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Application of Mathieu functions for the study of non-slanted reflection gratings

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Abstract

In this work we present an analysis of non-slanted reflection gratings by using exact solutions of the second order differential equation, derived from Maxwell equations, in terms of Mathieu functions. The results obtained by using this method will be compared to those obtained by using the well known Kogelnik's Coupled Wave Theory which predicts with great a ccuracy the response of the eff ciency of the zeros and f rst order for volume phase gratings, for both ref ection and transmission gratings.

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Key words: diffraction grating, Mathieu equation, volume holograms

1. Introduction

The study of the interaction of electromagnetic radiation with diffractive elements has received much attention in the literature [1-7]. In particular, several theoretical models have been proposed to accurately describe the behaviour of diffraction gratings of different kind. The attention posed on these structures is in part due to the fact that a sinusoidal diffraction grating is the simplest periodic structure that can be recorded on a photosensitive material. Therefore the basic problem in volume holography theory is to describe accurately the properties of this kind of structures. A usual way to calculate the efficiencies of the different orders that propagate in the volume grating is to solve Maxwell equations for the case of an incident plane wave on a medium where the relative dielectric permittivity varies. Although the idea seems clear and precise, in the literature there are a great number of models that allow solving the problem.

One of the most predictive and popular theories to calculate the efficiency of the orders that propagate inside a diffraction grating is the Kogelnik's Coupled Wave Theory [1]. This theory has the advantage over other theories in that, in spite of being mathematically simple, it predicts very accurately the response of the efficiency of the zero and first order for volume phase gratings. Nonetheless, the accuracy decreases when either the thickness is low or when over-modulated patterns (high refractive index modulations) are recorded in the hologram. In these cases, the coupled wave theory (CW) allowing for more than two orders or the rigorous coupled wave theory (RCW) [4-5] which doesn't disregard second derivatives in the coupled wave equations as does CW, are needed.

Although exact predictions can be obtained by using the RCW it is still interesting to work with analytical expressions in order to calculate the efficiency of the different orders that propagate inside the hologram. Analytical expressions give a deeper understanding of the physical processes than numerical solutions do. In addition, by direct inspection of the analytical expressions a clearer interpretation of how the different parameters influence in the efficiency of the different orders is got. In this work the efficiencies of the zero and first order are obtained by solving the second order differential equation, from Maxwell equations, applied to a non-slanted reflection in terms of Mathieu functions. The results obtained by using this method will be compared to those obtained by Kogelnik's coupled wave theory showing good agreement.

2. Theory

Consider a plane electromagnetic wave incident onto a periodic non-magnetic medium, which dielectric constant varies in form:

$$\varepsilon_r = \varepsilon_{r0} + \varepsilon_{r1}\cos(Kz) \tag{1}$$

The treatment is done only for TE polarization, but can be extended to TM polarization. In this case the function E (z) for the electric field inside the medium verifies the following differential equation:

$$\frac{d^2E}{dz^2} + \left[k_0^{\ 2}\left(\varepsilon_{r0} + \varepsilon_{r1}\cos(Kz)\right) - K_x^{\ 2}\right]E = 0 \tag{2}$$

Where:

$$k_0 = \frac{2\pi}{\lambda} \tag{3}$$

Being λ the wavelength in vacuum.

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If θ_1 is the angle of incidence and θ_2 is the angle between the wave vector and the normal to the substrate of refractive index n_2, then the following parameters can be defined:

$$K_x = n_1(\omega/c)\sin\theta_1 = n_2(\omega/c)\sin\theta_2 \tag{4}$$

$$q_1 = n_1(\omega/c)\cos\theta_1 \tag{5}$$

$$q_2 = n_2(\omega/c)\cos\theta_2 \tag{6}$$

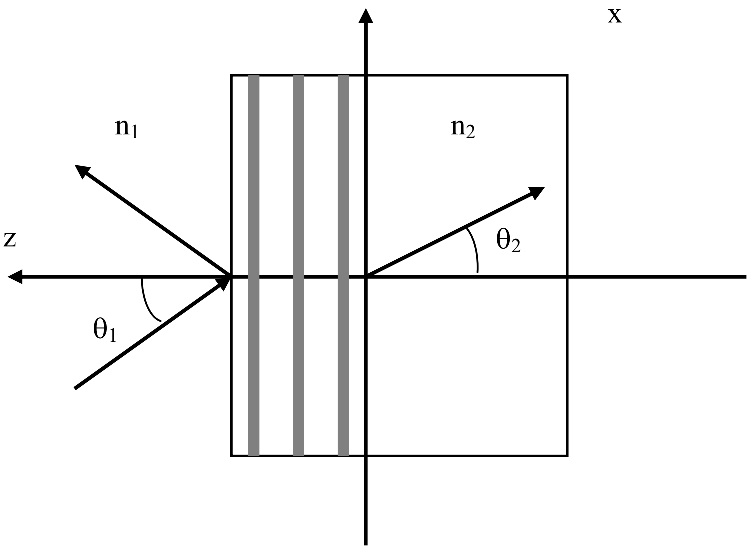

Figure 1.- Nonslanted reflection grating

The electric field in the first medium can be expressed as the superposition of an incident wave and a reflected wave of the form:

$$E^I(z) = \exp(jq_1 z) + r_s \exp(-jq_1 z) \tag{7}$$

While in the second medium only the transmitted wave exist:

$$E^{II}(z) = t_s \exp(jq_1 z) \tag{8}$$

Now suppose that f(z) is a so lution of the differential equation with a unit amplitude transmitted wave:

$$E^{II}(z) = \exp(jq_1 z) \tag{9}$$

with initial conditions:

$$E(0) = 1 \tag{10}$$

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$$\frac{dE}{dz}(0) = jq_2 \tag{11}$$

The boundary conditions in z = d imply:

$$E(d) = \exp(jq_1 d) + r_s \exp(-jq_1 d) \tag{12}$$

$$\frac{dE}{dz}(d) = jq_1 \exp(jq_1 d) - jq_1 r_s \exp(-jq_1 d) \tag{13}$$

Given the linearity of Maxwell's equations we have:

$$t_s f(d) = \exp(jq_1 d) + r_s \exp(-jq_1 d) \tag{14}$$

$$t_s \frac{df}{dz}(d) = jq_1 \exp(jq_1 d) - jq_1 r_s \exp(-jq_1 d) \tag{15}$$

From equations (14) and (15) we can obtain the amplitudes of the reflected and transmitted waves based on the solution f(z) with initial conditions (10) and (11):

$$t_s = \frac{2q_1 \exp(jq_1 d)}{-j\frac{df}{dz}(d) + q_1 f(d)} \tag{16}$$

$$r_s = \frac{\frac{df}{dz}(d) - jq_1 f(d)}{\frac{df}{dz}(d) + jq_1 f(d)} \exp(2jq_1 d) \tag{17}$$

It is now necessary to obtain a solution of the differential equation with initial conditions (10) and (11).

In this case the differential equation can be solved in terms of Mathieu functions. If we call

$$a = \frac{4(-K_x{}^2 + k_0{}^2 \varepsilon_{r0})}{K^2} \tag{18}$$

and

$$q = \frac{-2k_0{}^2 \varepsilon_{r1}}{K^2} \tag{19}$$

The function f(z), solution of the differential equation (2) with initial conditions (10-11) has the form:

$$f(z) = \frac{sm(a,q,Kz/2)(2jq_2 cm(a,q,0) - K \cdot cmp(a,q,0))}{K(cmp(a,q,0)sm(a,q,0) - cm(a,q,0))} + \\ + \frac{cm(a,q,Kz/2)(-2jq_2 sm(a,q,0) - K \cdot smp(a,q,0))}{K(cmp(a,q,0)sm(a,q,0) - cm(a,q,0))}$$

And its derivative:

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$$f^{'}(z) = \frac{cmp(a,q,Kz/2)(-2jq_2 sm(a,q,0) + K \cdot smp(a,q,0))}{2cmp(a,q,0)sm(a,q,0) - 2smp(a,q,0)cm(a,q,0)} + \\ + \frac{smp(a,q,Kz/2)(2jq_2 sm(a,q,0) - K \cdot cmp(a,q,0))}{2cmp(a,q,0)sm(a,q,0) - 2smp(a,q,0)cm(a,q,0)}$$

Where cm (a , q, z) is the even Mathieu function sm (a, q, z) the odd Mathieu function, cmp (a, q, z) and smp (a, q, z) the corresponding derivatives.

3. Results and discussion

To validate the theoretical model previously developed we will now conduct a comparison between the results obtained using the m odel (classical differential theory, TDC) with those obtained by the coupled wave theory of Kogelnik (TK). A simulation for a non-slanted reflection grating with a grating period of 0.22 μm is presented, the average refractive index was supposed to be n_0 = 1.63 and an index modulation of 0,015, the incident wavelength was assumed to be of 633 nm. Figure 2 shows the diffraction efficiency as a function of the angle for a grating of thickness d = 22 μm. As shown in the figure the degree of agreement between the two theories is quite good indicating the validity of the model.

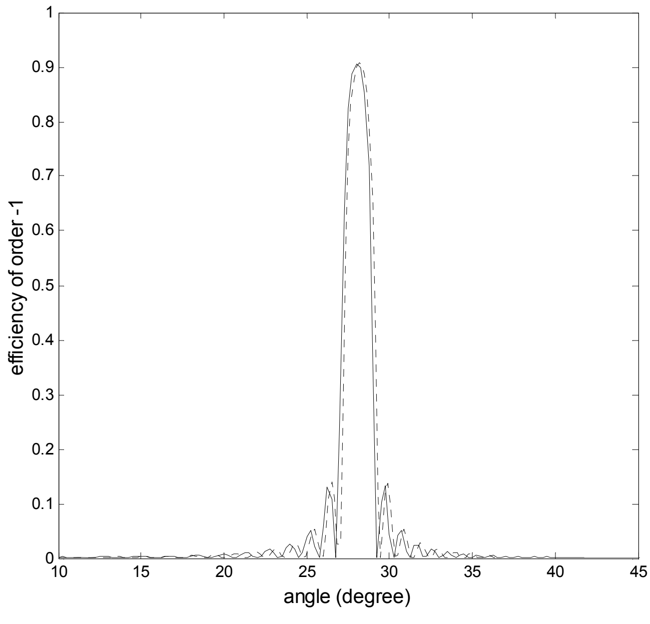

Figure 2.- Comparison of Kogelnik's Coupled Wave Theory with the method proposed in this work. Dotted line: Kogelnik's Theory; Continuous line: method of this work.

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4. Conclusions

A solution of the second order differential equation obtained from Maxwell equations for TE describing a non-slanted reflection grating in terms of Mathieu functions is presented. The model is rigorous in the sense that no approximations are made. The results obtained by this method were compared to those obtained by using Koglenik's coupled wave theory showing a good agreement between both simulations, and thus validating the model proposed.

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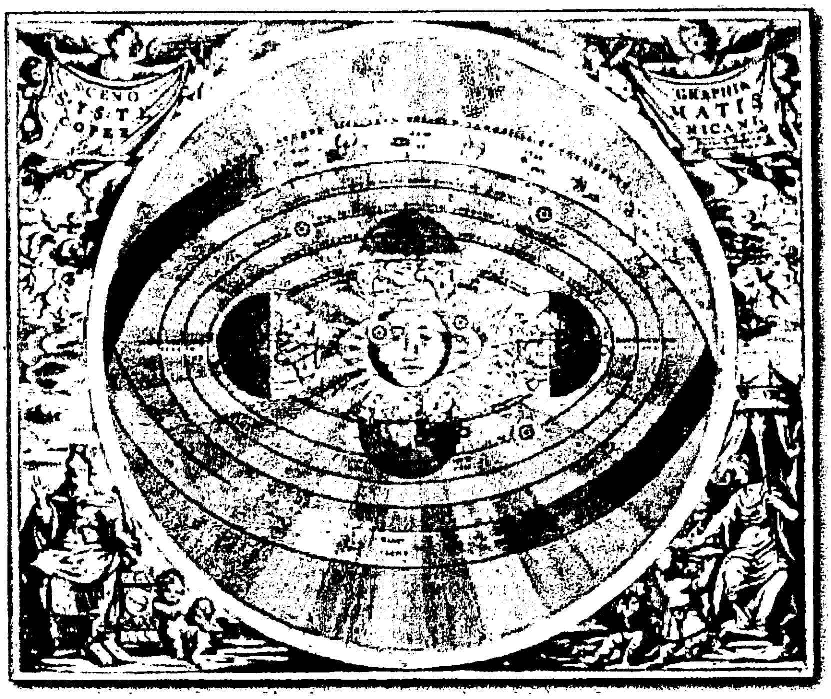

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