Fundamentals of Physics in Engineering I

Unit 3.- WORK AND ENERGY

• Introduction

Work and energy are two of the most important concepts in physics and also in everyday life. In physics, a force performs work when it acts on an object which moves a distance and a component of the force acts along the line of motion of the object. The concept of energy is closely linked to that of work. When a system performs work on another system, energy is transferred between the two systems. There are many forms of energy. Kinetic energy is the energy an object possesses due to its motion. Potential energy is energy which results from position or configuration, such as the distance between a body and the earth. Thermal energy is the kinetic energy resulting from the random movement of atoms and molecules within a system and is closely related to temperature. One of the fundamental laws of nature is the law of conservation of energy. If the energy of a system is conserved, its total energy remains constant, but some of it may be transformed from one form to another.

• Work and power

The *work W* performed by a force **F** that acts on a body moving along a trajectory is defined by the integral:

$$W = \int_1^2 \mathbf{F} \cdot d\mathbf{r}$$

In the simple case of a constant force and distance $\Delta\mathbf{r}$ along a straight line, the work is given by the scalar product:

$$W = \mathbf{F} \cdot \Delta\mathbf{r}$$

For a force that varies along one dimension (for example, along the *X* axis):

$$W = \int_{x_1}^{x_2} F_x(x) \cdot dx$$

The unit of work in the SI is the Joule (J).
The *power P* is the speed at which a force performs work:

$$P = \frac{dW}{dt}$$

The power of a force **F** performing work on an object moving at a velocity **v** is given by:

$$\mathbf{P} = \mathbf{F} \cdot \mathbf{v}$$

In the SI, power is measured in watts (W).

• Kinetic energy. Kinetic energy theorem

The *kinetic energy* E_c of a body of mass *m* moving at a velocity v is:

$$E_c = \frac{1}{2} m v^2$$

Kinetic energy is the energy resulting from motion. The *kinetic energy theorem* states that the work done by a force acting on a body is equal to the change in kinetic energy of the body:

$$W = \frac{1}{2} m v_{final}^2 - \frac{1}{2} m v_{inicial}^2 = E_{c,final} - E_{c,inicial}$$

that is:

$$W = \Delta E_c$$

• Conservative forces and potential energy

A force is *conservative* if the work done in moving a particle around a closed trajectory is zero. In addition the work done by a conservative force is independent of the path taken and depends solely on the initial and final points.
The work done by the weight of a body near the earth's surface is:

$$W = - mg(y_2 - y_1)$$

and it is independent of the trajectory connecting the initial and final points. Such a force is conservative.
The *potential energy* E_p depends solely on position. Two examples of potential energy are gravitational potential energy:

$$E_p = mgy$$

and elastic potential energy due to compression or stretching of a spring:

$$E_p = \frac{1}{2} k x^2$$

In the case of a conservative force, the work *W* and potential energy E_p are related by the equation:

$$W = - \Delta E_p$$

and the force **F** and potential energy E_p by the equation:

$$\mathbf{F} = -\mathbf{grad} E_p = \nabla E_p$$

which in the one-dimensional case may be written as:

$$F_x = -\frac{dE_p}{dx}$$

The motion of an object may be represented by a potential energy graph. The points of equilibrium may be identified on this graph.

• Conservation of mechanical energy

The sum of the kinetic and potential energy of a system is called *mechanical energy E:*

$$E = E_c + E_p$$

If there are no external forces acting on the system and all the internal forces are conservative, the total mechanical energy of the system remains constant:

$$E = E_c + E_p = cte.$$

that is, between two states, initial 1 and final 2:

$$E_{c,1} + E_{p,1} = E_{c,2} + E_{p,2}$$

The total energy of the system E_{sist} is the sum of the different types of energy. One way of transferring the energy (absorbed or ceded) of a system is to exchange work with the surroundings. If this is the only source of transferred energy, the law of conservation of energy is written as:

$$W_{ext} = \Delta E_{sist}$$

W_{ext} is the work done on the system by external forces and ΔE_{sist} is the variation in total energy of the system. This is *the work-energy theorem.*