Three Essays on Dynamic Processes and Information Flow on Social Networks

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To my family.
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Chapter 1

Introducción

Con la constitución de la economía de mercado moderna en el siglo XIX, científicos sociales pensaban que la producción y asignación de los bienes económicos se realizarían principalmente por los procesos impersonales del Mercado. Por ejemplo, Polanyi (1957) describe en detalles como los diferentes mercados del siglo XIX se crearon bajo la ruptura de las estructuras personales de la economía tradicional. Por otro lado, en la segunda parte del siglo XX científicos que pertenecían al grupo de investigación de la nueva sociología económica han demostrado que el capital social sigue teniendo una gran relevancia en la economía moderna (véase por ejemplo Coleman (1988), Bourdieu (1990)). El análisis de las redes sociales es una de las posibles maneras de estudiar como las relaciones interpersonales influyen en las variables económicas. En su libro clásico Granovetter (1995) estudia el papel de las redes sociales para facilitar la búsqueda satisfactoria de trabajo. En su investigación empírica encuentra que los trabajadores extensivamente utilizan sus conexiones sociales cuando buscan empleo, estos métodos informales de búsqueda de trabajo son menos costosos. Además, a través de estos es más probable encontrar un trabajo de mayor remuneración que en el mercado formal. Los resultados de Granovetter han sido confirmado y extendidos por muchos investigadores, véase por ejemplo Marsden y Gorman (2001) para un resumen amplio del tema. El uso de las nuevas tecnologías de comunicación hacen todavía más evidente la importancia de las redes sociales: nuevas formas virtuales de conexiones aparecen en internet, facilitando la obtención de bases de datos de gran tamaño sobre dichas conexiones (Barabasi (2002)).

La investigación de las redes sociales por los economistas empezó en los años 1990. Podemos diferenciar entre dos tipos de estudios. El primer tipo estudia la formación de las redes sociales por individuos racionales que basan sus decisiones sobre los vínculos en el análisis de
CHAPTER 1. INTRODUCCIÓN

los costes y beneficios. Este tipo de modelos complementan la investigación por físicos (véase por ejemplo Duncan y Watts (1998), Barabasi y Albert (1999)): ellos estudian la formación de las redes sociales por modelos mecánicos aplicando métodos estadísticos. Por ejemplo, Jackson y Wolinsky (1996) demuestra que la red formada por individuos racionales muchas veces no es eficiente en el sentido económico, ya que no maximiza el bienestar de sus miembros. Esto ocurre porque ciertas externalidades están presentes en las decisiones sobre los vínculos: la decisión de un agente directamente influye los beneficios y por lo tanto las decisiones de otros individuos. Otro importante ejemplo es Bala y Goyal (2000) que estudia el caso cuando los beneficios de un vínculo depende del número de los agentes que pueden ser alcanzados por ese vínculo siendo el beneficio más bajo si el agente conectado está más lejos en la red. Bala y Goyal (2000) caracterizan las estructuras de las redes formadas en el equilibrio y los diferentes procesos que llevan a un sistema a alcanzarlos.

El segundo tipo de los modelos económicos sobre las redes sociales estudia los efectos de las redes (network effects) en diferentes contextos económicos. Los efectos de las redes significan que la decisión de un individuo sobre algo o el bienestar del dicho agente está influido por los decisiones de los demás que estén conectados a él en la red social. La definición de los efectos de las redes sociales es muy parecida a la definición de externalidades y efectos de grupos (peer effects): todos esos conceptos formulan la misma idea de que los individuos directamente se influyen el uno al otro cuando toman decisiones. La característica que diferencia el estudio de los efectos de las redes sociales de los demás estudios utilizando externalidades es que las estructuras de las redes sociales determinan en gran medida el comportamiento de los individuos. Por ejemplo, Ballester et al. (2006) demuestra que en un juego de complementariedad local y sustitutibilidad global, el esfuerzo que los agentes deciden está determinado por la posición del individuo en la red. En concreto, su acción es una función de su centralidad en la red social. Así que el bienestar agregado de la sociedad también depende de la estructura de la red.

Estos tipos de efectos están presentes en muchas aplicaciones. Un ejemplo es la provisión de bienes públicos locales. Bramoullé y Kranton (2007) estudia una situación en la que un individuo puede decidir tomar una acción que beneficia a todos sus vecinos en la red social, de tal forma que si el individuo realiza esa acción ya no es óptimo para sus vecinos realizar la misma acción. Los autores analizan cuántos individuos proveen el bien público y cómo será la estructura de los individuos que proveen el bien público dependiendo de la estructura de la red social. Otra cuestión importante que los autores estudian es si los individuos toman la
decisión optima para el conjunto de la población, es decir, si la cantidad de bienes públicos proporcionada por los agentes maximiza el bienestar de la sociedad. Otro ejemplo de este tipo de artículos es Calvó-Armengol y Jackson (2004) que estudia como los individuos encuentran trabajo por sus conexiones sociales, en este caso, por sus vecinos en la red. Si un individuo no tiene trabajo, no va a transmitir ninguna oferta a sus vecinos, ya que él mismo aprovechará dichas ofertas para volver al mercado de trabajo. Esto conlleva a que sus vecinos también estén más tiempo desempleados. Consecuentemente, este efecto de la red da lugar a grupos en la sociedad interconectados únicamente entre ellos, creando minorías o grupos que permanecen desempleados por un largo periodo de tiempo.

Otros ejemplos de este tipo de efectos de las redes se puede encontrar en el campo de difusión de innovaciones y tecnología (Foster y Rosenzwerg (1995)), colaboración científica (Goyal et al. (2006)), en los seguros informales y colaterales (Karlan et al. (2009)), en la colaboración de investigación y el desarrollo entre empresas (Goyal y Moraga-González (2001)) entre otros. Algunos de estos artículos son teóricos y otros empíricos. Los trabajos teóricos desarrollan modelos para entender cómo la estructura de la red social influye al comportamiento de los individuos y como esos efectos se agregan al nivel de la sociedad entera. Por otro lado, los artículos empíricos intentan estimar la magnitud de este tipo de efectos de las redes sociales o desarrollan nuevos métodos estadísticos para estudiar este problema (véase por ejemplo Munshi (2003), Bramoullé et al. (2009) o Topa (2001)).

La presente tesis contribuye al análisis teórico de los efectos de las redes sociales. El primer contexto donde investigo los efectos de las redes es el papel de las redes en la búsqueda de trabajo. Las cuestiones más importantes que intento responder son las siguientes. En un contexto de trabajadores y empleos heterogéneos, cuál es el efecto de la presencia de las redes sociales en el emparejamiento de los trabajadores y empresas de diferentes características. ¿Las redes sociales mejoran o empeoran el emparejamiento de los trabajadores con los puestos donde pueden trabajar con mayor eficiencia? ¿Es el mercado formal de trabajo o la red social más eficiente en producir parejas de alta productividad (good matches)? ¿Bajo qué condiciones pagan los puestos obtenidos por las redes sociales un mayor salario que los trabajos encontrados a través del mercado formal? Cuando hablamos de trabajos encontrados por el mercado formal, nos referimos a que el trabajador consigue su puesto a través de los anuncios en los periódicos o en internet, o lo obtiene por una solicitud directa a la empresa o por otros métodos independientes de sus contactos sociales. El segundo contexto investigo los efectos de las redes
sociales en la evolución de la norma cooperativa en la sociedad cuando los individuos están conectados por una red social. Aquí nuestras principales cuestiones se refieren a cuáles son las estructuras de las redes sociales que apoyan más la emergencia de la cooperación. Para ello consideramos un modelo donde los individuos aprenden de sus vecinos sobre la reputación de sus oponentes en el juego del dilema de los prisioneros. También estudio el efecto de las limitaciones de la memoria de los individuos en dicha tasa de cooperación. Posteriormente, describo estas contribuciones con más detalles.

En el capítulo 3 analizo el impacto de las redes sociales en el desajuste (“mismatch”) del mercado de trabajo, es decir, la tasa de empleados que trabajan en puestos donde no explotan toda su productividad. En realidad, tanto los trabajadores como los puestos de trabajo son heterogéneos con respecto a sus características lo que implica que el mismo trabajador puede tener diferentes niveles de productividad cuando trabaja en diferentes puestos. La eficiencia de la economía depende de una manera crucial en la capacidad del mercado de trabajo para asignar a los trabajadores puestos donde puedan llevar a cabo su tarea con la mayor eficiencia. Dado que una fracción importante (ca. 30-60%) de los puestos son obtenidos por conexiones sociales, la cuestión es como la presencia de las redes sociales afecta a la eficiencia del mercado de trabajo con respecto a crear parejas (trabajador-empresa) productivas. La mayoría de la literatura anterior se centra en el impacto de las redes en la tasa de desempleo de la sociedad. Por ejemplo, Calvó-Armengol y Zenou (2005) considera una red donde todos los individuos tienen el mismo número de conexiones y obtiene que si la conectividad de la red sobrepasa un valor crítico, la incorporación de más conexiones en la red aumenta la tasa de desempleo. Así que un grafo más conectado implica que los individuos tienen menos probabilidad de encontrar trabajo. Ese efecto viene de que hay un cierto tipo de congestión en el flujo de la información si hay muchas conexiones en la red. Por otro lado, en el modelo de Ioannides y Soetevent (2006), los individuos son heterogéneos con respecto a sus conexiones. En este contexto, los resultados indican que más conexiones implican una tasa de desempleo más bajo.

En lugar de la tasa de desempleo, mi artículo estudia los efectos de las redes sociales en el nivel de “mismatch” de la sociedad. Analizo un modelo de búsqueda de trabajo tipo Mortensen-Pissarides donde supongo que existen dos tipos de trabajadores y la economía consiste de dos sectores. Supongo que cada tipo de trabajadores tiene un sector donde su productividad es alta (good match) y otro sector en el que su productividad es baja (mismatch). Los trabajadores pueden obtener información sobre los nuevos puestos en el mercado o bien por sus vecinos en
la red social. El mercado es modelado como un proceso de llegada estocástico donde introduzco un parámetro que simboliza la eficiencia del mercado, es decir, la probabilidad de que un puesto obtenido en el mercado formal es del sector donde el individuo tiene alta productividad. También supongo que la llegada directa de ofertas a los agentes empleados no es estocástica sino que ellos exclusivamente conocen las ofertas del sector donde actualmente trabajan. Este último es un supuesto válido en un contexto donde los empleados son informados sobre los nuevos puestos ofrecidos por sus empresas.

En cuanto a la red social supongo que los trabajadores que tienen trabajo traspasan la información sobre nuevos puestos a sus vecinos desempleados sin considerar el tipo del emparejamiento que resulta si su vecino empieza a trabajar en dicho puesto. Técnicamente, un empleado transmite la información a un contacto desempleado que elige aleatoriamente entre sus vecinos desempleados. Este comportamiento puede ser interpretado como favoritismo, ya que los trabajadores recomiendan ofertas de trabajo sin considerar la productividad del candidato. Este favoritismo puede ser racional para el empleado incluso cuando hay una pérdida de su reputación al recomendar un trabajador de baja productividad. Como los links pueden ser usados en muchos contextos, su pérdida de reputación puede ser compensada por otros beneficios obtenidos a través del vínculo establecido con el individuo recomendado. También en el futuro el individuo recomendado puede devolverle el favor y recomendar a dicho trabajador en otros puestos de trabajo.

Si este tipo de favoritismo provoca “mismatch” o no depende de una característica de la red social: la probabilidad de que un vínculo conecte dos trabajadores del mismo tipo. Esta probabilidad es conocida como el nivel de homofilia (homophily) y es un parámetro exógeno en mi modelo. Utilizando dicha probabilidad supongo que los vínculos de cada individuo son seleccionados en forma aleatoria e independiente. Ha sido demostrado en muchos contextos que los individuos tienden a estar más conectados con otros de similares características. La dimensión de similitud puede ser en cuanto a edad, religión, etnicidad, educación, salario o comportamiento (McPherson et al. (2001)). En mi artículo, la similitud significa que dos agentes tienen una ventaja productiva en el mismo sector relacionado con la educación o profesión de los dos.

Mis resultados demuestran que si el nivel de homofilia crece en la sociedad, el emparejamiento de las empresas y trabajadores a través de los contactos sociales es más eficiente. El efecto de homofilia es más significativo si el “mismatch” es más bajo en la sociedad. Los
vínculos entre individuos parecidos aumentan la probabilidad de obtener una oferta de alta productividad si y solo si esos individuos están empleados en puestos de alta productividad también (good match). Este último resultado es consecuencia del supuesto de que un agente empleado tiene acceso a las nuevas ofertas de su sector de empleo actual.

También obtengo que para cualquier nivel de eficiencia del mercado formal siempre existe un valor crítico del parámetro de homofilia tal que si la homofilia sobrepasa este valor, la presencia de la red social en el proceso de emparejamiento decrece el nivel de “mismatch” en comparado a una economía donde solo el mercado formal opera. Al contrario, si el nivel de homofilia es demasiado bajo, la red social aumenta el “mismatch”. También estudio cómo cambia este valor crítico de homofilia con otros parámetros del modelo. Mis resultados indican que el valor crítico decrece si la probabilidad de despido de los individuos que trabajan en puestos de baja productividad es mayor a la de los individuos trabajando en puestos de alta productividad. Lo mismo pasa si hay contactos profesionales en la red social. Este tipo de contactos siempre transmiten ofertas de alta productividad porque supongo que para ellos la pérdida de reputación no está compensada por otros beneficios. Estos dos factores disminuyen el nivel de “mismatch” en la sociedad, haciendo la llegada de ofertas mediante la red social más eficiente. Por tanto, menos homofilia es necesaria para mantener el mismo nivel de eficiencia. Por el contrario, si la eficiencia del mercado formal sube, el valor crítico de homofilia sube también ya que comparamos la eficiencia de la red social a un mercado más eficiente. Además, demuestro que si la red social está más conectada, el nivel de “mismatch” crece o decrece dependiendo de que el nivel de homofilia esté por debajo o por encima del valor crítico, respectivamente.

Estos resultados ofrecen una nueva explicación a la ambigüedad encontrada en la literatura empírica sobre si la búsqueda de trabajo a través de las redes sociales o mediante el mercado formal proporcionan un mayor salario esperado para el buscador de trabajo. En un estudio que compara diferentes países Pellizzari (2010) encuentra que en Austria, Bélgica y los Países Bajos la red social paga una prima de salario sobre el mercado formal. Al contrario, en Grecia, Italia, Portugal y el Reino Unido la red social paga unos salarios más bajos comparado al mercado. Kugler (2003) y Dustmann et al. (2010) también encuentra una prima salarial para la red, por otro lado, Bentolila et al. (2010) obtiene un descuento en el salario para la red social. Mi modelo explica esa evidencia mixta basándose en el nivel de la homofilia y los diferentes factores que influyen el valor crítico de está. Mi modelo sugiere que podemos encontrar un salario esperado más alto para la búsqueda vía redes si la homofilia es suficientemente alta y/o los trabajadores
utilizan más contactos profesionales.

En el capítulo 4 trato la cuestión de salarios esperados con más detalles y explico dicha ambigüedad observada de otra manera. Construyo un modelo de empresas heterogéneas y trabajadores homogéneos donde nuevas ofertas aparecen a una tasa de llegada exógena. Mi modelo está basado en una versión de Calvo-Armengol y Jackson (2004, 2007). Los agentes constantemente cambian entre diferentes estados, de empleado a desempleado. Si un desempleado tiene conocimiento de una vacante, la ocupa. Por otro lado, los individuos empleados traspasan las ofertas a sus vecinos en la red social. Uno de los supuestos es que un trabajador puede tener dos tipos de trabajos: donde recibe un salario bajo o un salario alto. En realidad se puede observar que los trabajadores supuestamente iguales reciben sueldos diferentes, luego el supuesto sobre la existencia de una varianza salarial es válido. Para introducir este supuesto, específico un proceso de nuevas vacantes que permite correlaciones entre el estado de un empleado y el tipo del puesto del que el individuo tiene conocimiento. Por ejemplo, un trabajador que reciba sueldos altos es más probable que tenga información sobre nuevos puestos parecidos (de sueldo alto). Este supuesto implica que el desempleado conectado a individuos en mejor posición se beneficia más de sus conexiones: tiene más probabilidad de obtener un trabajo de sueldo alto. Este fenómeno ha sido observado por sociólogos (véase por ejemplo Lin (1999)).

La cuestión más importante que investigo en este capítulo es determinar si la búsqueda por redes sociales o en el mercado formal asegura un mejor salario esperado a los buscadores de trabajo. Identifico la búsqueda por el mercado formal con la llegada directa de ofertas. Por otro lado, la búsqueda a través de la red social significa que el buscador de trabajo recibió la oferta indirectamente: por uno de sus vecinos en la red social.

Mis resultados demuestran que cuando la correlación entre el estado de un empleado y el tipo de las ofertas del cual tiene conocimiento es suficientemente alta, las redes sociales proveen puestos mejores pagados en término medio que el mercado formal. Si los nuevos puestos de alto sueldo tienden a llegar a los individuos que ya reciben un salario alto, van a traspasar esas ofertas a sus vecinos. Los individuos menos remunerados van a tener una probabilidad más baja de recibir ofertas con mayor salario directamente, así que no van a utilizar esas ofertas para mejorar sus posiciones. Por lo tanto, el salario esperado de la búsqueda por redes sociales será más alto que el del mercado formal. Si la mencionada correlación incrementa, el desempleo disminuye, ya que a través de la red más vacantes llegaran al desempleado y estas no serán aprovechadas por otros empleados menos remunerados. Si esa correlación es cero, es decir, las
ofertas llegan a la red social aleatoriamente, el mercado formal siempre paga salarios más altos que la red social.

También demuestro que siempre existe un valor crítico de dicha correlación donde el mercado formal y la red social ofrecen el mismo salario esperado. Este valor crítico cambia con los otros parámetros del modelo. Obtengo que si hay más ofertas vacantes o relativamente hay más ofertas nuevas de salario alto, el valor crítico decrece. Por el contrario, el valor crítico crece, si la tasa de despido crece o la red social está más conectada. Entonces mi modelo demuestra que si la economía está en época de bonanza, la red social es más probable que provea un mayor salario que el mercado formal. En este caso los vecinos de un desempleado son más probables a tener puestos de salario alto y así a traspasar las nuevas ofertas de mayor remuneración. Si las ofertas de sueldo alto son distribuidas aleatoriamente entre los vecinos de un desempleado, todavía hay una alta probabilidad de que se traspasen al desempleado. Por el contrario, si los vecinos de un desempleado son desempleados o reciben salarios bajos, las ofertas vacantes de mayor salario deben ser distribuidas más sistemáticamente entre los pocos empleados de salario alto para que esta llegue al desempleado. Entonces, en tiempos de recesión la correlación debe ser más alta de tal modo que la red social ofrezca un sueldo esperado más alto.

En cuanto a la metodología usada en este capítulo, obtengo mis resultados utilizando dos tipos de análisis. Primero resuelvo el modelo analíticamente a través de un tipo de aproximación estocástica, que la literatura se llama mean-field analysis. Este análisis se basa en establecer un sistema de ecuaciones diferenciales utilizando el supuesto que el estado de los individuos es asignado aleatoriamente en cada periodo de tiempo. Se supone que la probabilidad de que un individuo está en un estado es igual a la frecuencia del mismo estado en el nivel de la población. Por ejemplo, la probabilidad de que un agente está desempleado es igual a la tasa de desempleo. El estado de los diferentes individuos se toma de forma independiente de la dicha distribución. Los supuestos de este tipo de análisis no cumplen en un sistema real, así acompaña el análisis con simulaciones donde el estado de los individuos no es aleatoriamente asignado sino continuamente evoluciona. Las simulaciones también permiten que investigue el proceso en redes con diferentes estructuras. Obtengo que mis resultados sobre los salarios de los diferentes métodos de la búsqueda de trabajo son válidos independientemente de la metodología utilizada o la estructura de la red social.

En el capítulo 5 yo y mis coautores, Friederike Mengel y Jaromir Kovarik, investigamos el impacto de las redes sociales en otro contexto: la evolución de la norma cooperativa. En un
juego del dilema de prisionero los jugadores recibirían un pago más alto si todos cooperarán comparado los pagos del equilibrio de Nash donde los dos desertan. Este equilibrio aparece porque cuando solo uno de los jugadores coopera, el otro puede abusar ("free-rider") de su co-operación y recibir un mayor pago. Sin embargo, la predicción del juego-teórico no coincide con la conducta observada en la realidad: entre animales y entre humanos hay muchos ejemplos de cooperación. Hay varias maneras de explicar la diferencia entre la predicción teórica y la empírica. Los más populares están basados en la reputación directa o indirecta. En el caso de los modelos de reputación directa los agentes se encuentran repetitivamente para jugar el dilema de prisionero y los jugadores reciprocan la cooperación del otro jugador por cooperar o la defección del otro jugador por desertar. En el caso de los modelos de reputación indirecta se supone que los mismos jugadores no se encuentran dos veces. La cooperación emerge porque los jugadores cooperan para ganar reputación frente a la población entera, lo que induce a que los oponentes futuros cooperen con ellos. En el modelo de Nowak y Sigmund (1998), los agentes aumentan sus puntuaciones de imagen (reputación) cuando cooperan, estas puntuaciones son observadas por toda la población o por una fracción de la población que se elige aleatoriamente. Este mecanismo críticamente depende de la observabilidad de la reputación de los nuevos oponentes.

Tanto los modelos de reputación directa como los de indirecta suponen que los individuos tienen capacidades cognitivas para recordar las interacciones anteriores y decidir sobre la reputación de los oponentes. Nuestro modelo investiga los efectos de una de esas capacidades mentales, la memoria. Suponemos que los individuos tienen memoria limitada para recordar las interacciones anteriores y por eso disponen de información limitada sobre la reputación de los demás. Además suponemos que los agentes están conectados por una red social y que interactúan y aprenden de sus vecinos directos en la red. Así suponemos que los individuos utilizan su propia experiencia y la experiencia de sus vecinos para formar una opinión sobre sus oponentes al jugar el dilema del prisionero. Al introducir la red social como una fuente de información nos apartamos del supuesto poco realista de Nowak y Sigmund (1998). Puesto que estos asumen que la reputación de un agente es observable para todo el mundo (o mejor dicho que todos los individuos tienen la misma oportunidad de observar la reputación de alguien).

Además, suponemos que existen tres tipos de preferencias en la población: altruistas, desertores y cooperadores condicionales. Los altruistas siempre cooperan, los desertores siempre desertan. Los cooperadores condicionales cooperan si y solo si la reputación de oponente sug-
iere que es probable que va cooperar. Esos tipos no son observables. La evolución elige entre esos tipos donde la aptitud de un agente está determinada por su pago en el dilema del prisionero.

Estudiamos si la cooperación puede emerger en este tipo de contextos de memoria limitada y si una memoria más larga siempre implica una tasa de cooperación más alta. Además, analizamos el efecto de la estructura de la red social en la aparición de la norma cooperativa y la longitud de la memoria necesaria para la cooperación. Simulamos el modelo en una red de mundo pequeño originalmente introducida por Watts y Strogatz (1998). En este tipo de redes se puede cambiar fácilmente la conectividad y la transitividad de la red social, que son propiedades importantes de la red social en nuestro modelo, como próximamente voy a describir con más detalle.

Nuestros resultados demuestran que si la memoria es muy corta, la cooperación no puede emergir en ningún tipo de red social. La evolución de la cooperación críticamente depende de la capacidad de los cooperadores condicionales para distinguir entre altruistas y desertores. Si la memoria es demasiado corta, los cooperadores condicionales simplemente no tienen suficiente información para hacer esa distinción. Si la memoria es más larga, sobre 5-10 periodos, la cooperación emerge y la tasa de cooperación es significativamente alta. Por el contrario, si la memoria es todavía más larga, observamos que la tasa de cooperación empieza a decrecer otra vez. Entonces obtenemos que la tasa de cooperación no es monótona en la longitud de la memoria y existe una longitud óptima de la memoria donde el nivel de cooperación es máximo. El bajón de la cooperación en el caso de una prolongada memoria ocurre porque las deserciones de los primeros periodos son recordadas durante un periodo demasiado largo. Al principio de las simulaciones no hay ninguna historia del juego para recordar, lo que favorece a los desertores, que pueden explotar a los altruistas, aumentando así su peso en la población. Estas primeras deserciones son recíprocadas por los cooperadores condicionales que causan que tanto los desertores como los cooperadores condicionales empeoren su reputación. Como la memoria es muy larga, esa mala reputación es recordada durante mucho tiempo, causando más deserciones de los cooperadores condicionales. De este modo, se forma una trampa de deserción.

También podemos observar que la transitividad local de la red social es necesaria para la emergencia de la cooperación si los agentes interactúan y aprenden de sus vecinos directos. Transitividad local o “clustering” significa que es probable que dos vecinos de un individuo sean también vecinos directos. Si el “clustering” es alto, es más probable que
un individuo aprenda de la reputación de sus oponentes futuros utilizando la información que reciba de sus vecinos. Su vecinos directos también son vecinos en la red y por eso también interactúan uno con el otro.

Nuestros resultados son robustos a la introducción de mutaciones al proceso de selección. En el modelo básico suponemos que la selección está exclusivamente basada en las aptitudes (fitness) de los individuos. En el análisis de robustez introducimos mutaciones: en el proceso de la selección con alguna probabilidad pequeña no comparamos las aptitudes (fitness) de los individuos sino que asignamos un tipo aleatorio al individuo. De ahí que en esta versión del modelo, los distintos tipos de individuos puedan reaparecer en la población. Demostramos que nuestros resultados se mantienen en este contexto. Del mismo modo, nuestros resultados son robustos también si permitimos la posibilidad de que los individuos no solamente interactúen con sus vecinos directos sino que también lo hagan con los individuos más distantes en la red social.

En el modelo básico suponemos que todos los agentes tienen la misma longitud de memoria. Como una extensión consideramos el caso en el que los individuos son heterogéneos con respecto a la memoria. Incluimos 3 tipos de cooperadores condicionales que son diferentes según la memoria: un tipo tiene memoria corta, otro tiene la memoria óptima según los resultados anteriores, y otro tiene memoria muy larga. Aquí la evolución también elige entre los tipos de memoria según su rendimiento al jugar el dilema del prisionero. El análisis anterior ha demostrado que existe una memoria óptima desde el punto de vista de la población: es la memoria que apoya más la evolución de la cooperación. Cuando los individuos son heterogéneos con respecto a la memoria, la cuestión es si existe una memoria óptima desde el punto de vista de los individuos: es la memoria que permite al agente su mejor rendimiento/actuación comparado con otros tipos de agentes que tienen otra longitud de memoria. En este caso encontramos que los diferentes tipos con respecto a la memoria sobreviven más o menos en la misma fracción en la población. En particular, no encontramos que una memoria más larga tenga más beneficios para el individuo. Así que tanto desde el punto vista de la población como del individuo podemos rechazar la hipótesis de que una memoria más larga sea mejor.

Nuestros resultados son nuevos en el sentido que, según nuestro conocimiento, somos uno de los primeros en estudiar el papel de la memoria en la evolución de la cooperación en modelos basados en reputación. Algunos modelos ya habían investigado el papel de la memoria (véase por ejemplo, Qin et al. (2008), Hauert y Schuster (1997), Kirchkamp (2000)). Por ejemplo, Qin
et al. (2008) investigan el efecto de la memoria en el caso cuando los agentes están colocados en una celosía (lattice). Para la mayoría de las especificaciones de los parámetros encuentran que la densidad de cooperadores aumenta si los agentes tienen una memoria más larga. Cox et al. (1999) explora si la memoria sobre las interacciones pasados puede compensar el hecho que los individuos no tienen información sobre uno del otro. Su modelo es diferente del nuestro en muchos aspectos, entre ellos: los agentes pueden elegir con quien interactuar en el juego, están asignados una longitud de memoria aleatoriamente, existen cuatro tipos de preferencias. Generalmente estos modelos obtienen que una memoria más larga siempre implica una tasa más alta de cooperación. Nuestro modelo muestra un caso interesante donde una memoria larga es perjudicial para la evolución de la cooperación.

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Chapter 2

Introduction

With the rise of modern market economy in the 19th century, social scientists started to think that the production and allocation of economic goods would be dominantly carried out by impersonal market processes. For example, Polanyi (1957) describes with great detail how different markets of the 19th century economy were created by building down the personalized structures of the traditional economy. However, in the second half of the 20th century, scientists belonging to the research program of new economic sociology pointed out that social capital and social embeddedness are still important in a market economy (see e.g. Coleman (1988), Bourdieu (1990)). Social networks analysis is one of the possible ways to think of how interpersonal relations influence economic outcomes. In his seminal book Granovetter (1995) analyzed the role of social contacts in finding a job. In his empirical study he found that workers extensively use their social contacts to look for jobs, that informal job search methods are less costly and they are more likely to result in a job offer and provide high paying jobs than the formal market. The results of Granovetter have been confirmed and extended by many researchers, see e.g. Marsden and Gorman (2001) for an extensive summary. The usage of new communication technologies makes even more evident the importance of social networks: new virtual types of connections appear on the internet and it becomes possible to collect large scale data about these linkages (Barabasi (2002)).

The study of social networks by economists has started in the 1990s. We might differentiate between two broad types of studies. The first type investigates the formation of social networks by rational agents who base their linking decisions on cost-benefit analysis (see e.g. Jackson and Wolinsky (1996), Bala and Goyal (2000)). This literature complements the network formation models proposed by physicists which are normally based on statistical mechan-
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ical processes (see e.g. Duncan and Watts (1998), Barabasi and Albert (1999)). The second type studies the network effects in different economic contexts such as diffusion of innovations (Foster and Rosenzweig (1995)), public good provision (Bramoullé and Kranton (2007)), job search (Calvó-Armengol and Jackson (2004)), scientific co-authorship (Goyal et al. (2006)), informal insurance and social collateral (Karlan et al. (2009)), R & D collaboration (Goyal and Moraga-González (2001)) among others. Some of these papers are theoretical: they develop models to understand how the social structure influences the behavior of the individuals and how these effects aggregate to the performance of the whole network. The notion of network effects is conceptually related to externalities and peer effects: all these concepts formulate the same idea that individuals directly influence each other when making decisions. However, the social networks analysis realizes that the individual choices depend on the network structure. For example, Ballester et al. (2006) shows that in a game of local complementarity and global substitutability, the effort choice of an agent is determined by her centrality in the social network. In this way, the aggregate welfare of the society depends on the network structure. In addition to the theoretical papers, other studies estimate the magnitude of network effects or develop new econometric techniques to this end, see e.g. Munshi (2003), Bramoullé et al. (2009), Topa (2001).

The present thesis contributes to the theoretical analysis of network effects. The first context where I study network effects is the role of social contacts in job search. The main questions I deal with are the following. Does the presence of social networks help the matching of workers to appropriate jobs in a context of heterogeneous workers and jobs? Is the market or the social networks more efficient in producing good matches? Under what conditions do the jobs found through social contacts pay higher wages than the ones obtained on the market? The second area where I investigate the network effects is the evolution of cooperative norms in the society when individuals are connected by an underlying network structure. Here I ask which network structures support the emergence of cooperation in a model of indirect reputation and what the effects of memory limitations are in such a context. In what follows I describe these contributions in more details.

In Chapter 3 I analyze the impact of social networks on labor market mismatch. In reality both workers and jobs are heterogeneous with respect to their characteristics which implies that the same worker can reach different levels of productivity when working in different jobs. The efficiency of the economy depends on the ability of the labor market to match the workers to
jobs where they can the most exploit their abilities. Given that important fraction (ca. 30-60%) of the jobs are obtained via social connections, the question arises how social networks influence the efficiency of the labor market with respect to creating "good" matches. Previous literature mostly focused on the impact of networks on the unemployment rate in the society. For example, Calvó-Armengol and Zenou (2005) considered a graph where every worker has the same number of connections. They obtained that if the connectivity of the network (i.e. the number of links workers have) surpasses a certain threshold, the addition of further links increases the unemployment rate. According to the results of Ioannides and Soetevent (2006), however, if the connections of the workers are allowed to be heterogeneous, higher connectivity implies lower unemployment rate.

Instead of the unemployment rate, my article investigates the impact of social networks on the mismatch level in the society. I build a Mortensen-Pissarides search model with two types of workers and two types of jobs. I assume that each worker is highly productive in one of the sectors (good match) while her productivity is lower in the other (mismatch). Workers may obtain information about openings through the market and via their neighbors in the social network. I model the market as a stochastic arrival process where I parameterize the matching efficiency of the market, i.e. the probability that a job obtained on the market is a good match. I also assume that the direct job arrival to employed workers is not random but they hear about the new openings of the sector of their actual employment.

Regarding social networks I assume that employed workers transmit job information to their unemployed friends without considering the type of the resulting match. Technically, they forward offers to a randomly chosen unemployed contact of theirs. This behavior can be interpreted as favoritism since workers recommend each other to jobs irrespective of productivity. Whether this kind of favoritism leads to mismatch is determined by the probability that a link connects similar or different workers. This probability is called the homophily level and I treat it as an exogenous parameter. I assume that the links of each individual are randomly drawn in an independent way using this probability. It has been shown in many contexts that individuals tend to be more connected to others similar to them than to different individuals. The dimension of similarity can be age, religion, ethnicity, education, income or behavior (McPherson et al. (2001)). In my paper, similarity means to have productivity advantage in the same sector which is related to education.

My results show that as the homophily level increases in the society, the matching of firms
and workers through the social network becomes more efficient. The effect of homophily is higher when the mismatch level is lower. Connections to similar agents rise the probability of hearing about high productivity offers as long as the similar agents are employed in "good matches". This follows from the assumption that employed agents hear about the jobs of the sector of their actual employment.

I also obtain that for any market efficiency level there exists a threshold in the homophily parameter such that if the homophily is higher than that, the presence of social networks in the matching process decreases the mismatch level compared to an economy where only the market operates. On the contrary, if the homophily is too low, the social network is creating extra mismatch. I analyze how this critical homophily level changes with other parameters of the model. My results show that the homophily threshold decreases if bad matches are separated at higher rate than good matches. The same happens if there are professional contacts in the model who forward only high productivity offers to their contacts caring about their reputation toward the employer. These two factors decrease the mismatch level in the society and hence, make the network arrival more efficient. Thus less homophily is needed to reach the same efficiency level. On the contrary, if the market efficiency increases, the homophily threshold increases as well since we compare the efficiency of the network to a better performing market process. Moreover, I show that as the network becomes more connected, the mismatch rises or decreases depending if the homophily level is below or above the critical value, respectively.

These results offer a new explanation why the empirical literature finds an ambiguous result on whether social networks or the formal market provide higher expected wages for a job searcher. In a cross-country study Pellizzari (2010) finds that in Austria, Belgium and the Netherlands social networks pay a wage premium over the market while in Greece, Italy, Portugal and the United Kingdom a wage discount is found. Kugler (2003) and Dustmann et al. (2010) also obtain a wage premium while Bentolila et al. (2010) estimate a wage discount. My model rationalizes this mixed evidence based on the homophily level and the different parameters which affect the homophily threshold. Especially, my model suggests that we find a wage premium for the social contacts if the homophily is high enough and/or the workers rely more on their professional contacts.

In chapter 4 I deal with the question of expected wages in more detail and give another explanation of the observed ambiguity. I build a model of heterogeneous jobs and homogeneous workers where job openings and separations happen at exogenous rates. My model is based on a
version of Calvo-Armengol and Jackson (2004, 2007). Agents constantly become aware of new job openings. Unemployed workers take the jobs. Employed workers take those jobs which improve their wages. Otherwise, they pass the information to their neighbors in the social network. I assume that workers can be employed in low or high paying jobs. I specify an arrival process of vacancies which allows for correlations between the state of employed agents and the type of new openings they might hear about. For example, a worker employed in a high type job is more likely to hear about a similar new job than about a different one. This assumption is certainly true if workers hear about new openings of their current employers. One implication is that unemployed workers connected to individuals in better positions are more likely attain better status which phenomenon has been observed by sociologists (see e.g. Lin (1999)).

The main question of this chapter is what determines which search method provides higher expected wages for the job seeker: the formal market or the informal social network. I say that an individual finds a job on the formal market when she becomes directly aware of the job information. On the contrary, if the information reaches an unemployed through one of her neighbors, it means she used her social contacts.

My results show that when the mentioned correlation between the status of an employed and the type of the new opening she is likely to hear about is high enough, the social network provides higher wages on average than the formal market. If new high wage openings reach mostly those individuals who are already earning high wages, they will pass those offers to their neighbors. Low wage employed will have lower probability to hear about high wage jobs through the direct arrival, thus they cannot use those offers to improve their status. Hence, the expected wage of the network arrival will be high while that of the market low. If the mentioned correlation increases, the unemployment decreases since through the network more vacancies will reach unemployed workers and will not be taken by low wage employed individuals.

I establish that there always exists a critical value of this correlation for which the market and social networks give the same expected wage. I analyze how this threshold level moves if the other parameters of the model change. I obtain that as more jobs open up or higher fraction of the new jobs pays high wages, the critical value decreases. On the contrary, the threshold rises if jobs are separated at higher rates or the social network is more connected. Hence, my model predicts that in "good times" the social network is more likely to pay a wage premium over the market. In good times the neighbors of an unemployed worker are more likely to earn high wages themselves, hence they are more probable to pass high wage offers. If the high
wage offers are randomly distributed among the neighbors, they are still likely to be passed to the given unemployed contact. On the contrary, if the neighbors of an unemployed individual are mostly unemployed or earning low wages, high wage offers need to be given to the few high wage employed who will pass them to the unemployed contact. Hence, in bad times the correlation should be higher in order the networks give a wage premium.

Chapter 5 is a joint work with Friederike Mengel\textsuperscript{1} and Jaromir Kovarik\textsuperscript{2} which investigates the impact of social networks in another context: the evolution of cooperation. In a prisoner’s dilemma game the players would be better off if everybody cooperated compared to the Nash equilibrium where both players defect. This equilibrium arises because when only one of the players cooperate, the other can abuse (free-ride) her cooperation. However, the game-theoretic prediction does not coincide with the observed behavior: both among animals and humans there are many examples of cooperation. Usual ways of explaining the discrepancy between theory and empirics are based on direct or indirect reputation. In case of direct reputation models agents repeatedly meet to play the prisoner’s dilemma and players reciprocate cooperation by cooperating and defection by defecting. In case of indirect reputation models it is assumed that agents not necessarily meet repeatedly. They cooperate to gain good reputation toward the whole population which induces that future opponents will cooperate with them. In the model of Nowak and Sigmund (1998), agents increase their image scores when they cooperate, these scores are observed by everybody in the population\textsuperscript{3} This mechanism crucially depends on the observability of the reputation of new opponents.

Both direct and indirect reputation models assume that individuals have the cognitive abilities to remember past interactions and assess the reputation of others. Our model investigates the effects of one such cognitive ability, the memory. We assume that individuals have limited memory to recall past interactions and hence, have limited information about the reputation of others. In addition, we assume that individuals are connected by an underlying network structure and both interact with and receive information from their direct neighbors in the network. Thus, we assume that agents use their own experience and that of their neighbors to form opinion about others. By incorporating the social network as information channel, we depart from the unrealistic assumption of Nowak and Sigmund (1998) that reputation is commonly

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\textsuperscript{3}They also investigate the case when each interaction of an agent is observed by a randomly chosen subset of the population.
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Further, we assume that there are three preference types in the population: altruists, defectors and conditional cooperators. Altruists always cooperate, defectors always defect. Conditional cooperators cooperate only if the reputation of the opponent suggests that she is likely to cooperate. Types are not observable. Evolution selects among preference types.

We investigate the question whether cooperation emerges in such a context of limited memory and whether a longer memory spell always implies a higher rate of cooperation. Further, we study the effects of the network structure on the emergence of cooperation and on the memory length necessary for cooperation. We simulate the model on a small world network (Watts and Strogatz 1998).

Our results show that if the memory spell is very short, cooperation cannot emerge for any network structure. The emergence of cooperation depends on the conditional cooperators’ ability to distinguish between defectors and altruists. If the memory is too short, conditional cooperators do not have sufficient information to make this distinction. When we increase the memory spell to 5-10 periods, high rates of cooperation emerge. However, if the memory spell is even longer, we observe that cooperation starts to decrease again. Hence, we obtain that the cooperation is non-monotone in the length of memory and there exists an optimal memory spell which best supports the cooperation. The decrease of cooperation for very long memory happens because past defections are remembered too long. At the beginning of the simulations there is no history to recall, thus defectors can easily exploit altruists and soar in the population. These early defections are reciprocated by the conditional cooperators which causes that both defectors and conditional cooperators gain bad reputation. For very long memory spells this bad reputation is recalled and causes further defections by the conditional cooperators. Hence, a defection trap arises.

We also observe that network clustering is necessary for cooperation if agents both learn and interact with their direct neighbors. Clustering means that two neighbors of an individual are also likely to be direct neighbors in the social network. If clustering is high, an individual is more likely to learn about the reputation of her future opponents from their neighbors. Her direct neighbors are connected and hence learn about each other when they interact.

Our results prove to be robust to introduction of mutations into the selection process. In the baseline model, we assume that selection is based on fitness comparisons. In a robustness check, we allow for mutations: instead of imitating the type of another agent with higher fit-
ness, an agent is assigned a randomly chosen type with some probability. In this version of the model, types might reappear in the population. We show that the previously mentioned results go through to this case as well. In the same way, our findings are robust to allowing for the possibility that individuals not only interact with their direct neighbors but also with more distant individuals in the social network.

In the baseline model we assume that every agent has the same memory parameter. As an extension we consider the case when individuals are heterogeneous with respect to memory. We include 3 types of conditional cooperators which differ in memory. Here in addition to the preference types, evolution also selects the optimal memory length. The previous analysis established that there exists an optimal memory length from a population point of view: it supports best the evolution of cooperation. When individuals are heterogeneous with respect to memory, the question is whether there exists an optimal memory length from an individual point of view: it allows the agent to perform the best compared to other agents with different memory length. We find that different memory types survive in about the same fraction in a heterogeneous population. In particular, we do not find evidence that a longer memory length would be advantageous from an individual point of view.

Our findings are new in the sense that, to our knowledge, we are among the firsts studying the role of memory in reputation-based evolutionary models. Some models have already investigated the role of memory in other evolutionary contexts (see e.g. Qin et al. (2008), Hauert and Schuster (1997), Kirchkamp (2000)). Their general insight is that the rate of cooperation is increasing in the memory length. Our model shows an interesting case when longer memory is actually detrimental for cooperation.

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Chapter 3

Occupational Mismatch and Social Networks

3.1 Introduction

3.1.1 Motivation

Workers often use social contacts while searching for a job in addition to formal methods such as newspaper ads or direct application to employers. Different estimates find that 30-60% of the jobs are obtained through informal methods (see e.g. Blau and Robins (1990), Holzer (1987)). Economic literature mostly focused on the effects of social networks on the unemployment rate. Ioannides and Soetevent (2006) find that in a heterogeneous population better connected agents experience a lower likelihood to be unemployed. On the other hand, Calvó-Armengol and Zenou (2005) point out that, due to congestion in the information flow, a highly connected network might lead to a higher unemployment rate.

In this study, I investigate the impact of social networks on another labor market outcome, the mismatch level. I consider an economy of two types of workers and two sectors where a worker’s productivity is high if employed in one sector and is low in the other. I ask whether the presence of social connections as information channel improves the matching of workers to appropriate jobs. Regarding social networks, I assume that workers recommend each other to jobs irrespective of future productivity. This behavior can be interpreted as favoritism and is certainly attribute of family ties and close friendships.

Agents might have good reasons to recommend someone of low productivity for a job
despite possibly suffering reputation loss toward the employer. One such reason is that social ties are used in many contexts other than the labor market, for example risk sharing, and the reputation loss might be compensated by benefits along these other dimensions (Beaman and Magruder (2010)). Forwarding job offers contributes to the maintenance of such beneficial links. Another reason for passing not suitable offers is that such behavior is reciprocated in the future resulting in shorter unemployment spell for the individual (Bramoullé and Goyal (2009))

I build a Mortensen-Pissarides equilibrium search model (Pissarides (2000)) where matching of unemployed workers and vacancies can occur through two channels simultaneously present: the formal market and social networks. I look at the formal market as a random arrival process where I parameterize the efficiency of the market in producing good matches, i.e. the rate at which vacancies reach unemployed workers of high productivity on the market. This parameter could be seen as the intensity of directed search by unemployed and firms toward partners who fit their characteristics. On the other hand, I assume that the arrival to employed agents is not random, but they have direct access to information about the vacancies of the sector of their current job. Employed workers use the offers at their disposal to pass them to their unemployed social contacts.

In the model presented here the impact of social networks is determined by one characteristic of the network, the homophily level, i.e. the fraction of links that connect similar agents. It has been shown in many contexts that individuals tend to be more connected to others similar to them than to different individuals. The dimension of similarity can be age, religion, ethnicity, education, income or behavior (McPherson et al. (2001)). In this paper, similarity means to have productivity advantage in the same sector.

My results show that in a network with high homophily level the matching is more efficient and the impact of homophily is higher if the mismatch level is lower in the economy. Connections to similar agents rise the probability of hearing about high productivity offers as long as the similar agents are employed in "good matches". This follows from the assumption that employed agents hear about the jobs of the sector of their actual employment. For the same reason, an increase in the market efficiency has a positive feedback on the efficiency of the social networks.

I also find that for any market efficiency level there exists a critical homophily value such that if the homophily exceeds that, the presence of social networks decreases the mismatch compared to the case of a pure market economy. For a sufficiently homophilous network, the
addition of further links decreases the mismatch level in the society. On the contrary, if the homophily level is lower than threshold, the social networks creates additional mismatch. Hence, for some parameter values the network decreases the mismatch level even if the employed individuals do not direct the offers to their high productivity unemployed friends when choosing the receiver of the information. The here modelled random transmission is a lower benchmark regarding the efficiency of the social networks, thus I expect that real-world networks perform even better.

Further, when the mismatch level depends only on the market efficiency and the homophily level, the mentioned critical value is equal to the complete homophily, i.e. when every link connects similar agents. Hence, for less than complete homophily, the network always creates mismatch. However, when we introduce some other factors which decrease the mismatch, the homophily threshold decreases. I consider two such factors: when the bad matches have a higher separation rate than good matches and when professional contacts are present in the network. I assume that professional contacts recommend their neighbors only to good matches because they care more about their reputation toward the employers.

The intuition behind these findings is the following. If all neighbors of a given unemployed are of similar type to her, the average quality of the offers accessed through these contacts is determined by the mismatch level, i.e. the probability that a neighbor is employed in her high productivity sector over the probability to be employed in the low productivity sector. If this probability ratio is determined by the market efficiency level, the social networks and the market provide good matches at similar rates by complete homophily. On the other hand, if there are other factors which decrease the mismatch, the social network is more efficient by complete homophily and we require a lower homophily level to equalize the efficiency of the two search methods. Hence, for high homophily levels the social networks decreases mismatch compared to a pure market economy.

These results offer a new explanation why the empirical literature finds an ambiguous result on whether social networks or the formal market provide higher expected wages for a job searcher. In a cross-country study Pellizzari (2010) finds that in Austria, Belgium and the Netherlands social networks pay a wage premium over the market while in Greece, Italy, Portugal and the United Kingdom a wage discount is found. Kugler (2003) and Dustmann et al. (2010) also obtain a wage premium while Bentolila et al. (2010) estimate a wage discount. Based on the mismatch created by the different search methods, my model can recover both a
wage premium and a wage discount for the search via social contacts. According to my results, a wage premium is obtained if the homophily is high enough where the sufficient homophily level is influenced by the job separation rate difference between bad and good matches, by the market efficiency and the number of professional contacts used by the job searcher.

The paper is organized as follows. The next session relates my work to the existing literature. In Section 2 I describe the model, derive the matching function and deduce the equilibrium conditions. In section 3 I define the quantities of interest of the model. Section 4 contains my analytical results while in Section 5 I perform numerical analysis. In Section 6 I introduce professional contacts as additional information sources. Section 7 and 8 discuss and summarize the obtained results.

3.1.2 Literature review

The model presented here is closely related to Calvó-Armengol and Zenou (2005) who also analyze the impact of social network in a Mortensen-Pissarides framework. In a model of homogenous worker, they focus on the effects of network connectivity on the unemployment rate. Here I analyze the impact of homophily and connectivity on the mismatch level of the society and the expected wages of different search methods.

Bentolila et al. (2010) explicitly focus on the impact of social networks on mismatch. They model the occupational choice of a young worker entering the labor market and argue that connections in low productivity sectors create incentives to choose an occupation where she does not fully exploit her abilities. Their model is static in the sense that individuals choose which kind of job to look for only once and that the information access of social contacts is exogenously given. In the here presented continuous time dynamic model every agent moves between three states: unemployment, employment in low, high productivity job. This means that contacts transmit different information depending on their actual state. Hence, I explicitly model social networks as connections between agents. On the contrary, in Bentolila et al. (2010) neighbors transmit information about high productivity jobs at an exogenous rate and their state changes are not modelled.

There is a growing literature on the role of homophily in different processes on social networks (see e.g. Golub and Jackson (2009), Currarini, Jackson and Pin (2009)). Homophily is usually seen as a negative phenomenon since it is connected to segregation of different social groups (Moody (2001)). On the contrary, being connected to similar agents might also
CHAPTER 3. OCCUPATIONAL MISMATCH AND SOCIAL NETWORKS

mean more effective communication between individuals and easier access to relevant information.\(^1\) In my model a higher homophily level implies higher welfare for the individuals because through similar agents they can reach jobs where they earn high wages.

To my knowledge, the only paper in the context of homophily and labor markets is van der Leij and Buhai (2008) who show that homophily in the social network providing job information causes occupational segregation. This is because workers in the same group choose the same occupation to exploit the efficiency of the job arrival process which rises with homophily. My paper instead focuses on the impact of homophily on the mismatch level and the expected wage of job search via social contacts.

Many articles deal with the question whether the informal or the formal search methods provide higher wages for the job seekers. Montgomery (1991) argues that job referrals help to resolve the asymmetric information problem on the labor market and high ability workers tend to find jobs through informal methods. Hence, high wages are associated to the network method. Kugler (2003) points out that job referrals monitor the referred workers in the workplace who, in consequence, put more effort and get paid more. Hence, they obtain that social networks pay a wage premium over the market. On the contrary, some other papers show that the job finding via social contacts results in a wage discount. Bentolila et al. (2010) and Ponzo and Scoppa (2010) argue that family members recommend each other to jobs where their productivity is low which implies a wage discount for social contacts.

There are a couple of papers which are able to recover both a wage premium and a wage discount for the network search (Sylos-Labini (2004), Lipton (2010), Pellizzari (2010), Horváth (2010)). My paper relates the expected wages of the search methods to their ability to match workers to high productivity jobs. I show that even if contacts recommend each other to every job what they become aware of, the matching through social networks can be efficient if the homophily level is high enough. I point to different variables which influence the necessary homophily level, such as the market efficiency or the presence of professional contacts in the network.

\(^1\)In fact, there is evidence that links between similar agents tend to be more resistant to dissolution (McPherson et al. (2001)) indicating that these links are indeed advantageous.
3.2 Model

**Workers and Occupations.** There are two occupations (sectors) $s \in \{A, B\}$ and two types of workers $j \in \{A, B\}$ having $\pi_j = 0.5$ share in the population. Every worker is able to fill a job of any sector but $i$ workers have higher productivity in occupation $i$ (good match) than in occupation $j$ (bad match), $i \neq j, i, j \in \{A, B\}$. The productivity in a bad match is $p$ whereas in a good match $\bar{p}$, where $p < \bar{p}$. A worker’s productivity on a given job does not change over time. Unemployed workers earn unemployment benefits $b$ ($b < p < \bar{p}$).

**Observability of types.** I assume that when a firm and an unemployed meet, they both observe each other’s type, i.e. the type of the match. Partners accept bad matches which is rational if there are search frictions on the labor market and agents would have to wait too much for a consecutive match. They both incur costs of waiting: for the firm to maintain the vacancy costs $c$, while a worker is better off earning the wage corresponding to a bad match ($w_B$) than to earn the unemployment benefit ($b$).

This situation is more likely to be the case when the productivity of a bad and a good match are close enough. In fact, in a two sector model of heterogeneous workers Moscarini (2001) shows that workers whose productivity does not differ much between the two sectors search for and accept offers of both sectors. Thus I focus on the parameter range where the productivity difference is small enough and the firms pay sufficiently high costs of vacancy posting.

**Arrival of offers.** Regarding the arrival of offers to individuals, I assume that for each sector there is a technology which describes the rate of direct arrivals. In a model without social networks this technology would be the matching function itself, however here we need to take into account that the offers are passed along the links of the network. Hence, the matching function is derived from the assumptions on this transmission process.

I assume that there is a difference between the arrival of offers to employed and unemployed agents. Agents employed in sector $i$ might hear only about the offers of the same sector. This assumption reflects the idea that employed workers are likely to hear about job openings of their employers or related firms in the same sector. On the other hand, unemployed workers can find any kind of jobs when searching on the market, i.e. through direct arrival.

These assumptions imply that the set of direct receivers of offers of some sector $i \in \{A, B\}$ contains the following groups: unemployed of type $i$ ($u_i$), unemployed of type $j$ ($u_j$), employed in sector $i$ ($e'_i = e'_i + e'_j$) which might consists of workers of type $i$ ($e'_i$) and $j$ ($e'_j$).

The probability that some offer of sector $i$ reaches someone in the set of possible receivers
is given by:

\[ m_i = A\nu_i^{\eta} \]  

(3.1)

where \( \nu_i \) is the vacancy rate of the corresponding sector, \( A \) and \( \eta \) are technology parameters with the restriction \( 0 < \eta < 1 \). The assumption on the value of \( \eta \) implies decreasing return for the matches with respect to the vacancy rate: as there are more vacancies posted, the probability that a given vacancy is known by someone decreases \( (m_i/\nu_i = A\nu_i^{\eta-1}) \).

The probability that an unemployed of type \( i \) directly receives an offer of sector \( i \) is the following:

\[ P_M = \theta u_i / (\theta u_i + u_j + e^i A\nu_i^{\eta}) \]

where \( \theta u_i / (\theta u_i + u_j + e^i A\nu_i^{\eta}) \) is the probability that an unemployed of type \( i \) is chosen as receiver of the information among the candidate agents. The parameter \( \theta \) stands for the efficiency of the market in producing good matches. If \( \theta = 1 \), the direct arrival is uniform random. If \( \theta \to \infty \), exclusively the right unemployed group receives the job information. Therefore, \( \theta \) parameterizes to what extent firms and unemployed workers of fitting characteristics direct their search toward each other. Another interpretation can be that the institutions of the formal market (for example, agencies) facilitate the encounters of firms and workers and so the formation of highly productive matches.

In the same way, the probability that an unemployed of type \( j \) is directly reached by a vacancy of sector \( i \) is

\[ P_B = u_j / (\theta u_i + u_j + e^i A\nu_i^{\eta}) \]

The probability that an employed in sector \( i \) hears about an offer of the same sector is

\[ e^i / (\theta u_i + u_j + e^i A\nu_i^{\eta}) \]

Note that in this way I exclude the possibility that a mismatched individual improves her position by direct arrival. Moreover, I also excluded the possibility that they hear about better offers through their contacts: below I assume about the information transmission that workers having an unneeded offer consider only their unemployed friends as receiver of information. In this way I disregard the possibility of on-the-job search which simplifies the model in great extent since the job separation rate is exogenous and does not depend on the network structure.

**Job separation.** Jobs might be destroyed by a productivity shock which makes the productivity of the match so low that the continuation is not beneficial for the parties anymore. I
assume that the arrival of such a shock is more probable for bad matches since they have lower productivity than for good matches. Good matches are destroyed according to a Poisson process with parameter $\lambda$ and the process for bad matches is Poisson as well but with higher parameter: $\alpha \lambda$ where $\alpha \geq 1$.\footnote{Note that this formulation also admits an equal separation rate between bad and good matches. I analyze the model for equal and unequal separation rates as well.} This implies that good matches last longer time than bad ones, the difference is measured by parameter $\alpha$.

**Network structure and information transmission.** The underlying network structure is a regular random graph with degree $k$ (Vega-Redondo (2007)). In this type of graphs every worker has $k$ social contacts whom are randomly drawn from the population. The probability that at the two ends of a link two agents of similar type can be found is $\gamma$, consequently, that the two agents are of different types is $1 - \gamma$. $\gamma$ measures the homophily in the society, i.e. the tendency of individuals of similar characteristics to be connected. Throughout our analysis we focus on the values $\gamma > 0.5$.\footnote{$\gamma = 0.5$ would mean random connections, while $\gamma < 0.5$ would represent a tendency of dissimilar agents to be connected.}

When an offer arrives to an unemployed individual, she takes the offer (irrespective of her productivity on that job). When an employed agent has information about a vacancy, she passes the information to a randomly chosen neighbor of hers. Here we assume that the sender of the information does not direct the offer toward those unemployed neighbors who have high productivity on the given job. This behavior generates mismatch. If employed agents directed the offers toward high productivity unemployed, the network would generate less mismatch. Therefore my model describes a case when the efficiency of the network is at its lower bound.

If an employed agent having an offer does not have any unemployed neighbor, the offer is lost (the vacancy remains unfilled). Hence, the information might travel only one step in the network which largely facilitates the analysis.\footnote{The same assumption was used in Calvo-Armengol and Zenou (2005) and in Calvo-Armengol and Jackson (2004).}

The behavior that employed agents recommend their social contacts to jobs irrespective of productivity can be seen as favoritism. In this case, the referee cares only about the employment of her contacts and not about their performance on the job. I believe that this behavior might be relevant for family members, relatives or close friends. In Section 6 as an extension I introduce another type of relationship between agents assuming that employed workers pass information
only to high productivity workers on the given job.

For the derivation of the matching function I use the so-called homogeneous mixing assumption: the probability that an individual is in some state (unemployed, mismatched or employed in the right sector) is equal to the population frequency of that state and the states of different individuals are independently drawn. For example, the probability that a neighbor of an individual is unemployed is equal to the unemployment rate and is independent of the state of the individual. This assumption basically means that the links of the network are randomly drawn at each instant of time (Calvo-Armengol and Zenou (2005)).

**Timing.** I work in continuous time which implies that at some instant of time only one event can happen. Thus it cannot happen that an agent is employed and dismissed at the same instant of time, or that she receives a bad and a good offer at the same time. We can also exclude the possibility that in parallel more than one information source provides an offer for an unemployed. These implications of continuous time modelling to a large extent simplify the analysis.

### 3.2.1 Matching function

The technology defined in the previous section describes the number of "direct encounters" between vacancies and individuals. In this model, however, there is also the possibility to get information through social contacts: unemployed workers might hear about job openings through their employed friends. To construct the probabilities of job finding and vacancy filling we need to consider these two channels, direct and indirect. I define different probabilities for a bad and a good match to be formed.

**Bad matches**

A bad match occurs when an unemployed worker of type $i$ receives an offer of sector $j$ where $i, j \in \{A, B\}, i \neq j$.

First, I determine the probability that a given unemployed is chosen as the receiver of information when a given employed contact of hers is aware of a vacancy. This is the following:

$$Q_i = \sum_{s=0}^{k-1} \binom{k-1}{s} \bar{u}_i^s (1 - \bar{u}_i)^{k-1-s} \frac{1}{s+1} = \frac{1 - (1 - \bar{u}_i)^k}{\bar{u}_i k} \quad (3.2)$$

where $\bar{u}_i = \gamma u_i + (1 - \gamma)u_j$ is the average probability that a neighbor of an individual of type $i$ is
unemployed. The probability that a contact of an employed agent of type $i$ is unemployed is $\bar{u}$: her contact is either of type $i$ or type $j$ and in both cases he has to be unemployed which happens with probability $u_i$ or $u_j$, respectively. This is implied by the homogeneous mixing assumption: the state of an agent is randomly drawn using the population frequencies. $s$ is the number of competitors for the same information: the probability that $s$ other agents are unemployed apart from our given worker is $\bar{u}^{s}(1 - \bar{u})^{k-1-s}$. The probability that among these a given worker gets the information is $1/(1 + s)$.

Second, to get the probability that an unemployed individual of type $i$ receives information through her $k$ neighbors we need to take into account that each of these neighbors has to be employed in sector $j$ and has to be aware of a new opening. Taking this into account we have the following probability:

$$P_N^B = u_i \frac{k \ Av^j}{\theta u_j + u_i + e_j} (\gamma e^j_i Q_i + (1 - \gamma)e^j_j Q_j)$$

where $u_i$ is the probability that an agent is unemployed, she has $k$ contacts hence, $k$ chances to get information. $\gamma \frac{e^j_i \ Av^j}{\theta u_j + u_i + e_j}$ is the probability that a contact is of the same type $i$ ($\gamma$) and employed in sector $j$ ($e^j_i$) and is aware of a job opportunity of that sector. $(1 - \gamma) \frac{e^j_j \ Av^j}{\theta u_j + u_i + e_j}$ stands for the probability that a contact of the other type $j$ has a job offer of sector $j$. $Q_i$ and $Q_j$ are the probabilities that among the other unemployed contacts exactly a given one gets the job information.

The probability that a given unemployed agent of type $i$ finds a job in the other sector $j$ takes into account the direct and indirect arrival as well. In continuous time these two events cannot happen at the same time, thus we need to sum up the arrival rates of the different channels. So the probability that a bad match is formed is the following:

$$u_i q_B = P_M^B + P_N^B = u_i \left( \frac{Av^j}{\theta u_j + u_i + e_j} + k \frac{Av^j}{\theta u_j + u_i + e_j} (\gamma e^j_i Q_i + (1 - \gamma)e^j_j Q_j) \right) =$$

$$u_i \frac{Av^j}{\theta u_j + u_i + e_j} \left( 1 + (\gamma e^j_i Q_i + (1 - \gamma)e^j_j Q_j) \right)$$ (3.4)

This latter expression we can interpret in the following way: $\frac{Av^j}{u_i + \theta u_j + e_j}$ is the probability that a vacancy reaches one of the individuals and $u_i(1 + \gamma e^j_i Q_i + (1 - \gamma)e^j_j Q_j)$ is the probability that
this agent is an unemployed of type $i$ or some employed in sector $j$ who passes the information to an unemployed.

Using this expression, we may write down the bad job filling rate, i.e. the probability that a given vacancy of sector $j$ is filled by a bad match (an unemployed of type $i$):

$$q_B^F = \frac{u_i q_B}{v_j} \tag{3.5}$$

**Good matches**

A good match is formed when an unemployed worker of type $i$ gets an offer of type $i$ ($i \in \{A, B\}$). This might happen through direct arrival or by some social contact.

The probability that an unemployed worker of type $i$ receives an offer of sector $i$ through her contacts is given by:

$$P_N^G = u_i k \frac{A \theta_i}{\theta_i u_i + u_j + e^j} \left( (\gamma e^i Q_i + (1 - \gamma)e^j Q_j) \right) \tag{3.6}$$

The way of constructing this probability is exactly the same as in the case of bad matches. Here the social contacts have to be employed in sector $i$ in both cases of being of type $i$ or type $j$.

By aggregating the different sources of information (market and social contacts), we obtain the probability that a good match is formed:

$$u_i q_G = P_M^G + P_N^G = u_i \left( \frac{A \theta_i}{\theta_i u_i + u_j + e^j} + k \frac{A \theta_i}{\theta_i u_i + u_j + e^j} \left( (\gamma e^i Q_i + (1 - \gamma)e^j Q_j) \right) \right) \tag{3.7}$$

From here the job filling rate of a vacancy of sector $i$ might be expressed as

$$q_G^F = \frac{u_i q_G}{v_i} \tag{3.8}$$

### 3.2.2 Value functions

This section presents the value functions associated to the states of workers and firms. Workers might be unemployed ($U$) or employed in a bad match ($W_B$) or in a good match ($W_G$). Firms post vacancies which might be vacant ($V$) or filled by a worker of low ($J_B$) or high of productivity ($J_G$).
The worker

The discounted value of unemployment consists of the current value of unemployment benefits and the future value of possible employment which might be in the good or in the bad sector:

\[
\delta U = b + q_B(W_B - U) + q_G(W_G - U) \quad (3.9)
\]

where \( \delta \) is the discount rate (common to workers and firms) and \( q_B \) and \( q_G \) represent the job finding rates of a given unemployed, for a bad and a good match, respectively.

The discounted value of being employed in a bad match is the sum of the value of wages earned \( (w_B) \) and the possibility of job destruction which happens at a rate \( \alpha \lambda \) in case of bad matches.

\[
\delta W_B = w_B + \alpha \lambda (U - W_B) \quad (3.10)
\]

Similarly, the value function of being in a good match consists of the wages earned \( (w_G) \) and the future value of being unemployed, here the job destruction rate is \( \lambda \):

\[
\delta W_G = w_G + \lambda (U - W_G) \quad (3.11)
\]

The firm

Maintaining a vacancy is costly but it has the future benefits of becoming filled by a worker of either low or high productivity. The value function is given by:

\[
\delta V = -c + q_B^F(J_B - V) + q_G^F(J_G - V) \quad (3.12)
\]

where \( c \) is the cost of vacancy and \( q_B^F \) and \( q_G^F \) are the job filling rates for bad and good matches, respectively.

The value of a bad match comes from the productivity of the worker in a mismatched job \( p \) and the future value of job destruction while wages have to be paid for the worker:

\[
\delta J_B = p - w_B + \alpha \lambda (V - J_B) \quad (3.13)
\]

A good match has the same value except that now the productivity of the worker is higher: \( \bar{p} > p \) and that a different wage has to be paid:
\[
\delta J_G = \bar{p} - w_G + \lambda (V - J_G) \tag{3.14}
\]

Wages paid in a bad match are lower compared to the wage of a good match which compensates the firm for receiving a lower productivity.

I assume that firms can freely enter to the market when they find it beneficial to do so. The free-entry implies that the value of the vacancy reduces to zero: \( V = 0 \). Applying this equality, we can express \( J_B = \frac{p - w_B}{\delta + \alpha \lambda} \) and \( J_G = \frac{\bar{p} - w_G}{\delta + \lambda} \). Substituting these into the equation (3.12), we get the job creation equation:

\[
c = q_F \frac{p - w_B}{\delta + \alpha \lambda} + q_G \frac{\bar{p} - w_G}{\delta + \lambda} \tag{3.15}
\]

### 3.2.3 Wage determination

Wages are determined by the Nash bargaining procedure upon meeting by workers and firms. I assume that the type of the worker and the type of the job is common knowledge between the firm and the worker, so they condition their decision on the quality of the match.

The wage of a bad match is determined by the following problem:

\[
w_B = \arg\max_{w_B} (W_B - U)^{\beta} (J_B - V)^{1-\beta}
\]

where \( \beta \) is the bargaining power of the worker. This gives the usual first order condition (see the derivation in the Appendix):

\[
W_B - U = \beta (J_B + W_B - V - U)
\]

In the same way we obtain the FOC for the wage of a good match:

\[
W_G - U = \beta (J_G + W_G - V - U)
\]

Using the free-entry condition \( (V = 0) \) and the expressions for \( J_B \) and \( J_G \), we can derive the wages from the FOC’s, see the details in the Appendix.

The expressions for the wages are as in the original Pissarides model with the exception that now we have two of them, one for bad and one for good matches:

\[
w_B = \beta p + (1 - \beta)b + \beta c \frac{v}{u} \tag{3.16}
\]
\[ w_G = \beta \bar{p} + (1 - \beta)b + \beta c \frac{v}{\bar{u}} \] (3.17)

Note that the difference between the two wages is just the productivity difference multiplied by the bargaining power of the worker: \( \beta(\bar{p} - p) \).

### 3.2.4 Equilibrium

In this section, I summarize the equilibrium conditions of the model. The environment of the model guarantees that the equilibrium is symmetric. I have assumed that the two types of workers have equal shares in the population and they are equally homophilous. Moreover, all agents have the same number of connections. Thus if the firms of the two sectors post the same number of vacancies, the unemployed workers of the two groups have the same likelihood to find a bad or a good job. This ultimately implies that the firms in the two sector have the same incentives to post vacancies since the productivity difference between bad and good matches is the same for both sectors. Hence, we can look for the equilibrium in the following form:

- \( v_i = v_j \equiv v \)
- \( u_i = u_j \equiv u \)
- \( e_i^l = e_j^l \equiv e_B \)
- \( e_i^l = e_j^l \equiv e_G \)

Note that, we have some accounting identities as well: \( 1 - u_i = e_i^l + e_i^l \) and \( 1 - u_j = e_j^l + e_j^l \), so in the symmetric case: \( 1 - u = e_B + e_G \).

Applying these equalities, I write down the set of equations defining the equilibrium. First the job finding probabilities for a bad match:

\[ q_B(u, v, e_B) = \frac{A v^l}{\theta u + 1} \left( 1 + k(\gamma e_B + (1 - \gamma)(1 - u - e_B)) \frac{1 - (1 - u)^k}{uk} \right) \] (3.18)

Note that this probability is decreasing in \( u \) since a given unemployed is less likely to hear about a job offer from her neighbors if those have many other unemployed friends since they compete for the same information. \( q_B(u, v, e_B) \) is increasing in \( e_B \) if \( \gamma > 0.5 \) and decreasing if \( \gamma < 0.5 \). If most neighbors of an individual are of the same type and they are employed in the bad sector, they will transmit job information about the vacancies of this sector where the
individual’s productivity is low. The effect of the homophily parameter depends on the state of the economy. On the one hand, if the mismatch is high, homophily increases the probability of receiving a bad offer: same type agents are most probably employed in the bad sector and hence they can transmit offers of only that sector. On the other hand, if the mismatch is low, own type contacts tend to be in good employment, hence they pass information about good matches. The job finding probability increases with the number of connections: to have more contacts means more chance to gather job information.

Second, the probability that a good match is formed is the following:

$$ q_G(u, v, e_B) = \frac{A^n}{\theta u + 1} \left( \theta + k(\gamma(1 - u - e_B) + (1 - \gamma)e_B) \frac{1 - (1 - u)^k}{uk} \right) $$

(3.19)

The probability of receiving good job information is decreasing in $u$ for the same reason as in the case of receiving bad information. It also decreases in $e_B$ for $\gamma > 0.5$: if there are many agents in bad jobs they will learn mostly about offers of the bad sector hence the probability of receiving a good offer is lower. The effect of $\gamma$ again depends on the state of the economy. Homophily increases the probability to hear about a good offer if there are more people employed in the good sector than in the bad.

In the steady state of the model, the employment in- and outflows have to be equal for both good and bad matches. We have the following two turnover equations:

$$ \alpha \lambda e_B = uq_B(u, v, e_B) $$

(3.20)

$$ \lambda (1 - u - e_B) = uq_G(u, v, e_B) $$

(3.21)

$\alpha \lambda e_B$ is the mass of employed agents in the low productivity sector who looses their job. $uq_B(u, v, e_B)$ is the mass of unemployed agents who find employment in a low productivity sector. The second equation can be interpreted similarly for the good sector.

The Job Creation equation has been derived in the previous section, see equation (3.15):

$$ c = \frac{uq_B(u, v, e_B) p - w_B}{v} \frac{\delta + \alpha \lambda}{\delta + \alpha \lambda} + \frac{uq_G(u, v, e_B) \bar{p} - w_G}{v} \frac{\delta + \lambda}{\delta + \lambda} $$

(3.22)

where the wages are given by the following expressions:

$$ w_B = \beta p + (1 - \beta)b + \beta c\frac{v}{u} $$

(3.23)
\[ w_G = \beta \bar{p} + (1 - \beta) b + \beta c \frac{v}{u} \] (3.24)

The equilibrium is defined as follows.

**Definition 1** The equilibrium of the model is a triple \( \{ u^*, e^*_B, v^* \} \) satisfying the equations (3.20), (3.21), (3.22), (3.23), (3.24).

The following Lemma determines some general conditions which guarantee that a unique equilibrium exists in the model.

**Lemma 1** Define the arrival probabilities as functions of \( u \) and \( v \) as follows:

\[
q_B(u, v) = \frac{\alpha \lambda \bar{v}[u + (1 - \gamma)(1 - u)(1 - (1 - u)^k)]}{\alpha \lambda (1 + \theta u) u - \lambda \bar{v}(2 \gamma - 1) u(1 - (1 - u)^k)}
\] (3.25)

\[
q_G(u, v) = \frac{\lambda \bar{v}[u \theta + (1 - \gamma)(1 - u)(1 - (1 - u)^k)]}{\lambda (1 + \theta u) u - \lambda \bar{v}(2 \gamma - 1) u(1 - (1 - u)^k)}
\] (3.26)

Define the Beveridge curve as follows:

\[
BC(u, v) \equiv \alpha uq_G(u, v) + uq_B(u, v) - \alpha \lambda (1 - u) = 0
\] (3.27)

Define the Job Creation curve as follows:

\[
JC(u, v) \equiv \frac{uq_B(u, v) p - w_B}{\delta} + \frac{uq_G(u, v) \bar{p} - w_G}{\delta + \alpha \lambda} - c = 0
\] (3.28)

A unique equilibrium exists in the model if

1. \( \frac{\partial BC(u,v)}{\partial u} > 0 \) for \( \forall u \in [0, 1] \) and \( \forall v \in [0, 1] \)
2. \( \frac{\partial JC(u,v)}{\partial u} > 0 \) for \( \forall u \in [0, 1] \) and \( \forall v \in [0, 1] \)
3. \( \frac{\partial JC(u,v)}{\partial v} < 0 \) for \( \forall u \in [0, 1] \) and \( \forall v \in [0, 1] \)
4. \( \frac{\partial BC(u,v)}{\partial v} > 0 \) for \( \forall u \in [0, 1] \) and \( \forall v \in [0, 1] \)
5. \((1 + \theta)c > A \left( \frac{(1 - \beta)(p-b)}{\delta + \alpha \lambda} + \frac{\theta(1 - \beta)(p-b)}{\delta + \lambda} \right) - A \left( \frac{\beta c}{\delta + \alpha \lambda} + \frac{\theta \beta c}{\delta + \lambda} \right). \)
The Lemma defines the equilibrium conditions of the model as functions of only \( u \) and \( v \) and defines the Beveridge\(^6\) and Job Creation curves as they are usually used in the Diamond-Mortensen-Pissarides type models. The equilibrium of the model is determined by the intersection of these two curves which should be unique if the Beveridge curve is monotone decreasing while the Job Creation curve is monotone increasing in the \((u, v)\) space. The following Proposition identifies sufficient conditions on the model parameters which guarantee the monotonicity of these curves.

**Proposition 2** Assume that the following conditions on the parameters of the model hold:

1. \( \lambda \theta > A(2\gamma - 1)(\gamma - 1 + \theta) + \lambda(1 - \gamma)(1 + \theta) \)

2. \( \alpha \lambda > \alpha \lambda(1 - \gamma)(1 + \theta)) + A(-1 + 2\gamma) \gamma \)

3. \( (-1 + \eta)\lambda \theta + A(-1 + 2\gamma)k \geq 0, \text{ then } (-1 + \eta)\lambda(1 + \theta \tilde{u}) + A(-1 + 2\gamma)(1 - (1 - \tilde{u})^k) < 0 \)

\[ \text{where } \tilde{u} = 1 - \left[ \frac{(1-\eta)\lambda \theta}{kA(2\gamma - 1)} \right]^{\frac{1}{1-k}} \]

4. \( (1 + \theta)c > A \left( \frac{(1-\beta)(\eta-\beta)}{\delta+a\lambda} + \frac{\theta(1-\beta)(\beta-\beta)}{\delta+a\lambda} \right) - A \left( \frac{\beta c}{\delta+a\lambda} + \frac{\theta \beta c}{\delta+a\lambda} \right) \)

Then a unique equilibrium exists in the model.

### 3.3 Quantities of interest

I define the quantities of interest of the model. We can measure the level of mismatch by the fraction of employed in the bad sector compared to the fraction of employed in the good sector.

**Definition** The level of mismatch is defined as the fraction \( e_B/e_G \).

Note that in equilibrium this fraction is equal to \( q_B/(\alpha q_G) \), the fraction of arrival rates corrected by the separation rate difference. This follows from the division of the two equilibrium conditions (3.20) and (3.21).

For the comparison of the market and social networks with respect to "mismatch creation", I define the expected wages of the different search methods. I use the conditional expectation

\(^6\)This version of the Beveridge curve can be obtained using (3.20) and (3.21) by expressing \( e_B \) from the first and substituting it to the second equation.
of the wages upon arrival because the different methods have dissimilar arrival rates of offers, thus we need to normalize the expectations by this.

By concentrating on the conditional expectation upon arrival we disregard the differences in the arrival rate of offers between search methods. The focus on this statistics is also justified by the empirical literature on expected wages of the different search methods (Pellizzari (2010), Bentolila et al. (2010)): these papers estimate the same conditional expectation. The estimates are based on the ex-post relation between the earned wages of an employed individual and the search method she used to obtain her job. Clearly, this relation conditions on the arrival of offer through that method and the acceptance of the offer.

**Definition 2** The expected wage of the formal search is the conditional expectation of the wage which is obtained through direct arrival conditioned on the event that an offer has arrived:

$$\frac{P^M_B w_B + P^M_G w_G}{P^M_B + P^M_G} = \frac{\frac{\theta e_B}{1+\theta} w_B + \frac{\theta e_G}{1+\theta} w_G}{1+\theta} = \frac{w_B + \theta w_G}{1+\theta}$$

Hence if the market arrival is random ($\theta = 1$), i.e. "similar" workers and firms do not search for each other, in symmetric equilibrium the direct arrival provides good and bad offers with the same probability and the expected wage is just the average of the wages in good and bad matches. On the other hand, if $\theta > 1$, the expectation puts higher weight on good jobs. Note that in a symmetric equilibrium, $\theta$ means that upon arrival an offer coming through the market is $\theta$ times more likely to be of good type than of bad type.

As for the network channel we can define the wage expectation in a similar way.

**Definition 3** The expected wage of network search is defined as the conditional expectation of the wage obtained through the contacts conditional on the event that an offer has arrived through a contact:

$$\frac{P^N_B w_B + P^N_G w_G}{P^N_B + P^N_G} = \frac{(\gamma e_B + (1-\gamma)e_G)w_B + (\gamma e_G + (1-\gamma)e_B)w_G}{e_B + e_G}$$

Observe that to compare these expectations it is sufficient to compare the probability weight on the good wages which depends on the ratio of bad and good arrival rates through that channel ($P_{r_B^j}/P_{r_G^j}$). For example, in the case of the market, the weight on good matches
is just $\theta/(1 + \theta)$ which reciprocally depends on the fraction $1/\theta$. This fraction determines how much would be the mismatch in the society if only the market operated as information channel. In this case we could write:

$$\frac{e_B}{e_G} = \frac{q_B}{\alpha q_G} = \frac{\alpha \gamma}{1 + \theta \alpha} = \frac{1}{\alpha \theta}$$

Here we applied that the measure of mismatch is equal to the ratio of arrival rates corrected by $\alpha$ which would be equal to the last expression if only the market worked in the society. We can see that it also depends on $1/\theta$ as the probability weight.

Note that the weight on the good wage in the conditional expectation of the network arrival can be re-written as follows:

$$\gamma + (1 - \gamma) \frac{e_B}{e_G} \frac{\alpha}{1 + \alpha}$$

Its derivative with respect to the mismatch level ($e_B/e_G$):

$$\frac{1 - 2\gamma}{(1 + \alpha)} \frac{\alpha}{1 + \alpha} < 0$$

This derivative is negative if $\gamma > 0.5$: as the mismatch decreases, the social contacts are more likely to provide good offers.

The derivative of this weight with respect to $\gamma$ is

$$\frac{1 - \alpha}{1 + \alpha}$$

This derivative clearly increases if the mismatch level $e_B/e_G$ decreases. Hence, the factors which decrease the mismatch (e.g. market efficiency) increase the response of the network wage expectation to the homophily level.

The following Lemma relates the conditional wage expectations of the search methods to the mismatch level of the economy.

**Lemma 3** Mismatch level in the society can be written as:

$$\frac{e_B}{e_G} = \frac{1}{\alpha} \left[ \frac{P_r^M}{P_r^G} \mu + \frac{P_r^N}{P_r^G} (1 - \mu) \right]$$

where $\mu \in (0, 1)$ and $P_r^j$ is the probability that a search method $j \in \{N(etwork), M(arket)\}$ provides an offer of type $i \in \{B, G\}$.

**Proof** See Appendix.
We have seen that the conditional wage expectation of a search method depends on the ratio of bad and good arrival rates through that method. This lemma says that the mismatch of the economy can be written as the linear combination of these ratios between the two search methods (market and networks). An implication of this lemma is that whenever the expected wage of the network is higher than that of the market, the mismatch level is lower in the economy with social networks than it would be if only the market operated as information channel. In this case the presence of the network improves on the market arrival.

3.4 Results

Using the described model, I investigate whether the presence of social networks in the matching process increases or decreases the mismatch level of the society compared to a market economy. I show that even though individuals recommend each other to jobs irrespective of productivity (favoritism), for some parameter values the presence of networks still leads to an increase in the efficiency of the labor market. I also compare the performance of the market and the social contacts with respect to producing good matches which contributes to the literature on expected wage comparison between search methods. I also study what is the effect of the parameters of the network structure, i.e. the homophily rate and the connectivity of the network, on the efficiency of the matching process. Numerically I evaluate how these parameters affect other endogenous variables of the model such as the vacancy rate, unemployment rate and wages.

The first proposition describes the benchmark case in which the market arrival is uniform random and there is no difference between the separation rates of bad and good matches.

**Proposition 4** If the market arrival is uniformly random ($\theta = 1$) and the job separation rate is equal between bad and good matches ($\alpha = 1$),

(i) in equilibrium the fraction of workers employed in bad and good matches are equal ($e_B = e_G$),

(ii) the network provides bad and good matches with equal probability,

(iii) the homophily and the connectivity has no impact on the mismatch level.
The intuition behind this result is based on the assumption that employed individuals learn about the offers of their sector of occupation. If $\theta = 1$, the direct arrival provides good and bad jobs with equal probability. As for the network, if the employment rates in good and bad matches are the same, an employed of any type has equal probability to be employed in the two sectors. This implies that the arrival rate of information about bad and good matches through the contacts will be the same. Hence, both sources of information give bad and good offers at uniformly random. Now if, in addition, we have that bad and good matches have the same job separation rate ($\alpha = 1$), the employment rates are equal in good and bad matches. Hence, we have an equilibrium with $e_B = e_G$.

Homophily has no effect on the mismatch level since contacts of both own and different type have the same probability to provide information about bad and good matches. In this case the connectivity has no effect either since adding more neighbors per se does not change the average quality of offers arriving through social contacts.

Next, we move away from this benchmark case: we allow for a biased market arrival and good matches last longer time than bad ones.

**Proposition 5** If the market efficiency ($\theta$) or the separation rate difference between bad and good matches ($\alpha$) increase, the mismatch decreases. If $\theta > 1$ or $\alpha > 1$, the mismatch decreases as the homophily level ($\gamma$) increases.

As the market efficiency parameter rises, the market arrival provides good job information with higher probability, this implies that the mismatch level decreases. The same is the resulting effect if we increase the job separation rate of the bad matches: in equilibrium there will be less workers employed in low productivity jobs.

When a worker has higher probability to be employed in a good match than in a bad one ($e_G > e_B$), homophily decreases the mismatch level. In this case to be connected to a worker of own type is beneficial in the sense that she is likely to be employed in the good sector and thus to provide a job information about a high productivity job.

Thus a more homophilous network is more efficient in providing good job information. The following proposition states that for high homophily values the network actually can be more efficient than the market channel. This implies that it pays a wage premium over the market arrival and that the mismatch level is lower in the economy with social networks compared to a pure market economy.
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**Proposition 6** There exists a $\tilde{\gamma} > 0.5$ such that for $\forall \gamma > \tilde{\gamma}$, the social network provides higher expected wage than the market and the mismatch level is lower in the economy with social networks than in a pure market economy.

i If $\alpha = 1$, $\tilde{\gamma} = 1$,

ii if $\alpha > 1$, $\tilde{\gamma} < 1$,

iii $\tilde{\gamma}$ increases if $\alpha$ increases,

iv $\tilde{\gamma}$ is independent of the connectivity of the network ($k$).

This proposition identifies a threshold value of the homophily level for which the market and social networks are equally likely to provide good offers conditional on arrival. We have seen that both the mismatch and the expected wages of the search methods depend on the probability ratio that a search method provides a good offer over that it provides a bad one. When the homophily is complete ($\gamma = 1$), this probability ratio for the social network is equal to $e_G/e_B$. This is the probability that an agent of the same type is employed in a good job over the probability to be employed in a bad one, i.e. the inverse of the mismatch level. When bad and good jobs are separated at the same rate ($\alpha = 1$), the only exogenous factor which determines the mismatch level is the market efficiency: $e_G/e_B = \theta$. Hence, at $\gamma = 1$, the market and the social networks are equally efficient. We have seen that as the homophily level decreases, the likelihood that the social contacts provide a good offer decreases. Hence for $\gamma < 1$, the market is more efficient than the social network.

If bad matches are separated at higher rates ($\alpha > 1$), in equilibrium there will be more agents in good matches, hence the ratio $e_G/e_B$ is higher than in the case of $\alpha = 1$. This implies that for complete homophily, $e_G/e_B > \theta$, i.e. conditional on arrival the social networks provide good jobs at a higher rate than the market. When the homophily is lower, the efficiency of networks decreases and we find a threshold value of the homophily where the two methods are equally efficient. If the type of the connections of a worker are random ($\gamma = 0.5$), the market is always more efficient than the social network. When $\alpha$ rises, the mismatch decreases and for any $\gamma > 0.5$ it is more likely that the social contacts provide good offers. Thus, the efficiency of the market and social networks are equal for a lower homophily value, i.e. the threshold decreases.
The connectivity of the network does not affect the position of the homophily threshold. We can see that for $\gamma = \bar{\gamma}$, the mismatch level does not change with the connectivity, hence the efficiency of the network is unaffected. Recall that the mismatch level can be written as a linear combination:

$$\frac{e_B}{e_G} = \frac{1}{\alpha} \left[ \frac{Pr_B^M}{Pr_G^M} \mu + \frac{Pr_B^N}{Pr_G^N} (1 - \mu) \right]$$

Here both $Pr_B^M / Pr_G^M$ and $Pr_N^N / Pr_G^N$ do not change with the connectivity, only the weight on the network arrival changes (see the proof of Lemma 3). Given that these two ratios are equal at $\bar{\gamma}$, the mismatch stays at the same level even though the weight changes. The efficiency of the network depends on $Pr_N^N / Pr_G^N$ which does not change when the mismatch keeps constant, so the homophily threshold remains constant as well.

It is ambiguous how the threshold changes when the market efficiency ($\theta$) increases. First, such an increase makes the market wage expectation higher. Second, the mismatch decreases which makes the network arrival more efficient. Hence, theoretically we cannot decide which effect is more important. However, the numerical analysis presented below suggests that the threshold homophily level increases when the market becomes more efficient.

The following proposition describes the effect of the network connectivity on the mismatch level and the unemployment rate.

**Proposition 7** As the network connectivity $k$ rises,

i. the unemployment rate $u$ decreases,

ii. if $\gamma > \bar{\gamma}$, the mismatch decreases,

iii. if $\gamma < \bar{\gamma}$, the mismatch increases.

First, if the network is more connected, every agent has more chances to hear about new job openings. Hence, the information transmission on the labor market becomes more efficient: there is higher probability that a vacancy reaches an unemployed. The unemployment rate decreases.

Second, the effect of connectivity on the mismatch level depends on the homophily index. On the one hand, if the homophily is above the threshold identified by the previous proposition, the network channel is more efficient than the market. In this case, if we increase the connectivity, there will be more job offers arriving through the network and these offers are likely to
be of good type. Hence, the mismatch decreases. On the other hand, if the homophily is below the threshold, giving more weight to the network arrival generates more mismatch.

Hence, at least for high homophily values we can say that the presence of the network leads to higher welfare: it decreases the unemployment rate and increases the fraction of workers employed in high productivity jobs.

### 3.5 Numerical example

#### 3.5.1 Parameter values

Table 3.1 summarizes the parameter values. The bargaining power of the workers \( \beta \) is set to 0.5, a value mostly used in the literature (Mortensen and Pissarides (1999), Fontaine (2008)). The discount rate \( \delta \) is set to 0.988 which corresponds to a quarterly interest rate of \( r = 0.012 \) used by Shimer (2005). As for the quarterly separation rate \( \lambda \), I use the value of 0.1 as it was estimated by Shimer (2005). This corresponds to the separation rate of good matches and I assume that the separation rate of bad matches is double, \( \alpha = 2 \). We set the value of market efficiency \( \theta \) to 4 which means that an offer obtained on the market is four times more likely to be of good type than to be of bad type.

The productivity in a good match \( \bar{p} \) is normalized to 1. I assume that the productivity in a bad match is 15% less than the productivity in a good match, I set \( p = 0.85 \). These values of \( p \) and \( \bar{p} \) guarantee that bad matches are optimally accepted which can be checked using the conditions presented in Section 10.2 of the Appendix.

The elasticity of the matching function with respect to the vacancy rate (\( \eta \)) is set to be 0.3 which is the lower bound of the estimates summarized by Petrongolo and Pissarides (2001).

The two network parameters, the connectivity (\( k \)) and the homophily index (\( \gamma \)) are the parameters of interest and I vary them on a grid: \( \gamma \in [0.5, 1] \), \( k \in [1, 15] \). However, for the calibration exercise I use two benchmark values: \( k = 10 \), \( \gamma = 0.5 \).

The remaining three parameters, the efficiency parameter of the matching function (\( A \)), the cost of vacancy posting (\( c \)) and the value of the unemployment benefit are calibrated to match three real world statistics of the US economy: the unemployment rate being 5.67%, the market tightness (\( v/u \)) being 0.634 (both of them estimated by Shimer (2005)) and the unemployment benefit replacement rate\(^7\) being 55.67% in 2007 (see the OECD labor market statistics). These

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\(^7\)The replacement rate is the average unemployment benefit/wage ratio. In our model this is the unemployment
### Table 3.1: Parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>[0.5,1]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.988</td>
</tr>
<tr>
<td>$b$</td>
<td>0.471</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>1</td>
</tr>
<tr>
<td>$p$</td>
<td>0.85</td>
</tr>
<tr>
<td>$c$</td>
<td>0.363</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.1</td>
</tr>
<tr>
<td>$k$</td>
<td>[1,15]</td>
</tr>
<tr>
<td>$A$</td>
<td>0.482</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\theta$</td>
<td>4</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2</td>
</tr>
</tbody>
</table>

Note that for these parameter choices the conditions to guarantee a unique equilibrium (see Lemma 1) are satisfied and both firms and workers make optimal decisions to accept bad matches instead of waiting for a good match.

#### 3.5.2 Results

In this section I solve the model numerically using the parameter values of Table 3.1, figures can be found in the Appendix (Section 11). First, I illustrate the homophily threshold value which separates the different cases with respect to the relative performance of networks and market. If the homophily level is above this threshold, the presence of social networks decreases the mismatch compared to the level of a pure market economy. In this case, the market wage expectation is lower than the expected wage paid by the network channel. Figure 3.1 shows this threshold for different values of the market efficiency ($\theta$) and separation difference ($\alpha$) between bad and good matches. As it is shown by Proposition 6, if $\alpha = 1$, this threshold is equal to 1 for any value of $\theta$. As $\alpha$ gets higher, the threshold decreases: the network needs to be

\[ \text{benefit } b \text{ divided by the average wage } \frac{e_B + e_G}{w_B + w_G}. \]
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less homophilous to guarantee the same expected wage. On the contrary, if the market becomes more efficient, i.e. $\theta$ increases, the homophily threshold increases.

Second, we have seen in Proposition 7 that depending on the level of homophily, an increase in the connectivity of the network might increase or decrease the mismatch level in the society. Figure 3.2 gives two examples, one for $\theta = 2$ and one for $\theta = 4$. When the homophily is above the mentioned threshold, adding more connections decreases the mismatch. On the contrary, if the homophily is sufficiently low, a more connected network implies a higher mismatch level. Interestingly, in the case of a more efficient market ($\theta = 4$), the mismatch level is lower in general compared to the case of $\theta = 2$. However, the network needs to be more homophilous to decrease the mismatch.

Figure 3.3 shows the wage premium/discount of the jobs obtained through social contacts over the jobs found on the market. Here we use the baseline parameter values $\theta = 4$, $\alpha = 2$. These imply a homophily threshold of 0.886. We can see on the figure that if the homophily level exceeds this value, the network search gives a wage premium over the market. On the contrary, for lower homophily levels the market pays higher expected wages. We can also observe that even though the connectivity had a significant effect on the mismatch levels, it barely changes the wage premium values.

Figure 3.4 highlights the effects of the network structure on further endogenous variables of the model: unemployment rate, vacancy rate and wages. As indicated by Proposition 7, if the network becomes more connected, the unemployment rate decreases. More connections mean that workers and vacancies have more channels to meet each other on the labor market. The unemployment rate also decreases with the homophily level. Lower homophily implies lower mismatch and since I assume that good matches are separated at a lower rate, this causes a lower unemployment rate.

When the unemployment rate becomes lower, workers can claim higher wages. Their bargaining position gets better since it becomes easier for them to find a job and more difficult for the firms to find an unemployed worker. Hence, with the rise of connectivity or homophily the wages get higher as well. Note that the difference between the wage of good and bad matches is constant (see the expression of wages in equation (3.24) and (3.23)).

Both the increase of wages and decrease of unemployment rate makes less beneficial for the firms to open a vacancy. They will have to pay higher wages and face a lower job filling probability. In equilibrium, this implies a lower vacancy rate. Thus, as the network becomes
more connected or more homophilous, the vacancy rate decreases in the economy.\textsuperscript{8}

### 3.6 Extension

So far I have assumed that the individuals recommend each other to jobs irrespective of productivity. This behavior can be interpreted as favoritism. However, in reality individuals are endowed with social contacts who are less likely to do such favors, for example professional contacts or weak ties. Recommending someone of low productivity for a given job might cause reputation loss for the recommender and in case of weak ties this is not compensated by the value of the link toward the worker.\textsuperscript{9}

I investigate the impact of professional contacts on the mismatch level and the efficiency of social networks. Professional contacts forward good job offers, so I modify the arrival rate of good offers by introducing the parameter $P$ which stands for the individual’s endowment with such contacts:

$$q'_G(u, v, e_B) = \frac{A_y}{\theta u + 1} \left( \theta + P + k(\gamma(1 - u - e_B) + (1 - \gamma)e_B) \right) \frac{1 - (1 - u)^k}{uk}$$

(3.29)

$q'_G(u, v, e_B)$ differs from $q_G(u, v, e_B)$ only in the constant $P$. If $P = 0$, $q'_G(u, v, e_B) = q_G(u, v, e_B)$.

Now, we can differentiate between two types of network channel: family contacts who favor each other and transmit any kind of offers and the overall network which contains both family and professional contacts. Based on this distinction I look at two wage expectation: expected wage of the family channel as defined in Definition 3 and the expected wage of the overall network which is defined as follows.

**Definition** The expected wage of the overall network conditional on arrival is defined as

$$\frac{P^N_B w_B + P^N_G w_G}{P^N_B + P^N_G} = \frac{kQ(ye_B + (1 - \gamma)e_G)w_B + (P + kQ(ye_G + (1 - \gamma)e_B))w_G}{kQ(e_B + e_G) + P}$$

where $P^N_B = P^N_B$ since the arrival rate of bad offers does not change with the introduction of professional contacts. $P^N_G = P^N_G + P$ because the professional contacts increase the arrival rate of good jobs by $P$ compared to the family network.

\textsuperscript{8}Note that less vacancy means that the chances of a given vacancy to be filled increase. It also implies that workers have harder time to find jobs. These conditions imply that the bargaining position of the workers worsens. However, as we see this effect is not significant enough to reverse the trends outlined above.

\textsuperscript{9}Beaman and Magruder (2010) show that incentives put by the employer might be crucial whether incumbent workers recommend family members or professional contacts.
We have the following proposition.

**Proposition 8** If professional contacts are present \((P > 0)\), for any value of the market efficiency parameter \(\theta\),

(i) there exists a homophily value \(\bar{\gamma}_1\) such that for any \(\gamma \in (\bar{\gamma}_1, 1]\) the family network provides higher expected wage than the market,

(ii) \(\bar{\gamma}_1\) is decreasing in \(P\),

(iii) There exists an interval \((\bar{\gamma}_2; \bar{\gamma}_1]\) such that for any \(\gamma\) in this interval the market expected wage is higher than the family one but lower than the overall social network’s expected wage.

Given that the professional contacts only transmit good offers, their presence decreases the mismatch level in the society. Consequently, for high homophily values the "family" network is more efficient than the market. Here the effect of professional contacts is similar to the impact of separation rate difference \(\alpha\) presented above. The overall network also contains the professional contacts hence, it gives a wage premium over the market even for lower homophily values.

Proposition 8 suggests that we can differentiate three ranges of the homophily level: 1. both types of network give a wage premium over the market, 2. the overall network provides a wage premium but not the family network, 3. both social networks give a wage discount. These three regions are illustrated on Figure 3.5 for two values of the connectivity of family networks.

We can see that as the significance of the professional contacts increases both threshold values decrease. In this case the mismatch level decreases in the society which makes the network channel more efficient. Thus less homophily is needed to reach the efficiency of the market. However, as we increase the connectivity of the family network \((k)\), the impact of the professional contacts becomes smaller since their relative importance in the matching process decreases (see the right panel of Figure 3.5).

### 3.7 Discussion

The model presented here helps to rationalize the mixed cross-country evidence regarding the relative expected wages provided by the different job search methods (Pellizzari (2010)).
A wage discount of the network search is most likely to be found in countries where both the homophily level and the number of professional contacts are low. Wage premium is likely in countries where unemployed rely much on professional contacts and the homophily is sufficiently high as well. For intermediate values of homophily a likely situation is that the family contacts give a wage discount while the overall network gives a wage premium over the market.

I have shown that every time the social networks pays higher expected wages than the market, the mismatch level is lower in the society in the presence of the network channel than it would be in an economy where individuals exclusively rely on the formal market. The necessary homophily level to this positive effect of the network rises with the efficiency of the market and decreases with other factors that diminish the mismatch in the society. Hence for example, if the governmental employment agencies match the unemployed workers more efficiently to appropriate jobs, this may have a negative effect on the efficiency of the informal job finding channels. On the contrary, the increase of unemployment benefit may enhance the efficiency of informal methods. Marimon and Zilibotti (1999) show that an increase of the unemployment benefits makes the mismatch level lower since agents tend to wait more for appropriate job offers. According to my model such an increase also has a positive side-effect on the efficiency of the network channel, namely, it increases as well.

### 3.8 Conclusions

I investigate the impact of social networks on mismatch of workers to occupations is, whether networks perform better or worse than the market in this respect. In my model individuals have two types of relationships: family contacts who favor each other and recommend each other to jobs irrespective of productivity, and professional contacts who forward high productivity offers.

I show that if the probability that the family ties connect agents of similar type (homophily index) increases, the level of mismatch in the society decreases. This effect is larger when there are other factors in the society which decrease mismatch such as the presence of professional contacts or the fact that good matches last longer time than bad ones. If unemployed rely more on professional contacts, the mismatch level decreases.

Regarding the relative performance of market and networks I differentiate between the family network and the overall network which contains both types of relationships. I show that
whether the formal market performs better in producing good matches compared to any of these two networks is determined by the homophily level of the family network.

If there are professional contacts in the network or there is difference in the job separation rate between bad and good matches, for sufficiently high level of homophily the family provides a wage premium over the market. This is true for any efficiency level of the market since the market efficiency has a positive feedback on the social network. The necessary homophily level is increasing in the market efficiency and decreasing in the number of professional contacts and the extent of separation rate difference. I have also shown that whenever the network gives a wage premium, the mismatch in the society is lower than it would be if only the market operated as information channel. Hence favoritism is not necessarily bad from an efficiency point of view, the presence of social networks might lead to higher average productivity.

Interestingly, I obtain that if every family tie connects dissimilar agents, a relatively inefficient market provides a wage premium over the overall network. Thus, some homophily is necessary to get a wage premium for the overall network over the market arrival.

3.9 References

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CHAPTER 3. OCCUPATIONAL MISMATCH AND SOCIAL NETWORKS


3.10 Appendix

3.10.1 Calculation of wages

The problem to be solved is the following:

$$w_l = \text{argmax}_{w_l} (W_l - U)^{\beta} (J_l - U)^{1 - \beta}$$

where \(l \in \{B, G\}\)

FOC:

$$(J_l - V)^{1 - \beta} \frac{\partial (W_l - U)}{\partial w_l} + (1 - \beta)(J_l - V)^{-\beta} \frac{\partial (J_l - V)}{\partial w_l} (W_l - U)^{1 - \beta} = 0$$
Similarly to the literature, we assume that \( \frac{\partial U}{\partial w} = 0 \), by the free-entry condition \( V = 0 \). Further we have \( \frac{\partial W}{\partial w} = 1 \) and \( \frac{\partial J}{\partial w} = -1 \). This leads to the following simplified version of the FOC:

\[
\beta J = (1 - \beta)(W - U)
\]

Now we can substitute the expressions of the value functions. In the text we have seen that:

\[
J = \frac{p_l - w_l}{\delta + \lambda_l}
\]

where \( p_l \) is the corresponding productivity depending on the type of the match and \( \lambda_l \) is just \( \lambda \) for good matches while it is \( \alpha \lambda \) for bad matches.

Using the expressions for the value of employment for the worker (3.10) and (3.11), we get:

\[
W = \frac{w_l + \lambda_l U}{\delta + \lambda_l}
\]

Substituting into the FOC we can express the wages as the function of \( U \) and the parameters:

\[
w_l = \delta U + \beta(p_l - \delta U)
\]

What is left is to express \( \delta U \) as the function of the parameters. In the text we had:

\[
\delta U = b + q_B(W_B - U) + q_G(W_G - U)
\]

Using the FOC we substitute out \( (W - U) \) and we get:

\[
\delta U = b + \frac{\beta}{1 - \beta}(q_B J_B + q_G J_G)
\]

Using (3.22) and the relationship between the job filling and job finding probability \( (q^F_I = \frac{u q_I}{v}) \), we have the following:

\[
(q_B J_B + q_G J_G) = c \frac{v}{u}
\]

Hence we get the following for \( \delta U \):

\[
\delta U = b + \frac{\beta}{1 - \beta} c \frac{v}{u}
\]

Using this latter and substituting into the expression for wages:

\[
w_l = \beta p_l + (1 - \beta)b + \beta c \frac{v}{u}
\]

which is similar to the standard wage equation in the basic Pissarides model.
3.10.2 Optimality of decisions

It is optimal for the worker to accept a bad offer now if the discounted value of bad unemployment is higher than the value of staying unemployed. This is:

$$\delta U \leq \delta W_B \iff b + q_B(W_B - U) + q_G(W_G - U) \leq w_B + \lambda(U - W_B) \iff q_G(W_G - U) + (q_B + \lambda)(W_B - U) \leq w_B - b$$

The last inequality says that the difference between the bad wage and the unemployment benefit should be high enough.

Using the first-order condition of the wage bargaining problem (see Appendix) we can express this condition as a function of the endogenous variables. First, we write:

$$W_i - U = \frac{\beta}{1 - \beta} J_i = \frac{\beta}{1 - \beta} \frac{p_i - w_i}{\delta + \lambda}$$

where $i \in \{B, G\}$.

Second, using this relation we can express the optimality condition of the worker:

$$\frac{\beta}{1 - \beta} \left( q_G \frac{\bar{p} - w_G}{\delta + \lambda} + (q_B + \lambda) \frac{p - w_B}{\delta + \alpha \lambda} \right) \leq w_B - b$$

This condition can be evaluated at the equilibrium values of the endogenous variables. We will perform this check at the numerical analysis of the model.

Turning to the firms, we may write their optimality condition in the same way:

$$\delta V \leq \delta J_B \iff -c + q_B^F(J_B - V) + q_G^F(J_G - V) \leq p - w_B + \alpha \lambda (V - J_B) \iff -c + q_G^F \frac{\bar{p} - w_G}{\delta + \lambda} \leq (p - w_B) \left( 1 - \frac{(q_B^F + \alpha \lambda) \frac{1}{\delta + \alpha \lambda} }{\delta + \alpha \lambda} \right)$$

where the last relation uses the expression for $J_i$ from above and that the value of the vacancies is equal to zero. The condition says that it should have higher value to accept a bad match now than maintaining a vacancy and waiting for a subsequent match. Again this condition can be evaluated at the equilibrium.

3.10.3 Proofs

Lemma 1

**Proof** First, we define the equilibrium conditions of the model as a function of $u$ and $v$. 
We can use (3.20) to express $e_B = \frac{q_B}{\alpha \lambda}$. Substituting this identity into (3.18) and using that $e_G = 1 - u - e_B$, we can find $q_B(u, v)$:

$$q_B(u, v) = \frac{\alpha \lambda Av^\theta[u + (1 - \gamma)(1 - u)(1 - (1 - u)^k)]}{\alpha(1 + \theta u)u - Av^\theta(2\gamma - 1)u(1 - (1 - u)^k)}$$  \hspace{1cm} (3.30)

We may do a similar step to obtain $q_G(u, v)$. Using (3.21), we express $e_G = \frac{q_G}{\alpha \lambda}$. Substituting into (??) and using that $e_B = 1 - u - e_G$, we get:

$$q_G(u, v) = \frac{\lambda A v^\theta[u(1 - \gamma)(1 - u)(1 - (1 - u)^k)]}{\lambda(1 + \theta u)u - Av^\theta(2\gamma - 1)u(1 - (1 - u)^k)}$$  \hspace{1cm} (3.31)

To obtain the Beveridge Curve in the $(u, v)$ plane, we again write $e_B = \frac{q_B}{\alpha \lambda}$ and substitute it into (3.21):

$$BC(u, v) = \alpha u q_G(u, v) + u q_B(u, v) - \alpha \lambda (1 - u) = 0$$  \hspace{1cm} (3.32)

To get the Job Creation curve as a function of $u$ and $v$, we only need to substitute $q_B(u, v)$ and $q_G(u, v)$ into (3.22):

$$JC(u, v) = \frac{u q_G(u, v) p - w_B}{\delta + \alpha \lambda} + \frac{u q_B(u, v) \bar{p} - w_G}{\delta + \lambda} - c = 0$$  \hspace{1cm} (3.33)

The equilibrium of the model is the intersection of these two curves in the $(u, v)$ plane: $BC(u, v) = 0$ and $JC(u, v) = 0$.

Then if $\frac{\partial BC(u, v)}{\partial u} > 0$ and $\frac{\partial BC(u, v)}{\partial v} > 0$ for all $u \in [0, 1]$ and for all $v \in [0, 1]$, the Beveridge Curve is monotone decreasing in the $(u, v)$ plane ($\frac{dv}{du} = \frac{\partial BC(u, v)}{\partial u} \left(\frac{1}{\partial BC(u, v) / \partial v}\right)^{-1}$). While if $\frac{\partial JC(u, v)}{\partial u} > 0$ and $\frac{\partial JC(u, v)}{\partial v} < 0$ for all $u \in [0, 1]$ and for all $v \in [0, 1]$, then the Job Creation curve monotone increasing in the $(u, v)$ plane ($\frac{dv}{du} = \frac{\partial JC(u, v)}{\partial u} \left(\frac{1}{\partial JC(u, v) / \partial v}\right)^{-1}$). This implies that these curves can cross only once, i.e. if the equilibrium exists, it should be unique.

The next step is to show that the two mentioned curves actually cross. First, we demonstrate that as $u \to 0$, the Beveridge Curve lies above the Job Creation curve in the $(u, v)$ plane.

To this end we compute the following limits:

$$\lim_{u \to 0} q_B(u, v) = \lim_{u \to 0} \frac{\alpha \lambda Av^\theta[u + (1 - \gamma)(1 - u)(1 - (1 - u)^k)]}{\alpha(1 + \theta u)u - Av^\theta(2\gamma - 1)u(1 - (1 - u)^k)} = \frac{Av^\theta(\gamma - (1 + \gamma)(1 + k)(1 - u)^k)}{\alpha \lambda}$$

$$\lim_{u \to 0} q_G(u, v) = \lim_{u \to 0} \frac{\lambda A v^\theta[u(1 - \gamma)(1 - u)(1 - (1 - u)^k)]}{\lambda(1 + \theta u)u - Av^\theta(2\gamma - 1)u(1 - (1 - u)^k)} = \frac{Av^\theta(\gamma - (1 + \gamma)(1 + k))}{\alpha \lambda}$$
where we used the L’Hopital.

In the same way, we can write:

$$\lim_{u \to 0} q_G(u, v) = \lim_{u \to 0} \frac{\lambda A v^\eta [u \theta + (1 - \gamma)(1 - u)(1 - (1 - u)^k)]}{\lambda (1 + u \theta)u - A v^\eta (2\gamma - 1)u(1 - (1 - u)^k)} = \frac{A \lambda (-1 + \gamma + \theta - (-1 + \gamma)(1 + k)(1 - u)^k)}{\lambda + 2\lambda \theta u + \frac{\lambda (-1 + 2\gamma)(1 - u + (1 - u)^k(1 - u + ku))}{1 + u}}$$

Now, we write the Beveridge curve in the following way:

$$\alpha q_B(u, v) + q_G(u, v) - \frac{\alpha \lambda (1 - u)}{u} = 0$$

If we take the limit $u \to 0$, the last term of the left-hand side goes to infinity while the first two terms take a finite value. Hence, the equation can only hold if $v \to \infty$. Thus the solution of the equation of the Beveridge Curve at $u = 0$ is $v = \infty$.

Next, we see that the $(0, 0)$ point lies on the Job Creation curve. This becomes clear writing the Job Creation curve in the following way:

$$c_v = \frac{u q_B(u, v, e_B)(p - w_B)}{\delta + \alpha \lambda} + \frac{u q_G(u, v, e_B)(\bar{p} - w_G)}{\delta + \lambda}$$

If $u \to 0$, the expressions on the right-hand side go to 0. Hence, the left-hand side has to go to zero which is the case when $v \to 0$.

Second, we show that at $u = 1$ the Job Creation curve lies above the Beveridge Curve. If $u = 1$, only the direct arrival (the market) provides offers. Hence the arrival rates are:

$$q_B(v) = \frac{A v^\eta}{1 + \theta}$$

$$q_G(v) = \frac{\theta A v^\eta}{1 + \theta}$$

Looking at the Beveridge Curve evaluated at $u = 1$:

$$\alpha q_G(1, v) + q_B(1, v) = \alpha \frac{\theta A v^\eta}{1 + \theta} + \frac{A v^\eta}{1 + \theta} = 0$$

This equation can hold only if $v = 0$.

As for the Job Creation curve, it can be written at $u = 1$ as follows:

$$(1 + \theta) c = A v^\eta \left( \frac{1 - \beta (p - b)}{\delta + \alpha \lambda} + \frac{\theta (1 - \beta)(\bar{p} - b)}{\delta + \lambda} \right) - A v^\eta \left( \frac{\beta c}{\delta + \alpha \lambda} + \frac{\theta \beta c}{\delta + \lambda} \right)$$
If \( v \to 0 \), the expression on the right-hand side goes to infinity: the last term goes to zero, while the first tends to infinity since \( \eta < 1 \). Hence, \( v = 0 \) cannot be a solution of the equation. The right-hand side is decreasing in \( v \), so as we increase \( v \) from 0, it decreases. Thus, the solution of the equation in \( v \) is strictly positive.

If we assume that
\[
(1 + \theta)c > A \left( \frac{(1 - \beta)(p - b)}{\delta + \alpha \lambda} + \frac{\theta(1 - \beta)(\bar{p} - b)}{\delta + \lambda} \right) - A \left( \frac{\beta c}{\delta + \alpha \lambda} + \frac{\theta \beta c}{\delta + \lambda} \right)
\]

\( v \geq 1 \) cannot be the solution since at this point the right-hand side is already smaller than the left-hand side. By the monotonicity and continuity of the right-hand side in \( v \), this implies that there should be a solution of the Job Creation curve in the interval \((0, 1)\).

In sum, we have seen that the Beveridge Curve is above the Job Creation curve at the point \( u = 0 \) while the reverse is the case at \( u = 1 \). Hence, by the continuity and the assumed monotonicity of both curves, there should be an intersection of the two for some value of \( u^* \in (0, 1) \) and \( v^* \in (0, 1) \).

**Lemma 2**

We define \( P^j_i \) as the probability that method \( j \in \{M, N\} \) (standing for 'Market' and 'Networks') provides an offer of type \( i \in \{B, G\} \). As we have indicated above in the text, when we compare conditional expected wages of the search methods, in fact, we compare the ratios \( P^j_G/P^j_B \), \( j \in \{M, N\} \). This ratio is what would determine the mismatch if only method \( j \) operated in the society as information channel.

Here we show that the mismatch level of the society can be written as the linear combination of these ratios across the search methods. This fact implies that whenever the network performs better than the market in terms of expected wages, the mismatch is lower than it would be in the case if only the market provided information about the vacancies.

The mismatch level is given by the following expression:

\[
e_B/e_G = q_B/q_G = \frac{1}{\alpha} \left\{ \frac{p_B^M + p_B^N}{p_G^M + p_G^N} \right\} = \frac{1}{\alpha} \left\{ \frac{p_B^M}{p_G^M} + \frac{p_B^N}{p_G^N} \right\} = \frac{1}{\alpha} \left\{ \frac{p_B^M}{p_G^M} \frac{p_G^M}{p_G^G} + \frac{p_B^N}{p_G^N} \frac{p_G^N}{p_G^G} \right\} = \frac{1}{\alpha} \left\{ \frac{p_B^M}{p_G^M} \mu + \frac{p_B^N}{p_G^N} (1 - \mu) \right\}
\]

where \( \mu \in (0, 1) \). The first equality uses the equilibrium relations (3.20) and (3.21), the second equality uses that the good and bad arrival rates are sums of good and bad arrival rates
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by search methods (market and networks), respectively.

If the network provides higher expected wage upon arrival than the market, \( P_B^N / P_G^N < P_B^M / P_G^M \), implying that the mismatch, as the linear combination of these two ratios, is lower than the higher of the two (\( P_B^M / P_G^M \)).

**Proposition 1**

**Proof** To establish the result we show that the first three conditions of the proposition imply the first three conditions of lemma 1.

First, looking at the derivative of \( BC(u, v) \) as defined in the previous lemma:

\[
\frac{\partial BC(u, v)}{\partial u} = \frac{\partial u q_G(u, v)}{\partial u} + \frac{\partial u q_G(u, v)}{\partial u} + \alpha \lambda
\]

This derivative is positive if the derivative in the first two terms are positive. We compute the first derivative:

\[
\frac{\partial u q_G(u, v)}{\partial u} = A \lambda v \left( \lambda (\theta + (-1 + \gamma)(1 - (1 - u)^k)(1 + \theta)) - A(-1 + 2\gamma)(1 - (1 - u)^k)((-1 + \gamma)(1 - (1 - u)^k) + \theta) v^\eta \right)
\]

The last term in this derivative is positive (0.5 < \( \gamma < 1, u < 1 \)). So the whole derivative is positive if the first term is positive too which is the case if it’s denominator is positive:

\[
\lambda (\theta + (-1 + \gamma)(1 - (1 - u)^k)(1 + \theta)) - A(-1 + 2\gamma)(1 - (1 - u)^k)((-1 + \gamma)(1 - (1 - u)^k) + \theta) v^\eta > 0
\]

After rearranging:

\[
\lambda \theta > \lambda (-1 + \gamma)(1 - (1 - u)^k)(1 + \theta) + A(-1 + 2\gamma)(1 - (1 - u)^k)((-1 + \gamma)(1 - (1 - u)^k) + \theta) v^\eta
\]

The expression on the right-hand side is increasing in \( v \) since it is multiplied by positive terms ((-1 + \gamma)(1 - (1 - u)^k) + \theta > 0, since \( \theta \geq 1 \)). We can show that it is increasing in \( u \) as well, taking the derivative we obtain:

\[
A \eta (-1 + 2\gamma) \left( \theta - (-1 + \gamma)(-1 + (1 - u)^k) \right) (1 - (1 - u)^k) v^{-1+\eta}
\]
This derivative is positive if $\gamma > 0.5$ and $\theta \geq 1$ since $(1 - (1 - u)^k) > 0$ and $-1 < (1 - \gamma)(-1 + (1 - u)^k) < 0$.

Given that the right-hand side is increasing in $v$ and $u$, the inequality holds for every $u$ and $v$ if it holds for $u = 1$ and $v = 1$. Evaluating the right-hand side at this point we arrive to the condition:

$$\lambda \theta > \lambda(1 - \gamma)(1 + \theta) + A(-1 + 2\gamma)(-1 + \gamma + \theta)$$  \hspace{1cm} (3.34)

Now, we look at the derivative of $uq_B(u, v)$ wrt $u$:

$$\frac{\partial uq_B(u, v)}{\partial u} = \frac{A\alpha\lambda v^\theta (\alpha\lambda(1 + (-1 + \gamma)(1 - (1 - u)^k)(1 + \theta)) - A(-1 + 2\gamma)(1 - (1 - u)^k)(1 + (-1 + \gamma)(1 - (1 - u)^k))v^\theta)}{A\alpha\lambda v^\theta (\alpha(-1 + \gamma)\lambda(-1 + u)(1 + \theta u) + A(-1 + 2\gamma)uv^\theta) k(1 - u)^{k-1}} > 0$$

Again, the last term is positive, the first term is positive too if:

$$\alpha \lambda > -\alpha\lambda(-1 + \gamma)(1 - (1 - u)^k)(1 + \theta) + A(-1 + 2\gamma)(1 - (1 - u)^k)(1 + (-1 + \gamma)(1 - (1 - u)^k))v^\theta$$

Here the rhs is again increasing in $v$ and $u$, the derivative wrt $u$ is the following:

$$k(1 - u)^{-1+k} \left( \alpha(1 - \gamma)\lambda(1 + \theta) + A(1 - 2\gamma) \left( 1 - 2\gamma + 2(\gamma - 1)(1 - u)^k \right) v^\theta \right) > 0$$

Hence, we again need to evaluate the rhs at $u = 1$ and $v = 1$:

$$\alpha \lambda > \alpha\lambda(1 - \gamma)(1 + \theta) + A(-1 + 2\gamma)\gamma$$  \hspace{1cm} (3.35)

Thus conditions (3.34) and (3.35) guarantee that $BC(u, v)$ is increasing in $u$.

We can also see that if $\frac{\partial uq_B(u, v)}{\partial u}$ and $\frac{\partial uq_B(u, v)}{\partial u}$ are positive, the second condition of Lemma 1 also holds. Derivating $JC(u, v)$ wrt $u$:

$$\frac{\partial uq_B(u, v)}{\partial u} \left/ \frac{\partial q_B(u, v)}{\partial u} + \frac{1}{\delta + \alpha \lambda} \frac{\partial w_B}{\partial u} \right. + \frac{\partial uq_G(u, v)}{\partial u} \left/ \frac{\partial q_G(u, v)}{\partial u} + \frac{1}{\delta + \alpha \lambda} \frac{\partial w_G}{\partial u} \right.$$  

Note that the wages are decreasing in $u$. Thus conditions (3.34) and (3.35) guarantee the second condition of Lemma 1.

Next, we provide two conditions under which the third condition of Lemma 1 is satisfied. This conditions says that $\frac{\partial J_C(u, v)}{\partial v} < 0$, $\forall u \in [0, 1]$ and $\forall v \in [0, 1]$. Thus we write:
\[ \frac{\partial J C(u, v)}{\partial v} < 0 = \frac{\partial u q_B(u, v)}{\partial v} \frac{v - w_B}{\delta + \alpha \lambda} \frac{1}{v} \frac{\partial w_B}{\partial v} + \frac{\partial u q_G(u, v)}{\partial v} \frac{\bar{v} - w_G}{\delta + \lambda} \frac{1}{v} \frac{\partial w_G}{\partial v} \]

Given that the wages are increasing in \( v \), this derivative is negative if both \( \frac{\partial u q_B(u, v)}{\partial v} \) and \( \frac{\partial u q_G(u, v)}{\partial v} \) are negative. First, we look at the latter:

\[ \frac{\partial u q_G(u, v)}{\partial v} = \frac{A \lambda ((-1 + \gamma)P(-1 + u) + \theta u) v^{-2 + \eta} ((-1 + \eta) \lambda (1 + \theta u) + A(-1 + 2 \gamma) P v^2)}{u (\lambda + \lambda \theta u - A(-1 + 2 \gamma) P v^2)^2} \]

where \( P = (1 - (1 - u)^k) \).

Given that \((-1 + \gamma)P(-1 + u) > 0\), this derivative is negative if \((-1 + \eta) \lambda (1 + \theta u) + A(-1 + 2 \gamma) P v^2 < 0\). If \( \gamma > 0.5 \), the second term is positive, and the highest when \( v = 1 \). So we can write the condition as:

\[ f(u) \equiv (-1 + \eta) \lambda (1 + \theta u) + A(-1 + 2 \gamma) (1 - (1 - u)^k) < 0 \]  \hspace{1cm} (3.36)

Some properties of \( f(u) \):

\[ \frac{\partial f(u)}{\partial u} = (-1 + \eta) \lambda \theta + A(-1 + 2 \gamma) k (1 - u)^{-1 + k} \]

\[ \frac{\partial f(u)}{\partial^2 u} = -A(-1 + 2 \gamma) (-1 + k) k (1 - u)^{-2 + k} \leq 0 \]

\( f(u) \) is concave.

\[ f(0) = (-1 + \eta) \lambda < 0 \]

At \( u = 0 \), condition (3.36) is always satisfied.

\[ \left. \frac{\partial f(u)}{\partial u} \right|_{u=1} = (-1 + \eta) \lambda \theta < 0 \]

At \( u = 1 \), \( f(u) \) is decreasing in \( u \).

Hence, if \( f(u) \) is decreasing in \( u \) at \( u = 0 \), then \( f(u) \) is negative \( \forall u \in [0, 1] \). On the contrary, if \( f(u) \) is increasing in \( u \) at \( u = 0 \), then by concavity \( f(u) \) has a maximum on the interval (0,1).

Thus we require \( f(u) \) be negative at this maximum. This is stated by the following condition:

\[ \text{if} \quad \left. \frac{\partial f(u)}{\partial u} \right|_{u=0} = (-1 + \eta) \lambda \theta + A(-1 + 2 \gamma) k \geq 0, \text{then} \quad f(\bar{u}) < 0 \]
where \( \bar{u} = 1 - \left[ \frac{(1-\eta)\tilde{\theta}}{k\lambda(2\gamma-1)} \right]^{\frac{1}{\gamma}} \)

Second, we look at the derivative of \( q_B(u, v) \):

\[
\frac{\partial u q_B(u, v)}{\partial v} = \frac{Ax(1+\gamma)(-1+u)+u)^{2+\eta}(\alpha(-1+\eta)\lambda(1+\theta u)+A(-1+2\gamma)Pv^\eta)}{u(\alpha(\lambda+\lambda\theta u)-A(-1+2\gamma)Pv^\eta)^2} \]

The sign of this derivative depends on the sign of the following expression:

\[
g(u) \equiv \alpha(-1+\eta)\lambda(1+\theta u)+A(-1+2\gamma)Pv^\eta \]

Observe that for \( \alpha \geq 1 \), \( g(u) \leq f(u) \), \( \forall u \). Thus the condition which guarantee that \( f(u) \) is negative is sufficient in this case as well.

Finally, we can see that the condition \( \frac{\partial BC(u, v)}{\partial v} > 0, \forall u \in [0, 1] \) and \( \forall v \in [0, 1] \)

is always satisfied. This is so because both \( \frac{\partial u q_B(u, v)}{\partial v} \) and \( \frac{\partial u q_G(u, v)}{\partial v} \) are positive. Expressing these derivatives:

\[
\frac{\partial u q_B(u, v)}{\partial v} = \frac{Ax^2\eta\lambda^2(1+\gamma)(-1+u)+u(1+\theta u)w^{1+\eta}}{(\alpha(\lambda+\lambda\theta u)-A(-1+2\gamma)Pv^\eta)^2} > 0 \]

This derivative is positive since \( \gamma \leq 1 \) and \( u \leq 1 \).

And in the same way we have:

\[
\frac{\partial u q_G(u, v)}{\partial v} = \frac{Ax_\gamma^2(1+\theta u)(-1+\gamma)P(-1+u)+u(1+\theta u)w^{1+\eta}}{(\lambda+\lambda\theta u-A(-1+2\gamma)Pv^\eta)^2} \]

The last condition of the proposition is the same as the one in Lemma 1.

**Proposition 2**

**Proof** (i) We assume that \( \theta = 1, \alpha = 1 \). We substitute these parameter values into the equations of the arrival rates (3.18) and (3.29). Then we divide the two equilibrium conditions (3.20) and (3.21) and we get the following equation:

\[
\frac{q_B}{q_G} = \frac{1 + Qk(\gamma e_B + (1-\gamma)e_G)}{1 + Qk(\gamma e_G + (1-\gamma)e_B)} = \frac{e_B}{e_G} \]

where \( Q \) is defined as in (3.2). Rearranging this latter equality we obtain:

\[
e_B(1 + Qk(\gamma e_G + (1-\gamma)e_B)) = e_G(1 + Qk(\gamma e_B + (1-\gamma)e_G))
\]

\[
e_G - e_B + (1-\gamma)Qk(e_G^2 - e_B^2) = 0
\]
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\[(e_G - e_B)(1 + (1 - \gamma)Qk(e_G + e_B)) = 0\]

Since the second term is always positive, the multiplication can be zero only if \(e_G = e_B\). This is true for any \(\gamma\) and \(k\).

(ii) If we look at the good and bad arrival rates through social contacts in (3.3) and (3.6), we can see that if \(e_G = e_B\), they are equal for any value of \(\gamma\), \(u\) and \(k\). Hence, the network provides good and bad jobs at the same rate.

**Proposition 3**

**Proof** We again divide the two equilibrium conditions (3.20) and (3.21) and plug in the equations for the arrival rates (3.18) and (3.29) and we obtain the following:

\[
\frac{q_B}{\alpha q_G} = \frac{e_B}{e_G} = \frac{1 + Qk(\gamma e_B + (1 - \gamma)e_G)}{\alpha(\theta + Qk(\gamma e_G + (1 - \gamma)e_B))}
\]

(3.37)

where \(Q\) is defined in (3.2).

First, we see that the mismatch decreases with \(\theta\) and with \(\alpha\). Using Lemma (3), we can write the mismatch level as follows:

\[
\frac{e_B}{e_G} = \frac{1}{\alpha} \left[ \frac{P^M_B}{P^M_G} + \frac{P^N_B}{P^N_G}(1 - \mu) \right]
\]

(3.38)

where

\[
\frac{P^M_B}{P^M_G} = \frac{1}{\theta}
\]

and

\[
\frac{P^N_B}{P^N_G} = \frac{(\gamma e_B + (1 - \gamma)e_G)}{(\gamma e_G + (1 - \gamma)e_B)} = \frac{\gamma \frac{e_B}{e_G} + 1 - \gamma}{\gamma + (1 - \gamma) \frac{e_B}{e_G}}
\]

The derivative of this last expression with respect to \(e_B/e_G\) is negative if \(\gamma > 0.5\):

\[
-1 + 2\gamma \frac{(\gamma + \frac{e_B}{e_G} - \gamma \frac{e_B}{e_G})^2}{(\gamma + \frac{e_B}{e_G} - \gamma \frac{e_B}{e_G})^2}
\]

If \(\theta\) or \(\alpha\) increase, the right-hand side of equation (3.38) decrease. Thus the left-hand side, the mismatch has to decrease as well which makes the network arrival more efficient: \(P^N_B/P^N_G\) decreases. In turn, the mismatch level further increases until we reach the new equilibrium.

If \(\theta > 1\) or \(\alpha > 1\), \(e_G > e_B\) since the mismatch is decreasing in \(\theta\) and \(\alpha\) and we know from the previous proposition that for \(\alpha = 1\) and \(\theta = 1\) we have \(e_G = e_B\).

We can see that if \(e_G > e_B\), the mismatch level decreases in \(\gamma\). Looking at equation (3.37):

\[
\frac{q_B}{\alpha q_G} = \frac{e_B}{e_G} = \frac{1 + Qk(\gamma(e_B - e_G) + e_B)}{\alpha(\theta + Qk(\gamma(e_G - e_B) + e_G))}
\]
If we increase $\gamma$, the numerator decreases while the denominator increases. Hence, the mismatch decreases on the left-hand side which again makes the network arrival more efficient and the mismatch further decreases.

**Proposition 4**

**Proof** To compare the expected wage of the market arrival and the social networks, we need to compare conditional probability that the given search method provides a high productivity job. As we argued in the text, this depends on the probability ratio $P_G^j/P_B^j$, where $j$ stands for the search method (market or network). We can write:

$$\frac{P_G^M}{P_B^M} = \theta$$

$$\frac{P_G^N}{P_B^N} = \frac{\gamma e_G + (1 - \gamma) e_B}{\gamma e_B + (1 - \gamma) e_G} = \frac{\gamma + (1 - \gamma) e_B}{\gamma e_G + 1 - \gamma}$$

This latter ratio increases if the mismatch decreases (we showed in the proof of the previous proposition that $P_B^N/P_G^N$ was increasing in the mismatch level).

First of all, we look at the case when $\alpha = 1$ and $\gamma = 1$. Here the two search methods provide equal expected wages if:

$$\frac{P_G^M}{P_B^M} = \frac{P_G^N}{P_B^N} \iff \theta = \frac{e_G}{e_B}$$

Now, we see that this exactly is what the equilibrium conditions imply. Looking at the equilibrium condition (3.37) of the previous proof and substituting $\alpha = 1$ and $\gamma = 1$, we obtain:

$$\frac{e_B}{e_G} = \frac{1 + Qke_B}{\theta + Qke_G}$$

Solving this equation for $\theta$, we get that in equilibrium $\theta = e_G/e_B$. Hence, if $\alpha = 1$ and $\gamma = 1$, the two methods give the same wage expectation and generate mismatch to the same extent. By Lemma 3, we also know that the mismatch in this economy with social network coincides with the mismatch of a pure market economy. Since the performance of the network decreases with $\gamma$ if $\theta > 1$, for $\gamma < 1$ we have that the market gives higher expected wage and the presence of social networks increases the mismatch in the society.

Now, if $\alpha$ increases above 1, by the previous proposition we know that the mismatch level decreases. This leaves the market expectation unchanged, since it depends only on $\theta$, and makes the network arrival more efficient. If the mismatch decreases, $P_G^N/P_B^N$ increases. Hence at $\gamma = 1$
the network wage expectation is higher than that of the market. If \( \gamma \) decreases, \( \frac{P_G}{P_B} \) decreases. By \( \gamma = 0.5 \) the market is always more efficient than the network:

\[
\frac{P_G}{P_B} = 0.5e_G + 0.5e_B = 1 < \frac{P_M}{P_B}
\]

Thus if \( \gamma = 1 \), the network expectation is higher. \( \gamma = 0.5 \) the market expectation is higher.

Hence, we have that if \( \alpha > 1 \), there exists a threshold value of homophily \( \gamma \) where the wage expectations of the two search methods coincide.

By the same reasoning we have that \( \bar{\gamma} \) decreases if \( \alpha \) increases: the network expectation gets higher by the higher \( \alpha \), so we find equal network and market expectations by lower \( \gamma \).

Lemma 3 implies that when the network gives a higher expected wage than the market, the mismatch level is lower in the economy with social networks than in a pure market economy.

Next, we show that \( \bar{\gamma} \) is independent of the network connectivity \( k \). We again write the mismatch level as the linear combination of the probability ratios:

\[
\frac{e_B}{e_G} = \frac{1}{\alpha} \left[ \frac{P_M}{P_G} + \frac{P_N}{P_N(1 - \mu)} \right]
\]

where \( \mu = \frac{P_M}{P_G + P_M} \). We know that \( \frac{P_N}{P_B} \) does not depend on \( k \) directly. If \( k \) increases, only \( P_G^N = kQ(\gamma e_G + (1 - \gamma)e_B) \) reacts: it increases. However, at \( \bar{\gamma} \), \( \frac{P_N}{P_B} = \frac{P_M}{P_M} \). Hence, a change in \( \mu \) leaves the mismatch level unchanged. Thus, as \( k \) increases, \( \bar{\gamma} \) remains the same and the market and network expectations are equal by the same \( \gamma \) as before.

**Proposition 5**

**Proof** First we see the effect of connectivity on the unemployment rate. Looking at the job arrival rates \( q_B(u, v) \) and \( q_G(u, v) \) as they are defined in Lemma 1, we can see that both of them increase as \( k \) increases. For both the numerator increases while the denominator decreases. Hence, \( BC(u, v) \) increases for any \( u \). To get back to equilibrium \( (BC(u, v) = 0) \), we need to decrease \( v \) for every \( u \) (recall that \( \frac{\partial BC(u, v)}{\partial v} > 0 \)). Thus the Beveridge Curve moves to the left in the \((u, v)\) plane. As \( q_B(u, v) \) and \( q_G(u, v) \) increase, \( JC(u, v) \) gets higher as well for any value of \( u \). Thus to get equilibrium \( (JC(u, v) = 0) \), we need to increase \( v \) since \( \frac{\partial JC(u, v)}{\partial v} < 0 \). Thus the Job Creation curve moves up in the \((u, v)\) plane. Hence, the new equilibrium establishes at a lower unemployment rate.
Second, looking at the mismatch level, we again write it as the linear combination of the arrival ratios by search methods:

\[ \frac{e_B}{e_G} = \frac{1}{\alpha} \left[ \frac{P^M_B}{P^M_G} \mu + \frac{P^N_B}{P^N_G} (1 - \mu) \right] \]

where \( \mu = \frac{P^M_G}{P^M_G + P^N_G} \) and \( P^N_G = kQ(ye_G + (1 - y)e_B) \) with \( Q = \frac{1-(1-u)^k}{ku} \). As \( k \) increases, \( P^N_G \) increases: first directly, second through the decrease in \( u \) (\( Q'(u) < 0 \)). Note again that \( P^N_G/P^N_B \) does not depend on \( k \) directly. Thus as \( k \) increases, the weight on the network arrival \( 1 - \mu \) increases.

Then we have two cases. For \( \gamma < \tilde{\gamma} \), \( \frac{P^M_B}{P^M_G} < \frac{P^N_B}{P^N_G} \). Thus by putting higher weight on the network arrival, the mismatch increases which further increases \( P^N_B/P^N_G \) making the mismatch even higher. Hence, if \( \gamma < \tilde{\gamma} \), the connectivity increase makes the mismatch level higher. On the contrary, when \( \gamma > \tilde{\gamma} \), \( \frac{P^M_B}{P^M_G} > \frac{P^N_B}{P^N_G} \), by putting higher weight on the network arrival, the mismatch decreases making the network arrival more efficient: \( \frac{P^N_B}{P^N_G} \) decreases. Thus the connectivity increase makes the mismatch level lower.

Proposition 6

Proof The proof of the first two points is similar to the proof of Proposition 6 for the case of \( \alpha > 1 \). As \( P \) increases, the mismatch decreases similarly to the effect of \( \alpha \).

\( \tilde{\gamma}_1 \) is the threshold where the "family" networks’ expectation coincides with the market wage expectation. We know that for a given homophily level the overall networks’ wage expectation is always higher than that of the family network. This is because:

\[ \frac{P^T_G}{P^T_B} = \frac{\gamma e_G + (1 - \gamma)e_B}{\gamma e_B + (1 - \gamma)e_G} > \frac{\gamma e_G + (1 - \gamma)e_B}{\gamma e_B + (1 - \gamma)e_G} + \frac{P}{Qk(\gamma e_B + (1 - \gamma)e_G)} = \frac{P^T_G}{P^T_B} \]

Hence, at \( \tilde{\gamma}_1 \) the total network still gives a wage premium over the market. As we decrease \( \gamma \) from this level, the efficiency of the total network decreases. There is an interval \( (\tilde{\gamma}_2, \tilde{\gamma}_1) \) where the market gives higher expected wage than the family network but lower than the overall network. \( \tilde{\gamma}_2 \) is the homophily level where the total network’s wage expectation is equal to the market expectation:

\[ \theta = \frac{Qk(\tilde{\gamma}_2 e_G + (1 - \tilde{\gamma}_2)e_B) + P}{\tilde{\gamma}_2 e_B + (1 - \tilde{\gamma}_2)e_G} \]

From here:
\[
\bar{\gamma}_2 = \frac{Qke_B - \theta e_G + P}{(e_B - e_G)(\theta + Qk)}
\]

Note that \(\bar{\gamma}_2\) might be zero.

3.11 Figures

Figure 3.1: Threshold values of homophily for different values of market efficiency (\(\theta\)) and separation rate difference (\(\alpha\)).
Figure 3.2: The effect of network connectivity (degree) on the mismatch level.
Figure 3.3: Wage difference (network wage expectation - market wage expectation) for different network structures (connectivity and homophily).
Figure 3.4: The effect of the network structure (connectivity and homophily) on the unemployment rate (left above), vacancy rate (right above) and wages.
Figure 3.5: Homophily thresholds in the presence of professional contacts for two connectivity values of the "family" network ($k = 2$ and $k = 5$).
Chapter 4

Explaining Wage Premiums/Discounts Associated to Job Contact Networks

4.1 Introduction

It has been widely documented that workers use social contacts, that is friends, relatives and acquaintances, to find employment apart from searching on the formal market, i.e. responding advertisements, directly applying to employers. Different studies find that 30-60% of jobs are found using informal methods (e.g. see Blau and Robins (1990), Holzer (1987)). Looking at the empirical literature, it is much less clear whether using social contacts actually leads to better jobs for the unemployed than using the formal job search methods. Some estimates indicate that the informal job search gives a wage premium over the formal market (see e.g. Kugler (2003), Dustmann et al. (2010), Simon and Warner (1992)), while other studies find a wage discount (e.g. see Bentolila (2010)). In a cross country study, Pellizzari (2010) finds that social networks give a wage premium in Austria, Belgium and the Netherlands while a wage discount in Greece, Italy, Portugal and the United Kingdom.

The aim of this paper is to build a theoretical model that can explain the observed mixed evidence and predict under which circumstances jobs found through social contacts pay higher wages than the ones obtained on the formal market. I build a model of homogeneous workers and heterogeneous jobs: workers are either better or worse paid depending on the type of the job they find. In the economy jobs are constantly created and destroyed at exogenous rates (Calvó-Armengol and Jackson (2004, 2007)). I assume that when unemployed workers look for jobs on the market, they make a random draw out of the available jobs and thus the probability that
CHAPTER 4. EXPECTED WAGE OF JOB SEARCH METHODS

they find a high paying job is given by the exogenous probability that a new job is of high type. On the contrary, in the case of network search, the quality of jobs an unemployed can find depends on the status of their neighbors and hence it is endogenous. There are two ways how the status of contacts influences the type of jobs available through them. First, neighbors forward only those offers which they cannot use to improve their own situation, Second, I assume that employed workers have higher probability to hear about jobs which are of similar type to their employment than about vacancies of different type. This latter assumption can be interpreted as if employed workers had easier access to the information on new openings of their employer and than to the vacancies of other firms. Here I parameterize this correlation between employment status and information access.

Both of these assumptions imply that to be connected to workers employed in better jobs lead to better employment possibilities for an individual. This phenomenon has been described in Sociology both theoretically and empirically. Social resource theory states that an individual’s social capital consist of the attained status of their social contacts (see e.g. Lin (1999)). Empirical studies document that the social status, prestige and wages of the social contact used to obtain a job positively influences the status and wages attained by an individual (see e.g. Marsden and Hurlbert (1988) and Marsden and Gorman (2001)).

According to my findings, if the correlation between the status of an employed and the quality of jobs she is more likely to hear about increases, the expected wage of network search rises as well. If this correlation is sufficiently high, we observe a wage premium for network search over the wages obtained on the market. In this case, offers paying high wages mostly become known by workers who already earn high wages and hence instead of taking those offers they transmit them to their neighbors. On the contrary, when the mentioned correlation is low, employees earning low wages are likely to learn about offers of high wages and they take those offers. I show that if the arrival process is random, due to on-the-job search the network always gives a wage discount over the market.

I establish that there always exists a critical value of this correlation by which the expected wages paid by the informal search method coincide with that obtained on the market. This critical value becomes lower if there are more employment opportunities or if higher proportion of the new openings pays higher wages or if the job separation rate is lower in the economy. In these cases, it is more likely that the neighborhood of a given unemployed worker consists of employed agents earning high wages. Thus, if the offers are randomly distributed among these
neighbors, a new high wage offer has higher chance to be transmitted to the unemployed agent implying a lower threshold correlation. Hence, I predict that in "good times" we are more likely to observe a wage premium paid by the network search.

Moreover, I show that the critical correlation value increases with the connectivity of the social network, especially in the case when the employment possibilities are poor. I argue that the addition of new links rises more the probability that a low wage offer is transmitted by a social contact than the probability that a bad one is passed. The difference increases if there are less new openings or the new openings pay less wages.

I also obtain that as the correlation between employment status and offer quality rises, the employment rate increases and the fraction of workers earning high wages decreases. Thus the Rawlsian welfare rises in the economy and the utilitarian welfare might decrease if the wage differentials are high enough. The intuition behind this finding is that if the correlation is high, high wage offers reach mostly high wage employed workers who transmit those offers partly to their unemployed friends. Hence, these offers indirectly reach the unemployed workers with higher probability and are less likely to (directly) end up in the hands of low wage employed workers. This implies that the unemployment rate decreases. However, in some cases the high wage employed agent who receives the new offer does not have anyone among his neighbors to pass the offer to. Thus some of these vacancies remain unfilled. Consequently, the fraction of agents employed in high paying jobs becomes lower.

The presented model is placed in the literature on the impact of social networks on labor market processes. The closest models are Calvó-Armengol and Jackson (2004, 2007). They describe an economy where job arrivals and job separations happen at exogenous rates. Similarly to Calvó-Armengol and Jackson (2004), I assume that job information travels only one step in the network, i.e. if none of the direct neighbors of an employed worker is unemployed, the job information is not transmitted to more distant unemployed individuals. Further similarity is that the receiver of the job information is randomly chosen among the possible candidates in the direct neighborhood. Calvó-Armengol and Jackson (2007) extends their model to the case of heterogeneous wages and also generalizes the job transmission process. However, none of the versions studies the question under what conditions the job finding through social contacts leads to a wage premium over the market. The model presented here introduces the realistic possibility of correlation between the status of employed workers and the quality of jobs they hear about and studies the consequences of this assumption.
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Besides the mentioned empirical literature, the question of expected wages of the different job search methods has been studied theoretically as well. There are different arguments why the jobs obtained via social contacts should pay higher expected wages. Montgomery (1991) argues that job referrals help to resolve the asymmetric information problem on the labor market and high ability workers tend to find jobs through informal methods. Kugler (2003) points out that job referrals monitor the referred workers in the workplace who, in consequence, put more effort and get paid more.

On the contrary, some other articles argue that informal job search methods should pay lower wages than the formal market. Bentolila et al. (2010) points out that young workers accept jobs in sectors where their productivity is lower because using social contacts they find employment earlier. Ponzo and Scoppa (2010) shows that social contacts, especially family members, often recommend each other to jobs where they lack the qualifications which again leads to lower wages.

My model presents a unique framework where I can obtain both a wage premium and a wage discount for the network search over the market by simply changing the parameters of the model. I connect the performance of social contacts to the characteristics of the job arrival process and to the aggregate state of the economy. I show that in "good times" the network search is more likely to pay higher wages than the market.

Sylos-Labini (2004) can also recover both wage premiums and discounts. He proposes that when workers rely more on favoring family contacts, the network search gives a wage discount while when they rely on professional contacts it gives a wage premium. In another paper, I show that even if the social networks represent favoring relationships, it may pay higher wages than the formal market (see Horváth (2010)). I identify a critical level of homophily, i.e. the tendency of similar agents to be connected, which guarantees that social contacts are more efficient in matching workers to appropriate jobs than the formal market.

The paper is organized as follows. In Section 2, I describe the assumptions of the model, I write it up as a Markov process and simplify the problem using mean-field approximation. Section 3 contains the main analytical results while in section 4 I test the correctness of the approximation by simulating the model. In section 5 I show that the results are robust to introducing heterogeneity in the number of neighbors of the individuals. Section 6 concludes.

---

1Dustmann et al. (2010) and Simon and Warner (1992) also reason that the informal job methods are more informative for the employer.
4.2 Model

4.2.1 Direct job arrival

In a model of continuous time I assume an exogenous job arrival process. New vacancies open up at a constant rate $\alpha$. There are two types of jobs: low paying and high paying jobs. Low wage jobs pay $w_L$, high wage jobs pay $w_H$, being $w_H > w_L$. Workers are homogeneous and they can be employed in any of these jobs. Conditional on arrival a new opening is of low paying with probability $p_L$ and is of high paying with the complementary probability $p_H = 1 - p_L$.

The key element of the model is that offers are not randomly distributed in the society, but employed agents are more likely to hear about offers similar to their current job. This assumption is certainly true if employed workers are more likely to be aware of the openings of their own place of employment than be aware of other vacancies. On the contrary, unemployed agents are randomly matched to vacancies by the formal labor market taking into account the availability of the different jobs.

I implement these ideas in the following way. When a new job opens up, it becomes known by an unemployed agent with probability $u$ where $u$ is the unemployment rate. Among the unemployed agents the exact receiver is chosen at random. With the complementary probability $e = 1 - u$, i.e. the employment rate, an employed agent hears about the vacancy. Now for example, if the job is of high wage, the receiver of the information is an agent employed in a high paying job with the following probability:

$$h(e_H, e, \lambda) = \frac{e_H e}{\lambda e_H + e_L}$$

where $e_H$ is the fraction of the population employed in high wage jobs, $e_L$ is the fraction of the population in low wage jobs. Parameter $\lambda$ measures the mentioned correlation between the state of the employed who becomes aware of a new vacancy and the type of that vacancy. If $\lambda = 1$, there is no bias toward the high wage employed agents. New high wage jobs become known by a high wage employed with probability $h(e_H, e, 1) = e_H$ as if the arrival process were uniformly random. On the contrary, when $\lambda \to \infty$, $h(e_H, e, \infty) = e$, i.e. every time an employed agent is chosen as receiver of the new information, she should be employed in a high wage job. In this case, low wage employed have zero probability to hear about high wage jobs. The third

\[2\text{In the sequel, I refer to } \lambda \text{ as correlation parameter even though it's value ranges between 0 and } \infty \text{ instead of } -1 \text{ and } 1 \text{ which is the range of the correlation in the statistical sense.}\]
corner case is when $\lambda = 0$ which is more of theoretical interest. $h(e_H, e, 0) = 0$, i.e. high wage employed never get aware of high wage openings, they are distributed between the unemployed and low wage employed. We introduce a similar function for the case when the job opening is of low type. These probabilities are summarized in Table 4.1. Once the information receiver’s type is determined using the probabilities in Table 4.1, the exact agent is uniform randomly chosen within that category.

<table>
<thead>
<tr>
<th></th>
<th>Unemployed</th>
<th>Low wage employed</th>
<th>High wage employed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low wage offer arrives: $\alpha p_L$</td>
<td>$u$</td>
<td>$\frac{e L e L}{\lambda L e H + e H}$</td>
<td>$\frac{e H e H}{\lambda L e H + e H}$</td>
</tr>
<tr>
<td>High wage offer arrives: $\alpha p_H$</td>
<td>$u$</td>
<td>$\frac{e H e L}{\lambda H e H + e L}$</td>
<td>$\frac{e L e H}{\lambda H e H + e L}$</td>
</tr>
</tbody>
</table>

Table 4.1: Distribution of arriving offers to different types of agents

4.2.2 On-the-job search and transmission of job information

I assume that unemployed agents accept offers of any type. This behavior is rational if the wage difference $w_H - w_L$ is low enough such that waiting for a high wage offer is too costly when an individual can accept a low wage job.

Low wage employed workers can change their job to high wage if they hear about a vacancy. Thus they forward only low wage offers to their contacts. I assume that in this case they randomly choose an unemployed worker in their direct neighborhood in the social network. If they do not have any unemployed neighbor, the offer is lost, nobody takes up the job. Hence, the job information can travel only one step in the network. Similar assumptions have been used in Calvó-Armengol and Zenou (2005) and Calvó-Armengol and Jackson (2004).

High wage workers cannot upgrade their situations, so they forward any offer they become aware of. If they hear about a low wage offer, they pass it to one unemployed contact at random. When they learn of a high wage offer, they also consider the low wage friends: choose one contact at random from the pool of unemployed and low wage employed workers.

Hence the wage distribution of vacancies one unemployed can learn by using their contacts is determined by the composition of her neighborhood: having more high wage employed contacts increases the probability to get a high wage offer. This is even more so if we increase the value of the "correlation" $\lambda$, since in this case good offers arrive more often to high wage agents.

---

3 Note that the rows of the table add up to 1.
4.2.3 Network structure

In the economy $N$ agents are placed on an undirected graph defined by the $N \times N$ adjacency matrix $G$ where the element $g_{ij} = 1$ if individual $i$ and $j$ are directly connected and otherwise $g_{ij} = 0$. We have that $g_{ij} = g_{ji}$.

In the baseline model, I assume that the underlying network structure is a regular random graph (see Vega-Redondo (2007)). In this network every agent has $k$ direct neighbors. These $k$ contacts are randomly chosen from the population. As an extension and robustness check, I allow for heterogeneity among agents with respect to their number of neighbors. In particular, I analyze the model in the case of Poisson degree distribution.

4.2.4 Job separations

Job separations happen according to a Poisson arrival process with parameter $\beta e$, where $e$ is the employment rate in the economy. I assume that if the employment rate is lower, job separation happens less often. This corresponds to the idea that the economy needs to meet certain demands, so if the employment rate is already low, further dismissals have lower probability.

4.2.5 Markov process representation

In this section I represent the model as a Markov process with state space $\{U, L, H\}^N$: each of the $N$ agents can be in one of the states of unemployment ($U$), low wage employment ($L$) or high wage employment ($H$). There are two types of events that change the states of individuals: arrival of offers and job separation. The direct arrival of offers happens according to a Poisson process with parameter $\alpha$. The rate of offer arrivals through social contacts depends on the state of these contacts. Job separations happen according to a Poisson process with parameter $\beta e$.

Table 4.2 describes the transition probabilities between the states for an individual $i$ at any instant of time. I denote $N_i^S$, the set of neighbors of $i$ in state $S \in \{U, L, H\}$, i.e. $N_i^S := \{j|g_{ij} = 1 \land s_j = S\}$. $\eta_i^S$ is the cardinality of this set. Further, we denote the number of neighbors in state $U$ or $L$ as $\eta_i^{U+L} = |\{j|g_{ij} = 1 \land (s_j = U \lor s_j = L)\}|$.

The intuition behind the formulation of these probabilities is the following. Unemployed agents find employment either by direct arrival (on the formal market) or through their contacts. For example, a low wage offer opens up with probability $\alpha p_L$, this offer goes to an unemployed with probability $u$ and the probability that we choose a given unemployed is $1/N u$. In addition,
CHAPTER 4. EXPECTED WAGE OF JOB SEARCH METHODS

<table>
<thead>
<tr>
<th>Transition</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U \rightarrow L$</td>
<td>$A \equiv \frac{\alpha_p L}{N_u} + \sum_{j \in N_i^L} \alpha_p L \frac{e_{U j} L}{e_{L} + e_{U}} \frac{1}{N_e L} + \sum_{j \in N_i^H} \alpha_p H \frac{e_{U j} H}{e_{L} + e_{U}} \frac{1}{N_e H}$</td>
</tr>
<tr>
<td>$U \rightarrow H$</td>
<td>$B \equiv \frac{\alpha_p H}{N_u} + \sum_{j \in N_i^H} \alpha_p H \frac{e_{U j} H}{e_{L} + e_{U}} \frac{1}{N_e H}$</td>
</tr>
<tr>
<td>$U \rightarrow U$</td>
<td>$1 - A - B$</td>
</tr>
<tr>
<td>$L \rightarrow U$</td>
<td>$\frac{\beta}{N_e}$</td>
</tr>
<tr>
<td>$L \rightarrow H$</td>
<td>$C \equiv \alpha_p H \frac{e_{L U}}{e_{L} + e_{H}} \frac{1}{N_e L} + \sum_{j \in N_i^H} \alpha_p H \frac{e_{L j} H}{e_{L} + e_{H}} \frac{1}{N_e H}$</td>
</tr>
<tr>
<td>$L \rightarrow L$</td>
<td>$1 - \frac{\beta}{N_e} - C$</td>
</tr>
<tr>
<td>$H \rightarrow U$</td>
<td>$\frac{\beta}{N_e}$</td>
</tr>
<tr>
<td>$H \rightarrow L$</td>
<td>$0$</td>
</tr>
<tr>
<td>$H \rightarrow H$</td>
<td>$1 - \frac{\beta}{N_e}$</td>
</tr>
</tbody>
</table>

Table 4.2: Transition probabilities for an individual

an unemployed can hear about a low wage offer both through their low and high wage employed contacts. The probability that a given low wage agent is aware of a low wage opening is $\alpha_p L \frac{e_{L j} L}{e_{L} + e_{U}} \frac{1}{N_e L}$. She chooses a given unemployed randomly among her unemployed neighbors, so a given neighbor has $1/\eta_j^L$ probability to get the information. We need to sum up these probabilities over all the neighbors of $i$. We have a similar formulation for the case when a high wage employed neighbor transmits the information about a low wage offer.

High wage job offers are transmitted only by high wage employed agents. Hence, an unemployed can find a high wage job either on the market or through her high wage employed neighbors. The same is true for low wage employed workers. Any employed worker can lose her job with a probability $\beta/(N_e)$. High wage employed individuals never move to the state of low wage employment. Finally, it is possible that an agent does not change state at some moment of time.

There exists a unique limit distribution of the described Markov process and it is independent of the initial state. The existence of this distribution follows from the fact that every state of the process belongs to the same communication class. Any employed worker can lose her job at every instant of time and an unemployed always can find a job of any type through the direct arrival. Moreover, for any finite $\lambda$ there is always some non-zero probability that a low wage employed directly hears about a high wage job. Every agent stays in her actual state with some positive probability.

In Section 4, I am going to analyze the limit distribution of the Markov process by simu-
lations. Note that the analytical calculation of the limit distribution is complicated by the fact that the transition probability of a given individual is influenced by the state of her neighbors. Moreover, as it has been shown by Calvó-Armengol and Jackson (2004, 2007), the states of any two connected agents are correlated. Thus to simplify calculations, in my analytical solution I consider a mean-field approximation of the process.

### 4.2.6 Mean-field representation

The mean-field approximation assumes that the actual state of an agent at some instant of time is randomly drawn from the set \{U, L, H\} with the respective probabilities \{\(u, e_L, e_H\)\}, the so-called homogeneous mixing assumption. Thus the probability that an agent can be found in some state equals to the population frequency of the given state. Moreover, the states of different agents are independently drawn from this distribution. Hence, this approximation does not take into account that in the limit the states of connected agents are correlated. However, we still can approximate the movement of the aggregate variables \(u, e_L, e_H\) by the mean-field approach because the local interdependence of individual states is averaged out in a large enough population.

By applying the homogeneous mixing assumption, we basically restrict our attention to a representative individual who moves between states based on average transition probabilities. I summarize these probabilities in Table 4.3.

<table>
<thead>
<tr>
<th>Transition</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>U \to L</td>
<td>(A \equiv \frac{\alpha p_H}{Nu} + k e_L \alpha p_L e_L \frac{1}{Ne_L} \left[1 - \frac{(1-u)^k}{uk}\right] + k e_H \alpha p_L e_H \frac{1}{Ne_H} \left[1 - \frac{(1-e_H)^k}{uk}\right])</td>
</tr>
<tr>
<td>U \to H</td>
<td>(B \equiv \frac{\alpha p_H}{Nu} + k e_H \alpha p_H e_H \frac{1}{Ne_H} \left[1 - \frac{(1-e_H)^k}{uk}\right])</td>
</tr>
<tr>
<td>U \to U</td>
<td>(1 - A - B)</td>
</tr>
<tr>
<td>L \to U</td>
<td>(\frac{\beta}{Ne})</td>
</tr>
<tr>
<td>L \to H</td>
<td>(C \equiv \alpha p_H e_L \frac{1}{Ne_L} \left[1 - \frac{(1-e_H)^k}{uk}\right] + k e_H \alpha p_H e_H \frac{1}{Ne_H} \left[1 - \frac{(1-e_H)^k}{uk}\right])</td>
</tr>
<tr>
<td>L \to L</td>
<td>(1 - \frac{\beta}{Ne} - C)</td>
</tr>
<tr>
<td>H \to U</td>
<td>(\frac{\beta}{Ne})</td>
</tr>
<tr>
<td>H \to L</td>
<td>0</td>
</tr>
<tr>
<td>H \to H</td>
<td>(1 - \frac{\beta}{Ne})</td>
</tr>
</tbody>
</table>

Table 4.3: Transition probabilities for an individual in the mean-field case.

These probabilities have been derived from the probabilities of Table 4.2 using the homo-
geneous mixing assumption. First, I look at the case when an unemployed finds a low wage job through their contacts. She has $k e_L$ low wage employed contacts and $k e_H$ high wage employed contacts on average. Each of the low wage employed contacts has $\alpha p_L \frac{e_{e_L}}{e_{e_L} + e_{e_H}} \frac{1}{N e_L}$ probability to be aware of a low wage offer. Each of the high wage employed contacts has $\alpha p_L \frac{e_{e_H}}{e_{e_L} + e_{e_H}} \frac{1}{N e_H}$ probability to know about a low wage opening. These contacts choose a random unemployed worker among their neighbors as the receiver of the job information. The number of unemployed contacts is randomly drawn from the binomial distribution $B(k, u)$. Thus, the probability that a given unemployed contact receives the information is given by the following formula:\(^4\)

$$\sum_{i=0}^{k-1} \binom{k-1}{i} \frac{1}{i+1} u^i (1-u)^{k-i-1} = \frac{1 - (1-u)^k}{uk} \quad (4.1)$$

Second, we look at the case when an unemployed or a low wage employed individual find a high wage job through their high wage contacts. Any of these individuals has $k e_H$ high wage contacts on average. Any of these contacts has $\alpha p_H \frac{e_{e_H}}{e_{e_H} + e_{e_L}} \frac{1}{N e_H}$ probability to be aware of a high wage offer. When she has an offer, she chooses a random agent out of the pool of unemployed and low wage employed neighbors. The number of such neighbors is randomly drawn from the binomial distribution $B(k, u + e_L)$. Hence, a given unemployed or low wage employed has the following probability to be chosen as receiver of the information:

$$\sum_{i=0}^{k-1} \binom{k-1}{i} \frac{1}{i+1} (u + e_L)^i (1 - (u + e_L))^{k-i-1} = \frac{1 - e_H^k}{k(1 - e_H)} \quad (4.2)$$

Based on the mean-field transition probabilities of Table 4.3 we can derive the law of motion of the average quantities $u, e_L, e_H$. Given that $u + e_L + e_H = 1$, we know that $\dot{u} + \dot{e}_L + \dot{e}_H = 0$. Hence, it is sufficient to write up the law of motion of two of these quantities. I am going to work with $\dot{e} = -\dot{u}$ and $\dot{e}_H$.

The employment rate changes if any of the $N u$ unemployed workers finds a job of any type or any of the $N e$ employed workers loses their job. We have the following:

\(^4\)For a similar derivation see Calvó-Armengol and Zenou (2005).
We have the following:

\[
\frac{\dot{e}}{N_u} = N_u \alpha p_{lu} + N u e \alpha \frac{p_{lu}}{\lambda e_l + e_H} \frac{e e_l \lambda e_l}{1 - (1 - u)^k} + N u e \alpha p_{lu} \frac{e e_H \lambda e_l + e_H N e_H}{1 - (1 - u)^k}
\]

\[
= \alpha u + \alpha p_{LU} \frac{e e_L}{1 - \left(1 - u\right)^k} + \alpha p_{LU} \frac{e e_H}{\lambda e_l + e_H} \left(1 - (1 - u)^k\right) + \alpha p_{LU} \frac{e e_H}{\lambda e_l + e_H} \left(1 - (1 - u)^k\right) - \beta e_H
\]

After simplification we get that the employment increases if some offer directly arrives to an unemployed individual (first term in the last line) or if a low wage offer arrives to some employed who has at least one unemployed friend out of \(k\) neighbors (second term) or if a high wage offer arrives to a high wage employed worker who passes it to an unemployed friend (third term). The employment decreases if some worker loses her job.

The fraction of high wage employed agents changes if either an unemployed or a low wage employed finds a high wage job using any method and if a high wage employed looses her job.

We have the following:

\[
\frac{\dot{e}_H}{N_u} = N_u \alpha p_{HU} + ke_H \alpha p_{HU} \frac{e e_L}{\lambda e_H + e_H} \frac{1 - \left(1 - e_H\right)}{1 - \left(1 - e_H\right)} + N u e \alpha p_{HU} \frac{e e_H}{\lambda e_H + e_H} \frac{1 - \left(1 - e_H\right)}{1 - \left(1 - e_H\right)} - \beta e_H
\]

\[
= u \alpha p_H + \alpha p_H \frac{e e_H}{\lambda e_H + e_H} \left(1 - \left(1 - e_H\right)\right) + \alpha p_H \frac{e e_L}{\lambda e_H + e_H} \left(1 - \left(1 - e_H\right)\right) + \alpha p_H \frac{e e_H}{\lambda e_H + e_H} \left(1 - \left(1 - e_H\right)\right) - \beta e_H
\]

\[
= \alpha p_H \left(1 - \frac{e e_H}{\lambda e_H + e_H} \left(1 - \left(1 - e_H\right)\right)\right) - \beta e_H
\]

where in the second step we used that \(u + e_L = 1 - e_H\).

The simplified expression can be interpreted as follows. The fraction of high wage employed increases whenever a new high wage offer arrives (\(\alpha p_H\)) and it does not arrive to a high wage employed whose all neighbors are also high wage employed workers \(\left(1 - \frac{e e_H}{\lambda e_H + e_H} \left(1 - \left(1 - e_H\right)\right)\right)\).

The fraction of high wage employed workers decreases if among the employed individuals one earning high wages is separated from her job.
Summarizing, the dynamic system of the mean-field approximation is given by the following two equations:

\[
\dot{e} = \alpha u + \alpha p_L e (1 - (1 - u)^k) + \alpha p_H \frac{e \lambda e_H}{\lambda e_H + e_L} \frac{u(1 - e^k_H)}{(1 - e_H)} - \beta \tag{4.3}
\]

\[
e_H' = \alpha p_H \left(1 - \frac{e \lambda e_H}{\lambda e_H + e_L} e_H^k\right) - \beta \frac{e_H e}{e} \tag{4.4}
\]

### 4.3 Results

#### 4.3.1 Dynamics and steady state

In this section I show that the dynamical system defined by the mean-field equations has a unique stationary state \((e^*, e_H^*)\) which is a global attractor. Further, I show how the position of this steady state changes with the parameters of the model. This is summarized by the following proposition.

**Proposition 9** For any \(0 < \alpha, 0 < \beta, 0 < p_H < 1, \alpha p_H < \beta\), there exists a unique stationary state of the dynamics defined by (4.3) and (4.4) where \(0 < e^* < 1\) and \(0 < e_H^* < 1\). We have that:

(i) This stationary state is a global attractor.

(ii) \(e^*\) and \(e_H^*\) increase with \(k, \alpha\).

(iii) \(e^*\) and \(e_H^*\) decrease with \(\beta\).

(iv) As \(\lambda\) increases, \(e^*\) increases, while \(e_H^*\) decreases.

(v) \(e_H^*\) is increasing in \(p_H\), the effect of \(p_H\) on \(e\) is ambiguous.

The unique steady state of the system \(u^*, e_L^*, e_H^*\) corresponds to the unique invariant distribution of the Markov process. The invariant distribution determines the probability that an agent is in one of the states \(\{U, L, H\}\), using these probabilities we can derive the unique unemployment and employment rates.

When the arrival rate of offers \(\alpha\) rises or the frequency of job separations \(\beta\) diminishes, the (high wage) employment level increases.
CHAPTER 4. EXPECTED WAGE OF JOB SEARCH METHODS

As the number of neighbors $k$ increases, the efficiency of the arrival process increases since there is higher probability that an employed agent having an unneeded offer finds someone in the neighborhood who is interested in taking the job. Hence, the employment level increases as the network becomes more connected. Part of this increase is realized by the higher number of high wage employed. Note that Calvó-Armengol and Zenou (2005) obtain different result on regular graphs: there exists a critical value of $k$ which minimizes the unemployment rate. The reason is that in their model offers simultaneously arrive to different individuals which gives rise to congestion of offers in case of high connectivity: many offers are passed to the same unemployed agent who makes use only one of them and does not transmit the other ones further in the network. On the contrary, in my model offers sequentially arrive so this possibility is excluded. Higher connectivity facilitates the placement of unneeded offers.\footnote{Note that the congestion effect pointed out by Calvó-Armengol and Zenou (2005) would be diminished if the job information traveled more than one step in the network. Ioannides and Soetevent (2006) also challenge their non-monotonicity result: assuming arbitrary degree distribution they obtain that higher average connectivity implies higher employment.}

When the correlation parameter $\lambda$ rises, high wage offers more often arrive to high wage employed workers than to low wage employed. This has a positive effect on the employment level because in this way more high wage offers will be used by unemployed agents who get it from their high wage contacts. Low wage employed workers will not be able to use these offers to upgrade their situation. Another implication of a higher $\lambda$ is a lower rate of high wage employment. Some fraction of the offers transmitted by high wage employees will be lost when there is nobody among their direct neighbors who would need it. This does not happen when the high wage offers arrive directly to the low wage agents. Proposition 9 also shows that if there are more high wage jobs arriving ($p_H$ goes up), there will be more high wage employed agents. Concerning the effect on the total employment we have two forces to consider. First a negative one: since there are more (less) high (low) wage offers coming in, higher percentage of the new vacancies will be used by low wage agents to improve their situation, hence these jobs do not arrive to the unemployed workers. Second a positive effect: since there will be more high wage agents who pass all offers they get, unemployed agents have more chance to hear about a vacancy through social contacts. Thus there is a substitution between direct and indirect arrival. In general, we do not know which effect dominates the other.
Further, using the findings of Proposition 9 we can study the welfare effects of the different parameter changes. If we increase the correlation between the status of employed workers and the type of offers they hear about, the overall employment increases but the share of high wage employed decreases. The welfare effects of this change are different depending if we consider a Rawlsian or a utilitarian welfare concept. In this model the Rawlsian welfare can be measured by the unemployment rate, i.e. lower unemployment rate means higher welfare. In this respect, the increase of the correlation $\lambda$ augments the welfare of the society. However, the utilitarian welfare, i.e. the sum of individual utilities, might decrease if the correlation rises.

If unemployed workers earn zero and the individuals are risk-neutral, the utilitarian welfare is given by $Ne_L w_L + Ne_H w_H$. When $\lambda$ increases, $e = e_L + e_H$ rises but $e_H$ decreases. Hence, if $w_H$ is significantly higher than $w_L$, the utilitarian welfare decreases. We summarize this finding in the following Corollary.

**Corollary 1** If the correlation $\lambda$ increases, the Rawlsian welfare increases and the utilitarian welfare might decrease.

When there are more vacancies opened ($\alpha$ is higher), both the overall and the high wage employment rate rises leading to an increase in both the utilitarian and the Rawlsian welfare. If we increase the connectivity of the graph ($k$), we have the same effects. Hence, in my model the presence of the informal job finding possibility rises the welfare of the society. Higher job destruction has an opposite impact.

### 4.3.2 Wages

The main question of this article is how the expected wage of the network search relates to the expected wage of the formal search in this model. I associate formal search to the "direct" arrival of offers: an individual obtains a job if she learns it directly "by the arrival process". Network search means that the individual has learned about an opportunity indirectly, i.e. through one of her neighbors in the network. Below I compare how the wages from these two sources relate to each other.

I consider the conditional wage expectation upon arrival. I choose this option in order to measure the same quantity that has been used in the empirical literature. Estimation of the expected wage of a search method is based on the information obtained from employed individuals who the question "By what means were you first informed about your current job?".
Individuals are provided with possible answers containing social contacts (family, friends etc.) and different formal methods, e.g. by answering adverts in newspapers, employment agencies, direct application to firms etc. Based on this answer two groups of individuals are established and the average wages between these two groups are compared, controlling for the characteristics of the employee and the job. Hence, these wages are conditioned on the employment status of the individual. Moreover, I take the average wage among all employed workers who used a given search method independently whether they were unemployed or low wage employed before.

Using the mean-field approach, the conditional expected wage for the direct arrival can be written as follows:

\[
E(\text{direct } w | \text{useful offer arrives}) = \frac{w_L p_L u + w_H p_H (u + \frac{e_H}{e_H + e_L})}{p_L u + p_H (u + \frac{e_H}{e_H + e_L})} \quad (4.5)
\]

Direct offer arrives if any offer reaches an unemployed or a high wage offer goes to a low wage employed. We divide the probabilities by the probability of the condition: a useful offer directly arrives to some agent.

Regarding the network channel: a useful offer arrives through social contacts if a low or high wage offer reaches an unemployed or if a high wage offer is passed to a low wage employed. High offers are forwarded only by high wage workers while low offers by any employed. We have the following formulation for the mean-field case:

\[
E(\text{indirect } w | \text{useful offer arrives}) = \frac{w_L p_L e (1 - e^k) + w_H p_H \frac{e_L}{e_H + e_L} (1 - e^H)}{p_L e (1 - e^k) + p_H \frac{e_L}{e_H + e_L} (1 - e^H)} \quad (4.6)
\]

A low wage offer will be transmitted if it reaches an employed agent which happens with probability \(e\). This employed agent transmits it to some unemployed with probability \(1 - e^k\): the probability that not all of her \(k\) neighbors are employed. A high wage offer will be passed to an unemployed or low wage employed worker if it reaches a high wage employed which happens with probability \(e_H\). This high wage employed transmits it to someone if not all of her \(k\) neighbors are high wage employed workers. This event has \(1 - e^H\) probability.

In the following result we relate these two expected wages to each other.

**Proposition 10** There exists a critical value of \(\lambda\) such that \(\forall \lambda < \bar{\lambda}\), the expected wage of a direct offer is higher than the expected wage of an indirect offer, while \(\forall \lambda \geq \bar{\lambda}\), the expected wage of indirect arrival exceeds that of direct arrival. Moreover,
1. $\bar{\lambda}$ is decreasing in $\alpha$.

2. $\bar{\lambda}$ is increasing in $\beta$.

The intuition behind this result is the following: when we increase $\lambda$ the information about high wage vacancies more often goes to high wage employed who pass those offers to their contacts. The share of the forwarded low wage offers does not change since low and high wage employed pass them in the same way. Hence, the share of good offers transmitted to social contacts rises and with that the wage expectation increases. Low wage employed workers less frequently get high wage offers in the direct way, hence the weight of high wage arrival in the expectation of formal search decreases. Moreover, when the direct arrival is random ($\lambda = 1$) the market performs better than the network: the fact that low wage workers search on the job makes the quality of transmitted offers lower on average. On the contrary, when the "correlation" is perfect ($\lambda \to \infty$), the indirect arrival provides high wage jobs at higher rate than the direct arrival. Between these two corner values, we can find a threshold of $\lambda$ where the two expected wages coincide.

Regarding the comparative statics of this threshold, I find that when the economy is in good state, i.e. more vacancies open up or the job destruction rate is lower, the threshold value is lower. In this case, the neighborhood of a given individual consists of more high wage employed agents and less low wage employed or unemployed. Thus even if the arrival process is random ($\lambda \to 1$), there is higher chance that a high wage offer ends up by a high wage employed who will pass it to one of her contacts. On the contrary, when the economy is in bad situation, the neighborhood of an unemployed contains very few workers earning high wages. The new incoming high wage offers will be transmitted through the network only if they directly arrive to these few high wage employed what only a higher correlation can guarantee.

As for the impact of connectivity on the threshold correlation ($\bar{\lambda}$), we need to consider two effects. First, as the connectivity increases, both the employment and high wage employment rates increase which implies a decrease of the threshold value for the previously seen reasons. Second, there is a direct effect of connectivity on the expected wage of network search (see equation (4.6)). The weight on the high wage in this expectation depends on the probability ratio that a high wage offer arrives to the arrival of a low wage offer:

$$
\frac{p_H e^{\lambda e_H} (1 - e^L)}{p_L e^{1 - e^{k}}}
$$
The direct impact of connectivity on this ratio enters through the fraction:

$$\frac{(1 - e^k_p)}{(1 - e^k)}$$

A rise in the connectivity $k$ increases the denominator more than the numerator which implies that the performance of the network decreases with the number of neighbors. When $k$ increases by 1, the numerator increases by $\frac{1-e^k_p}{1-e^k}$ while the denominator by $\frac{1-e^k}{1-e^{k-1}}$. The rise of the denominator is higher because the function $f(x) \equiv \frac{1-x}{1-x^k}$ is increasing in $x \in [0, 1]$.\(^6\) My numerical results suggest that this second effect is larger and the critical correlation increases with the connectivity of the graph, especially if the job arrival rate is low or most of the new jobs pay low wages. Figure 4.1 plots the threshold correlation against the network connectivity.

Figure 4.1: The critical value of the correlation as a function of connectivity. Right panel: $\beta = 0.5, pH = pL = 0.5$, Left panel: $\beta = 0.5, \alpha = 0.8$.

The effect of the share of high wage offers $p_H$ on the critical correlation is ambiguous.

\(^6\)It’s derivative is $\frac{x^2(1+k-kx+x)}{(1-x^k)^2}$ whose sign depends on the sign of $(-1 + k - kx + x^k)$ which is positive (it is decreasing in $x$ and is weakly positive at $x = 1$).
If $p_H$ increases, there will be more workers employed in high wage jobs which would imply that the threshold correlation becomes smaller. However, the employment rate might decrease which would cause an increase of the critical value $\bar{\lambda}$. Numerical results suggest that this second effect is smaller than the first and the critical value of $\lambda$ decreases with $p_H$ (see Figure 4.2 and the left panel of Figure 4.1).

Figure 4.2: The critical value of the correlation as a function of $p_H$ for different values of $\alpha$ ($\beta = 0.5$).

These results shed new light on why and when we observe a wage premium or a discount for the network search compared to formal methods. First, the presented model suggests that the network search provides a wage premium in cases when the status of an individual and the quality of the vacancies she might hear about are highly correlated. Second, the required correlation depends on the actual state of the economy: in "bad times", when less vacancies open up or the wage distribution of new offers puts lower weight on the well-paid jobs or there are more jobs destroyed, the correlation should be higher. Hence, it is less likely to observe a wage premium of the network search and more likely to have a wage discount. Figure 4.1 indicates that the critical correlation level is higher when the connectivity of the network is higher as well and the impact of connectivity is higher in "bad times".
4.4 Computational analysis

To verify that the findings using the mean-field approximation technique are indeed properties of the original Markov process, I simulate the model. In the simulation the neighbors of an individual are not randomly re-drawn at each point of time. Thus, it allows for the correlation between the states of connected agents which was neglected by the mean-field approximation.

The simulation strategy is based on the existence of an invariant distribution of the Markov process: I simulate the model for sufficiently long time and use the fact that the proportional time spent in each state is equal to the corresponding probability in the invariant distribution (when the time goes to infinity). Hence, I can obtain the long-run value of different statistics derived from the invariant distribution by taking the time average of them using the data recorded during a simulation.

To be sure that I have chosen a sufficiently long period of time, I run the simulation for $2T$ periods and I compute the average statistics after period $T$ and $2T$. Given that the invariant distribution is independent of the initial conditions, the averages taken at the two points of time should coincide. In my simulations, this happens after $T = 300000$ periods.\footnote{After 300000 periods the difference of the two averages is smaller than 0.01.}

To determine the average wages of the two search methods, in each period when a new offer arrives to the economy, I save the type of the offer and the way (direct or indirect) it reaches an individual who can use that offer.

I simulate the model on regular graphs, i.e. every agent has the same number of links. The graph is a random network which is fixed at the beginning of a run: the $k$ neighbors of an agent are randomly drawn from the population and remain the same throughout the run. To exclude the effects of any arbitrary realization of this random network, I run the model on 30 different networks and I take the average of the statistics over this sample.

I run the simulations with $N = 1000$ individuals. In the initial state, each agent is unemployed, low or high wage employed with probability $1/3$. The choice of initial state has no effect on the results since the invariant distribution is independent of the initial conditions.

The baseline values of the parameters are the following: $\alpha = 0.55$, $\beta = 0.5$, $p_H = 0.5$, $k = 6$, $w_L = 1$, $w_H = 2$. $\lambda$ has been changed between 0 and 12 by step 1.

Figure 4.3 shows the average wage difference between the offers arriving through a contact (indirect arrival) and the jobs obtained on the market (direct arrival). I plot this statistics for different values of $\lambda$ and job arrival rate $\alpha$. In accordance with the analytical results, for low
values of the correlation $\lambda$, the social networks pay a wage discount over the market while for high values of $\lambda$ a wage premium is obtained. The critical correlation value decreases when the arrival rate of new offers $\alpha$ rises.

![Figure 4.3: Average wage difference between network and market arrival for different values of $\lambda$ and $\alpha$.](image)

We can observe a similar pattern on Figure 4.4 and 4.5. The wage difference between the network and market search is increasing in the correlation $\lambda$ and it goes from a negative to a positive value. The critical correlation value is higher if the social network is more connected and if the new job openings are less likely to pay high wages.

The simulations confirm all the results presented using the mean-field approach: the presence of correlation in the state of connected agents does not influence the findings regarding the relation of expected wages of the different search methods.

### 4.5 Graphs with heterogeneous degree distribution

In this section, I consider the case when individuals can be different with respect to their degree: I look at general random graphs instead of regular graphs. In this case, the network
Figure 4.4: Average wage difference between network and market arrival for different values of \( \lambda \) and \( p_H \).

is defined by an exogenous degree distribution: \( P(k) \) is the probability that an individual has \( k \) degrees, \( k \in [0, \infty) \). I run simulations when the degree distribution is Poisson.\(^8\) Note that in this case agents with different number of neighbors will experience different unemployment and employment probabilities. An individual with more neighbors will have lower unemployment probability since she has more information sources to become aware of a vacancy.

Figure 4.6 shows the expected wage difference between social contacts and the formal market when the degree distribution of the underlying network is Poisson with parameter \( \mu = 4 \). I plot the wage difference for different values of the job arrival rate \( \alpha \). The findings for regular graphs seem to generalize to the case of heterogeneous degree distribution. For low values of the correlation parameter \( \lambda \), the formal market pays higher expected wages, while for high values of \( \lambda \) we have the opposite. The critical correlation value decreases as the job arrival rate rises.

\(^8\)For each parameter value now I take a sample of 50 runs.
4.6 Conclusions

I analyze the relationship between the expected wages of two job search methods: social contacts and formal methods. Previous empirical (and theoretical) literature has found contradictory evidences regarding which of the two search methods pays higher expected wages. I show that social contacts provide higher expected wage if there is a high correlation between the status of an employed individual and the quality of the job offer she might hear about. This might be the case when employed workers mostly hear about the new openings of their own firm.

The critical value of this correlation, which guarantees a wage premium for the network search, depends on the parameters of the arrival process and the connectivity of the social network. In "bad times", when less vacancy opens, the new offers pay lower wages or the job destruction rate is high, the correlation has to be higher to obtain a wage premium of the network search. Hence, for a given correlation level the network search is likely to pay a wage discount in bad times. This "business cycle" argument can add to the explanation of the observed mixed evidences regarding the relative wages of the two search methods.
I have also obtained that the Rawlsian welfare of the society, measured by the negative of the unemployment rate, is the highest when the network search gives the highest wage premium. In contrast, the utilitarian welfare might be the highest in the opposite case, when the formal search gives the highest wage premium. This finding suggests that looking exclusively at the unemployment rate can be misleading regarding the welfare consequences of the presence of social networks in the labor markets: in a context of heterogeneous wages we might get different conclusions.

4.7 References

- Calvó-Armengol, A. and Jackson, M. O. (2004): The effects of social networks on em-

Figure 4.6: Average wage difference between network and market arrival in case of Poisson degree distribution with parameter $\mu = 4$, $b = 0.5$, $p_H = 0.5$. 

I have also obtained that the Rawlsian welfare of the society, measured by the negative of the unemployment rate, is the highest when the network search gives the highest wage premium. In contrast, the utilitarian welfare might be the highest in the opposite case, when the formal search gives the highest wage premium. This finding suggests that looking exclusively at the unemployment rate can be misleading regarding the welfare consequences of the presence of social networks in the labor markets: in a context of heterogeneous wages we might get different conclusions.

4.7 References

- Calvó-Armengol, A. and Jackson, M. O. (2004): The effects of social networks on em-


4.8 Appendix: proofs

4.8.1 Proposition 1

Proof The steady state of the system is defined by the following two equations:

\[ \dot{e} = -\beta e^* + \alpha (1 - e^*) + \alpha p_H e^* (1 - e^*) + \alpha p_H \frac{e^* \lambda e^*_H}{1 - e^*_H} (1 - e^*) - \beta e^* \equiv g(e^*_H, e^*) = 0 \]

\[ e^*_H = \alpha p_H \left( 1 - \frac{e^* \lambda e^*_H}{\lambda e^*_H + e^* - e^*_H} \right) - \beta e^*_H \equiv f(e^*_H, e^*) = 0 \]

State space of the dynamics is \( \Gamma := \{(e, e_H) : 0 \leq e \leq 1, e_H \leq e\} \).

First, we can see that the loci \( f(e_H, e) = 0 \) and \( g(e_H, e) = 0 \) cross only once. We have that \( g(e_H, 0) = \alpha p_H - \beta e_H = 0 \) which has a solution \( e_H = \frac{\alpha p_H}{\beta} \). Thus \( g(e_H, e) = 0 \) crosses the axis of \( e_H \). Second, we have that it is impossible that \( g(0, e) = 0 \), so the curve cannot cross the axis of \( e \) in the plane \((e, e_H)\).

Third, the locus \( g(e_H, e) = 0 \) changes monotonously when \( e \) increases starting from zero. To see this we write up the derivatives of \( g(e_H, e) \). Let \( h(e_H, e, \lambda) = \frac{e^*_H \lambda e^*_H}{\lambda e^*_H + e^* - e^*_H} \). The derivatives of \( h \) are the following:
\[ \frac{\partial h(e_H, e, \lambda)}{\partial e} = \frac{e_H^{1+\lambda} - 1}{(e_H - e)^2} > 0, \text{ if } \lambda > 1 \text{ and } \leq 0 \text{ if } \lambda \leq 1 \]

\[ \frac{\partial h(e_H, e, \lambda)}{\partial H} = \frac{e_H^{1+\lambda} - 1}{(e_H - e)^2} \geq 0 \]

\[ \frac{\partial h(e_H, e, \lambda)}{\partial \lambda} = \frac{(e_H - e)^2 e_H}{(e_H - e)^2(1+\lambda)^2} \geq 0 \]

Using these derivatives we can write up the derivatives of \( g(e_H, e) \):

\[ \frac{\partial g(e_H, e)}{\partial e_H} = -b + a p_H \left( -e_H^{1+\lambda} h(e_H, e, \lambda) - e_H^k \frac{\partial h(e_H, e, \lambda)}{\partial e_H} \right) \leq 0 \]

\[ \frac{\partial g(e_H, e)}{\partial e} = -ae_H^k p_H \frac{\partial h(e_H, e, \lambda)}{\partial e} \]

which is (weakly) positive if \( \lambda \leq 1 \) and negative if \( \lambda > 1 \).

Hence, for \( \lambda > 1 \) as we increase \( e, e_H \) has to decrease to have \( e_H = 0 \) (see Figure 4.7). For \( \lambda \leq 1 \) as we increase \( e, e_H \) has to increase to have \( e_H = 0 \) (see Figure 4.8). So, in both cases we have a monotone change.

Looking at \( f(e_H, e) = 0 \), we have that \( f(0, e) = -e(\beta + \alpha p_H) - \alpha p e^{k+1} + \alpha \) which is monotone decreasing in \( e \), and is positive at \( e = 0 \) and negative at \( e = 1 \), hence we have a root of this function for some \( e \in (0, 1) \). The derivative of \( f(e_H, e) \) with respect to \( e_H \) is the following:

\[ \frac{\partial f(e_H, e)}{\partial e_H} = \frac{1}{(-1 + e_H^2) e_H} \left[ \alpha p H e H (e_H - (1 + k) - k) \right] \]

\[ - \frac{1}{(-1 + e_H^2) e_H} \left[ \alpha p H (1 + e_H) e_H (1 + e_H^k) \right] \geq 0 \]

Here the term before the parenthesis is positive. The second term in the parenthesis is positive as well. The first term in the parenthesis is positive if \( (e_H + e_H^k (e_H - (1 + k) - k)) \geq 0 \). This function takes the value \( 0 \) at \( k = 1 \), and is increasing in \( k \). To see this latter we take the derivative with respect to \( k \): \( e_H^k (1 + e_H + (e_H - (1 + k) - k) \log e_H) \geq 0 \) if \( (1 + e_H + (e_H - (1 + k) - k)) \log e_H \geq 0 \). This latter is true since the expression on the right is monotone decreasing in \( e_H \) and it takes the value \( 0 \) at \( e_H = 1 \).

Similarly we can see that \( \frac{\partial f}{\partial e} \leq 0 \) (even though the last term is increasing in \( e \) for some values of \( e_H \), the increase is smaller than the decrease in the first term). Hence, as we increase \( e_H \) from zero, we need to increase \( e \) as well to stay on the curve \( \dot{e} = 0 \). We also have that \( f(e_H, 1) = -b \) is never zero, so the locus does not cross the line \( e = 1 \)

\[ 9 \text{It’s derivative with respect to } e_H \text{ is } \frac{(e_H - 1 + e_H^k)}{e_H} + (1 + k) \log e_H \]
This argument characterizes a single cross of the loci \( \dot{e} = 0 \) and \( e_H = 0 \) as it is shown in Figure 4.7 and 4.8. The cross cannot be on the boundary of the state space because \( e^* \neq e_H^* \) since this would imply that \( e_L^* = 0 \) which cannot be when \( p_L > 0 \).

Now, it is easy to see that this steady state is a global attractor. If \( e_H \) is higher (lower) for a given \( e \) than the value defined by \( \dot{e} = 0 \), we have that \( e \) increases (decreases). If \( e_H \) is higher (lower) for a given \( e \) than the value defined by \( \dot{e}_H = 0 \), we have that \( e_H \) decreases (increases).

Hence, the out of the equilibrium dynamics is such that the system converges to the equilibrium, as it is shown in the figures.

Now we turn to the comparative statics results. If \( k \) or \( \alpha \) increases, the \( \dot{e}_H = 0 \) curve moves up, while the \( \dot{e} = 0 \) locus shifts to the right, hence we have that the equilibrium values of \( e, e_H \) are higher than before. If \( \beta \) increases, the \( \dot{e}_H = 0 \) curve moves upward, while the \( \dot{e} = 0 \) locus moves to the left, the equilibrium quantities decrease. If \( \lambda \) increases, the \( \dot{e} = 0 \) locus moves to the right, while the \( \dot{e}_H = 0 \) locus shifts downward. This determines a new equilibrium at a higher level of \( e \) and a lower level of \( e_H \). When \( p_H \) increases, the locus \( \dot{e}_H = 0 \) moves upward, the new equilibrium value of \( e_H \) is higher than before. By an increase in \( p_H \), \( \dot{e} \) moves to the left because

\[
\frac{\partial f(e_H, e)}{\partial p_H} \leq 0.
\]

We take the derivative of \( f(e_H, e) \) with respect to \( p_H \) when \( \lambda \to \infty \), the highest value of the derivative regarding \( \lambda \):

\[
\frac{\partial m(e_H, e)}{\partial e} = -1 + e_H^k - e^k - 1 < 0.
\]

The sign of this derivative depends on the sign of \( m(e_H, e) \equiv (1 - e)(1 - e_L^k) + (e^k - 1)(1 - e_H) \).

The derivatives of this function:

\[
\frac{\partial m(e_H, e)}{\partial e} = -1 + e_H^k - e^{k-1}(e_H - 1)k
\]

For \( e = 0 \) this derivate takes the value \( e_H^k - 1 < 0 \).

\[
\frac{\partial^2 m(e_H, e)}{\partial^2 e} = -e^{k-2}(e_H - 1)(k - 1)k \geq 0
\]

since \( k \geq 1 \). Hence, \( m(e_H, e) \) is convex in \( e \). Then it is possible to differentiate two cases: 1. \( \frac{\partial m(e_H, e)}{\partial e} \bigg|_{e=1} < 0 \), 2. \( \frac{\partial m(e_H, e)}{\partial e} \bigg|_{e=1} \geq 0 \). In the first case, \( m(e_H, e) \) is decreasing in \( e \), thus takes it’s maximum value at \( e = 0 \), where \( e_H = 0 \) too. Here \( m(0, 0) = 0 \), hence, \( m(e_H, e) \leq 0 \). In the second case, there exists a minimum point of \( m(e_H, e) \) for some \( e \in (0, 1) \). So the maximum of
Figure 4.7: Phase diagram $\lambda > 1$

$m(e_H, e)$ with respect to $e$ can be taken by it’s corner values: $m(0, 0) = 0$ and $m(e_H, 1) = 0$ too. Hence, $m(e_H, e) \leq 0$.

Thus $p_H$ increases, $e_H = 0$ moves upward and $\dot{e} = 0$ moves to the left. Hence, in the new equilibrium $e_H$ increases but we don’t know whether $e$ increases or decreases.

Hence, we cannot assess the final effect of $p_H$ on the employment level.

4.8.2 Proposition 2

Proof First, I show that for $\lambda = 1$ we have that the direct wage is higher, while for $\lambda = \infty$, the indirect one. For $\lambda = 1$, $\frac{e \cdot e_H}{n \cdot e + e_L} = e_H$. To compare the two expected wages, we can relate the ratio of the probability of high and low wage offer arrivals (direct-indirect):

$$\frac{p_H(1 - e_H)}{(1 - p_H)(1 - e)} - \frac{p_H e_H (1 - e_H^k)}{(1 - p_H)e (1 - e^k)} \geq 0$$

That is:

$$\frac{(1 - e^k)e}{1 - e} - \frac{(1 - e_H^k)e_H}{1 - e_H} \geq 0$$

This difference is indeed positive given that the function $g(x) \equiv \frac{(1-x)}{1-x}$ is increasing in $x \in [0, 1]$. The derivatives of $g(x)$ are the following:
First, we can see that \( g(x) \) is convex. The denominator of the second derivative is negative, thus the function is convex if the numerator is negative too:

\[
-2 + x^{k-1} \left(k + k^2(-1 + x)^2 + 2x - kx^2\right) < 0
\]

To see this we take the derivative of this latter expression with respect to \( x \):

\[
k \left(-1 + k^2\right)(-1 + x)^2x^{-2+k} \geq 0
\]

Hence, the numerator is (weakly) negative, if it is (weakly) negative for \( x = 1 \) where it takes the value 0. Hence, for \( x \leq 1 \), \( g(x) \) is (weakly) convex.

This implies that \( g(x) \) is increasing in \( x \) if it is increasing at \( x = 0 \):

\[
\left. \frac{\partial g(x)}{\partial x} \right|_{x=1} = 1
\]

Thus, \( g(x) \) is increasing in \( x \) which implies that for \( \lambda = 1 \) the market expectation is higher than the network wage expectation.
CHAPTER 4. EXPECTED WAGE OF JOB SEARCH METHODS

Note that for $\lambda = 0$, we have the same relation.

For $\lambda = \infty$, $\frac{e_{1h}}{e_{1h} + e_L} = e$. Here we can show that the weight on the high wage in the indirect expectation is higher than in the direct expectation:

\[
\frac{p_H(1 - e^k_H)}{p_L e(1 - e^k) + p_H e(1 - e^k_H)} > \frac{u p_H}{u p_L + u p_H}
\]

using that $p_L = 1 - p_H$:

\[
\frac{p_H(1 - e^k_H)}{(1 - e^k)(1 - p_H) + p_H(1 - e^k_H)} > p_H
\]

That is:

\[
p_H(1 - e^k_H) > p_H - p_H^2 - e^k p_H + e^k p_H^2 + p_H^2 - e^k p_H = p_H(1 - e^k) + p_H^2(e^k - e^k_H)
\]

Equivalently

\[
p_H(e^k - e^k_H) > p_H^2(e^k - e^k_H)
\]

which is true given that $p_H > p_H^2$ if $p_H < 1$.

To establish that there exists a threshold value of the correlation $\lambda$, it is enough to show that the direct expected wage decreases in it and the indirect wage increases. We divide the probability of the arrival of a high wage offer with that of a low wage offer, in case of the formal search we have:

\[
\frac{p_H(u + \frac{e_{1h}}{e_{1h} + e_L})}{p_L u}
\]

which is clearly decreasing in $\lambda$. In case of the indirect wage:

\[
\frac{p_H \frac{e_{1h}}{e_{1h} + e_L} (1 - e^k_H)}{p_L e(1 - e^k)}
\]

which is increasing in $\lambda$ since $\frac{e_{1h}}{e_{1h} + e_L}$ is increasing in $\lambda$.

The critical value can be expressed by equalizing these two ratios:

\[
\frac{p_H(1 - e^k_H) \frac{e_{1h}}{e_{1h} + e_L}}{p_L e(1 - e^k)} = \frac{p_H(u + \frac{e_{1h}}{e_{1h} + e_L})}{p_L u}
\]

After rearranging and expressing $\lambda$, we have the following:

\[
\bar{\lambda} = \frac{(e - e_H)(1 - e^k)}{e_H(1 - e)(e^k - e^k_H)}
\] (4.11)
Regarding the effect of parameters on the critical value, we can see that \( \bar{\lambda} \) is decreasing in \( e \) and \( e_H \).

First, we take the derivate with respect to \( e_H \):

\[
\frac{\partial \bar{\lambda}}{\partial e_H} = \frac{(-1 + e^k)(-e^{1+k} + e_H^k(e + ek - e_Hk))}{(-1 + e)e_H^2(e^k - e_H^k)^2}
\]

The sign of the derivative is determined by the sign of \( A(e) \equiv (-e^{1+k} + e_H^k(e + ek - e_Hk)) \). We can see that this term is decreasing in \( e \), after simplification it’s derivative with respect to \( e \) is:

\[
-\left(e^k - e_H^k\right)(1 + k)
\]

which is negative since \( e > e_H \). Hence, \( A(e) \) takes it’s maximum at \( e = 0, \) where \( A(0) = -ke_H^{k+1} \): it is negative. Hence, the derivative of \( \bar{\lambda} \) with respect to \( e_H \) is negative.

Second, we show that \( \bar{\lambda} \) decreases in \( e \) as well. First we take the following limits:

\[
\lim_{e \to 1} \bar{\lambda} = \frac{(-1 + e_H)k}{e_H\left(-1 + e_H^k\right)}
\]

\[
\lim_{e \to 0} \bar{\lambda} = \frac{1}{e_H^k}
\]

We can see that the value of the limit by \( e = 0 \) is higher than that by \( e = 1 \):

\[
\frac{1}{e_H^k} \geq \frac{(-1 + e_H)k}{e_H\left(-1 + e_H^k\right)}
\]

We multiply both size by \( e_H \) and after rearranging, we get:

\[
1 - ke_H^{k-1} + e_H^k(k - 1) \geq 0
\]

This is true since at \( e_H = 1 \) the left-hand side is equal to 0 and the left-hand side is decreasing in \( e_H \), it’s derivative with respect to \( e_H \) is \((-1 + e_H)e_H^{-2+k}(-1 + k)k \leq 0 \).

Thus \( \bar{\lambda} \) is increasing in \( e \) if it is monotone in \( e \) in the interval \((0, 1)\). To see this we show that the derivative of \( \bar{\lambda} \) with respect to \( e \) cannot be zero on this interval.

We express the derivate:

\[
\frac{\partial \bar{\lambda}}{\partial e} = \frac{e_H^k\left(e(-1 + e_H) + e^k(e - ee_H - (-1 + e)(e - e_H)k)\right) + e^k\left(e^{1+k}(-1 + e_H) + e^2k + e_Hk - e(-1 + e_H + k + e_Hk)\right)}{(-1 + e)^2ee_H(e^k - e_H^k)^2}
\]
It can be zero if the numerator is zero. We elaborate on the numerator:

\[
\begin{align*}
& e_H^k \left( e(-1 + e_H) + e^k(e - ee_H - (-1 + e)(e - e_H)k) \right) + e^k \left( e^{1+k}(-1 + e_H) + e^2 k + e_H k - e(-1 + e_H + k + e_H k) \right) \\
& \quad = e_H^k \left( (1 - e_H)(e^{k+1} - e) - e^k e_H - 1)(e - e_H) \right) + e^k \left( (e_H - 1)(e^{k+1} - e) + k(e - 1)(e - e_H) \right) \\
& \quad = (1 - e_H)(e^{k+1} - e)(e_H^k - e^k) + k(e - 1)(e - e_H)e^k (1 - e_H^k)
\end{align*}
\]

This latter expression can be zero in the following cases:

1. \( e_H = e \)
2. \( e_H = 1 \Rightarrow e = 1 \)
3. \( e = 0 \Rightarrow e_H = 0 \)
4. \( e = 1 \)

Since \( e_H \leq e \), the second and third cases are sub cases of the first. We know that as long as \( p_L > 0 \), \( e_L > 0 \) and \( e \neq e_H \). Hence the derivative can be zero only at the boundary \( e = 1 \).

In sum, we have obtained that as \( \alpha \) increases or \( \beta \) decreases, \( e_H \) and \( e \) increases (by Proposition 1) which makes the critical value \( \lambda \) lower.
Chapter 5

Limited Memory Can Be Beneficial for the Evolution of Cooperation

*Joint work with Jaromir Kovarik and Friederike Mengel*

5.1 Introduction

Human cooperation is often supported through mechanisms such as direct or indirect reputation building (Hammerstein [9]). Direct reputation is built through own experience (Axelrod [1], Trivers [28]), while indirect reputation is also learned through communication with others (Nowak and Sigmund [17], Sommerfeld et al. [25]). Cognitive abilities, such as memory and language, seem crucial for the emergence of large scale cooperation through reputation-based mechanisms (see e.g. Silk [24]). Ex ante it seems that the larger human memory capacity the better the chances for effective reputation mechanisms to develop, simply because being able to process more information should allow agents to learn more effectively about other agent’s types. This is all the more so, if agents rely on indirect reputation-building as well, which can increase the effectiveness of reputation mechanisms, but also has a larger demand for memory capacity than first-hand experience. Sufficient memory capacity thus seems a key requirement for allowing reputation-based mechanisms to successfully establish cooperative societies (Trivers [28], Nowak and Sigmund [17]).

Since Miller’s [16] "magical number seven +/- two" it has been widely accepted, though,
that human working memory is very limited (Cowan [4]).\footnote{In a related framework, Milinski and Wedekind [15] find working memory capacities of the same order of magnitude to be used by human subjects in an experiment.} Hence, if larger memory provides an evolutionary advantage either individually or at the group level (by building more cooperative social structures through more effective use of reputation based mechanisms), why don’t we observe an evolution toward larger working memories in humans?

In this study we ask whether there is some evolutionary advantage of limited memory capacity (other than the obvious saving of energy or complexity costs). In particular we analyze the effect of working memory on the evolution of cooperation and show a case in which societies with strongly limited working memory outperform others with larger memory. Of course working memory is not the only kind of memory humans use and other types of memory (such as episodic memory) seem important for cooperation as well. We discuss this in some more detail below.

Agents in our evolutionary model are arranged on a network and interact in a Prisoner’s Dilemma. The prisoner’s dilemma is an abstraction typically used to capture certain aspects of real-life interactions. Agents learn from their own experience and that of their neighbors in the network about the past behavior of others and use this information to choose actions which maximize their preferences over payoffs in the Prisoner’s dilemma. Each agent can only process information from the last $h$ interactions. Our agents are heterogenous in terms of their preferences and evolution selects among preference types. Evolutionary fitness is determined by the payoffs in the Prisoner’s dilemma. We show that if memory is too short, cooperation does not emerge in the long run. A slight increase of memory length to around 5-10 elements, though, can effectively build up largely cooperative societies. Longer memory, on the other hand, can entrap populations in full defection. Quite interestingly the optimal memory length from our model coincides with empirical evidence on actual human memory in cognitive sciences and psychology.\footnote{We discuss some caveats below.}

Assessing reputation through "image scoring" (Nowak and Sigmund [18], [17]), where individuals are assigned either a good or bad score according to their past play, has shown to stabilize cooperation when people interact with each partner only once. In this paper, people can meet repeatedly. Therefore, to assess reputation we combine image scoring with direct experience from past interactions and systematically vary the number of past interactions our agent remembers. Moreover, due to the role of the interaction structure agents may be unaware
of the reputation of a partner and the same individual may enjoy different reputation scores for different people. These elements of social interactions relate to memory constraints in obvious and important ways. Both longer time intervals between potential cooperative encounters and having to recall the past play of multiple opponents are challenges for reputation-based mechanisms (Milinski and Wedekind [15], Stevens and Houser [26]).

The role of working memory for the evolution of cooperation has already attracted some attention in the literature. The general insight seems to be that longer memory is beneficial (Hauert and Schuster [10], Kirchkamp [13]). Qin et al [22] have investigated the effect of memory if agents are arranged on a square lattice. They have found that the density of cooperators was enhanced by an increasing memory effect for most parameters. Janssen [11] asks whether the frequency of receiving positive and negative feedback in reputation systems can explain observed levels of cooperation. In his model, if memory is too short cooperation does not survive, whereas too large memory spans can lead to a modest decline of cooperation (see Figure 1 in Janssen [11]). His model analyzes the efficiency of reputation systems in e-Bay auctions and to this aim he uses a version of prisoner’s dilemma where agents have the option to leave. This modification changes the strategic structure of the game. Our model investigates the original version of the prisoner’s dilemma and focuses on the evolution of preference types. Cox et al. [5] explores whether memory about past interactions can compensate the fact that people do not have information about others. Their model differs from ours in many aspects: agents choose whom to interact with, they are randomly assigned a memory length, there are four preference types, among other differences. According to their results agents endowed with longer memory tend to survive on the long run.

The paper is organized as follows. In Section 2 we present the model. In Section 3 we discuss our results regarding optimal memory. In Section 4 we consider populations with heterogeneous memory and discuss individual level selection of both memory and preference types. Section 5 is dedicated to a discussion of our results and of some of the modeling assumptions. Some additional tables, information about simulations as well as extensions of the model can be found in an Appendix.

3See also Ohtsuki and Iwasa [19].
5.2 The Model

The prisoner’s dilemma

There is a large, but finite population of individuals, \( i \in \{1, 2, \ldots, N\} \), who are repeatedly matched to play a prisoner’s dilemma game. The payoff of an agent when she chooses action \( a_i \) and her opponent chooses \( a_j \) is given by

\[
\begin{array}{c|cc}
\text{\( a_i \backslash a_j \)} & C & D \\
C & a & 0 \\
D & 1 & d \\
\end{array}
\]

(5.1)

with \( 1 > a > d > 0, d > 1 - a \).

We assume that there are three types in the population:

- **D**: Defectors who always defect. Preferences of defectors could be represented by the payoffs given in the table above or they could have even stronger preferences for defection. Essential is that they always defect.

- **CC**: Conditional Cooperators cooperate whenever their beliefs about the probability that their current round match cooperates is large enough (at least \( \frac{1}{2} \) given the parameters used in our main simulation). This is the optimal behavior of e.g. an agent whose preferences are represented by matrix (5.1) above but who suffers a psychological payoff loss of \( w \in [1 - a, d] \) each time she defects.

- **A**: Altruists always cooperate. This is the optimal behavior of an agent whose preferences are represented by (5.1) but who suffers an even higher psychological payoff loss of \( w \geq d \) each time she defects.

Note that in principle there could be many more strategies and preference types. The strategies we consider are those that are (i) anonymous, (ii) deterministic and (iii) monotone in the cooperation rate in the population. We also exclude strictly decreasing strategies, which are not empirically observed and easily outperformed by other monotone strategies. The strategies (preference types) we consider are also those most often found in experimental studies (see Fischbacher et al.[6] or Grimm and Mengel[8]).

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4Mengel [14] discusses these three preference types in more detail.

5Cooperation yields higher expected "payoffs" for such an agent whenever \( r \cdot a > r \cdot (1 - w) + (1 - r) \cdot (d - w) \), where \( r \) is the probability that the current round match cooperates.
Matching: Agents are organized on a fixed undirected network, which mediates who meets whom in the population. In each period agents are randomly drawn to play the Prisoner’s Dilemma game (5.1). Once an agent \( i \) is called to play the game, she is matched with one of her direct neighbors \( j \) in the network. For each player, there is an equal probability to meet each of her neighbors. Denote the degree of node \( i \), i.e. the number of neighbors \( i \) has by \( d_i \). Then the probability for \( i \) to interact with \( j \) given that \( i \) has been chosen as a row player is the following:

\[
P_{ij} = \begin{cases} 
\frac{1}{d_i} & \text{if } i \text{ is a neighbor of } j \\
0 & \text{otherwise}
\end{cases}
\] (5.2)

Note that players with more neighbors might be matched more frequently implying that these agents gain more experience than others. Hence to avoid confounding effects of the degree (i.e. number of neighbors) of a player on evolutionary fitness, our definition of fitness below will only consider the payoffs from interactions, in which the agent is drawn first, i.e. acts as agent \( i \). This assumption is not realistic, but allows us to disentangle the effects from the heterogeneity of agents in terms of network degree and the heterogeneity stemming from the different preferences. In this paper we are interested in the latter.

Reputation and beliefs: We assume that agents have bounded working memory and are only able (or willing) to remember and (simultaneously) process information from the last \( h \) periods.\(^6\) They then form beliefs about the behavior of their match using their experience from these last \( h \) interactions. In addition, they can use information they get from their direct neighbors in the network.

Denote by \( \gamma_{ij}(h) \) the number of times that \( j \) cooperated with \( i \) in \( i \)'s last \( h \) interactions divided by all interactions they (\( i \) and \( j \)) had among \( i \)'s last \( h \) interactions. If there was no interaction between \( i \) and \( j \) among \( i \)'s last \( h \) interactions (which are all those that \( i \) can cognitively process), then we set \( \gamma_{ij}(h) < 0 \) as a matter of convention. Denote by \( \beta_{ij}(h) \) the average of the respective statistics among \( i \)'s neighbors: the number of times \( j \) cooperated with a neighbor of \( i \) divided by the number of times they interacted in the neighbor’s last \( h \) interactions. Hence \( \beta \) refers to information from neighbors (where all neighbors are weighted equally) and \( \gamma \) refers to own experience. Again we set \( \beta_{ij}(h) < 0 \) if none of \( i \)'s neighbors has interacted with \( j \) in their

---

\(^6\)We make no claim as to whether agents use only \( h \) periods to form their beliefs because (i) they are not able to remember or process more period or (ii) because they are not willing to do so because processing additional information is too cognitively costly given its additional information content. Hence the limitation of \( h \) may also come from a very sophisticated trade off between costs and benefits.
last \( h \) interactions. The reputation that player \( j \) has for player \( i \) (what \( i \) thinks about \( j \)) at time \( t \) is then given by

\[
 r'_{ij} = \begin{cases} 
 \lambda \gamma_{ij}(h) + (1 - \lambda) \bar{\beta}_{ij}(h) & \text{if } \gamma_{ij}(h) \geq 0 \land \bar{\beta}_{ij}(h) \geq 0 \\
 \gamma_{ij}(h) & \text{if } \gamma_{ij}(h) \geq 0 \land \bar{\beta}_{ij}(h) < 0 \\
 \bar{\beta}_{ij}(h) & \text{if } \gamma_{ij}(h) < 0 \land \bar{\beta}_{ij}(h) \geq 0 \\
 \bar{\sigma}^{t-1} & \text{if } \gamma_{ij}(h) < 0 \land \bar{\beta}_{ij}(h) < 0
\end{cases}
\] (5.3)

In words, the reputation that \( j \) enjoys with \( i \) is a weighted average of her direct reputation with \( i \) (\( \gamma_{ij}(h) \)) and her indirect reputation communicated from \( i \)'s neighbors (\( \bar{\beta}_{ij}(h) \)). A high value of \( \lambda \) means that she relies mostly on her own experience and low \( \lambda \) that she forms judgements, based on the information from others. If \( i \) has not met \( j \) in the last \( h \) periods but at least one of her neighbors has, she relies on neighbors’ experience, and vice versa. If nothing is known about \( j \) (i.e. if neither \( i \) nor any of her neighbors have information about \( j \)) there is "no reputation". In this case the agents use the average rate of cooperation in the population in period \( t - 1 \) (\( \bar{\sigma}^{t-1} \)), which is assumed to be always known to all players. \( i \)'s reputation for \( j \), \( r'_{ji} \) is built analogously.

One could imagine that in a more sophisticated model agents form reputations by also considering the behavior or type of \( j \)'s matches (i.e. they may not judge defecting against a cooperator and defecting against a defector in the same way). Below we explain several reasons why we do not make this assumption here.

**Selection.** Our model is set in the tradition of the indirect evolutionary approach [2]. In these models, evolution selects among preference types: altruists, conditional cooperators and defectors. We are interested in which of the three types survive evolutionary selection, when cooperation will survive and how this depends on memory constraints. Evolutionary success (fitness) is measured by per interaction payoffs of the agents. As explained above we use the last interaction payoffs agents get as row player (agent \( i \) above), such that more connected agents do not per se enjoy higher fitness. The results are robust to considering more periods for fitness.

The selection process is modeled as follows. In each period \( k \) agents are called randomly for selection. For each of these agents, say agent \( i \), another agent \( j \neq i \) is randomly chosen from the population and their fitness is compared.\(^7\) If \( j \) has higher fitness, the former agent \( i \) adopts

\(^7\) An alternative approach would be to compare the payoff of agent \( i \) to that of one of her neighbors [20]. In such a case, cooperation can stabilize thanks to local clusters of cooperators. Our model of local interaction, but global
\( j \)’s type. Note that this implies that only the order of payoffs matter, not the particular numerical values of the parameters \( a \) and \( d \) given in table 5.1. This process could be interpreted as cultural evolution, which could take place e.g. via imitation learning. In this case the randomly chosen agent could be thought of as a cultural role model for our agent (see e.g. Mengel [14]). Since types are not observable (other than through reputation), also type changes can only be learned over time.

**Networks.** In the simulations, we use small-world networks (Watts and Strogatz [30]). These networks are generated from a one-dimensional lattice, where agents have links to their neighbors up to a distance \( \rho \), the connection radius. Starting from this network, each link is rewired with probability \( \theta \). As \( \theta \) tends to 0, we get a regular lattice, while as \( \theta \) tends to 1 a random network is obtained. As \( \theta \) becomes positive but small, distances shorten dramatically, while the clustering coefficient is almost unaffected (see Fig. 1 showing the network structure by these three values of \( \theta \)). Since short distances and high clustering are typical for real-life social networks [29, 7], we focus on these "small-world transition" values [30]. More details on small-world networks can be found e.g. in the textbooks by Vega-Redondo [29] or Goyal [7].

Figure 5.1: Watts-Strogatz [30] small-world networks \((N = 16 \text{ and } \rho = 2)\) for three rewiring probabilities: \( \theta = 0 \) (regular lattice, left), small but positive \( \theta \) (small-world network, center), and \( \theta = 1 \) (random network, right), see Vega-Redondo (2007) p. 59.

The benchmark parameter values used for the numerical analysis are summarized in Table 1. In the Appendix we provide further details of the simulations.
### Table 5.1: Benchmark parameter values.

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<th>Value</th>
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<tr>
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</tr>
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<tr>
<td>$\theta$</td>
<td>$0, 0.01, 0.05, 1$</td>
</tr>
</tbody>
</table>

### 5.3 Results

Fig. 2 summarizes the computer simulations of the model. There is a non-trivial relation between steady state cooperation levels and memory length. Cooperation is rare for low memories. However, as the memory constraint increases from 2 to 5, there is a dramatic increase of cooperation. Hence, only a slight improvement of cognitive capacities of agents can build up cooperative societies. The stark increase of cooperation is followed by a relatively stable level for memory constraints between 5 and 10. For larger memory levels, cooperation starts to decrease. It seems to be optimal for a population if its members have (strong) bounds on their memory constraints. In fact, in (almost all) our simulations the optimal memory level is around 5-10 periods (see Appendix).

The intuition behind the simulation results is as follows. If memory is too low, conditional cooperators cannot effectively learn the type of their opponents and hence are prone to exploitation by defectors. This is true as long as the amount of defection in the population does not exceed the critical level above which conditional cooperators will start to defect. In this case everyone in the society will defect except for some altruists who can be exploited by defectors in any case.

For intermediate levels of memory, conditional cooperators can prevent the extinction of altruists. Since, due to larger memory, conditional cooperators can differentiate between defectors and cooperative individuals, cooperation can survive.

But why is even larger memory detrimental? As memory gets larger, cooperation is sustained by conditional cooperators. Long memory enables them to distinguish between altruists
and defectors, but now there is a drawback. Initially, defectors have a selective advantage over altruists because before reputations are established both types are treated equally by conditional cooperators. This means that there is a point at which the system consists of quite many defectors. This temporary selective advantage of defectors has long-lasting consequences now. The reason is that if there are many defectors which are identified as such, conditional cooperators will quite often defect against such defectors. But since memory is very long this will not easily be forgotten. Conditional cooperators will earn themselves a bad reputation among each other and will start defecting among themselves. Hence large memory traps the system in full defection.

Note that this effect would persist even if agents form reputations (of, say, agent $j$) conditional on the past behavior or type of $j$’s matches. Remember first that types are unobservable.\(^8\)

\(^8\)If they were observable, then it is well known that cooperation can survive.
Hence, it is impossible to know the type of j’s past matches. In addition, it is impossible to infer whether a defecting agent defects because he is a defector or a conditional cooperator who has no information about her match and "bad beliefs" about her environment. Hence in order to condition their behavior on the type of the opponent agents would need a theory about how other agents form their beliefs about their matches. In our model this would require agents to sample not only from their own and their neighbors experience about j’s behavior, but also try to find out what j knows about themselves via joint neighbors. This is not only cognitively extremely complex, but also impossible if agents have only limited information about the network.

The results are robust if we allow for heterogeneity in agent’s memory by modifying the model as follows: Assume that each agent remembers all her interactions of the last h periods where each period consists of N interactions in the population, but people are drawn to play with replacement. In this case, some agents can be chosen more often than others. This means that on average agents remember h interactions but some agents may remember more and others less (see Section 6.1 for details). Allowing for heterogeneity in this sense does not affect our results (see Section 6.2).

Our findings are also robust to the introduction of mutations in the following sense. Assume that there is a small probability that a type chosen for selection, rather than adopting the type of the individual she compares with, adopts a random type. This does not change our results qualitatively. The analogue of Fig. 2 with mutations is reported in the Appendix (see Section 6.4). The results are furthermore robust to changes of other parameters of the model and modifications of model attributes (see Appendix). We show in Section 6.2 that the results are robust even if we allow for agents being matched with others located further away in the social network.

Finally one could ask whether the results of the paper could also be obtained from a simpler version of the model. Additional simulation results illustrate that if we remove any mechanism (direct reputation, indirect reputation, or network-based meetings) from the model the long-run levels of cooperation decline dramatically or disappear completely. If only direct reputation is removed, there are certain patterns of an optimal memory length. However, they are less salient and the levels of cooperation lie well below the magnitudes in Fig. 5 (see Section 6.3).9

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9The additional simulations can be found in a longer version of the article, which can be downloaded from http://www.ivie.es/downloads/docs/wpasad/wpasad-2010-25.pdf.
5.4 Evolution of Memory

In order to check whether the limited levels of memory are optimal from the point of view of individual selection within groups, we inject into the model conditional cooperators with different working memory capacities. We choose three memory levels: (i) $h = 2$ (corresponding to full defection states in Fig. 1), (ii) $h = 7$ (optimal memory levels), and (iii) $h = 15$ (decreasing levels of cooperation). The selection process still works at the level of types, but each memory level is treated as a different type here. We observe very high cooperation levels in the long run, as long as the networks exhibit the small-world property (Watts and Strogatz, [30]). Defectors have a hard time surviving in these environments (between 2 and 7% of population), as opposed to conditional cooperators who tend to survive, irrespective of their memory level (more than 85%). There is larger heterogeneity of population structures in the long run. Populations, where at least some agents have (optimal) bounded memory, do better than populations where all agents have very long memory. The most frequent population structure is the coexistence of altruists and all types of conditional cooperators in the long run (71, 64 and 39% of the cases for $\theta = 0, .01,$ and $.05$, respectively).

Hence, in terms of individual selection there are no strong evolutionary pressures favoring any of the memory sizes. The optimal memory capacity is not evolutionary costly within groups, while from a population viewpoint bounded memory emerges clearly as being optimal. Thus, optimal memory may be an outcome of evolutionary conflicts across groups (Boyd and Richerson [3]). Again, introducing mutations into the model does not affect our conclusions (see Appendix).

5.5 Discussion

Our results provide some new viewpoints on the co-evolution of cooperation and memory or more loosely speaking on why evolution did not make us "infinitely" smart. Undoubtedly, there are other obvious reasons (such as energetic/reasoning costs) for why human memory is limited, but we show one example where long memory need not even be optimal in the absence of such costs. In line with the group selection literature [3], limited memory in our model can emerge from the conflict of two populations endowed with different memory capacity. This is enhanced by the fact that within-groups there are no evolutionary pressures on larger memories.

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10See Section 6.5 in the Appendix for details of these simulations.
to develop as our analysis makes precise. Since human memory capacity is in reality limited, our results are suggestive for future research studying how and whether human cognitive capacities, such as memory, may have co-evolved with cooperation. Tomassello [27] for example argues that human capacities such as communication have evolved jointly with cooperation.

Note that there are multiple kinds of memory and other forms of memory (such as short-term/working vs. long-term memory, episodic memory or recognition memory) may play a role for the evolution of cooperation. In the present model, we abstract from possible interactions among different memory types. While e.g. episodic memory may be important for the evolution of cooperation, it seems that working memory is essential as well. However there is little in the model that hinges on this definition of memory. The essential insight is that with larger memory there can be drawbacks to reputation based systems because an initial fitness advantage of defectors will entrap conditional cooperators in defection not only against defectors, but also among themselves. This essential insight seems to go through even if we had modelled e.g. episodic memory instead of working memory capacity. In future research, however, it could be very interesting to model how these different types of memory interplay in fostering cooperative relations.

We have also found that there is a certain memory threshold, under which large-scale cooperation does not emerge. Primatologists agree that primates have lower working memory capacity than humans (Kawai and Matsuzawa [12], Premack and Premack [21]). How much cooperation there is in other animals is an area of controversy (see e.g. the discussion in Silk [24]). Interestingly, though, some evolutionary psychologists believe that the era in which our ancestors seem to have surpassed the working memory capacity of today’s primates coincides with the rise of more complex forms of social organization around 80,000 years ago [23], which would naturally require different levels of organization and cooperation. Since longer memory provides no evolutionary advantage in terms of individual selection and is detrimental in terms of group selection, there may be good reasons why human memory is strongly limited. Of course, human memory capacity also has non-social functions and potential trade-offs have to be taken into account. There is large scope for future research in this area.
5.6 Appendix

5.6.1 Details of the simulations

In this section we provide further details regarding the simulations of the model. We describe the sequence of events happening in a run of the computer program:

1. Initialize the model
   
   (a) Generate a graph of $N$ agents: a small-world network with parameters: $\rho$ and $\theta$
   
   (b) Assign types: each agent’s initial type is uniform randomly drawn from the type space $\{A, CC, D\}$
   
   (c) Set initial belief to $2/3$ (Conditional Cooperators play ‘Cooperate’ in the first period)

2. In each period $t = 1, 2, 3...$
   
   (a) Interactions: repeat $N$ times
      
      i. draw a random agent $i$ from the set of all $N$ agents with $[\text{without}]$ replacement\(^{11}\)
      
      ii. draw an opponent $j$: a random agent among $i$’s neighbors
      
      iii. determine $j$’s reputation for $i$: as it is described in the section ’ Reputation and beliefs’
      
      iv. determine $i$’s optimal action based on $j$’s reputation
      
      v. determine $i$’s reputation for $j$
      
      vi. determine $j$’s optimal action based on $i$’s reputation
      
      vii. realize payoffs based on the two players’ action
      
      viii. assign $i$’s payoff as her fitness
      
      ix. change $i$’s memory: new information: $j$’s action
   
   (b) Count how many agents cooperated in the position of agent $i$
   
   (c) Selection: $k$ times

---

\(^{11}\)We ran 2 distinct sets of simulations. One where we draw agents with replacement and one where we draw agents without replacement. Drawing without replacement ensures that each agent remembers exactly the last $h$ interactions. This corresponds to the benchmark model in the main text. Drawing with replacement allows for heterogeneity because some agents may be drawn repeatedly and hence remember more than $h$ interactions. On average, though, agents will remember $h$ interactions also in that case. See Section 6.2.
i. draw a random agent $i$

ii. draw a random agent $j$

iii. if $j$'s fitness is higher than $i$'s fitness, $i$ adopts $j$'s type

(d) Update memory: for every agent: delete the oldest information (from period $t-h$)

(e) Update the default belief using the number of cooperating agents

In the baseline case we run the model 100 times. Figure 2 and Table 2 show the average of the steady state cooperation rate over the 100 runs.

### 5.6.2 Extensions

**Meeting strangers: Can cooperation still survive?**

In the benchmark model, people only interact with their first-order neighbors. This assumption can be too extreme, since in real life people meet more socially distant individual or even complete strangers. In such a case, the combination of limited memory and casual meeting of unknown individuals may suggest that cooperation cannot arise. We explore an alternative setup here. We assume that people can meet *anybody* in their component, being the probability to interact with a particular player proportional to the shortest "geodesic distance" that separates the two players. Geodesic distance between any two nodes is the minimum number of links that connects the two nodes.

The probability for $i$ to interact with $j$ given that $i$ has been chosen as row player is the following:

$$
P_{ij} = \begin{cases} 
    e^{-\alpha d_{ij}} & \text{if } d_{ij} > 0 \\
    \frac{e^{-\alpha d_{ij}}}{\Sigma_{k \neq i} e^{-\alpha d_{ik}}} & \text{if } d_{ij} = 0 \text{ or } d_{ij} = \infty
\end{cases}
$$

Observe that the higher $\alpha$ the more likely it is to meet neighbors and the more unlikely to meet distant individuals. Hence, $\alpha$ parametrizes the effect of the network for matching. As $\alpha \to \infty$, everybody meets exclusively her neighbors (the original model in the main text). If $\alpha = 0$ matching is completely random and cooperation does not survive. If $d_{ij} = \infty$ (agents in disconnected components), $P_{ij} = 0$.

Fig. 3 shows the average of the steady state cooperation rate over the 100 runs.\(^\text{12}\) For a

---

\(^{12}\)In the computer program we change step 2.(a).ii: we draw the opponent based on the distance using the described function.
given values of \( h \) and \( \theta \), the cooperation increases monotonically in \( \alpha \). As \( \alpha \) gets higher, agents are matched most frequently with agents from the neighborhood, which limits the number of possible opponents and facilitates the learning about their types. More importantly, the overall pattern of cooperation as a function of memory is robust to changes in the locality of matching.

Figure 5.3: Effect of locality of encounters (\( \rho = 4 \)).

**Heterogeneity**

In the benchmark model, people are drawn to play the game *without* replacement. This ensures that each agent interacts only once per period. Hence all agents remember their last \( h \) interactions. In this section, we allow for heterogeneity in the number of interactions people remember. To this aim, in each round we draw the agents *with* replacement. As a result, some people might have more and some less than \( h \) interactions in the last \( h \) periods. This creates heterogeneity with respect to the amount of information people have to assess reputation of their future opponents. Figure 4 shows that the results still hold under this specification of the model.
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5.6.3 Removing mechanisms

We also analyze whether cooperation emerges if we remove one of the model mechanisms (direct reputation, indirect reputation or network-based meetings). Fig. 5 shows the simulation results. If agents rely only on own experience ($\lambda = 1$), cooperation rates are very low for all memory values. If individuals only use the information of their neighbors ($\lambda = 0$), cooperation emerges but it is dramatically lower than in the baseline case. If we remove the matching role of the social network (maintaining both reputation mechanisms), agents are matched randomly and cooperation never emerges (non-reported in Fig. 5).

5.6.4 Mutations

We introduce the possibility of mutations into the selection process. With a probability 0.02 the randomly chosen agent’s fitness is not compared to the fitness of someone else, but she adopts a type uniform randomly.\footnote{This modifies the computer program at the point 2.(c).} This way, every behavioral type has the chance to be

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**Figure 5.4:** Cooperation rates by heterogeneous memories ($\rho = 4$). $\theta = 0$ (blue line), $\theta = 0.01$ (green), $\theta = 0.05$ (red), $\theta = 1$ (light blue). Confidence intervals are reported in Table 5.
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Figure 5.5: Cooperation rates when mechanisms are removed reintroduced to the population.

In this case, the model does not converge to absorbing states. Hence we run the model for 30000 periods and computed the average cooperation rate over the 30000 periods as the outcome of one simulation. We run 100 such simulations and take the average cooperation rate over this sample. The results are shown in Fig. 6 and Table 6 for different values of memory ($h$) and network structure parameter ($\theta$). Cooperation is slightly larger around the optimal memory levels than in the figure in the main text. This causes a starker decrease of cooperation levels as memory grows larger. As a result, the findings in the main text are more salient with mutations. Most importantly, we can conclude that the results are robust against the inclusion of mutations.

5.6.5 Evolution of memory

To see whether limited memory is optimal from an individual point of view, we introduce 5 types into the population: Altruists (A), Defectors (D), and Conditional Cooperators with levels of memories, $h = 2$ ($CC_2$), $h = 7$ ($CC_7$) and $h = 15$ ($CC_{15}$) each initially representing 20% of the population. Table 2 lists the average steady-state distributions of 100 runs of each parameter specification.
Figure 5.6: Average cooperation rates in the case of mutations for different memory values and rewiring probabilities.

Table 3 shows the results of the same exercise when there is possibility for mutation in the selection process. With a probability 0.02 the randomly chosen agent’s fitness is not compared to the fitness of someone else, but she adopts a type uniform randomly.

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Table 5.2: Evolution of memory.
## Table 5.3: Evolution of memory with mutations (2%)

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### 5.6.6 Tables: Group-Level Cooperation

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Table 4. Av. rates of cooperation (without mutation) and 95% conf. intervals
(see Fig. 2 in the main text).
### Table 5. Average rates of cooperation (without mutation) and 95% confidence intervals in the case of 'heterogeneity of memory' (See Fig. 4.)

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Table 5. Average rates of cooperation (without mutation) and 95% confidence intervals in the case of 'heterogeneity of memory' (See Fig. 4.)
### Table 6. Av. rates of cooperation with mutation (and 95% conf. intervals, see Figure 6).

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Bibliography


