Full-Wave Analysis and Applications of EBG Waveguides Periodically Loaded with Metal Ridges

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Abstract—In this paper, a fast and accurate full-wave analysis tool of passive structures based on Electromagnetic-Bandgap (EBG) waveguides periodically loaded with metal ridges is proposed. For this purpose, a very efficient Integral Equation (IE) technique is followed to model the planar steps involving arbitrary waveguides. The well-known Boundary Integral - Resonant Mode Expansion (BI-RME) method is used to obtain the modal chart of the ridged waveguides. In order to show the advantages of this tool, a periodically loaded E-plane filter with improved stopband performance is analyzed and compared to standard implementation. Dispersion relations are also derived and used as guidelines for designing an EBG fifth-order low-pass filter.

Index Terms—Modal analysis, Integral equations, Electromagnetic bandgap (EBG), periodic ridged waveguides.

I. INTRODUCTION

Periodic structures are currently receiving a renewed interest due to their potential use in the microwave and millimeter-wave frequency bands. For filtering applications, such structures are reported to offer reduced physical size and improved stopband performance [1], [2]. These features are direct consequence of the slow waves propagating inside periodic structures, which present a reduced phase velocity and wavelength. Therefore, the physical lengths of half-wavelength resonators, when periodically loaded, are considerably reduced. Furthermore, due to the dispersion behavior of slow waves, an improved stopband performance can be achieved. The E-plane waveguide technology, periodically loaded with reactive obstacles in form of metal ridges, offers a convenient way for realizing EBG passive waveguide structures (see [2] and [3]).

This paper proposes a novel method for the fast and rigorous full-wave analysis of passive structures based on Electromagnetic-Bandgap (EBG) waveguides periodically loaded with metal ridges. For such purpose, an Integral Equation (IE)-based method, originally outlined in [4] for rectangular waveguides, is updated to cope with arbitrarily shaped geometries. This method makes use of the modal chart of ridged waveguides, which is determined following a revisited version (see [5]) of the classical and well-known BI-RME technique.

In order to validate the new analysis tool proposed in this paper, we have considered two practical applications of EBG ridged waveguides. First, an E-plane filter periodically loaded with metal ridges is successfully analyzed, as

well as compared in terms of physical size and out-of-band response to a standard E-plane topology. Then, an EBG low-pass ridge filter, designed from the pass- and stopbands derived from the corresponding dispersion diagram, is discussed.

II. THEORY

An example of a periodically loaded structure considered in this paper, in particular an E-plane filter with periodic metal ridges, can be seen in Fig. 1.

Fig. 1. Three-dimensional layout of a periodically loaded E-plane filter.

For analysis purposes, it is first required to know accurately the complete modal chart of the ridged waveguides present in the structure of Fig. 1. Following the well-known BI-RME method, the electric field at a generic observation point $\mathbf{r}$ inside the arbitrary (ridged) waveguide can be obtained by:

$$\mathbf{E}(\mathbf{r}) = -j\eta k \int_{\sigma} \overline{\mathbf{G}}_e(\mathbf{r}, \mathbf{s}', k) \cdot \mathbf{J}_{\sigma}(l') dl' \quad (1)$$

where the meaning of all involved variables, dyadic and vector functions can be seen in [5]. Splitting $\overline{\mathbf{G}}_e$ and $\mathbf{J}_{\sigma}$ into its transversal and longitudinal components, and imposing the corresponding boundary conditions (i.e. $\mathbf{E}_t(\mathbf{r}) \cdot \hat{\mathbf{t}}(l) = 0$, and $E_z(\mathbf{r}) = 0$) on σ , two integral equations are obtained for the transversal electric field $\mathbf{E}_t$ (TE modes) and the longitudinal electric field E_z (TM modes). Then, both integral equations are solved via the Galerkin version of the MoM, where the chosen basis and testing functions are overlapping piece-wise parabolic splines. Applying the Galerkin approach, a generalized/standard eigenvalue problem is obtained for the TE/TM case. The

solutions for these problems provide as eigenvalues the cutoff wavenumbers, and as eigenvectors the modal expansion coefficients and the amplitudes of the transversal and longitudinal components of the unknown current density.

An advantage of the BI-RME technique is that, without hardly additional computational effort, the coupling coefficients between the ridged waveguide and the standard rectangular contour enclosing the ridged profile can be easily computed (see [6]). The coupling integrals between the modes of the ridged waveguide and the ones of the rectangular waveguide are defined as follows:

$$I_{pq} = \int_S \mathbf{e}_p^{(1)} \cdot \mathbf{e}_q^{(2)} dS \quad (2)$$

where $\mathbf{e}_p^{(1)}$ and $\mathbf{e}_q^{(2)}$ are, respectively, the normalized electric modal vectors of the rectangular and ridged waveguides, being S the cross-section of the smaller (ridged) guide.

In order to obtain a full-wave characterization of the planar junction between two waveguides, a very efficient method based on an integral equation technique, originally described in [4] for dealing with rectangular waveguides, has been properly updated. The objective of this technique is to obtain a Generalized Impedance Matrix (GIM) representation of each planar waveguide junction. A remarkable contribution of this method is the distinction made between accessible and localized modes: accessible modes are those used to connect transitions, while localized modes are only used to describe the electromagnetic fields in the junction. Imposing the boundary condition at the junction plane, and after some mathematical manipulations (see details in [4]), it is possible to obtain the following integral equation:

$$\mathbf{h}_n^{(\delta)}(s) = \iint_{cs(2)} \mathbf{M}_n^{(\delta)}(s') K(s, s') ds' \quad (3)$$

where s and s' are the observation and source points, respectively, $K(s, s')$ is the kernel of the integral equation, $\mathbf{M}_n^{(\delta)}(s')$ are unknown vector functions which define the magnetic field in the junction, and $(\delta) = (1), (2)$ indicates the suitable region of the junction. Finally, the solution of the integral equation can be performed using the Galerkin procedure, where the unknown vector functions are expanded in terms of the smallest waveguide modes. Proceeding in this way, we can compute the impedance parameters of the generalized Z -matrix as follows:

$$\begin{aligned} Z_{m,n}^{(\delta,\gamma)} &\triangleq \iint_{cs(2)} \mathbf{M}_n^{(\gamma)}(s') \cdot \mathbf{h}_m^{(\delta)*}(s') ds' \\ &= \sum_{q=1}^{M^{(\gamma)}} \alpha_q^{(n,\gamma)} \iint_{cs(2)} \mathbf{h}_q^{(2)} \cdot \mathbf{h}_m^{(\delta)*} ds' \end{aligned} \quad (4)$$

On the other hand, once the generalized Z matrix is obtained, the full-wave two-port $ABCD$ parameters of

the unit cell of the periodic structure can be numerically derived:

$$\mathbf{P} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \quad (5)$$

where:

$$\mathbf{A} = \mathbf{Z}_{11} \cdot \mathbf{Z}_{21}^{-1} \quad (6)$$

$$\mathbf{B} = \mathbf{Z}_{11} \cdot \mathbf{Z}_{21}^{-1} \cdot \mathbf{Z}_{22} - \mathbf{Z}_{12} \quad (7)$$

$$\mathbf{C} = \mathbf{Z}_{21}^{-1} \quad (8)$$

$$\mathbf{D} = \mathbf{Z}_{21}^{-1} \cdot \mathbf{Z}_{22} \quad (9)$$

In a periodic structure of period p , a generic component $F(z)$ of the electromagnetic field must satisfy the Floquet condition [7]:

$$F(z + p) = e^{-\gamma p} F(z) \quad \forall z \quad (10)$$

where γ is the propagation constant of the Floquet mode. Imposing the Floquet condition at the unit cell, we obtain:

$$\begin{pmatrix} \mathbf{V}_1 \\ \mathbf{I}_1 \end{pmatrix} = \mathbf{P} \begin{pmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{pmatrix} = e^{-\gamma p} \begin{pmatrix} \mathbf{V}_2 \\ -\mathbf{I}_2 \end{pmatrix} \quad (11)$$

The right-hand side of equation (11) can now be recognized as the classical canonical form:

$$[\mathbf{P}] \cdot [\mathbf{x}] = \lambda \cdot [\mathbf{I}] \cdot [\mathbf{x}] \quad (12)$$

where the eigenvalues and, consequently, the propagation constants, can be straightforwardly determined using well-established routines. At a given frequency, once the eigenvalues λ have been obtained, the real and imaginary parts of the Floquet propagation constant $\gamma = \alpha + j\beta$ are directly derived by:

$$\alpha = -\frac{\ln|\lambda|}{p} \quad \beta = -\frac{\angle\lambda}{p} \quad (13)$$

In the canonical problem (12), the number of eigenvalues (λ) obtained is equal to the order of the matrix $\mathbf{P}$ in the left-hand side. These eigenvalues are, in general, complex and appear as pairs corresponding to forward and backward directions of propagation of the same mode. Pass-bands and stop-bands are characterized, respectively, by the condition as to whether or not the periodic structure supports at least one propagating Floquet mode within the considered frequency range.

III. RESULTS

In order to demonstrate the practical application of an EBG ridged waveguide structure, a two-resonator periodic filter (proposed in [2]) has been first considered, as well as compared with an equivalent standard E-plane topology. The layout and dimensions of these two filters (which are implemented with standard WR-90 input/output waveguides, i.e. $a = 22.86$ mm and $b = 10.16$ mm, and considering a metal insert thickness $t = 0.10$ mm) are shown in Fig. 1 and Fig. 2.

Fig. 2. Views of the metal inserts for the standard E-plane filter in (a) and the periodically loaded E-plane filter in (b). The dimensions for the standard E-plane filter are: septa lengths $L_{s1} = L_{s3} = 0.31$ mm and $L_{s2} = 1.68$ mm, resonator length $L_{r1} = L_{r2} = 20.10$ mm, total length of filter 42.50 mm. The dimensions for the periodically loaded E-plane filter are: septa lengths $L_{s1} = L_{s3} = 1.20$ mm and $L_{s2} = 4.20$ mm, resonator length $L_{r1} = L_{r2} = 8.0$ mm, ridge dimensions $s_1 = b = 10.16$ mm and $s_2 = 1.0$ mm, ridge lengths $l_1 = 1.0$ mm and $l_2 = 0.5$ mm (period $p = 1.5$ mm), total length of periodically E-plane filter 22.40 mm.

The homogeneous rectangular waveguides acting as resonators of the standard E-plane filter (see Fig. 2 (a)) are replaced with periodic structures consisting of a cascade of ridged waveguides (see Fig. 2 (b)). The periodically loaded filter is very similar to the standard E-plane topology, but instead of having a homogeneous waveguide of length L_r between the two septa of length L_s (see Fig. 2 (a)), a cascade of ridged waveguides of equal length are placed inside the resonator section (see Fig. 2 (b)).

In Fig. 3 (a), the E-plane standard filter response obtained with our full-wave software package is well compared with Ansoft HFSS v10.0 results. Exploiting the symmetry of the structure, the analysis method has required to consider 80 accessible modes, 250 basis functions and 400 kernel terms. The complete simulation of the electrical response has only taken a CPU effort of 0.11 s per frequency point¹.

In Fig. 3 (b), the periodically loaded E-plane filter response obtained with our full-wave software package is successfully compared again with Ansoft HFSS v10.0 data, as well as with measurements of a manufactured filter prototype from [2]. Exploiting again the symmetry of the structure, in this case our full-wave analysis method has required to consider 80 accessible modes, 250 basis functions and 500 kernel terms. The complete simulation of the electrical response has only taken a CPU effort of 0.35 s per frequency point (177 s in total). As it can be seen in both figures, the periodic loaded filter has better selectivity and wider stop-band response than its standard

Fig. 3. Electrical responses of the E-plane standard filter in a), and of the periodically loaded E-plane filter in b).

E-plane counterpart. Furthermore, the periodic filter is 47% shorter than the standard filter, because the lengths of the periodically loaded resonators are near halved.

Fig. 4 compares the dispersion relation for the TE_{10} mode of the standard WR-90 guide and for the EBG ridged waveguide with dimensions shown in Fig. 2 (b). As it can be seen, due to the slow wave effect, the dispersive behavior of the EBG waveguide is significantly more linear in the 8-9 GHz bandwidth than for the standard case.

Additionally, the dispersion diagram that can be obtained with the theory described in the previous section is useful for the practical design of waveguide filters. Analyzing the pass- and stop-bands (gaps) in the dispersion diagram of a unit cell, it is possible to generate a finite structure with a very similar electrical response. For instance, the layout of an EBG fifth-order low-pass filter, originally proposed in [3], can be seen in Fig. 5. This filter is composed of a cascade of five ridged waveguides. The dimensions of the unit cell have been selected in order to obtain a stop-band from 11 GHz, which corresponds

¹All the CPU times reported in this work were obtained with a Pentium IV platform at 3.2 GHz with 512-MB RAM.

Fig. 4. Dispersion relation for the rectangular WR-90 waveguide (dashed line) and E-plane EBG waveguide (solid line) in the same housing of length $p = 1.5$ mm, with $s_2 = 1.0$ mm, $l_1 = 1.0$ mm and $t = 0.1$ mm

Fig. 5. Layout of the EBG fifth-order low-pass filter. The dimensions are: $L_w = 6.0$ mm, $L_{r1} = L_{r5} = 1.0$ mm, $L_{r2} = L_{r3} = L_{r4} = 2.0$ mm, ridge dimensions $b = 10.16$ mm $s_2 = 1.0$ mm, and $t = 0.1$ mm.

(as shown in Fig. 6) to the starting frequency of the first band gap of the periodic structure. The number of unit cells of the finite structure defines the order of the filter, the attenuation in the stop-band and the out-of-band response. It should be noted that the finite structure is not completely periodic, since the first and last ridge loadings have been slightly modified in order to match the EBG waveguide impedance to the TE_{10} -mode impedance of the input waveguide. In Fig. 7, the EBG fifth-order low-pass filter response provided by our analysis tool is successfully compared with Ansoft HFSS v10.0 results.

IV. CONCLUSION

An efficient and accurate tool for the full-wave analysis of passive structures based on Electromagnetic-Bandgap (EBG) ridged waveguides has been presented. For the fast and rigorous analysis of these filters, an advanced modal method combining BI-RME and IE techniques has been outlined. This novel tool has been successfully applied first to the analysis of a compact periodically loaded (with metal ridges) E-plane filter with an improved stop-band response. Then, the dispersion behavior of EBG waveguides are derived and used for the practical design of an EBG low-pass filter.

Fig. 6. Dispersion behavior (β - f) for EBG periodically loaded waveguides. The dimensions of the unit cell are $p = 8$ mm, $l_r = 2$ mm, and $s_2 = 1$ mm. The circles are taken from [3].

Fig. 7. Electrical responses of the EBG fifth-order low-pass filter.

REFERENCES

- [1] J. Sor, Y. Quian and T. Itoh, "Miniature Low loss CPW periodic structure for filter applications," *IEEE Trans. Microwave Theory & Tech.*, vol. 49, no. 12, pp. 2336-2341-1504, December 2001.
- [2] G. Goussetis, and D. Budimir, "Novel periodically loaded E-plane filters", *IEEE Microwave and Wireless Comp. Lett.*, vol. 13, no. 6, pp. 193-195, June 2003.
- [3] G. Goussetis, A. Faresidis and P. Kosmas, "Efficient analysis, design, and filter applications of EBG waveguide with periodic resonant loads", *IEEE Trans. Microwave Theory & Tech.*, vol. 54, no. 11, pp. 3885-3892, November 2006.
- [4] G. Gerini, M. Guglielmi and G. Lastoria, "Efficient integral formulations for admittance or impedance representation of planar waveguide junctions", *IEEE MTT-S Int. Microwave Symp. Dig.*, vol. 3, pp. 1747-1750, June 1998.
- [5] S. Cogollos, S. Marini, V. E. Boria, P. Soto, A. Vidal, H. Esteban, J. V. Morro, and B. Gimeno, "Efficient modal analysis of arbitrarily shaped waveguides composed of linear, circular and elliptical arcs using the BI-RME method," *IEEE Trans. Microwave Theory & Tech.*, vol. 51, no. 12, pp. 2378-2390, December 2003.
- [6] P. Arcioni, "Fast evaluation of modal coupling coefficients of waveguide step discontinuities", *IEEE Microwave Guided Wave Lett.*, vol. 6, no. 6, pp. 232-234, June 1996.
- [7] R. E. Collin, *Field Theory of Guided Waves*, New York: McGraw-Hill, 1960.