able to exclude individuals from consuming the public good. This does not work with non-excludable public goods

Third, there is no decentralized mechanism for obtaining prices

In practice, the level of a PG is determined by voting on bundles of PGs and taxes
Voting: Brief Discussion

- PG is financed with fixed taxes $\tau^h G$. Individuals vote on $G$ but not on $\tau^h$

- Preferences over $G$ given by $U^h(\gamma^h - \tau^h G, G)$

- A voting equilibrium is a level $G^*$ that cannot be defeated by (pairwise) majority voting by any other alternative $G$

- We know from Condorcet Paradox that majority voting does not always lead to a stable outcome

- Moreover, Arrow’s Theorem says that this is a general problem (not restricted to majority voting)
Single-Peaked Preferences

- Restrict the space of preferences (one of the assumptions of Arrow’s Theorem is that all preferences are allowed)

- Two assumptions:
  1. Unidimensional space
  2. Preferences over $G$ are “single-peaked”
Median Voter Theorem

- With SP preferences, majority voting rule always leads to a voting equilibrium.

- The voting equilibrium coincides with the preferred level of the voter whose preferred level of PG spending is at the median of the distribution.

- To solve, compute the preferred spending level of each individual, $G^h$.

- Majority voting selects the median of distribution of $G^h$. 
In general, the median voter equilibrium is not Pareto efficient:

- Suppose $\tau^h = 1/H$ for all $h$
- Voting outcome: $MRS(G^{med}) = 1/H$
- Samuelson rule: $\sum_h MRS(G^h) / H = 1/H$

Difference between median and mean determines degree of inefficiency

Existence problems with more than one dimension
Private provision leads to under-provision of PGs. However, voluntary contributions to PGs are a large fraction of GDP (in the US, private donations to charities are 2% of GDP)

We study a positive theory of private provision of PGs

Two issues:
- Government provision may crowd out private donations
- Government cannot use LSTs to finance PG

Main reference is Bergstrom, Blume and Varian (1986)
Neutrality result: Redistributions of income among givers that do not change set of givers, do not change the level of privately provided PGs.

Since only a small set of consumers contribute to the public good, assuming interior solutions may be misleading.

LSTs on givers completely crowd out private contributions.

Redistributions that tend to equalize incomes may actually reduce the level of the PG (for instance, if rich people donate more).
Model

- Individual $h$ consumes $X_h$ of private good and donates $G_h \geq 0$ to the PG
- Total amount of PG is $G = \sum G_h$
- Utility is $U_h(X_h, G)$ and wealth is $Y_h (X_h + G_h = Y_h)$. $G_{-h}$ is the sum of all contributions but $h$’s contribution
- A Nash equilibrium is a vector $(G_h^*)$ such that for each $h$, $(X_h^*, G_h^*)$ solves:

$$\max_{\{X_h, G_h\}} U_h(X_h, G_h + G_{-h}^*)$$

s.t. $X_h + G_h = Y_h$, and $G_h \geq 0$
Alternative problem

- Each consumer chooses implicitly the equilibrium level of $G$
- She can decide $G_h = 0$, in which case $G = G_{-h}$ or she can choose $G_h > 0$ which implies $G > G_{-h}$
- The problem can be rewritten as:

$$\max_{\{X_h, G\}} U_h(X_h, G)$$

s.t. $X_h + G = Y_h + G^*_{-h}$, and $G \geq G^*_{-h}$

- Similar to an ordinary consumer choice problem. In figure, point B represents initial endowments. Consumer chooses a point on AB. At the optimal choice, $G^*_h$ is the vertical distance between $G^*$ and $G^*_{-h}$
Consumer choice

Public good (G)

A

G_{-h}^*

Private good

Slope -1

Y_h

Y_h + G_{-h}^*

Public Goods

I. Iturbe-Ormaetxe (U. of Alicante)
Consumer choice

Public good (G)

Private good

A

G*

G-h*

Y_h-G_h*

Y_h

Y_h+G-h*

B

I. Iturbe-Ormaetxe (U. of Alicante)
Consumer choice

If $G = G_{-h}^*$ (or $G_{-h}^* = 0$)

Private good

Public good (G)

$G_{-h}^*$

$G^*$

$G_{h}^*$

$Y_h - G_{h}^*$

$Y_h$

$Y_h + G_{-h}^*$

$A$

I. Iturbe-Ormaetxe (U. of Alicante)
Now suppose government introduces LSTs $t_h$ on each individual $h$.

The revenue is used to finance expenditure on PG: $T = \sum t_h$.

Optimization problem of individual $h$ is now:

$$\max U(X_h, G_h + G^*_h + T)$$

s.t. $X_h + G_h = Y_h - t_h$, and $G_h \geq 0$.

Call $Z_h = G_h + t_h$ total contribution of $h$. 
Since $X_h + G_h + G_{-h} + T = Y_h - t_h + G_{-h} + T = Y_h + Z_{-h}$ and $G_h + G_{-h} + T = G$, we can rewrite the problem as:

$$\max U(X_h, G)$$

s.t. $X_h + G = Y_h + Z_{-h}$, and $G \geq Z_{-h} + t_h$

Total PG provision is unchanged

Each person simply reduces voluntary provision by $t_h$
Complete Crowd-out

\[ \begin{align*}
Y_h - t_h - G_h^* & \quad Y_h \\
Y_h & \quad Y_h + Z_h \\
G^* & \quad Z_h
\end{align*} \]
Complete Crowd-out
1. Total amount of the PG independent of the distribution of income among givers

2. When all individuals have the same preferences and are separable in $X$ and $G$, all givers have the same level of private consumption in equilibrium regardless of their incomes

$$u^1_X(x, G) = u^1_G(x, G) = u^2_G(x, G) = u^2_X(x, G)$$

$$\Rightarrow x_1 = x_2$$

3. As the size of economy gets large, the proportion of individuals who give to the PG goes to zero (Andreoni 1988)
As the economy grows large, the fraction of agents contributing to the public good goes to zero.
Only the very richest individuals will contribute and average giving decreases to zero.
This contrasts with observations about charitable contributions in the US and elsewhere.
BBV Assumptions

1. The set of givers does not change. If the set changes, the transfer neutrality breaks down: a tax increase $T$ results in no private contribution from individuals with $G^h < T$, but contributions increase on net terms.
   - If some taxes are collected from non-givers, although private contributions may change, $G$ must increase.

2. Ignores direct utility from giving: $U(X^h, G^h, G)$. This is in Andreoni (1990) “warm glow” model.
   - Stigler and Becker (1977) critique: should not modify preferences to explain behavior.

3. Ignores prestige/signalling motives.
The motivation is that free-riding is not pervasive. It is found that, in experiments, individuals tend to cooperate.

Also, crowding-out is not complete. In fact, taxes tend to increase the level of public goods.

What Andreoni does is to introduce public good contribution as an argument in the utility function. Now individuals feel better because they give.

Before (BBV), individuals enjoyed giving only indirectly through the effect on the level of the public good. In a large society this effect will be small. Now there is a “warm glow” from giving.

This allows to explain cooperation in public goods provision and incomplete crowding-out.
Two approaches

- The empirical literature tries to measure the degree of crowding-out and also to estimate the income and price effects on giving.
- There are two approaches, to study field data and to run experiments in the lab.
- Both are interesting, although experiments may fail to capture some motives for giving like warm glow, social prestige...
- Kingma (1989) studies individual donations to public radio stations in the USA. He has a sample with 3,451 individuals and 63 different radio stations.
- There were 1,783 contributors and 1,668 non-contributors. Average contribution was $45 in 1986.
The equation to estimate is:

\[ D_i = \beta_0 + \beta_3 G_i + X_i \gamma + \epsilon_i, \]

where \( D_i \) is individual contribution, \( G_i \) is total level of support from other sources (in thousands of dollars) and \( X_i \) are controls.

The estimation he gets is \( \hat{\beta}_3 = -0.015 \). An increase of \$10,000 in government funds (i.e., an increase of 10 in \( G_i \)) reduces \( D_i \) by \(-0.015 \times 10 = 0.15\), that is, a reduction of 15 cents of a dollar.

Given that the average public radio station has 9,000 members, the total crowd-out is \( 0.15 \times 9000 \), that is, \$1,350. The net increase in funding is \$8,650. The ratio of crowding-out is very low.
A problem with this paper is that it is likely that government support is not exogenous.

That is, it may happen that those stations that do not receive private contributions get government support (reverse causality).

This creates a spurious negative correlation between government support and individual contributions.
One possible solution when the dependent variable (private donations) affects one of the other covariates (government support) is to look for an instrument.

Hungerman (2005) studies crowding-out of church-provided welfare by government welfare spending (food stamps, Medicaid, etc.).

He uses as an instrument the 1996 welfare reform act. This law changed eligibility criteria for welfare services from legal residency to legal citizenship.

There was a sudden change in government spending. See how large is the increase in private contributions.
The technique is diffs-in-diffs: to compare churches in communities with larger shares of non-citizens relative to other churches before and after the law change.

Hungerman finds that decreases in government expenditure lead to significant increases in church activity.

The estimated crowding-out effect falls between 20 and 38 cents of a dollar.
Experiments in the lab

- This is the classical experiment by Marwell and Ames (1981)
- Groups of 5 individuals, each with 10 “tokens”
- Each individual can invest tokens in an individual account or in a group account
- One token in my individual account gives me $1, while one token in the public account gives 50 cents to every individual in the group
- Utility for a given individual $i$ is:

  \[ u_i = (10 - g_i) + \frac{1}{2} \sum_{j=1}^{5} g_j \]
Since $\frac{\partial u_i}{\partial g_i} = -(1/2) < 0$, it is a dominant strategy to contribute 0 tokens to public good (free-ride)

- Individual $i$ gets utility $u_i = 10$
- The Pareto efficient outcome is to contribute the 10 tokens to the public good
- Individual $i$ gets utility $u_i = 25$
Marwell and Ames, findings

- They run the experiment with different groups of individuals. Found between 40% and 60% of tokens contributed to the public good.
- In the 12th replication, the group of subjects contributed an average of 20%, significantly less than all other groups.
- This was a group of first-year graduate students in Economics. The title of the paper is: “Economists Free-Ride, Does Anyone Else?”
- Marwell and Ames also asked subjects two questions:
  1. What is a “fair” investment in the PG?
  2. Are you concerned about “fairness” in making your investment decision?
Answers by noneconomists to the first question: 75% said “half or more” of the endowment, and 25% answered “all”. Almost all of them answered “yes” to the second question.

Economists: More than 1/3 either refused to answer the first question. Those who answered were more likely to say that little or no contribution was “fair”.

Economists were about half as likely as others to say that they were “concerned with fairness” when making their decisions.

Problems: high-school and undergraduates versus graduate students; age; gender composition.

When the game is repeated with a fixed set of players, it is typically found that public good contribution levels fall over time. This is found by Isaac, McCue and Plott (1985).
Financing public goods

- Another problem with the Samuelson rule is that governments cannot use lump sum taxes because of redistributional concerns.
- With distortionary taxes, the total cost of providing public goods will be higher than the production cost. Then, the optimal level of the public good will be lower than in the first best.
- We follow Atkinson and Stern (1974).
Model

- There is a large number of identical individuals
- Utility depends on consumption \((c)\), labor \((l)\) and public good \((G)\):

\[
u(c, l, G) = c - \frac{l^{k+1}}{k+1} + v(G),\]

where \(v(\cdot)\) is concave and \(k > 0\)
- Prices of \(c\) and \(G\) are 1
- Individuals do not contribute to the public good because their contributions have a negligible effect on the level of the public good
- Individual constraint is \(c = wl(1 - \tau) - R\), where \(R\) is a LST. Taxes are \(T = \tau wl + R\)
The consumer has to choose $l$ to maximize:

$$wl(1 - \tau) - R - \frac{l^{k+1}}{k+1} + \nu(G)$$

The solution is $l = w^e(1 - \tau)^e$, where $e = 1/k$ is the elasticity of $l$ with respect to $(1 - \tau)$.

The level of public good is simply the value of tax revenue:

$$G = wl\tau + R$$
When the government can use LSTs, the problem is:

\[
\max_{R,\tau} W = wl(1 - \tau) - R - \frac{lk^{k+1}}{k+1} + \nu(wl\tau + R)
\]

The FOC of \( R \) is \( \nu'(G) = 1 \), this is the Samuelson rule.

We also have that, in the first best, \( \tau^* = 0 \). To see this, we check that \( \frac{\partial W}{\partial \tau} \) is:

\[
-wl + wlv'(G) + w\tau v'(G) \frac{\partial l}{\partial \tau} = w\tau v'(G) \frac{\partial l}{\partial \tau}
\]

Then, \( \frac{\partial W(\tau^*)}{\partial \tau} = 0 \) implies that \( \tau^* = 0 \).
Now government cannot use LSTs \((R = 0)\). Government solves:

\[
\max_{\tau} W = w(1 - \tau) - \frac{l^{k+1}}{k+1} + \nu(wl/\tau)
\]

The FOC (using the Envelope Theorem) is:

\[
0 = \frac{\partial W}{\partial \tau} = -wl + \nu'(G)\left(wl + w\tau \frac{\partial l}{\partial \tau}\right)
\]

Note that the (additional) term \(w(1 - \tau)\frac{\partial l}{\partial \tau} - l^k\) is zero.
Note that $\frac{\partial l}{\partial \tau} = -\frac{\partial l}{\partial (1-\tau)}$. Then:

$$wl = v'(G) \left( wl - w\tau \frac{\partial l}{\partial (1-\tau)} \frac{(1-\tau)}{l} \frac{l}{(1-\tau)} \right)$$

We call $e = \frac{\partial l}{\partial (1-\tau)} \frac{(1-\tau)}{l}$ the elasticity of labor supply with respect to $(1-\tau)$.
Dividing by $wl$ we get:

$$1 = v'(G) \left[ 1 - \frac{\tau}{1-\tau} e \right]$$

- We have the additional term $\frac{\tau}{1-\tau} e$ that was not in Samuelson rule
- Since the term in brackets is lower than 1, $v'(G) > v'(G^*)$, and then $G < G^*$ ($v$ is concave)
- How far is $G$ from $G^*$ depends on $e$
Heterogeneous individuals

- Things are more complicated when individuals have different preferences
- Gaube (2000) proves that with redistributive preferences, it may happen that $G > G^*$
- The reason is that when the public good benefits mainly the poor, over-provision is a way of redistributing more