Water distribution networks optimization considering uncertainties in the demand nodes

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Abstract

The design of water distribution networks (WDN) can be formulated as an optimization problem. The objective function, normally, is the network cost, given by the installation cost, which depends on the pipe diameters and by the operation cost, given by the pumping costs associated to the network, which depends on the hydraulic pumps necessary in the system. The water demand can be variable in the network nodes and this variability can be modeled by a finite set of scenarios generated by a normal distribution. In the present paper a disjunctive Mixed Integer Nonlinear Programming (MINLP) formulation optimization problem is proposed to model the design of WDN under uncertainties in the nodes demand. Flow directions are considered unknown and a deterministic approach is used to solve the problem in three steps. Firstly, the problem is solved considering only a nominal value to each uncertain parameter. In the second step, the problem is solved for all the scenarios, being the scenario independent variables fixed to the solution achieved in the first step, which is a deterministic solution. Finally, all the scenarios are solved without fixing any variable value, in a stochastic approach. Two case studies were used to test the model applicability and global optimization techniques were used to solve the problem. Results show that the stochastic solution can lead to optimal solutions for robust and flexible WDN, able to work under distinct conditions, considering the nodes demand uncertainties.

Introduction

Water supply systems are fundamental in industrial processes and urban centers. These systems involve the water catchment, the distribution network and the pumping station. The water distribution network (WDN) is composed by the reservoirs of treated water, piping connecting the reservoir to the demand nodes, hydraulic pumps and, most of the time, loops among the demand nodes. These components are responsible by the most important part of the network cost.

The design of WDN is an important research field and the number of papers published in this subject is increasing. The synthesis of WDN can be formulated as an optimization problem, involving, mainly, the minimization of the network installation cost, which depends on the pipeline diameters. Normally, the flow directions are considered known in the demand nodes loop formed in the network. The optimization problem can have a nonlinear programming (NLP) or a mixed integer nonlinear programming (MINLP) formulation and stochastic and deterministic approaches have been used to solve the problem.

In some situations, it can exist variations in the water nodes demand in distinct periods of operation and these kinds of uncertainties need to be considered in the problem formulation. In the present paper an optimization model is proposed for the synthesis of WDN considering uncertainties in the nodes demand. The problem has an MINLP formulation and the flow directions are considered unknown and are treated as optimization variables. The objective function is the WDN total cost, considering the installation costs, which depends on the pipe diameters and the annualized pumping cost, due to the use of hydraulic pumps in the network. Disjunctive programming is used to reformulate the problem and linearization techniques are used to avoid problems with the nonlinear equations of Darcy-Weisbach and Hazen-Williams, used in the hydraulic calculations. No additional software or hydraulic simulator is used, once all the velocity and pressure drop calculations are included in the model. The water demand variability is modeled as a set of finite scenarios generated by a normal distribution in Excel. GAMS environment is used to implement the developed model, in three steps. First, only a unique nominal value for each uncertain parameter is used. In the sequence, for all scenarios, the independent variables are fixed to the solution achieved in the first step. It corresponds to a deterministic approach. Finally, the problem is solved for all scenarios, without fixing variables (stochastic solution). Two case studies are used to test the developed model and global optimization techniques are used to achieve the problem solution.

Literature review

The design of WDN can be formulated as an optimization problem. The WDN installation cost is the most used objective function. Normally, the problem is formulated as the minimization of the installation cost, related to the tube diameter, subjected to a set of constraints involving mass balances in the demand nodes, energy balances if network loops are present, pressure and velocity limits. It is considered a set of commercial tubes with proper costs and rugosity coefficients to be chosen, aiming to minimize the total WDN cost. It is usual to solve the hydraulic equations using additional software. EPANET (Rossman, 2000) is the most used hydraulic simulator. Distinct formulations have been used, as Linear Programming (LP), Nonlinear Programming (NLP), Mixed Integer Linear Programming (MILP) and Mixed Integer Nonlinear Programming (MINLP). Optimization models with MINLP formulations are more representative for real problems and, recently, the use of this kind of optimization models by research groups have been increased. In general, global optimality cannot be ensured due to the nonlinear and nonconvex behavior of the model.

In a recent paper, Mala-Jetmarova et al. (2018) presented a detailed review of types of WDN optimization problems and methods used to solve the problem. Designs of new WDNs, expansion and rehabilitation of existing water distribution systems, strengthening, design timing, parameters uncertainty, water quality and operational considerations were reviewed. As pointed by the authors, different deterministic and stochastic approaches have been used to solve the optimization problem. Stochastic approaches are used in large scale problems, where deterministic approaches normally fail. Some of important methods used are Particle Swarm Optimization (PSO), in Ezzeldin et al. (2014), Surco et al. (2017) and Surco et al. (2021), Genetic Algorithms (GA), in Savic and Walters (1997) and Kadu et al. (2008), Harmony Search (HS), in Geem (2006), Ant Colony Optimization (ACO), in Zecchin et al. (2006), Simulated Annealing (SA), in Cunha and Sousa (1999) and Honey Bee Mating Optimization (HBMO), in Mohan and Babu (2009).

There are less papers focusing on deterministic approaches to solve the WDN optimization problem. It is because the intrinsic limitations of deterministic solvers in getting trapp in local optima solutions in nonlinear problems and in the difficulties in using global optimality methods in large scale problems. However, important advances have been published in this important research field. Bragalli et al (2012) proposed, to the optimization of WDN with fixed topologies, a nonconvex continuous Non-Linear Programming (NLP) relaxation and an MINLP search approach. Raghunathan (2013) used linearization techniques and global optimization considering tailored cuts in MINLP formulation problems. D'Ambrosio et al (2015) presented a complete review of Mathematical Programming approaches in the optimization of WDN considering the notion of the network design and the network operation. Caballero and Ravagnani (2019) proposed an MINLP model considering unknown flow directions in the network loops and used global optimization techniques to solve the problem. Cassiolato et al. (2019) used Generalized Disjunctive Programming to reformulate an MINLP model, developed by Surco et al (2017), using a Big M approach. In Cassiolato et al. (2020), a hull reformulation was proposed in the problem and the model was solved, reducing the relaxation gap and improving the overall numerical performance. As an extension, Cassiolato et al. (2023) a Mixed Integer Non-Linear Programming (MINLP) model was developed to the synthesis of WDN considering the minimization of the WDN total cost, given by the sum of installation and operational costs. Cassiolato et al. (2022a) considered in the model unknown flow directions and SBB and BARON solvers were used to achieve the problem solution. Cassiolato et al. (2022b) considered installation and energy costs, with unknown flow directions.

Balekelayi and Tesfamariam (2017) reviewed three approaches to the WDN synthesis, the use of deterministic and non-gradient methods and real time optimization and compared some population-based algorithms to solve the problem for a case study.

As mentioned before, the majority of the published papers use non-deterministic approaches and consider fixed and known flow directions and a hydraulic simulator to solve the velocities and pressures calculation. In real WDN, variations in nominal values can occur and these variations can influence in the optimum network operation conditions, causing an unappropriated behavior. So, the evaluation of the uncertainties in distinct operation periods is a recent and important research field. The uncertainties in the demand nodes or in the tubes rugosity due to the use in long times are problems that need to be considered in the final stage of the WDN design. Branisavljevimc et al. (2009) used a Genetic Algorithm to find optimal solutions considering uncertainties in the water nodal demand by a Monte Carlo simulation. Sivakumar et al. (2015) studied the uncertainties in the tube rugosity and evaluated the tube flowrate and the different pressures between two adjacent nodes. Dongre and Gupta (2017) considered uncertainties in the water demand and in the tubes rugosity using fuzzy logic. Geranmehr et al. (2019) also used a fuzzy model to evaluate uncertainties in the nodes demand in the reservoir and in the rugosity coefficient. Calvo et al. (2018) considered non-correlated functions of log-normal probability distributions to the management of valves. Salcedo-Díaz et al. (2020) modeled the uncertainties in the nodes demand by a set of correlated scenarios generated by a Monte Carlo simulation, assuming a log-normal probability distribution.

In the present paper, the existence of uncertainties in the nodes demand in the synthesis of WDN is considered. The optimization model has an MINLP formulation and disjunctive programming is used to deal with integer variables relating the tubes diameter, cost and rugosity coefficient. The objective function is the total WDN installation cost and the constraint are mass balances in the demand nodes, energy balances in the network loops and velocity and pressure limits. No additional software is required for the hydraulic calculations and the model was coded in GAMS and a deterministic approach is used to solve the problem. The uncertainties in the nodes demand are thought as a set of correlated scenarios generated by a Monte Carlo simulation, assuming a log-normal probability distribution. Two case studies were used to test the applicability of the developed model.

Optimization model

The system is modeled as a set of reservoirs and node demands, described by their elevation and expected demand values, and a set of pipes with initial and final nodes, pipe length and chosen from a set of commercial diameters. To each diameter is associated a cost per length and a specific rugosity coefficient. Between the demand nodes it can exist closed loops. For each demand node there is a minimum pressure limit and the velocity in the tubes is between an upper and a lower bounded.

The design of the WDN is thought as an optimization problem with MINLP formulation, in which the objective function to be minimized is the network installation cost, subject to a set of algebraic constraints composed by a mass balance in each node, pressure difference between two adjacent nodes, considering the existence of loops, the equation for the volumetric flow rate in each pipe and Hazen-Williams equation for the pressure loss calculation, forming a nonlinear equations system. Complete the constraints set the inequalities for the velocity inside the tubes and pressure in the demand nodes limits. Disjunctive programming is used to determine the optimal WDN topology, with the attribution of binary variables and linear equations.

The uncertainties in the water demand nodes are modeled using a finite set of scenarios sampling from a probability distribution. The problem must be solved in three stages. In the first stage the problem is solved without considering the existence of uncertainties. The decisions taken in the first stage before considering the uncertainties are given for the design variables. In the second stage, the decisions taken after considering the uncertainties allow to calculate operational variables. At last, in the third stage, the decisions are given by the design and operational variables.

The indexes, sets, variables and parameters are described as:

Indexes	
i, j	Demand node
K	Available diameter
S	Scenario
Sets	
D	Available commercial diameters (*k*)
$\mathrm{E}_{i,j}$	There exist a pipe between node *i* and node *j* (*i-j*)
N	Demand nodes (*i, j*)
S	Scenarios (*s*)
Parameters	
$CostD\left(D_k\right)$	Cost per length of pipe with diameter D_k [$/m]
$d_{i,s}$	Water demand for node *i* in the scenario *s* [volume/time]
D_k	Available commercial diameter *k* [m]
e_1	Annual interest ratio [%]
$Ep_{i,j}^{min}$ and $Ep_{i,j}^{max}$	Minimum and maximum values for the pump energy in pipe *i-j* [m]
FAI	Annualization factor for the installation cost [year^{-1}]
h_i	Node *i* elevation [m]
$L_{i,j}$	Pipe *i-j* length [m]
n_a	Design life time [year]
P_i^{min}	Minimum pressure in node *i* [m]
$prob_s$	Probability of occurrence of scenario *s* [%]
$q_{i,j}^{min}$ and $q_{i,j}^{max}$	Minimum and maximum values for the volumetric flowrate in pipe *i-j* [m^3/s]
R_k	Rugosity coefficient in pipe with diameter D_k [non-dimensional]
$v_{i,j}^{min}$ and $v_{i,j}^{max}$	Minimum and maximum values for the velocity in pipe *i-j* [m/s]
α	Hazen-Williams numerical conversion factor [depends on the system being used]
β e γ	Hazen-Williams equation coefficients [non-dimensional]

Indexes	
$\Delta P_{i,j}^{min}$ and $\Delta P_{i,j}^{max}$	Minimum and maximum values for the pressure loss in the pipe *i-j* [m]
Boolean variables	
$W_{i,j}^{1}$	True, if water flows from node *i* to node *j* or false, on the contrary
$W_{i,j}^{2}$	True, if water flows from node *j* to node *i* or false, on the contrary
$Y_{i,j,k}$	True, if in the pipe *i-j* diameter D_k is selected or false, on the contrary
Binary variables	
$w_{i,j}^{1}$	1, if water flows from node *i* to node *j* or 0, on the contrary
$w_{i,j}^{2}$	1, if water flows from node *j* to node *i* or 0, on the contrary
$y_{i,j,k}$	1, if in the pipe *i-j* diameter D_k is selected or false, on the contrary
Variables	
$Cp_{i,j,s}$	Pump in the pipe *i-j* annualized operational cost in scenario *s* [$/year]
$Cost_{i,j}$	Pipe *i-j* cost [$]
$Diam_{i,j}$	Pipe *i-j* diameter [m]
$E_{i,j,s}^{pow}$	Pump energy in pipe *i-j* in scenario *s* [kW]
$Ep_{i,j,s}$	Pipe *i-j* pump in scenario *s* [m]
$Ep_{i,j,s}^{1}\left(Ep_{i,j,s}^{2}\right)$	Equal to $Ep_{i,j,s}$ if water flows from node *i* (*j*) to node *j* (*i*) in scenario *s*
$expTAC$	Expected total annual cost [$/year]
$P_{i,s}$	Pressure in node *i* in scenario *s* [m]
$q_{i,j,s}$	Volumetric flowrate in pipe *i-j* in scenario *s* [m^3/s]
$q_{i,j,s}^{1}\left(q_{i,j,s}^{2}\right)$	Equal to $q_{i,j,s}$ if water flows from node *i* (*j*) to node *j* (*i*) in scenario *s*
$\bar{q}_{i,j,s}$	Logarithm of $q_{i,j,s}$ in pipe *i-j* in scenario *s*
$Rug_{i,j}$	Rugosity coefficient in pipe *i-j* [nondimensional]
TAC_s	Total annual cost in scenario *s* [$/year]

Indexes	
$v_{i,j,s}$	Water velocity in pipe *i-j* [m/s]
$v^1_{i,j,s}\left(v^2_{i,j,s}\right)$	Equal to $v_{i,j,s}$ if water flows from node *i* (*j*) to node *j* (*i*) in scenario *s*
$\overline{v}_{i,j,s}$	Logarithm of $v_{i,j,s}$ in pipe *i-j* in scenario *s*
$\Delta P_{i,j,s}$	Pressure loss in pipe *i-j* in scenario *s* [m]
$\Delta P^1_{i,j,s}\left(\Delta P^2_{i,j,s}\right)$	Equal to $\Delta P_{i,j,s}$ if water flows from node *i* (*j*) to node *j* (*i*) in scenario *s*
$\Delta \overline{P}_{i,j,s}$	Logarithm of $\Delta P_{i,j,s}$ in pipe *i-j* in scenario *s*

The WDN is evaluated by its total annual cost (TAC), given by the annual installation cost plus the annual pump energy cost. For each scenario $s \in \mathrm{S}$, a value for TAC_s is calculated and, to evaluate the performance of the WDN under uncertainties in a unique metric, the expected value for TAC is minimized, given by:

$$E[TAC] = \sum_{s \in \mathrm{S}} prob_s \cdot TAC_s$$

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In this equation $prob_s$ is the inverse of the number of generated scenarios, being used the same probability of occurrence for all scenarios.

The model constraints are the algebraic equations and inequalities that must be solved in each scenario, which constitute a nonlinear system.

Mass balance in each demand node:

$$\sum_{j \in \mathrm{E}_{j,i}} \left(q^1_{j,i,s} - q^2_{j,i,s}\right) - \sum_{j \in \mathrm{E}_{i,j}} \left(q^1_{i,j,s} - q^2_{i,j,s}\right) = d_{i,s}, \forall i \in \mathrm{N} \text{and} s \in \mathrm{S}$$

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Energy balance in the network loops:

$$P_{i,s} + h_i + Ep^1_{i,j,s} - Ep^2_{i,j,s} = P_{j,s} + h_j + \Delta P^1_{i,j,s} - \Delta P^2_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$$

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Being $P_i^{min} \leq P_{i,s}$, for all $i \in \mathrm{N}$ and $s \in \mathrm{S}$.

The choice of the pipe diameter depends on its cost, rugosity coefficient, volumetric flowrate and pressure loss, given by the Hazzen-Williams equation. Logarithms can be used to linearize the last two equations. The exclusive disjunction can be thought:

$$\underset{k \in \bar{D}}{\vee} \left[\begin{array}{c} Y_{i,j,k} \\ Diam_{i,j} = D_k \\ Cost_{i,j} = L_{i,j} CostD\left(D_k\right) \\ Rug_{i,j} = R_k \\ \bar{v}_{i,j,s} = \bar{q}_{i,j,s} - \ln\left(\frac{\pi}{4} D_k^2\right) \\ \Delta\bar{P}_{i,j,s} = \ln\left(\alpha L_{i,j}\right) + \beta \bar{q}_{i,j,s} - \ln\left(R_k^{\beta} D_k^{\gamma}\right) \end{array} \right], \forall i,j \in E_{i,j} \text{ and } s \in S$$

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This disjunction can be written using a convex hull reformulation (Grossmann and Lee, 2003). The binary variable $y_{i,j,k}$ associated to the pipe *i-j* with diameter D_k, for all $k \in D$, is equal to 1 if in the pipe *i-j* the diameter D_k is selected and 0, on the contrary. In this way:

$\boldsymbol{Diam_{i,j} = \sum_{k \in D} D_k y_{i,j,k}, \forall i,j \in E_{i,j}}$	**(5)**
$Cost_{i,j} = \sum_{k \in D} L_{i,j} CostD\left(D_k\right) y_{i,j,k}, \forall i,j \in E_{i,j}$	(6)
$Rug_{i,j} = \sum_{k \in D} R_k y_{i,j,k}, \forall i,j \in E_{i,j}$	(7)
$\bar{v}_{i,j,s} = \bar{q}_{i,j,s} - \sum_{k \in D} \ln\left(\frac{\pi}{4} D_k^2\right) y_{i,j,k}, \forall i,j \in E_{i,j} \text{ and } s \in S$	(8)
$\Delta\bar{P}_{i,j,s} = \ln\left(\alpha L_{i,j}\right) + \beta \bar{q}_{i,j,s} - \sum_{k \in D} \ln\left(R_k^{\beta} D_k^{\gamma}\right) y_{i,j,k}, \forall i,j \in E_{i,j} \text{ and } s \in S$	(9)
$\sum_{k \in D} y_{i,j,k} = 1, \forall i,j \in E_{i,j}$	(10)

The original variables can be found by exponentiation:

$e^{\bar{v}_{i,j,s}} = v_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} e s \in \mathrm{S}$	(11)
$e^{\bar{q}_{i,j,s}} = q_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} e s \in \mathrm{S}$	(12)
$e^{\Delta \bar{P}_{i,j,s}} = \Delta P_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} e s \in \mathrm{S}$	(13)

The flow direction in each pipe is given by the exclusive disjunction:

$$\begin{bmatrix} W^1_{i,j} \\ v_{i,j,s} = v^1_{i,j,s} \\ q_{i,j,s} = q^1_{i,j,s} \\ \Delta P_{i,j,s} = \Delta P^1_{i,j,s} \\ Ep_{i,j,s} = Ep^1_{i,j,s} \\ v^{min}_{i,j} \le v_{i,j,s} \le v^{max}_{i,j} \\ q^{min}_{i,j} \le q_{i,j,s} \le q^{max}_{i,j} \\ \Delta P^{min}_{i,j} \le \Delta P_{i,j,s} \le \Delta P^{max}_{i,j} \\ Ep^{min}_{i,j} \le Ep_{i,j,s} \le Ep^{max}_{i,j} \end{bmatrix} \underline{\vee} \begin{bmatrix} W^2_{i,j} \\ v_{i,j,s} = v^2_{i,j,s} \\ q_{i,j,s} = q^2_{i,j,s} \\ \Delta P_{i,j,s} = \Delta P^2_{i,j,s} \\ Ep_{i,j,s} = Ep^2_{i,j,s} \\ v^{min}_{i,j} \le v_{i,j,s} \le v^{max}_{i,j} \\ q^{min}_{i,j} \le q_{i,j,s} \le q^{max}_{i,j} \\ \Delta P^{min}_{i,j} \le \Delta P_{i,j,s} \le \Delta P^{max}_{i,j} \\ Ep^{min}_{i,j} \le Ep_{i,j,s} \le Ep^{max}_{i,j} \end{bmatrix}, \forall i,j \in \mathrm{E}_{i,j} \text{ and } s \in \mathrm{S}$$

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This disjunction can be written using a convex hull reformulation. The binary variable $w^1_{i,j}$ is equal to 1 if water flows from node *i* to node *j* and 0, on the contrary, and the binary variable $w^2_{i,j}$ is equal to 1 if water flows from node *j* to node *i* e 0, on the contrary. In this way:

$\boldsymbol{v_{i,j,s} = v^{1}_{i,j,s} + v^{2}_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}}$	**(15)**
$v^{min}_{i,j} w^{1}_{i,j} \leq v^{1}_{i,j,s} \leq v^{max}_{i,j} w^{1}_{i,j}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(16)
$v^{min}_{i,j} w^{2}_{i,j} \leq v^{2}_{i,j,s} \leq v^{max}_{i,j} w^{2}_{i,j}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(17)
$q_{i,j,s} = q^{1}_{i,j,s} + q^{2}_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(18)
$q^{min}_{i,j} w^{1}_{i,j} \leq q^{1}_{i,j,s} \leq q^{max}_{i,j} w^{1}_{i,j}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(19)
$q^{min}_{i,j} w^{2}_{i,j} \leq q^{2}_{i,j,s} \leq q^{max}_{i,j} w^{2}_{i,j}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(20)
$\Delta P_{i,j,s} = \Delta P^{1}_{i,j,s} + \Delta P^{2}_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(21)
$\Delta P^{min}_{i,j} w^{1}_{i,j} \leq \Delta P^{1}_{i,j,s} \leq \Delta P^{max}_{i,j} w^{1}_{i,j}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(22)
$\Delta P^{min}_{i,j} w^{2}_{i,j} \leq \Delta P^{2}_{i,j,s} \leq \Delta P^{max}_{i,j} w^{2}_{i,j}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(23)
$Ep_{i,j,s} = Ep^{1}_{i,j,s} + Ep^{2}_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(24)
$Ep^{min}_{i,j} w^{1}_{i,j} \leq Ep^{1}_{i,j,s} \leq Ep^{max}_{i,j} w^{1}_{i,j}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$	(25)
$Ep^{min}_{i,j} w^{2}_{i,j} \leq Ep^{2}_{i,j,s} \leq Ep^{max}_{i,j} w^{2}_{i,j}, \forall i,j \in \mathrm{E}_{i,j} \text{eand} s \in \mathrm{S}$	(26)
$w^{1}_{i,j} + w^{2}_{i,j} = 1, \forall i,j \in \mathrm{E}_{i,j}$	(27)

The uncertainties in the demand nodes imply in the use of possible pumps which could be necessary to satisfy the water demands in the network, depending on the scenario being evaluated. The pump energy located in the pipe *i-j* in scenario *s* and the pump annualized operation cost are given by:

$$E^{pow}_{i,j,s} = \frac{9.81}{0.82} q_{i,j,s} Ep_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$$

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$$Cp_{i,j,s} = 0.24 \cdot 8{,}000 E^{pow}_{i,j,s}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$$

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The pump efficiency is 0.82, the energy cost per kWh is 0.24 and the number of total hours of pumping is 8,000.

The total annual cost in each scenario is composed by the pipe installation annual cost and by the pumps annual operation cost, given by:

$$TAC_s = \sum_{i,j \in \mathrm{E}_{i,j}} \left(FAI \cdot Cost_{i,j} + Cp_{i,j,s} \right), \forall s \in \mathrm{S}$$

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The annualization factor of the installation cost for the extension of the WDN design in n_a years, subject the annual interest rate e_1, is:

$$FAI = \frac{e_1 \left(1 + e_1\right)^{n_a}}{\left(1 + e_1\right)^{n_a} - 1}$$

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As a result, the complete optimization model for the WDN synthesis considering uncertainties in the demand nodes and unknown flow directions is:

min	$\sum_{s\in S} prob_s \cdot TAC_s$
s. a	$\sum_{j\in E_{j,i}}\left(q^1_{j,i,s}-q^2_{j,i,s}\right)-\sum_{j\in E_{i,j}}\left(q^1_{i,j,s}-q^2_{i,j,s}\right)=d_{i,s}, \forall i\in N \text{ and } s\in S$
	$P_{i,s}+h_i+Ep^1_{i,j,s}-Ep^2_{i,j,s}=P_{j,s}+h_j+\Delta P^1_{i,j,s}-\Delta P^2_{i,j,s}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$x_{i,j}=\sum_{k\in D} D_k y_{i,j,k}, \forall i,j\in E_{i,j}$
	$Diam_{i,j}=\sum_{k\in D} D_k y_{i,j,k}, \forall i,j\in E_{i,j}$
	$Cost_{i,j}=\sum_{k\in D} L_{i,j} CostD\left(D_k\right) y_{i,j,k}, \forall i,j\in E_{i,j}$
	$Rug_{i,j}=\sum_{k\in D} R_k y_{i,j,k}, \forall i,j\in E_{i,j}$
	$\bar{v}_{i,j,s}=\bar{q}_{i,j,s}-\sum_{k\in D}\ln\left(\frac{\pi}{4}D_k^2\right)y_{i,j,k}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$\Delta\bar{P}_{i,j,s}=\ln\left(\alpha L_{i,j}\right)+\beta\bar{q}_{i,j,s}-\sum_{k\in D}\ln\left(R_k^{\beta}D_k^{\gamma}\right)y_{i,j,k}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$\sum_{k\in D} y_{i,j,k}=1, \forall i,j\in E_{i,j}$
	$e^{\bar{v}_{i,j,s}}=v_{i,j,s}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$e^{\bar{q}_{i,j,s}}=q_{i,j,s}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$e^{\Delta\bar{P}_{i,j,s}}=\Delta P_{i,j,s}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$P_i^{min}\le P_{i,s}, \forall i\in N \text{ and } s\in S$
	$E^{pow}_{i,j,s}=\frac{9.81}{0.82}q_{i,j,s}Ep_{i,j,s}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$Cp_{i,j,s}=0.24\cdot 8{,}000E^{pow}_{i,j,s}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$TAC_s=\sum_{i,j\in E_{i,j}}\left(FAI\cdot Cost_{i,j}+Cp_{i,j,s}\right), \forall s\in S$
	$v_{i,j,s}=v^1_{i,j,s}+v^2_{i,j,s}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$v^{min}_{i,j}w^1_{i,j}\le v^1_{i,j,s}\le v^{max}_{i,j}w^1_{i,j}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$v^{min}_{i,j}w^2_{i,j}\le v^2_{i,j,s}\le v^{max}_{i,j}w^2_{i,j}, \forall i,j\in E_{i,j} \text{ and } s\in S$
	$q_{i,j,s}=q^1_{i,j,s}+q^2_{i,j,s}, \forall i,j\in E_{i,j} \text{ and } s\in S$

min	$\sum_{s \in \mathrm{S}} prob_s \cdot TAC_s$
	$q_{i,j}^{min} w_{i,j}^{1} \leq q_{i,j,s}^{1} \leq q_{i,j}^{max} w_{i,j}^{1}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$
	$q_{i,j}^{min} w_{i,j}^{2} \leq q_{i,j,s}^{2} \leq q_{i,j}^{max} w_{i,j}^{2}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$
	$\Delta P_{i,j,s} = \Delta P_{i,j,s}^{1} + \Delta P_{i,j,s}^{2}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$
	$\Delta P_{i,j}^{min} w_{i,j}^{1} \leq \Delta P_{i,j,s}^{1} \leq \Delta P_{i,j}^{max} w_{i,j}^{1}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$
	$\Delta P_{i,j}^{min} w_{i,j}^{2} \leq \Delta P_{i,j,s}^{2} \leq \Delta P_{i,j}^{max} w_{i,j}^{2}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$
	$Ep_{i,j,s} = Ep_{i,j,s}^{1} + Ep_{i,j,s}^{2}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$
	$Ep_{i,j}^{min} w_{i,j}^{1} \leq Ep_{i,j,s}^{1} \leq Ep_{i,j}^{max} w_{i,j}^{1}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$
	$Ep_{i,j}^{min} w_{i,j}^{2} \leq Ep_{i,j,s}^{2} \leq Ep_{i,j}^{max} w_{i,j}^{2}, \forall i,j \in \mathrm{E}_{i,j} \text{and} s \in \mathrm{S}$
	$w_{i,j}^{1} + w_{i,j}^{2} = 1, \forall i,j \in \mathrm{E}_{i,j}$

The three-stages procedure for considering uncertainties in WDN nodes demand can be described as:

i) In the first step the expected TAC is minimized with a probability of occurrence of 100% for the original scenario. In this step the optimum design is achieved considering a unique nominal value for each uncertain parameter. In this case, the annual pumping cost is zero.

ii) In the second step the expected TAC with probability of occurrence in all scenarios is minimized but with fixed values for the discrete variables, given by the solution obtained in the first step. In this case, the annual installation cost is fixed. The solution found in this stage is called deterministic solution.

iii) In the third stage the TAC is minimized with probability of occurrence in all scenarios with no fixed values for the variables. In this way, the optimal design considering uncertainties in the nodes demand is achieved. The solution found in this stage is called stochastic solution.

Case Studies

To test the applicability of the developed optimization model for the synthesis of WDN considering uncertainties in the nodes demand, two case studies from the literature are used. In both cases, the parameters considered for the Hazen-Williams equation are $\alpha = 10{,}667$, $\beta = 1{,}852 \text{and} \gamma = 4{,}871$. The scenarios are generated by using a normal distribution in Excel. The model was coded in GAMS and global optimization techniques were used to solve the problem. The lifetime of the WDN design is considered to be 20 years and the interest rate is 5% per year. Hydraulic pumps are considered if it is necessary to attend the demand in some node in each one of the generated scenarios.

Case study 1

The first benchmark problem is known as Two loop WDN, and is presented in Fig. 1. The network has 8 pipes linking 7 nodes demand and 2 loops involving the nodes. The reservoir is considered the first node and its demand is the summation of all other nodes. The water velocity must be bounded by 0.3 and 3 m/s and the minimum acceptable pressure for all nodes is 30 m. The pipe diameter is selected from a set of available commercial diameters, given in Table 1. The rugosity coefficient is 130 for all pipes.

For this network, 30 scenarios were generated, with expected values and standard deviation for the water demand in each one of the nodes given in Table 2. The problem has 4435 variables, being 128 discrete variables. The deterministic solutions found by fixing the design variables, according to the problem solution without assuming variations in the demand nodes is compared with the stochastic solution obtained without fixing the variables, following the solution of the problem assuming the variation of water demand in the nodes.

Table 3 presents results achieved by the diameters and pipe flow directions and the expected total annual cost, according to each solution. Results shown that the WDN design by the stochastic approach reduce the expected TAC in more than 2.6% in comparison to the deterministic solution. As the annual installation cost is greater in the stochastic solution, the reduction in the expected TAC is due, mainly, to the necessity of using pumps to satisfy the nodes demand in some scenarios in the deterministic design. Table 3 presents the differences between the stochastic and deterministic solutions in some pipe diameters. It is related to the water velocity in the pipes and to the use of energy of piping. To satisfy the necessary water demand, the greater the selected diameter the greater the water velocity. It implies in more use of energy.

Table 1
– Available diameters for the *Two loop* WDN

Diameter (m)	Cost ($/m)	Diameter (m)	Cost ($/m)
0.0254	2	0.3048	50
0.0508	5	0.3556	60
0.0762	8	0.4064	90
0.1016	11	0.4572	130
0.1524	16	0.5080	170
0.2032	23	0.5588	300
0.2540	32	0.6096	550

Table 2
– Water demand in the nodes for the *Two loop* WDN

Water demand (m^3/h)	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Expected value	100	100	120	270	330	200
Standard deviation	4.88	3.92	8.13	18.29	31.52	15.42

Table 3
– Results for the *Two loop* WDN

Pipe	Deterministic solution	Stochastic solution
1	0.4572 (1–2)	0.4572 (1–2)
2	0.2540 (2–3)	0.3556 (2–3)
3	0.4064 (2–4)	0.3556 (2–4)
4	0.1016 (4–5)	0.0254 (4–5)
5	0.4064 (4–6)	0.3556 (4–6)
6	0.2540 (6–7)	0.0254 (6–7)
7	0.2540 (3–5)	0.3556 (3–5)
8	0.0254 (7 – 5)	0.3048 (5–7)
$expTAC$ ($/year)	35,757.05	34,798.27

Case study 2

The second case study is a real WDN situated in Brasil and is known as the Modified Grande Setor WDN. Figure 2 presents the network topology, with 8 pipes linking 7 demand nodes, with 2 loops and a water reservoir, represented by the seventh node. Its demand is given as the summation of all other nodes. The water velocity must be between 0.2 and 3 m/s and the acceptable minimum pressure is 25 m for all nodes. The pipe diameters are selected from a set of available commercial diameters, given in Table 4.

In this case study, 20 scenarios were generated with the expected values and the standard deviation for the water demand in each node presented in Table 5. The problem has 2981 variables, being 96 discrete variables. As in case study 1, the deterministic solution obtained fixing the design variables, according to the problem solution without assuming variability in the water nodes demand is compared with the stochastic solution obtained without fixing the variables, following the problem solution assuming the variation in the nodes demand.

Table 6 presents the results obtained for the diameters and the pipe flow directions and the expected total annual cost, according to each solution. Results shown that the stochastic solution reduce the TAC in 1.2% when compared to the deterministic solution. It is due, mainly, to the necessity of using pumping to satisfy the water nodes demand in some scenarios in the deterministic solution. As the installation cost is greater in the stochastic solution, the reduction in the expected TAC is due to the pumping system to satisfy the nodes demand in some scenarios of the deterministic solution.

Table 4
– Available diameters for the Modified Grande Setor WDN

Diameter (m)	Cost (US$/m)	Rugosity coefficient	Diameter (m)	Cost (US$/m)	Rugosity coefficient
0.1084	23.55	145	0.3662	158.93	130
0.1564	31.90	145	0.4164	187.50	130
0.2042	43.81	145	0.4666	218.12	130
0.2520	59.30	145	0.5180	257.80	130
0.2998	76.12	145	0.6196	320.15	130

Table 5
– Water demand for the Modified Grande Setor WDN

Water demand (L/s)	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6
Expected value	84.29	47.78	80.32	208.60	43.44	40.29
Standard deviation	8.05	6.85	11.28	19.93	3.35	7.41

Table 6
– Results for the Modified Grande Setor WDN

Pipe	Deterministic solution	Stochastic solution
1	0.6196 (7 – 1)	0.6196 (7 – 1)
2	0.2520 (1–2)	0.2998 (1–2)
3	0.1084 (2–3)	0.2042 (2–3)
4	0.2998 (4 – 3)	0.2998 (4 – 3)
5	0.6196 (1–4)	0.6196 (1–4)
6	0.2520 (1–5)	0.2520 (1–5)
7	0.1084 (5–6)	0.1084 (5–6)
8	0.2520 (4–6)	0.2520 (4–6)
$expTAC$ (US$/ano)	150,877.70	148,942.16

Conclusions

In the present paper an optimization model with MINLP formulation was developed using disjunctive mathematical programming for the synthesis of water distribution networks considering uncertainties in the nodes demand and unknown flow directions. The main objective is the minimization of the total annual cost, composed by the annual installation cost and the pumping energy cost. The water demand variation was modeled as a set of finite scenarios generated from a normal distribution in Excel. The model was implemented in GAMS and the global optimization solver BARON was used.

The problem was solved in three stages. First, only a unique nominal value for each uncertain parameter was used. Second, for all scenarios, in which the independent scenarios variables are fixed to the solution obtained in the first step (deterministic solution). Third, for all scenarios, no variable is fixed (stochastic solution). Two case studies were used to test the applicability of the developed model and results shown that under uncertainties, the stochastic solution improve the deterministic one.

The optimization problem of WDN optimization with uncertainties in the nodes demand and unknown flow directions is not trivial, due to the complexity inherent to the nonlinearity of the system. It is necessary a finite set of applications of the developed model, according to the number of generated scenarios to the WDN evaluation.

Declarations

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Competing Interests

The authors have no relevant financial or non-financial interests to disclose.

Author Contribution

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by all authors. The first draft of the manuscript was written by Gustavo H. B. Cassiolato and authors Mauro A. S. S. Ravagnani, Raquel Salcedo-Diaz and Ruben Ruiz-Femenia commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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Figures

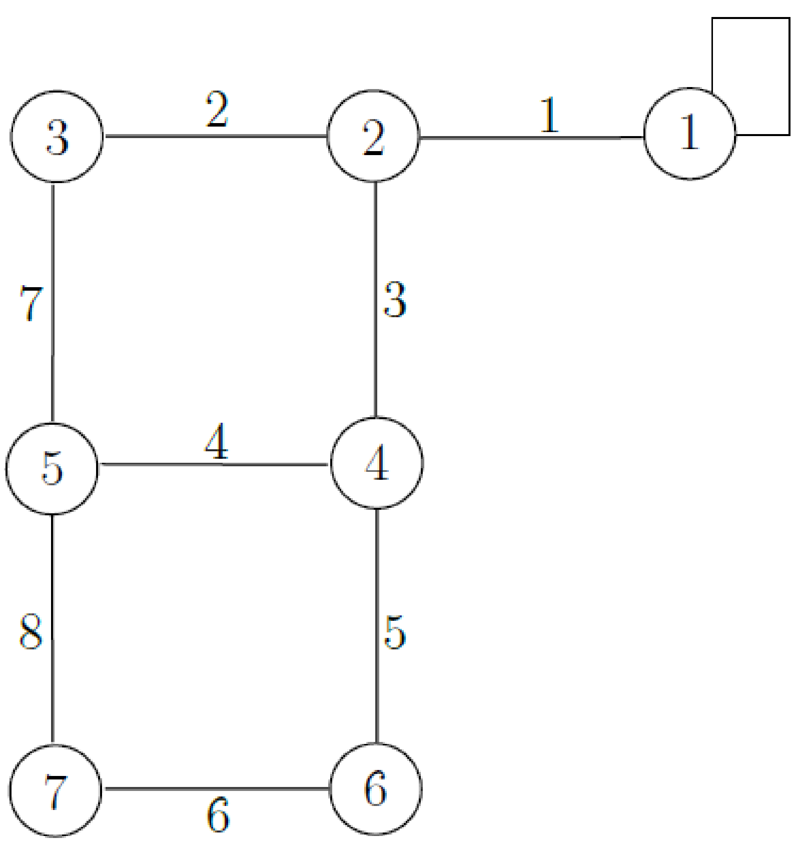

Figure 1

Two loop WDN

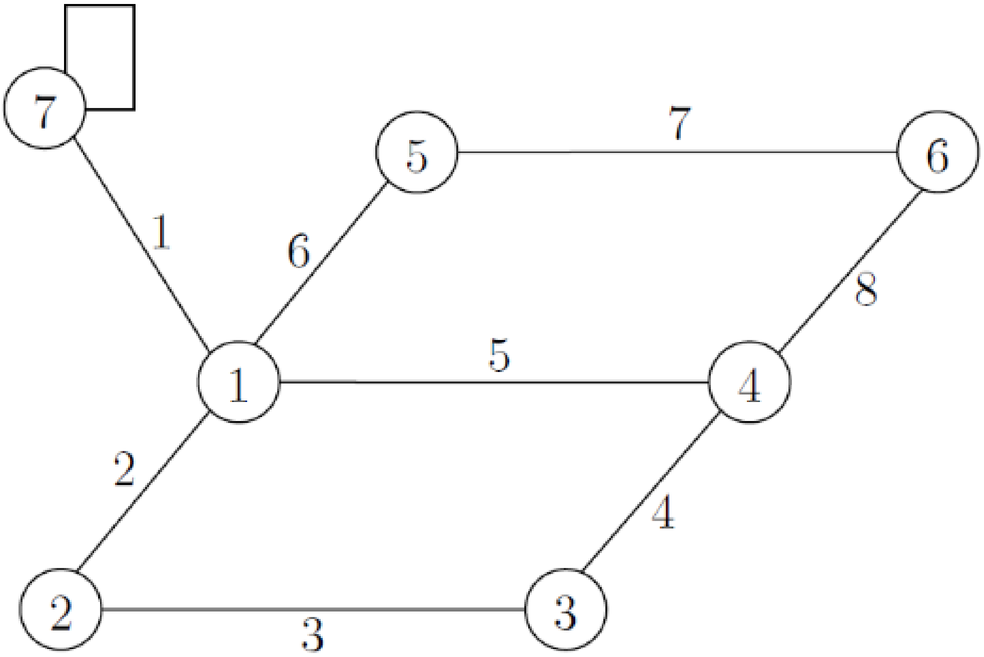

Figure 2

Modified Grande Setor WDN