Public Economics
Taxation I: Incidence and Efficiency Costs

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Winter 2012
Tax Composition, 2009

France  Germany  Spain  United Kingdom  United States

Direct (individuals)  Corporate  Social security contributions  Indirect taxes
Main functions of taxation

- Main functions of taxation: raising revenue, redistributing income, and correcting externalities
- According to Stiglitz, there are 5 desirable characteristics of any tax system: economic efficiency, administrative simplicity, flexibility, political responsibility, and fairness
- Here we deal mostly with **efficiency**. Most taxes change relative prices, distorting price signals, and altering the allocation of resources. By efficiency we mean to reduce distortions as much as possible
- A tax is nondistortionary if and only if there is nothing an individual or firm can do to alter his tax liability (a **lump-sum tax (LST)**)
Other characteristics

- **Administrative simplicity** means to avoid complex tax systems. For instance, the number of pages in the instruction book of the US income tax increased from 84 pages in 1995 to 142 pages in 2005 and to 189 pages in 2012.

- **Flexibility**: the tax system should respond easily (automatically) to changed economic circumstances.

- **Political responsibility**: individuals should be aware of the taxes they are paying. This is important because the government can sometimes misrepresent the true costs of the services and who bears the costs (related to tax incidence).

- **Fairness**: the tax system should treat fairly different individuals.
Definition

- Tax incidence studies the effects of tax policies on prices and on welfare distribution.

- What happens to market prices when a tax is introduced or changed?

- For example, some people say the government should tax capital income because it is concentrated among the rich.

- Neglects general equilibrium price effects:
  - Tax might be shifted onto workers.
  - If capital taxes, less savings and capital flights, capital stock may decline, driving return to capital up and wages down.
  - Some argue that capital taxes are paid by workers and therefore increase income inequality.
Limits of theory

- Tax incidence is an example of positive analysis

- Theory is informative about signs and comparative statics but inconclusive about magnitudes
  
  - Incidence of cigarette tax: elasticity of demand with respect to price is crucial
  
  - Labor vs. capital taxation: mobility of labor, capital are critical
Partial Equilibrium: Assumptions

1. **Two good economy**
   - Only one relative price → partial and general equilibrium are same
   - A good approximation to a multi-good model if:
     - the market being taxed is “small”
     - there are no close substitutes/complements in the utility function

2. **Tax revenue is not spent on the taxed good**
   - It is used to buy untaxed good or thrown away

3. **Perfect competition among producers**
Two goods: $x$ and $y$

Government levies an **excise tax** on good $x$ (levied on a quantity). Compare to an **ad-valorem tax**, a fraction of prices (e.g. VAT)

Call $p$ the pretax price of $x$ and $q = p + t$ the tax inclusive price of $x$. The consumer pays $q$ and the producer gets $p$

Good $y$, the numeraire, is untaxed
Partial Equilibrium: Demand and Supply

- Consumer has wealth $Z$ and utility $u(x, y)$

- Price elasticity of demand is $\varepsilon_D = \frac{\partial D}{\partial p} \frac{q}{D(p)}$

- Price-taking firms: They use $c(S)$ units of $y$ to produce $S$ units of $x$

- Cost of production increasing and convex: $c'(S) > 0$ and $c''(S) \geq 0$

- Profit at pretax price $p$ and level of supply $S$ is $pS - c(S)$

- Supply function for $x$ implicitly defined by $p = c'(S(p))$

- Price elasticity of supply is $\varepsilon_S = \frac{\partial S}{\partial p} \frac{p}{S(p)}$
The equilibrium condition:

\[ Q = S(p) = D(p + t) \]

defines an equation \( p(t) \)

- We want to compute \( \frac{dp}{dt} \), the effect of a tax increase on price
- First some graphical examples
Tax Levied on Producers

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Incidence and Efficiency Costs

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Source: Gruber (2007)
Tax Levied on Producers

(a)

Price per gallon ($P$)

$P_1 = $1.50

Quantity in gallons ($Q$)

$Q_1 = 100$

(b)

Price per gallon ($P$)

$P_2 = $1.80

Quantity in gallons ($Q$)

$Q_2 = 90$

Consumer burden = $0.30

Supplier burden = $0.20

Source: Gruber (2007)
Tax Levied on Consumers

\[ P_2 = $1.30 \]
\[ P_1 = $1.50 \]
\[ Q_1 = 100 \]
\[ Q_2 = 90 \]

\[ D_1 \]
\[ S \]
\[ D_2 \]
\[ $1.00 \]
\[ $0.50 \]

Incidence and Efficiency Costs

Source: Gruber (2007)
Perfectly Inelastic Demand

- $P_2 = $2.00
- $P_1 = $1.50
- $Q_1 = 100$

Source: Gruber (2007)
Perfectly Elastic Demand

\[ P_1 = \$1.50 \]
\[ Q_1 = 100 \]
\[ Q_2 = 90 \]

\[ D \]
\[ S_1 \]
\[ S_2 \]

$0.50
Price per gallon (P)
Quantity in billions of gallons (Q)

$1.00
Supplier burden

Source: Gruber (2007)
Effect of Elasticities

(a) Tax on steel producer

(b) Tax on street vendor

Source: Gruber (2007)
Formula for Tax Incidence

- To calculate $\frac{dp}{dt}$ we apply the IFT to the equilibrium condition:

$$D(p + t) - S(p) \equiv 0.$$ 

- We get (on producer price):

$$\frac{dp}{dt} = - \frac{\partial D}{\partial p} - \frac{\partial S}{\partial p} = \frac{\varepsilon_D}{\varepsilon_S - \varepsilon_D} \leq 0$$

- To find incidence on consumers price:

$$\frac{dq}{dt} = 1 + \frac{dp}{dt} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D} \geq 0$$
1 – excess supply of E created by imposition of tax

2 – re-equilibration of market through producer price cut

\[ dp = E / \left( \frac{\partial S}{\partial p} - \frac{\partial D}{\partial p} \right) \]

\[ \Rightarrow dp/dt = \frac{\partial D}{\partial p} / \left( \frac{\partial S}{\partial p} - \frac{\partial D}{\partial p} \right) \]

\[ E = dt \times \frac{\partial D}{\partial p} \]
In most countries, social security payroll taxes are split between employees and employers. However, economists think that workers bear the entire burden of the tax because employers pass the tax on in the form of lower wages. Tax incidence falls on employees only.

We present a partial equilibrium model. Consider a simple model of the labor market.

The wage tax is $\tau$, the net wage is $w$, and the gross tax is $W = (1 + \tau)w$. 
Competitive case

- Let $D(w)$ and $S(w)$ be the labor demand and supply, respectively. Equilibrium is:
  \[ D((1 + \tau)w) \equiv S(w) \]

- Differentiate with respect to $\tau$:
  \[ D'(dw(1 + \tau) + wd\tau) \equiv S'\, dw \]

- Take $\tau = 0$ and divide by $wd\tau$:
  \[ D'(\frac{dw}{wd\tau} + 1) \equiv S' \frac{dw}{wd\tau} \]

- Using the fact that $\frac{dw}{wd\tau} = \frac{d\ln w}{d\tau}$:
  \[ \frac{d\ln w}{d\tau} = \frac{D'}{S' - D'} = \frac{\epsilon_D}{\epsilon_S - \epsilon_D} < 0, \]

  where $\epsilon_D = \frac{wD'}{D}$ and $\epsilon_S = \frac{wS'}{S}$
Economic incidence

- We see that the **economic** incidence depends on the behavior of labor demand and supply.
- A larger supply elasticity means a smaller fall in the net wage to workers. A large demand elasticity has the opposite effect.
- Since the gross wage is $W = w(1 + \tau)$, we get:
  \[
  \frac{d \ln W}{d\tau} = \frac{\epsilon_S}{\epsilon_S - \epsilon_D}
  \]

  - The gross wage increases more when demand is relatively less elastic than supply.
- Finally, the effect on employment is:
  \[
  \frac{d \ln D}{d\tau} = \frac{\epsilon_S \epsilon_D}{\epsilon_S - \epsilon_D} = \frac{1}{\frac{1}{\epsilon_D} - \frac{1}{\epsilon_S}}
  \]
Each side of the market tries to avoid the tax by changing behavior. If $\epsilon_S$ is large, there is a smaller fall in the net wage. Extreme cases are $\epsilon_S = 0$ and $\epsilon_D = -\infty$. Full incidence on workers.

Important lesson: in markets with no impediments to market clearing, the economic incidence of the tax depends only on behavior ($\epsilon_S$ and $\epsilon_D$) and not on legislative intent (statutory incidence).

The general rule is that the more inelastic side of the market bears the greater part of the tax burden.
The above analysis neglects effects of taxes on other prices, neglecting the possibility of substitution between commodities. For example, higher taxes on labor may induce firms to substitute labor with capital. Also leave aside question of what is done with revenue raised by taxes. Seminal paper on general equilibrium theory of taxation is Harberger (1962).
1. Two outputs \((X_1, X_2)\) and two factors \((L, K)\)
2. Fixed total supplies of factors (short-run, closed economy)
3. Separate firms produce \(X_1\) and \(X_2\)
4. Constant returns to scale in both sectors
5. Firms are perfectly competitive (profits are zero)
6. Single representative consumer who owns the firms and has homothetic preferences
7. Zero initial taxes
Harberger Model: Setup

- Production in sectors 1 (bikes) and 2 (cars):
  \[ X_1 = F_1(K_1, L_1) = L_1 f(k_1) \]
  \[ X_2 = F_2(K_2, L_2) = L_2 f(k_2) \]

  with full employment conditions \( K_1 + K_2 = K \) and \( L_1 + L_2 = L \)

- Factors \( K \) and \( L \) fully mobile. In equilibrium returns must be equal:
  \[ w = p_1 F_{1L} = p_2 F_{2L} \]
  \[ r = p_1 F_{1K} = p_2 F_{2K} \]

- Demand functions for goods 1 and 2:
  \[ X_1 = X_1(p_1 / p_2) \] and \[ X_2 = X_2(p_1 / p_2) \]

- System of ten equations and ten unknowns: \( K_i, L_i, p_i, X_i, w, r \)
Introduce small tax \( d\tau \) on capital income in sector 2 \((K_2)\)

All equations the same as above except \( r = (1 - d\tau)p_2F_{2K} \)

Linearize the 10 equations around initial equilibrium to compute the effect of \( d\tau \) on all 10 variables \((dw, dr, dL_1, ...)\)

Labor income = \( wL \) with \( L \) fixed, \( rK \) = capital income with \( K \) fixed

Therefore change in prices \( dw/d\tau \) and \( dr/d\tau \) describes how tax is shifted from capital to labor

Changes in prices \( dp_1/d\tau, dp_2/d\tau \) describe how tax is shifted from Sector 2 to Sector 1
1. **Substitution effects**: capital bears incidence

- There is a reduction of capital demand in the taxed sector (Sector 2)
- Aggregate demand for $K$ goes down
- Because total $K$ is fixed, $r$ falls. This implies that $K$ bears some of the burden
2. **Output effects**: capital may not bear incidence

- There is a reduction in demand for output in taxed sector (Sector 2) because of its tax-induced increase in relative prices \( p_2 / p_1 \)

- Therefore demand shifts toward sector 1

**Case 1**: \( K_1 / L_1 < K_2 / L_2 \) (1: bikes, 2: cars)

  - Taxed sector (Sector 2) is more capital intensive, so aggregate demand for \( K \) goes down

  - Output effect reinforces substitution effect: \( K \) bears the burden of the tax

**Case 2**: \( K_1 / L_1 > K_2 / L_2 \) (1: cars, 2: bikes)

  - Untaxed sector (Sector 1) is more capital intensive, aggregate demand for \( K \) increases

Substitution and output effects have opposite signs, labor may bear
3. **Substitution + Output = Overshifting effects**

- **Case 1:** $K_1/L_1 < K_2/L_2$
  - Can get overshifting of tax, $dr < -d\tau$ and $dw > 0$
  - Capital bears more than 100% of the burden if output effect sufficiently strong
  - Taxing capital in sector 2 raises prices of cars → more demand for bikes, less demand for cars
  - With very elastic demand (two goods are highly substitutable), demand for labor rises sharply and demand for capital falls sharply
  - Capital loses more than direct tax effect and labor suppliers gain
3. **Substitution + Output = Overshifting effects**

- **Case 2**: $K_1/L_1 > K_2/L_2$
  - Possible that capital is made better off by capital tax
  - Labor forced to bear more than 100% of incidence of capital tax in sector 2
  - Example: Consider tax on capital in bike sector: demand for bikes falls, demand for cars rises
  - Capital in greater demand than it was before $\rightarrow$ price of labor falls substantially, capital owners actually gain
  - Bottom line: taxed factor may bear less than 0 or more than 100% of tax.
Harberger Two Sector Model

- Theory not very informative: model mainly used to illustrate negative result that “anything goes”

- More interest now in developing methods to identify what actually happens

- Harberger analyzed two sectors; subsequent literature expanded analysis to multiple sectors

- Analytical methods infeasible in multi-sector models. Instead, use numerical simulations to investigate tax incidence effects after specifying full model

- Pioneered by Shoven and Whalley
Criticism of CGE Models

- Findings very sensitive to structure of the model: savings behavior, perfect competition assumption
- Findings sensitive to size of key behavioral elasticities and functional form assumptions
Open Economy Application

- Key assumption in Harberger model: both labor and capital perfectly mobile across sectors

- Now apply framework to analyze capital taxation in open economies, where capital is more likely to be mobile than labor

- One good, two-factor, two-sector model

- Sector 1: small open economy where $L_1$ is fixed and $K_1$ mobile

- Sector 2: rest of the world $L_2$ fixed and $K_2$ mobile

- Total capital stock $K = K_1 + K_2$ is fixed
Small country introduces tax on capital income ($K_1$)

After-tax returns must be equal:

$$r^* = F_{2K} = (1 - \tau) F_{1K}$$

Capital flows from 1 to 2 until returns are equalized; if 2 is large relative to 1, no effect on $r^*$

Wage rate $w_1 = F_{1L}(K_1, L_1)$ falls when $K_1$ reduces because $L_1$ is fixed

Return of capitalists in small country is unchanged; workers in home country bear the burden of the tax

Taxing capital is bad for workers!
Mobility of $K$ drives the previous result

Empirical question: is $K$ actually perfectly mobile across countries?

Still an open question. It is found increasing movement of capital over time

General view: cannot extract money from capital in small open economies

Example: In Europe tax competition has led to lower capital tax rates
Government raises taxes for one of two reasons:

1. To raise revenue to finance public goods
2. To redistribute income

To generate 1 euro of revenue, the welfare of those taxed is reduced by more than 1 euro because taxes distort incentives and behavior.

Crucial question: how to implement policies that minimize these efficiency costs

- This framework for optimal taxation can be also adapted to study transfer programs, social insurance, etc.
- Start with positive analysis of how to measure efficiency cost of a given tax system.
Most basic analysis of efficiency costs is based on Marshallian surplus

Two critical assumptions:

1. Quasilinear utility (no income effects)
2. Competitive production
Partial Equilibrium Model: Setup

- Two goods: $x$ and $y$

- Consumer has wealth $Z$, utility $u(x) + y$, and solves
  \[
  \max_{x,y} u(x) + y \text{ s.t. } (p + t)x(p + t, Z) + y(p + t, Z) = Z
  \]

- Firms use $c(S)$ units of the numeraire $y$ to produce $S$ units of $x$

- Marginal cost of production is increasing and convex:
  \[
  c'(S) > 0 \text{ and } c''(S) \geq 0
  \]

- Firm’s profit at pretax price $p$ and level of supply $S$ is
  \[
  pS - c(S)
  \]
Model: Equilibrium

- With perfect optimization, supply function for $x$ is implicitly defined by the marginal condition

\[ p = c'(S(p)) \]

- Let $\eta_S = p \frac{S'}{S}$ denote the price elasticity of supply

- Let $Q$ denote equilibrium quantity sold of good $x$

- $Q$ satisfies:

\[ Q(t) = D(p + t) = S(p) \]

- Consider effect of introducing a small tax $dt > 0$ on $Q$ and surplus
Excess Burden of Taxation

Source: Gruber (2007)
Excess Burden of Taxation

Price per gallon ($P$)

- $P_2 = $1.80
- $P_1 = $1.50

Quantity ($Q$)

- $Q_2 = 90$
- $Q_1 = 100$

$EB$

Source: Gruber (2007)
Efficiency Cost: Marshallian Surplus

- Excess burden (EB) is the area of a triangle: \( EB = \frac{1}{2} dQ dt \)
- From \( Q = S(p) \rightarrow dQ = S'(p) dp \):
  \[
  EB = \frac{1}{2} S'(p) dp dt = \frac{1}{2} \frac{pS'}{S} \frac{S}{p} dp dt
  \]
- Differentiating the equilibrium condition: \( dp = \left( \frac{\eta_D}{\eta_S - \eta_D} \right) dt \)
- We plug into the equation above:
  \[
  EB = \frac{1}{2} \frac{pS'}{S} \frac{S}{p} \left( \frac{\eta_D}{\eta_S - \eta_D} \right) (dt)^2 = \frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D} pQ \left( \frac{dt}{p} \right)^2
  \]
- Dividing by tax revenue (\( R = Q dt \)), get expression of deadweight burden per euro of tax revenue:
  \[
  \frac{EB}{R} = \frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D} \frac{dt}{p}
  \]
If \( p = 1, \eta_S = 0.2, \eta_D = -0.5, \frac{EB}{R} \approx -0.07 \, dt \) (7 cents per euro of tax)

\[
EB = \frac{1}{2} \frac{\eta_S \eta_D}{\eta_S - \eta_D} pQ \left( \frac{dt}{p} \right)^2
\]

1. Excess burden increases with square of tax rate

2. Excess burden increases with elasticities (if either \( \eta_S = 0 \) or \( \eta_D = 0 \), \( EB = 0 \))
EB Increases with Square of Tax Rate

\[ \text{Source: Gruber (2007)} \]
EB Increases with Square of Tax Rate

\[ P \]

\[ Q \]

\[ P_2 \]

\[ P_1 \]

\[ Q_2 \]

\[ Q_1 \]

\[ S_1 \]

\[ S_2 \]

\[ B \]

\[ A \]

\[ C \]

$0.10

Source: Gruber (2007)
EB Increases with Square of Tax Rate

Source: Gruber (2007)
Comparative Statics

(a) Inelastic Demand

(b) Elastic demand

Source: Gruber (2007)
With many goods, formula suggests that the most efficient way to raise tax revenue is:

1. Tax relatively more the inelastic goods (e.g. medical drugs, food)

2. Spread taxes across all goods so as to keep tax rates relatively low on all goods (broad tax base)
Marshallian surplus is not a good measure of welfare with income effects

We drop quasilinearity and consider an individual with utility:
\[ u(c_1, \ldots, c_N) = u(c) \]

Individual program:
\[ \max_c u(c) \text{ s.t. } qc \leq Z \]
where \( q = p + t \) denotes vector of tax-inclusive prices and \( Z \) is wealth

Labor can be viewed as commodity with price \( w \) and consumed in negative quantity
The Lagrangian for individual’s problem is:

\[ \mathcal{L} = u(c_1, \ldots, c_N) + \lambda (Z - q_1 c_1 - \ldots - q_N c_N), \]

where \( \lambda \) is the Lagrange multiplier

First order condition in \( c_i \):

\[ \frac{\partial u}{\partial c_i} = \lambda q_i \]

From these conditions we get (implicitly):

- \( c_i(q, Z) \): the Marshallian (or uncompensated) demand function
- \( v(q, Z) \): the indirect utility function
Useful Properties

- Multiplier on budget constraint $\lambda = \frac{\partial v}{\partial Z}$ is the marginal utility of wealth.

- Give additional wealth $dZ$ to consumer. The effect on utility is:
  
  \[ du = \sum_i \frac{\partial u}{\partial c_i} dc_i = \lambda \sum_i q_i dc_i = \lambda dZ \]

- Roy’s identity ($c_i = -\frac{\partial v}{\partial q_i} / \frac{\partial v}{\partial Z}$) implies $\frac{\partial v}{\partial q_i} = -\lambda c_i$

- Welfare effect of a price change $dq_i$ same as reducing wealth by:
  
  \[ dZ = c_i dq_i \]
Path Dependence Problem

- Initial price vector $q^0$

- Taxes levied on goods → price vector now $q^1$

- Change in Marshallian surplus is defined as the integral:

$$ CS = \int_{q^0}^{q^1} c(q, Z) dq $$

- With one price changing, this is area under the demand curve

- Problem: $CS$ is **path dependent** with $> 1$ price changes

- Consider change from $q^0$ to $\tilde{q}$ and then $\tilde{q}$ to $q^1$:

$$ CS(q^0 \rightarrow \tilde{q}) + CS(\tilde{q} \rightarrow q^1) \neq CS(q^0 \rightarrow q^1) $$
Example of path dependence with taxes on two goods:

\[ CS_1 = \int_{q_1^0}^{q_1^1} c_1(q_1, q_2^0, Z) dq_1 + \int_{q_2^1}^{q_2^0} c_2(q_1^1, q_2, Z) dq_2 \quad (1) \]

\[ CS_2 = \int_{q_2^0}^{q_2^1} c_2(q_1^0, q_2, Z) dq_2 + \int_{q_1^0}^{q_1^1} c_1(q_1, q_2^1, Z) dq_1 \quad (2) \]

For \( CS_1 = CS_2 \), need \( \frac{dc_2}{dq_1} = \frac{dc_1}{dq_2} \)

With income effects, this symmetry condition is not satisfied in general.
Path-dependence problem reflects the fact that consumer surplus is an ad-hoc measure.

- It is not derived from utility function or a welfare measure.

Question of interest: how much utility is lost because of tax beyond revenue transferred to government?

- Need units to measure “utility loss”

- Introduce expenditure function to translate the utility loss into euros (money metric)
Expenditure Function

- Fix utility at $U$ and prices at $q$
- Find bundle that minimizes expenditure to reach $U$ for $q$:
  \[ e(q, U) = \min_c q c \text{ s.t. } u(c) \geq U \]
- Let $\mu$ denote multiplier on utility constraint
- First order conditions given by:
  \[ q_i = \mu \frac{\partial u}{\partial c_i} \]
- These generate Hicksian (or compensated) demand functions:
  \[ c_i = h_i(q, u) \]
- Define individual’s loss from tax increase as
  \[ e(q^1, u) - e(q^0, u) \]
- Single-valued function $\rightarrow$ coherent measure of welfare cost, no path dependence
But where should $u$ be measured?

Consider a price change from $q^0$ to $q^1$

Initial utility:
$$u^0 = v(q^0, Z)$$

Utility at new price $q^1$:
$$u^1 = v(q^1, Z)$$

Two concepts: compensating ($CV$) and equivalent variation ($EV$) use $u^0$ and $u^1$ as reference utility levels
Compensating Variation

- Measures utility at initial price level \((u^0)\)

- Amount agent must be compensated in order to be indifferent about tax increase

\[
CV = e(q^1, u^0) - e(q^0, u^0) = e(q^1, u^0) - Z
\]

- How much compensation is needed to reach original utility level at new prices?

- \(CV\) is amount of ex-post cost that must be covered by government to yield same \textit{ex-ante} utility:

\[
e(q^0, u^0) = e(q^1, u^0) - CV
\]
Equivalent Variation

- Measures utility at new price level

- Lump sum amount agent willing to pay to avoid tax (at pre-tax prices)
  \[ EV = e(q^1, u^1) - e(q^0, u^1) = Z - e(q^0, u^1) \]

- \( EV \) is amount extra that can be taken from agent to leave him with same ex-post utility:
  \[ e(q^0, u^1) + EV = e(q^1, u^1) \]
Goal: derive empirically implementable formula analogous to Marshallian EB formula in general model with income effects

Existing literature assumes either

1. Fixed producer prices and income effects
2. Endogenous producer prices and quasilinear utility

With both endogenous prices and income effects, efficiency cost depends on how profits are returned to consumers

Formulas are very messy and fragile
Efficiency Cost Formulas with Income Effects

- Derive empirically implementable formulas using Hicksian demand ($EV$ and $CV$)

- Assume $p$ is fixed $\rightarrow$ flat supply, constant returns to scale

- From Micro I we know that $\frac{\partial e(q,u)}{\partial q_i} = h_i$, and so:

$$\int_{q^0}^{q^1} h(q,u) dq = e(q^1, u) - e(q^0, u)$$

- If only one price is changing, this is the area under the Hicksian demand curve for that good

- Recall that optimization implies:

$$h(q, v(q, Z)) = c(q, Z)$$
Compensating vs. Equivalent Variation

Figure 2.2. Compensating and equivalent variations.

Source: Auerbach (1985)
Compensating vs. Equivalent Variation

Figure 2.2. Compensating and equivalent variations.

Source: Auerbach (1985)
Compensating vs. Equivalent Variation

Figure 2.2. Compensating and equivalent variations.

Source: Auerbach (1985)
Marshallian Surplus

Figure 2.2. Compensating and equivalence variations.

Source: Auerbach (1985)
Path Independence of EV, CV

- With one price change:

\[ EV < \text{Marshallian Surplus} < CV \]

but this is not true in general

- No path dependence problem for EV and CV measures with multiple price changes

  - Slutsky equation:

\[
\frac{\partial h_i}{\partial q_j} = \frac{\partial c_i}{\partial q_j} + c_j \frac{\partial c_i}{\partial Z}
\]

  - Hicksian Slope  \hspace{1cm}  Marshallian Slope  \hspace{1cm}  Income Effect

  - Optimization implies Slutsky matrix is symmetric: \( \frac{\partial h_i}{\partial q_j} = \frac{\partial h_j}{\partial q_i} \)

- Therefore the integral \( \int_{q_0}^{q_1} h(q, u) \, dq \) is path independent
Deadweight burden: change in consumer surplus less tax paid

Equals what is lost in excess of taxes paid

Two measures, corresponding to $EV$ and $CV$:

\[
EB(u^1) = EV - (q^1 - q^0)h(q^1, u^1)
\]
\[
EB(u^0) = CV - (q^1 - q^0)h(q^1, u^0)
\]
Source: Auerbach (1985)
Incidence and Efficiency Costs

Source: Auerbach (1985)
In general, $CV$ and $EV$ measures of $EB$ will differ.

Marshallian measure overstates excess burden because it includes income effects.

- Income effects are not a distortion in transactions.
- Buying less of a good due to having less income is not an efficiency loss; no surplus foregone because of transactions that do not occur.

$CV = EV = $ Marshallian DWL only with quasilinear utility.
Implementable Excess Burden Formula

- Consider increase in tax $\tau$ on good 1 to $\tau + \Delta \tau$

- No other taxes in the system

- Recall the expression for $EB$:

$$EB(\tau) = [e(p + \tau, U) - e(p, U)] - \tau h_1(p + \tau, U)$$

- Second-order Taylor expansion:

$$MEB = EB(\tau + \Delta \tau) - EB(\tau)$$

$$\approx \frac{dEB}{d\tau}(\Delta \tau) + \frac{1}{2}(\Delta \tau)^2 \frac{d^2EB}{d\tau^2}$$
Harberger Trapezoid Formula

\[
\frac{dEB}{d\tau} = h_1(p + \tau, U) - \tau \frac{dh_1}{d\tau} - h_1(p + \tau, U)
\]

\[
= -\tau \frac{dh_1}{d\tau}
\]

\[
\frac{d^2EB}{d\tau^2} = -\frac{dh_1}{d\tau} - \tau \frac{d^2h_1}{d\tau^2}
\]

- Standard practice in literature: assume \( \frac{d^2h_1}{d\tau^2} = 0 \) (linear Hicksian)

\[
\Rightarrow MEB \approx -\tau \Delta \tau \frac{dh_1}{d\tau} - \frac{1}{2} \frac{dh_1}{d\tau} (\Delta \tau)^2
\]

- Formula equals area of “Harberger trapezoid” using Hicksian demands
Without pre-existing tax, obtain “standard” Harberger formula:

\[ EB = -\frac{1}{2} \frac{dh_1}{d\tau} (\Delta \tau)^2 \]

Observe that first-order term vanishes when \( \tau = 0 \)

A new tax has second-order deadweight burden (proportional to \( \Delta \tau^2 \) not \( \Delta \tau \))

Bottom line: need compensated (substitution) elasticities to compute \( EB \), not uncompensated elasticities

Empirically, need estimates of income and price elasticities
With multiple goods and fixed prices, the excess burden of introducing a tax $\tau_k$ is:

$$EB = -\frac{1}{2} \tau_k^2 \frac{dh_k}{d\tau_k} - \sum_{i \neq k} \tau_i \tau_k \frac{dh_i}{d\tau_k}$$

- There are additional effects on other markets
- This formula is complicated to compute because we need all cross-price elasticities
- In practical applications, many times the term on the right is ignored
- Goulder and Williams (2003) prove that this may underestimate the excess burden
Taxes distort many economic decisions: labor supply, savings, and risk-taking among others.

According to theory only, income taxes have an ambiguous effect on labor supply. Recall that the theoretical effect of wages is also ambiguous: higher wages make labor more attractive relative to leisure (substitution effect), but also increase the demand for leisure (income effect), if leisure is normal.

Consumer has utility $U(C, L)$, $C$ : consumption, $L$ : labor. Non-labor income is $R$, wage before tax is $w$, the tax rate is $t$. The budget constraint is:

$$C \leq (1 - t)(wL + R) \equiv sL + M,$$

where $s = (1 - t)w$ and $M = (1 - t)R$. We are assuming a proportional tax that applies to all sources of income.
Increasing $t$

- Increasing $t$ has three effects:
  1. It reduces $M$. If leisure is normal, this increases labor supply (income effect)
  2. It reduces $s$, with the same effect as in 1 (income effect)
  3. The reduction in the relative price of labor ($s$), makes labor less attractive, reducing labor supply (substitution effect)

- To see the overall effect:
  \[
  \frac{\partial L}{\partial t} = \frac{\partial L}{\partial s} \frac{\partial s}{\partial t} + \frac{\partial L}{\partial M} \frac{\partial M}{\partial t} = -w \frac{\partial L}{\partial s} - R \frac{\partial L}{\partial M}
  \]

- Using the Slutsky equation:
  \[
  \frac{\partial L}{\partial s} = S + L \frac{\partial L}{\partial M}
  \]

  where $S$ is the Slutsky term which is positive (on labor, negative on leisure) because holding utility fixed there is just a substitution effect
The final effect is:

\[ \frac{\partial L}{\partial t} = -w \left( S + L \frac{\partial L}{\partial M} \right) - R \frac{\partial L}{\partial M} = -wS - (wL + R) \frac{\partial L}{\partial M} \]

- The first term is the substitution effect which is negative.
- If leisure is normal the second term is positive.
- Since we have income multiplying in the second term, we should expect that the overall effect will be negative for the poor and positive for the rich. This says that the negative effect of taxation on labor supply will be observed mainly on the poor.
Fixed hours of work

- Suppose workers can only choose between work a fixed numbers of hours $L$ or not working at all.
- Consider a simplified utility function where $U(C, L) = u(C) - v(L)$, with $v(0) = 0$
- If an individual decides to work $L$, consumption is $C = (1 - t)(wL + R)$. If she does not work, then $C = (1 - t)R$. An individual will work if and only if:
\[
 u((1 - t)(wL + R)) - v(L) - u((1 - t)R) \geq 0
\]
- What matters is not the marginal rate, but the average rate. The derivative of this term with respect to $t$ is:
\[
 -(wL + R)u'((1 - t)(wL + R)) + Ru'((1 - t)R)
\]
- If the term $xu'(x)$ is increasing in $x$, the above term will be negative implying that participation will be decreasing with $t$
Summary of empirical results

- Small elasticities for prime-age males
- Larger elasticities for workers who are less attached to labor force: married women, low skilled workers, retirees
- Responses driven mainly by extensive margin
- Extensive margin elasticity between 0.2 and 0.5
- Intensive margin (hours) elasticity close to 0
- Overall, the compensated elasticity seems to be small implying that the efficiency cost of taxing labor income is low
- Problems: there are many other behavioral responses to increasing taxation and not only hours of work. There is a literature on this (elasticity of taxable income). We will go back to this