Alternative approach to fit irregular corneas

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Background

Scheimpflug cameras and topographers based in Placido rings permit accurate estimation of corneal surfaces
Background

Corneal height data ➔ Model ➔ Wavefront analysis

- Zonal approach.
  - B-Splines
    - Well suited for fitting complex-shaped surfaces
    - Not related with aberrations

- Modal approach.
  - Zernike polynomials
    - Direct relation with Seidel aberrations
    - Not precise when describing highly irregular corneas
COMBINATION
ZONAL + MODAL approaches

Zonal Zernike fitting of corneal height data

Zernike coefficients computed in overlapping local areas

- Diminishing the influence of smooth areas over irregular zones and vice versa.
- Limiting the influence of the peripheral irregularities over the central corneal area, thus giving accurate reconstruction of the central optical zone.
Modal approach. Zernike polynomials

\[ W(x_u, y_v) \approx \sum_{j=0}^{p-1} c_j Z_j(x_u, y_v) \]

Least-squares method

\[ C = \left( Z^T Z \right)^{-1} Z^T W \]
Least-squares method

$$W(x_u, y_v) \approx \sum_{j=0}^{p-1} c_j Z_j(x_u, y_v)$$

$$C = \left( Z^T Z \right)^{-1} Z^T W$$

Modal approach. Zernike polynomials

$$Z = \begin{bmatrix} \sum_{j=0}^{p-1} Z_j(x_1, y_1) & \ldots & \sum_{j=0}^{p-1} Z_j(x_n, y_1) \\ \ldots & \ldots & \ldots & \ldots \\ \sum_{j=0}^{p-1} Z_j(x_1, y_n) & \ldots & \sum_{j=0}^{p-1} Z_j(x_n, y_n) \end{bmatrix}$$
Zonal Zernike fitting

Auxiliary $a \times b$ matrices of size $N \times N$, unmasking $M \times M$ zone

$$W_{a,b}(u,v) \simeq \begin{cases} \sum_{j=0}^{p-1} c_{j}^{(a,b)} Z_{j} \left( \rho_{(u,v)}, \theta_{(u,v)} \right); \\ 0; \end{cases}$$
Zonal Zernike fitting

Modal

Modal+Zonal

(NxN)

(MxM)

One point of the surface may belong to different local regions
Zonal Zernike fitting

Reconstructed surface: Mean at each point

\[ L(u, v) = \frac{\sum_{a=1}^{N-(M-1)} \sum_{b=1}^{N-(M-1)} W_{a,b}(u, v)}{\sum_{a=1}^{N-(M-1)} \sum_{b=1}^{N-(M-1)} O_{a,b}(u, v)} \]

Points in the central zone are evaluated \( M^2 \) times

\( M = 21 \text{ px} \Rightarrow 441 \text{ times} \)
Zonal Zernike fitting

Number of elements in the central zone

\[ f_N(M) = M^2 \left[ N - 2(M - 1) \right]^2 \]

From the derivative:

\[ M_{opt} = \text{round} \left( \frac{N+2}{4} \right) \]
Results

- Height data analysis of surfaces.
  - Irregular surface
  - Keratoconus from Pentacam

Root mean square deviation (RMSD)

\[ RMSD = \sqrt{\frac{\sum_{i=1}^{N'} (w_i - g_i)^2}{N'}} \]

- Validate \( M_{\text{opt}} \). Two masks of different sizes
Irregular surface: Sphere+Franke’s function

\[fr(x, y) = \frac{3}{4}\exp\left[\frac{-\left((9x - 2)^2 + (9y - 2)^2\right)}{4}\right] + \frac{3}{4}\exp\left[\frac{-\left((9x + 1)^2 + (9y + 1)^2\right)}{49} + \frac{(9y + 1)^2}{10}\right] + \frac{1}{2}\exp\left[\frac{-\left((9x - 7)^2 + (9y - 3)^2\right)}{4}\right] + \frac{1}{5}\exp\left[\frac{-\left((9x - 4)^2 + (9y - 7)^2\right)}{10}\right]\]
RMSD Sphere+Franke’s function

Pupil diameter = 4 mm  \( N=41 \Rightarrow M_{\text{opt}}=11 \)
Differences Sphere+Franke’s function

**Modal**: difference with 120 Zernike polynomials

**Zonal mask 21 px**: difference with 96 Zernike polynomials

**Zonal mask 11 px**: difference with 43 Zernike polynomials
Real irregular surface: Keratoconus

Keratoconus height data - sphere
RMSD keratoconus

Pupil diameter = 4 mm  \( N=41 \Rightarrow M_{\text{opt}}=11 \)
Differences keratoconus

Modal: difference with 178 Zernike polynomials

Zonal mask 21 px: difference with 196 Zernike polynomials

Zonal mask 11 px: difference with 217 Zernike polynomials
Conclusions

- Better results than modal fit for low order Zernike polynomials.
- The central surface part is better evaluated than the outer parts, since calculation is more intensive in this zone and not affected by peripheral irregularities.
- Diminishes the influence of smooth areas over irregular zones and vice versa.
http://web.ua.es/es/ocivis/