1. Find the eigenvalues and eigenspaces of the following matrices and determine if they are diagonalizable:
   
   (a) \[
   \begin{pmatrix}
   4 & 5 \\
   -1 & -2
   \end{pmatrix}
   \]
   
   (b) \[
   \begin{pmatrix}
   1 & 0 \\
   1 & -2
   \end{pmatrix}
   \]

2. Let \( A \) be invertible. Show that \( \lambda \) is an eigenvalue of \( T \) if and only if \( \lambda \neq 0 \) and \( \lambda^{-1} \) is an eigenvalue of \( A^{-1} \).

3. Find the minimal polynomials of
   
   \[A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\]
   
   \[B = \begin{pmatrix} 2 & 0 \\ 3 & -1 \end{pmatrix}\]
   
   \[C = \begin{pmatrix} 0 & 1 & 3 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}\]

4. Find an \( n \times n \) matrix with fundamental polynomial \( x^2 \).

5. Prove that \( A \) is invertible if and only if its fundamental polynomial has a nonzero constant term.

6. Let \( A \in \mathbb{R}^{n \times n} \) have \( n \) distinct eigenvalues. Show that \( A \) is diagonalizable.

7. Find the possible eigenvalues of a matrix \( A \) such that \( A^2 = A \).

8. Show that a \( 2 \times 2 \) real symmetric matrix is diagonalizable.