Beliefs, Epistemic Regress and Doxastic Justification

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Abstract

By justification we understand what makes a belief epistemologically viable: generally this is considered knowledge that is true. The problem is defining this with a higher degree of precision because this is where different conflicting conceptions appear. On the one hand, we can understand justification as what makes it reasonable to acquire or maintain a belief; on the other, it is what increases the probability that the belief is true. This work tries to prove that beliefs depend on other beliefs that are epistemically justified and that such beliefs are the result of (i.e., they arise from) our privileged intuition of reality. For this, we examine the concept of epistemic regress. Epistemic reasons authorize a proposition P to be the conclusion of an argument in which such reasons function as premises and are vulnerable to epistemic regress. The three most important approaches to epistemic regress are Infinitism, Coherentism and Foundationalism.

Keywords Basic propositions · Belief system · Derived beliefs · Derived propositions · Epistemic justification · Infinite regress · Knowledge · Perceptual beliefs · Substantive beliefs

1 Introduction

By justification we understand what makes a belief epistemologically viable: generally this is considered knowledge that is true. The problem is defining this with a higher degree of precision. Perhaps the most fundamental problem in epistemology is that of offering a satisfactory characterization of the notion of knowledge. The *locus classicus* of all the subsequent discussion, Plato's *Theaetetus*, already told us: for something to be considered as knowledge, it must be a belief of a certain subject, in addition it must be true and finally it must satisfy a third condition, which the Platonic text never makes very clear to us. Plato's philosophical tradition has crystallized the idea that such a condition corresponds to the requirement that the question be justified in having and maintaining this belief. What does it mean to say that an agent (subject) *S* has a justification for his belief that *P*, that is, *S*

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believes that a certain propositional content *P* is true? Suppose that our agent *S* believes that it is cold where he is located at the precise moment when he has this belief. Someone asks *S*: *how do you know it's cold in this place right now? S's* obvious answer is: *because I feel the cold in my body and I'm shivering.* His epistemic justification for sustaining his belief in the cold is, in this case, to appeal to his momentary perceptions. Certainly, there are other underlying beliefs supporting *S's* belief; for example, the belief that what he is feeling is cold and not something that closely resembles it, the belief that cold can make him sick, and many other beliefs. The interlocutor of *S*, although he does not hear him mention these other beliefs, probably based on his other beliefs about *S*, will attribute these and other beliefs to him and will be satisfied with the justification provided by *S*. We can then complete the classical characterization of knowledge by adding the condition of epistemic justification to the other two and affirm that all three conditions are necessary and together sufficient for a certain state of mind of agent *S* to be characterized as knowledge. However, although it seems evident from the previous example, what we should take as justification for a certain belief is the epistemological notion that has generated the most problems and deservedly the one that has led to most discussion. There are several questions that philosophers have tried to answer regarding epistemic justification. For example, what should be the strength of the justification between a belief and its justifications? This issue has separated the fallibilists from the so-called infallibilists.

(a) The former have maintained that such a relationship cannot be so strong as to make it impossible for a justified belief to be false. As proof of the plausibility of their thesis, fallibilists will say: "*no one in their right mind is going to try to deny that our best scientific theories constitute knowledge; however, it is most likely that it will come to show in the future that strictly speaking they are not true.*" The belief systems that make up scientific theories are ultimately justified by experience. Such justification we will call inductive since the truth of the set of observation propositions that justify a theory does not guarantee the truth of the theory. It is always possible that all observation propositions that support a theory are true and the theory itself is false.

(b) The infallibilists, for their part, have tried to defend an Aristotelian notion of knowledge according to which being justified makes a belief immune to the possibility of being false. The most cited example of infallible knowledge is Mathematics. Mathematicians accept as justification of a mathematical theorem (T) at least one proof, and the latter connects the theorem in question with other theorems only by means of deductive transitions, and ultimately with the axioms of the calculation to which it belongs *T*. Since deductive transitions preserve truth and if all mathematical axioms are also true, then the mathematical justification of a theorem automatically guarantees its truth. It is impossible, therefore, that a mathematical belief is mathematically justified but at the same time false.

Although the infallibilist position has lost quite a bit of ground due to the enormous success obtained by empirical knowledge in controlling, predicting, and explaining natural phenomena, such a position enjoyed enviable prestige among most Greek philosophers and early modern philosophers. In contrast to Greco-Roman antiquity, to the Middle Ages and to the first modern philosophers of the rationalist tradition, for whom the paradigm of knowledge was represented by mathematics, currently it is very difficult if not impossible to defend some types of infallibilism. One could say that only certain knowledge deserves the name of knowledge and at the same time reserve the terms *belief* or *opinion* for any

type of fallible knowledge. But if we do not want to fall into the dogma of terminological stipulation, we are obliged to give good reasons to deny knowledge to the best scientific theories. And the general opinion is unanimous that there are no such good reasons.

Another question that the concept of justification has raised is that of its nature. The question is: should all the justifiers of a certain belief be mental states that can serve as justifying reasons for such a belief or can something external to the domain of mental justify a certain belief of the subject *S*? Possible answers to this question have opposed internalists to externalists. An epistemological internalist is going to insist that justification for our beliefs have to be mental states with propositional content of a certain type. For example, other beliefs or states that have a strong representational component, that is: states whose purpose is to represent the world as being in a certain way. A perception, for example, has a fairly strong representational component and, as such, could serve as a justification for our beliefs according to the internalist. Clarification is important because there are internalisms and externalisms of other types. The most famous recent dispute between internalists and externalists is the one in the field of the constitution of the propositional content of our mental states and our declarative sentences. Internalists claim that such content is constituted regardless of what the subject of such content is in the world, while externalists maintain that such content is constituted by causal relationships between things in the outside world and our thoughts. The idea is that we could not have thoughts about a horse, for example, if we did not live in a world where there is in fact such an animal. Internalist explanations of justification are generally classified into two broad groups:

(1) On the one hand, we have the *Foundationalism* approach, which affirms that the justification of any belief must always and necessarily refer us to a set of basic beliefs (*substantive beliefs*) and *basic propositions*, whose justification makes use of intrinsic properties of these beliefs, such as, for example, their evidence. Substantive beliefs[1] are like geometry's axiom. They basic important beliefs in a belief system (Usó-Doménech et al., 2016). They include believing the statement "God exists", in France the right to strike, and in the USA the right to bear arms. Derived beliefs are like the theorems that follow from substantive beliefs and could include religious observances, participating in demonstrations, and negative attitudes to gun control. Perhaps the best-known example of the Foundationalist way of approaching knowledge is given to us by Euclidean

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[1] The abstract belief level *(BS)* is formed by a set of elements denominated substantive beliefs S forming the unquestionable truths of the system (axioms) and a set of derived beliefs D, formed from substantive beliefs. Substantive beliefs constitute the axioms of the system, while many of derived beliefs will constitute their theorems.

In the same sense as Christianity or Islam or Judaism, cannot be credited with the possession of Articles of Faith, many attempts have indeed been made at systematizing and reducing to a fixed phraseology and sequence the contents of the Jewish religion (Scholem, 1941). However, these have always lacked one essential element: authoritative sanction on the part of a supreme ecclesiastical body. In addition, for this reason they have not been recognized as final or regarded as having a universally binding force. However, to a certain extent incorporated in the liturgy and utilized for purposes of instruction, these formulations of the cardinal tenets of Judaism carried no greater weight than that imparted to them by the fame and scholarship of their respective authors. None of them had a character analogous to that given in the Church to its three great formulas (the so-called *Apostles' Creed, the Nicene or Constantopolitan Creed, and the Athanasian Creed*), or even to the *Kalimat AsShahadat* of the Muslims. None of the many summaries from the pens of Jewish philosophers and rabbis have been invested with similar importance and prominence. The reasons for this relative absence of official and obligatory creeds are easily ascertained. The most widely spread and popular of all creeds is that of *Maimonides*, embracing the thirteen articles. Why he chose this particular number has been a subject of much discussion. Some have seen in the number a reference to the thirteen attributes of God. Probably the choice of the number has no significance. His articles are:

geometry. The highest-level Euclidean theorems are supported by lower-level theorems and these, in turn, are based on other even lower-level theorems until we reach propositions that justify or admit proof. These are in Euclid's terminology the definitions, the postulates and the axioms. In the Euclidean case these propositions are so basic that we must consider them as self-evident.

(2) The second important group of internalist explanations of justification is what has been known as Coherentism. The Coherentists reject the idea of dividing the set of beliefs of an agent S into those that are substantive (those that are deemed self-evident) and those that are not, the derivative or inferential beliefs. They insist that we should not distinguish between beliefs whose justification is relational—that is, whose justification consists of a type of relationship that these beliefs hold with other beliefs in Agent S's belief system—and those beliefs whose justification depends only on themselves. For

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Footnote 1 (continued)

s_1 **Principle I:**	*To know the existence of the Creator.*
s_2 **Principle II:**	*The unity of God.*
s_3 **Principle III:**	*The denial of physicality in connection with God.*
s_4 **Principle IV:**	*God's Antiquity.*
s_5 **Principle V:**	*That God, blessed be He, is worthy that we serve Him, to. Glorify Him, to make known His greatness, and to do His. Commands.*
s_6 **Principle VI:**	*Prophecy.*
s_7 **Principle VII:**	*The prophetic capacity of Moses our Teacher, peace be. upon him.*
s_8 **Principle VIII:**	*That the Torah is from heaven [God].*
s_9 **Principle IX:**	*The completeness of the Torah.*
s_{10} **Principle X:**	*That God knows man's actions and does not remove His. eye from them.*
s_{11} **Principle XI:**	*That God gives reward to he who does the. commandments of the Torah and punishes those that. transgress its admonishments and warnings.*
s_{12} **Principle XII:**	*The era of the Messiah.*
s_{13} **Principle XIII:**	*Resurrection of the dead.*

Let $S = \{s_1, s_2,\ldots,s_n\}$ be the set of substantive beliefs and $D = \{d_1, d_2,\ldots,d_m\}$ the set of derived beliefs, such that $BS = S \cup D = \{s_1, s_2, \ldots, s_n, d_1, d_2, \ldots d_m\}$. There is no belief or empty belief that we will represent by Ø. Set BS forms a *belief sequence* because it is an ordered list of objects. It contains *terms or beliefs* and the number of terms is called the *length* of the sequence. Order matters, and the exact same terms can appear multiple times at different positions in the belief sequence. BS forms a finite sequence with terms in the set BS because it is a function from $\{s_1, s_2, \ldots, s_n, d_1, d_2, \ldots d_m\}$ to BS.

Hypothesis 1: *Beliefs are not products of reason or of abstract and logical thought.*

Hypothesis 2: *In the origin of any belief system there is always a supernatural system of beliefs.*

Hypothesis 3: *Derived beliefs become substantial beliefs with the passage of time, giving origin to a more or less ample body of substantive beliefs, that is to say, a religion.*

Hypothesis 4: *When belonging to the Ideological Doxical Superstructure, the set S of substantive beliefs will be "ideal", that is to say, merely abstract.*

Hypothesis 5: *Substantive and derived sets form a graphed text having a topological structure which represents the way in which the individual organizes semantic content, concepts and propositions in its cognitive structure through subsumption, differentiation and integration.*

the Coherentist, epistemic justification is always relational. A belief or belief system is justified as long as it is consistent, namely: as long as none of your beliefs contradicts other beliefs in the system. An example of the Coherentist way of approaching mathematical systems is given by David Hilbert's formalism. According to Hilbert, the essential epistemological property of such systems is consistency, and this is just what the fundamental theorists must look for, viz.: they must look for consistency tests for all mathematical systems starting with the system of first order arithmetic with identity. One of the most serious problems of Coherentism is that the unique demand for consistency does not guarantee anything about the central objective of the search for knowledge, which is to achieve true representations of the world. Ultimately, many consistent systems of propositions can be as far from the truth as you like. This is one of the reasons that motivated philosophers to look for epistemologies that mix elements of Coherentism and Foundationalism.

The externalist diverges from the internalist in affirming that a belief can be justified when it is in a certain type of relationship with states of affairs in the world that are outside the domain of reasoning of an agent *S*. This relationship can be a causal type between events in the world and one or more of our beliefs, but it can also be a covariance relationship between the types of beliefs that we form in certain circumstances and the cognitive processes in *S* that result in the formation of such types of beliefs. Those who defend the first type of explanation are said to have a causal approach to knowledge. Among those who opt for an alternative conception to the so-called causal theory of knowledge, the most disputed are the reliabilists. These epistemologists argue that a belief or set of beliefs is justified when it results from a reliable cognitive mechanism, that is: a cognitive mechanism that produces true beliefs most of the time. To avoid the possibility that this 'most of the time' is interpreted randomly, we must clarify that for the reliabilists the idea of a reliable process or mechanism for the production of beliefs corresponds to the following: when operating in optimal operating conditions such mechanisms produce true beliefs most of the time. As an example of a reliable cognitive mechanism, let's take human vision. When applied to medium-sized objects and not very distant from the perceiving subject, and also when such an object is supposedly normal, vision is generally one of the senses responsible for the formation of true perceptions and beliefs. On the other hand, when applied to objects very distant from the perceiving subject, or even when used in abnormal conditions for the general cognitive apparatus—for example, when under the influence of some hallucinogenic drug—it generally produces false beliefs. Thus it is said that in the first set of circumstances vision is reliable, while in circumstances such as those described in the second case, vision is no longer a reliable cognitive mechanism. Applied generally to scientific knowledge, this is to say that the conditions specifying the operation of any experiment must be given in sufficient detail that they can be precisely replicated.

In this work, we will adopt the internalist position, adding one more mode, the Infinitist, to the two that we have described (Coherentist and Foundationalist).

2 Beliefs and Perceptual Beliefs

Gershenson (2001) distinguished between relative being and absolute being. Relative being is finite and dependent on the Subject and therefore different in each individual. Absolute being is observer-independent, infinite and not materialistic. Materialism is

relative, and it is uncertain whether beings are only materialistic, because we are uncertain whether matter is absolute. We can only speculate about which things are absolutely infinite, because we are not ourselves absolutely infinite. Subjects describe beings from a relative point of view and so cannot say that something is absolutely true or false as they are relative to their contexts. The beings we describe consist in a conjunction of absolute being and relative being, and with confusion that arises from the need to define and explain something that is at the same time both absolute and relative, infinite and finite. This is what Subjects do daily. Intrinsic meaning arises for beings when we give them meaning, and then they change into signs.

A system is defined as impure when its elements include Objects and Subjects:

(1) Human beings are Subjects or agents. We can distinguish the Subject as observer (subjectively outside the system) and, the Subject by definition, within the system as an Object.
(2) As relative beings, Objects are significances Nescolarde-Selva & Usó-Doménech, 2014), and the results of Subject's perceptual beliefs about absolute beings as material or energetic objects with certain characteristics.

Linguistic codes and sub-codes that reflect attitudes, assumptions, beliefs, practices and values give context to the sense linguistic signs acquire in use. Knowledge of these rules facilitates text interpretation systems by reducing the possible relations between signifier and significance (Nescolarde-Selva & Usó-Doménech, 2014; Usó-Doménech & Nescolarde-Selva, 2012).

Definition 1 Denotation *(d-s)* is the obvious literal definition of the significance of a sign.

Definition 2 Connotation *(c-s)* includes socio-cultural and individual associations, ideologies depending on the belief systems, and emotional meanings belonging to the Subject's psychology, and they are indirectly determined by the contextual Semiotic Environment.

Both connotations and denotations depend on learned codes. Significance begins with an individual sign and acquires other meanings beyond the sign itself. There are different orders and levels of meaning or significance:

(1) Denotation is the first order of significance with a sign that is a signifier with significance.
(2) Connotation is a second-order significance using as its signifier the denotative sign (with signifier and significance) and having an additional associated significance.

This distinction makes connotation a sign derived from the signifier of a denotative sign, with the primary denotation leading to a chain of connotations.

Consequence 1 A signifier Σ or significance s functions in a way depending on the level that it operates. For this reason significance is a level of context; also it can be a signifier at another level.

As an example, an agent *S* receives two types of semiotic stimuli:

(a)   A signifier from *S*'s own processes.
(b)   A signifier of a transmitted semiotic stimulus or a connotation in the form of a signifier of a significance.

This example shows we need to distinguish two types of signifier:

Definition 3 An A-signifier *(A-Σ)* or a first order signifier is a signifier inherent in the referring context of beings, processes or phenomena.

Definition 4 A B-signifier *(B-Σ)*, is a second order signifier or connotation, and signifies a significance s.

Connotation *B-Σ* has a truth value $v(B\text{-}Σ) = 1$ and also having simultaneously a connotative or relative truth value.

Example 1 We perceive solar light rays with signifier Σ and a significance s for light, the Sun, etc., and these concepts have a truth value $v(s) \in [0, 1]$, with a relative signifier *(B-Σ)* with a truth value $v(B\text{-}Σ) = 1$.

Consequence 2 Varying signifier forms can generate different connotations.

Definition 5 The significance of a sign associated with a signifier emerges after passing through a filter or sieve, which we will call the doxastical filter consisting of the language and the belief system *BS*.

Definition 6 Belief systems *BS* (Usó-Doménech, Nescolarde-Selva, 2016; Usó-Doménech et al., 2022) consist of structures of interrelated deontic norms that may vary according to the degree to which the beliefs are systemically related.

Note 1 A belief system can be considered as a network of inferences.

Perceived Reality is constructed by means of systems of signs that are determined by means of Belief systems *BS*. Peirce (1974) (1931–1958) demonstrated that the semiotic process has been half-full culturally, that is to say, an agent S within a certain Belief system *BS* cannot understand a sign without implicitly using a socially learned system that assimilates and filters his perception to existing cognitive structures. Similarly, the sign classification in closed typologies is determined by the way the sign is used in the Belief system. Further, signs may be iconic or symbolic depending on the belief context and can only be classified with reference to the agent *S's* intentions and his belief contexts (Chandler, 1998). Within semiotic processes signs are open to uses and interpretations within any language *L* and a Belief system *BS* (Eco, 1973; Chandler, 1998).

Let *BS* be a Belief system and *L* be a language each forming a doxastical filter.

Definition 7 Each significance s is function of the components of a doxastical filter, that is to say, the Belief system and its associated language, so that *s = f(BS, L)*. This function *f* we will call the significance function.

Consequence 3 The set of possible senses or significances that vary depending on contexts and Agents is the definition of the connotation *(c-s)* of a sign.

Consequence 4 The most stable and apparently verifiable connotation is Denotation.

Consequence 5 The sign always has a connotation if the agent *S* interprets it as something that interposes, and in which case denotation is only the dominant connotation and interpreted as the true meaning of a being, process or text.

Consequence 6 The denotative truth value or the relative truth value of denotation will be $v(B\text{-}\Sigma)=1$.

Consequence 7 All the B-signifiers included in the conceptual space of the perception of Reality (context) will be denotative and will have for the agent *S* a denotative truth value equal to 1.

We call the systemic significance *s*, which is a denotative significance. We call ζ the set of signifier (signs) of Reality and ζ_Σ the set of systemic signifiers, that is to say, the set of signs that have been put in by the agent *S* establishing the system borders, so that $\zeta_\Sigma \subset \zeta$.

Definition 8 The denotative systemic significance (*d*-significance) is s_Σ if it is a function defined in ξ, $\xi_\Sigma \subset \xi$ so that if $\xi_\Sigma \subset \xi$ then $s_\Sigma(\xi_\Sigma) \subset \xi_\Sigma$.

Note 2 The denotative systemic significance *(d-s)* is the significance of the absolute beings *O*.

Note 3 The concept of relative beings coincides with the concept of denotative systemic significance *(d-s)*.

Let *S* be a subject, and *O* an object under specified conditions. Maddy's conditions (Maddy, 1990, 1996) for physical perceptions are the following:
Agent *S* perceives *O* if:

(1) *O* exists. It is an absolute being.
(2) *S* has perceptual beliefs *pB* about *O*, in terms of an appropriate sort of concepts. One talks about the concept of a physical object or relative being, rather than talking about a belief about a physical object. This is based on the assumption that having a concept of a physical object entails that one has a belief about the physical object.
(3) *O* causes *S*'s beliefs *B* about *O*.

Definition 9 The significances, that are consequence of the perceptual beliefs *pB* on the part of the Subject *S* of an object *O* with certain characteristics *C*, are called perceptual significances (*p*-significance) and denoted as *ps*.

Let *R* be a set of reasons such that $r \in R$, and *BS* be a set of beliefs such that $b \in BS$.

The most widely adopted foundation theories among epistemologists today are causal theories (Neta, 2011; Sylvian, 2016). In general terms, they maintain that a belief b is founded on a reason r only if r caused b in a non-deviant way, i.e., either r caused the formation of belief b, or r is the cause that currently supports belief b (or both). According to Moser (1991), for an agent S's belief b to be founded on reason r, S must be aware of r [*de re awareness*] (i.e., a direct, non-propositional awareness) of the fact that r supports P and, additionally, S must be in a dispositional state such that if he focused his attention only on his reasons for P (keeping everything else the same), he would focus on r.

Let $\mapsto$ be a direct cause and $\xrightarrow{t}$ be the current cause. We will adopt a scale in which the variables assume the value 1 if they have a causal influence on belief b of S and the value 0 if they do not. Thus, the set of the values of all direct causes to R other than 0 from belief b from S means assuming that such causes have an underlying causal influence on belief b from S.

Definition 10 The belief b of an agent S is based on reasons R if and only if, at time t:

(1) $\exists S / \forall r \in R, \forall b \in BS \rightarrow r \mapsto b$.
(2) $\exists S / \forall r \in R, \forall b \in BS \rightarrow r \xrightarrow{t} b$.
(3) In the case of the set of values of all direct causes other than R for belief b from S collectively that have a 0 value, then for this set of values S does not believe b, and all $r \in R$ are fixed at their present value.

Definition 11 A belief b is founded on a set of reasons R only if

(1) R rationally supports believing b.
(2) R explains why agent S believes b.

Definition 12 A meta-belief is a belief about another belief. The belief b_0 of an agent S is founded on the ratio r only if S has the meta-belief b_1 that r is a good reason for b_0.

3 Epistemic Regress and Infinite Regress

For Lewis (2003) there is an undeniable presentation of the physical object (or more generally, the causal origin of our perception) in our intuition that should be investigated. However, we should not mix that "*presentation*" (a term that we should always quote) with the metaphor of "*reflection*" in the mind of the outside world "*as it is.*" This metaphor implies, at least, that we could speak with certainty of the relationship between the physical object and the represented, which constitutes an ontological thesis explicitly repudiated by Lewis as metaphysically overloading an epistemological question. For Lewis there is a single immovable epistemological hypothesis in this regard: that our empirical knowledge implies some relationship with the external world. But Lewis knows well that we cannot exceed the limits of the a priori categories. These categories are not Kantian for Lewis, indicating that for Lewis they admit modification and do not enclose our contact with the external world

in a capsule of transcendental idealism. However, we only have our categories to provide an explanation of "*what is given*" and that is an obstacle to a supposed "*direct treatment*" of the physical-intuition object correlation. In other words, Lewis was well aware that "*epistemic regress*" is a problem (and not a minor one, but a central one) of epistemology.

What would be the answer to the question about how to justify our beliefs (perceptual beliefs) about physical objects and situations? When faced with this question, we have two options in principle:

(a) We can try to prove that these beliefs depend on others that are epistemically justified.
(b) We can try to prove that such beliefs are the fruit (i.e., they are born from) of our privileged intuition of reality.

Definition 13 The argument of epistemic regress indicates a progressive dependence on more and more unjustified assumptions, which in turn are supported by more and more unjustified assumptions, etc.

Option (a) is affected by the epistemic regress argument and this leads us either to a coherentist perspective or to embrace (b). But (b) is also not a solution, since the evidence we seek is itself another belief about the objective correlation of belief and intuition. So whatever option is selected, (a) or (b), it takes us away from a non-circular justification of belief. From this perspective, (a), despite appearing less reasonable, would ultimately be the most appropriate option. A justification based on coherence provides us with at least some pragmatic foundation that, despite leading to some form of social idealism, nevertheless provides us with a basis for knowledge. Thus, it could be said that the two options mentioned represent, respectively, the admission of the failure of the fundamentalist program in epistemology (a) and the unwarranted confidence in it (b).

The history of philosophy shows us that most of the philosophical problems are not solved, and that the positions that are conceived with a view to facing them constitute hardly better or worse ways of dealing with them. The problem of epistemic regress is no exception, as it has been a challenge for any theory of epistemic justification.

The *locus classicus* of the formulation of the problem of epistemic regress dates back to the "*Outlines of Pyrrhonism*" of Sextus Empiricus. In that text, the second of the skeptical tropes or 'Modes of Agrippa' for the suspension of the trial has to do, in the voice of Sixth Empiric, with the problem of infinite regress. Sextus Empiricus (1993, I, pp. 166–167) (Sienkiewicz, 2019) makes the following argument:

> "*By the regress ad infinitum, we justly maintain that what is adduced as proof of the proposed question requires further proof, and this in turn another, and thus ad infinitum, which results in the suspension [of assent], in the extent to which we lack a principle for our argument.*"

Thus, there are basically two reasons for rejecting infinity: (a) that our minds are finite and (b) that the regress of reasons has no beginning. Since circular reasoning is not admissible, we can imagine a *reductio*-type argument, which is apparently correct, the conclusion of which says that there is some basic knowledge or substantive belief, which is, that some of our beliefs reach the level of knowledge even when they do not. We have, but we can't even have reasons for it. Such an argument is as follows:

(1) Assumption for the *reductio*: All knowledge is the product of reasoning that transits from premises to conclusions.
(2) Either the series of premises ends upon reaching a first conclusion, or this does not happen.
(3) If there were no first premise, it would not be possible to achieve any knowledge.
(4) In case there is a first premise, either it has appeared before as part of the series, or it has not.
(5) If in fact it has appeared before, then there is a proposition that is being used as part of the evidence provided in favor of itself and, thus, circular reasoning has been produced, so such reasoning cannot give rise to knowledge.
(6) If it has not appeared before, it follows that it has simply been assumed to be true. But it is impossible for such assumptions to produce knowledge.
(7) Therefore, if all knowledge were the product of reasoning, knowledge would not exist.
(8) There is knowledge.
(9) Consequently, not all knowledge is the product of reasoning.

For Klein (2005) in epistemic regress there are three features:

(1) What requires proof or assurance are certain kinds of propositions, namely, non-obvious propositions.
(2) To give guarantees for the truth of a proposition P is to give reasons in favor of its truth.
(3) Agent *S* gives reasons in favor of *P* to increase the credibility of *P*.

The formulation of the problem is analyzed in detail by Cling (2008) in the following terms:

Let P_1, P_2,....be variables for propositions and $\underset{\text{sup}\,port}{\longrightarrow}$ be the function of supporting.

Theorem 1 *There is an epistemic regress when the following conditions* 1, 2, 3, *which are independently plausible*, *are put together.*

(1) It is possible that there is a P_1 and a P_2 such that P_2 supports P_1. $\Diamond \exists P_1, P_2 / P_1 \underset{\text{sup}\,port}{\longrightarrow} P_1$
(2) Necessarily, for all P_1 and P_2, P_2 supports P_1 only if there is a P_3 such that P_3 supports P_2. $\forall P_1, P_2,$ *if* $\exists P_3 / P_3 \underset{\text{sup}\,port}{\longrightarrow} P_2 \Rightarrow \Box P_2 \underset{\text{sup}\,port}{\longrightarrow} P_1.$
(3) Necessarily, if it is necessity that for all P_1 and P_2, P_2 supports P_1 only if P_1 is the first component of a sequence of propositions $(P_1, P_2,...)$ each component of which has a successor that supports it, then it is not possible that there is a P_1 and a P_2 such that P_2 supports P_1.

Proof In other words, by making a proposition satisfy at the same time what is described in 1, 2 and 3, an inconsistency arises that implies regress. Indeed, (1) only states that it is possible that for any proposition there is another proposition that supports it. (2) is, a regress generator. (3) is the missing proposition so that (1)-(3) are jointly inconsistent: if (2) and (3) are true (1) it must be false, that is, there cannot be, for each proposition, another proposition that supports it.

Definition 14 An infinite regression or infinite regress is a series of propositions that arises if the truth of proposition P_1 requires the support of proposition P_2, the truth of proposition P_2 requires the support of proposition P_3 …, and the truth of proposition P_{n-1} requires the support of the proposition P_n with n tending to infinity. $P_1 \xleftarrow[Jus]{} P_2 \xleftarrow[Jus]{} ... \xleftarrow[Jus]{} P_{n-1} \xleftarrow[Jus]{} P_n \xleftarrow[Jus]{} ....$

A distinction is made between infinite regressions that are "vicious" and those that are not.

Definition 15 A regression is vicious if in an attempt to solve a problem, the same problem is included in the series, if it continues on this path, the initial problem recurs and then becomes infinitely recurring and cannot be solved.

Infinite regress in consciousness is the formation of an infinite series of "*internal observers*", as when we ask ourselves the question of who is observing the departure of the neural correlates of consciousness in the study of subjective consciousness. Libertarian free will affirms that human actions have no cause and are consciously chosen, that is, they are not random. This raises a serious question: What are these conscious decisions based on? Since they cannot be based on anything (since the possibility that decisions are random is excluded), this question can be asked for each subsequent answer or answers to it, thus forming an infinite regression (Fischer, 2005).

4 Propositional and Doxastic Justification

The types of reasons that fall prey to the problem of epistemic regress are epistemic reasons. These are the kinds of reasons that authorize a proposition P to be the conclusion of an argument in which such reasons function as premises. Now, what makes these reasons epistemic is that they are in accordance with the preferred epistemic end described in a doxastic key: that is, to reach towards true beliefs and avoid false beliefs. Indeed, reasons are reasons to believe, leaving aside the idea that there can be justification of propositions without epistemic agents for whom those propositions are believed. Thus, the problem of epistemic regress is a problem expressed in propositional terms, but with doxastic scope. This may be because as the justification of beliefs depends on the justification of propositions, the problem of doxastic epistemic regress would be dependent on the problem of propositional epistemic regress. Bergmann (2007) considers that, while propositional justification could extend to infinity, the same does not happen with doxastic justification, which must stop. Turri (2010) also questions this relationship of dependency, and he considers, on the contrary, that propositional justification must be explained in terms of doxastic justification, and not the other way around. This conception of justification follows because justification is the responsibility of the subject agent S, having followed appropriate epistemic behavior according to certain deontic norms or duties such as adjusting their belief to the available evidence, not ignoring the evidence, not being careless or biased in collecting evidence, etc. The epistemic behavior must be guided by the search for truth and not by other factors such as interest, convenience or desire. According to the deontological conception, only this is relevant for epistemic justification (Alston, W. 1988).

Example 2 Daniel's belief that number seventeen will win at roulette just because today is the seventeenth and it is his birthday, is from this point of view, an irresponsible belief, because he likes the idea that the ball will stop at that number without adequate evidence and, consequently the idea is not epistemically justified: Daniel is failing to fulfill his rational obligation to form his beliefs in an adequate way.

This conception of justification suffers, according to many authors, from a problem that makes it unfeasible. Talking about deontic norms and when it is appropriate or permissible to believe, and when we are guilty if we do so, presupposes that believing or not believing is something that is under our control. It assumes that, in at least some cases, we can decide whether or not to believe something in particular. But is this so? Can we believe against the evidence we have or believe without any evidence? As much as I try, I can't believe it is night now. I can't because I'm seeing the sunshine. But it seems that I could not believe it either if, even though I had not had this experience against it, I still did not have any evidence in favor, and I simply wanted the sun to shine. According to the involuntary conceptions of belief, which are the majority (Williams, 1973), belief is not voluntary, and if we cannot believe at will, what is the use of establishing deontic norms that determine when we should believe and when we should not? If belief is not voluntary, it is something we have and not something we do, then how can we be held accountable?

This type of objection has been very influential in the perception and consideration of this conception of justification. To the point that authors such as Alvin Plantinga (1993) have chosen to replace the term "justification", which they consider linked to such approaches, by the term "warrant", which would not have such implications. There is, however, something interesting in this conception that can serve to resist such an objection. Although belief does not depend on our will, the epistemic behavior from which it results does depend on it and, regarding this behavior, deontic norms do seem to make sense. It is up to us to investigate more or less, and give more or less attention to the available evidence, etc. We could say that the will plays, with respect to belief, a role similar to that it has in perception. Although we cannot see what we want once we have looked, it is in our power to look in one direction or another (or to close our eyes). In the same way, the will plays a role in obtaining or examining the evidence, even if it ends up determining the perceptual beliefs. But is it enough that the agent is responsible in forming his belief for that belief to be justified? Isn't it possible to conceive an agent who was perfectly responsible but completely misguided regarding the truth?

On the other hand, any theory of epistemic justification is connected with the problem of knowledge. In the standard version defended by Peter Klein (1999) the position seeks to account for the so-called reflex reflective knowledge, namely, the knowledge that can only be obtained by responsible epistemic agents through careful examination of their beliefs. In this sense, Klein maintains that such a subject must respect two principles: the Principle of Avoiding Circularity and the Principle of Avoiding Arbitrariness.

Definition 16 An evidential ancestor is defined as a reason that appears previously in the justification chain.

Let S be an agent and x, y be two things. Let J be the knowledge operator $S(J)x$ reading "*S justifies x*" and $\xleftarrow{\text{evidence}}$ be the evidence support $x\left(\xleftarrow{\text{evidence}}\right)y$ reading "*y is an evidential ancestor of x*" or "*y evidently supports x*".

Principle of Avoiding Circularity For all x, if agent S, has a justification for x, then for all y, if y is an evidential ancestor of x for S, then x is not an evidential ancestor of y for S. $(\forall x, y, \exists S/S(J)x) \wedge \left(x\left(\xleftarrow[evidence]{} \right)y \right) \rightarrow \left(y\neg\left(\xleftarrow[evidence]{} \right)x \right).$

Let r_i be a reason, cause, explanation, or justification for an action or event x, $i = 1,2,...,n,$ such that rx reading *"r explains or is the cause of x"*.

Principle of Avoiding Arbitrariness For all x, if an agent S has a justification for x, then there is a reason r_1 for x, available for S; and there is a reason r_2 to r_1 available for S, etc. $\left(\forall x, \exists S/S(J)x, \exists r_1/r_1x \right) \wedge \left(\exists r_2/r_2r_1 \equiv r_2r_1x \right) \wedge .....\left(\exists r_n/r_nr_{n-1}...r_2r_1x \right).$

Klein (2007a) does not consider the regress problem as an intellectual problem but as a practical problem for responsible agents, that is, what structure of reasons provides a good model to identify propositions that are worth believing.

If justification is an epistemic value, that is, if it is something we desire for our beliefs, it is, precisely, because it is indicative of truth. We are supposed to want our beliefs to be justified because then they are more likely to be true, then between justification and truth there must be some kind of link. However, that link cannot be too strong, reaching the point that being justified would necessarily imply being true. On the contrary, most authors accept that a justified belief can be false. Sometimes all the available evidence points to the fact that something is a fact and, therefore, it is reasonable to believe that this is so, even if it turns out that it was not and that our belief is false. But on the other hand, the link between justification and truth must not be excessively weak, because if it is, if the justification does not suppose any indication of the truth of the belief, if it does not increase the probability that it is true, it loses all its value. And this is a problem that affects the deontological conceptions of justification, although not only them. Imagine two agents S_1 and S_2 deceived by the Cartesian evil genius, one of them S_1 behaves responsibly, attends carefully to evidence, does not yield to laziness or interest, etc. The other S_2 is not epistemically responsible, and he conducts his investigations carelessly. According to deontology, S_1 would be justified and S_2 not. Although it is true that such a verdict seems a rather intuitive answer, it nevertheless has the problem that we have been announcing: it leaves justification without its role of driving the truth, since in neither case will the beliefs of the agent in question come close to the truth.

There are authors who argue that evidence should be given to the propositional contents of beliefs, and others who argue that evidence should be provided to belief states. Regardless of the conditions that must be met for a question to be guaranteed or proven at this time, if it is understood that by proving a question, we are justifying it, this double characterization of the "*proposed question*" leads us to examine the distinction between propositional justification and doxastic justification.

Definition 17 Non-obvious propositions are whose truth has not been established, or on which we would acknowledge that there is no peer agreement about their truth.

Definition 18 A proposition P is available to an agent S only if there is an epistemically credible way in which S can come to believe that P, in accordance with current epistemic practices.

Definition 19 The propositional justification for a proposition P is an emergent property that it possesses by virtue of its belonging to an infinite chain, without repetitions, of reasons for *P*, available to agent *S*.

Let $|x(f(x))|$ be a satisfaction function, where x is the referent that satisfies a certain function $f(x)$. Let B the belief operator.

Definition 20 Propositional justification is the justification of a proposition, that is, of proposition P as believed by *S*. Then $\exists S \land \exists P / |S(f(S))| \Rightarrow f(S) = S(B)P$.

What gives propositional justification to a proposition P is the emergent property that it possesses by virtue of its belonging to a chain of available reasons.

Definition 21 Doxastic justification is the justification of belief, that is, of belief *b* as believed by *S*. Then $\exists S \land \exists b \in SB / |S(f(S))| \Rightarrow f(S) = S(B)b$.

To possess a doxastically justified belief an agent S must hold a meta-belief that there is a chain of reasons available for its belief.

As Woudenberg and Messter (2014) argue, the propositional/doxastic distinction is relevant because it leads to two ways of understanding the problem of epistemic regress: epistemic regress of propositional justification and epistemic regress of doxastic justification. If Woundenberg and Messter are correct, both types of regresses would require a different solution. Klein (2007a) formulates the distinction in the following terms: "*As the expression ''propositional justification'' implies, such justification is an epistemic property of propositions rather than a property of belief states. We can say that a proposition, h, is propositionally justified for S just in case there is an epistemically adequate basis for h that is available to S regardless of whether S believes that h, or whether S is aware that there is such a basis, or whether if S believes that h, then S believes h on that basis. […] Beliefs, i.e., belief states, are the bearers of doxastic justification. A belief that h is doxastically justified for S when and only when S is acting in an epistemically responsible manner in believing that h. (…) For a self-conscious epistemic agent—an agent who practices what she preaches—what constitutes a responsibly held epistemic belief will depend upon what that agent thinks is required for a proposition to be justified for her.*"

Pryor (2001) makes a formulation similar to that of Klein, although without labeling the two types of justification and with the purpose of highlighting two different forms of epistemic evaluation within the debate between internists and externalists. Alston (2005, p. 18) characterizes the distinction in the following terms:

"*A proposition, P, may be said to be justified for S provided S is so situated that if he were to make use of that situation to form a belief that P, that belief would be justified. (S can be justified vis-à-vis P in this sense even if one does have a justified belief that P. It is just that in attributing justification in this sense to S one is leaving it open whether S has a belief that P). In the doxastic sense one is justified in believing that P provided one has a belief that P which is justified.*"

Certainly, these formulations do not seem entirely the same, at least as far as doxastic justification is concerned. Indeed, all three authors could agree that propositional justification is an epistemic property of propositions, such that a proposition P has that property for any agent *S*, regardless of whether agent *S* believes that *P*; but in relation to doxastic

justification, we see that, for Alston and Pryor, this can be understood as an epistemic property of beliefs, whose justification depends on whether its propositional content is adequately justified. Although doxastic justification is also an epistemic property of beliefs for Klein, it is conceived as a matter of believing based on what agent S thinks is required for proposition P to be justified.

Klein (1999, 2007a) makes explicit the distinction between propositional justification and doxastic justification and develops the debate on the structure of epistemic justification based on the type of justification considered by different theories. For this author, the debate brings together on one hand, Foundationalism, Coherentism and Infinitism in relation to propositional justification, and from another, foundational, coherentist and infinitist positions in relation to doxastic justification. Analyzes like Klein's are illuminating since, unfortunately, the positions on the structure of epistemic justification are not always clearly stated enough to determine what type of problem is being solved. Lack of precision could lead to mixing, without any qualms, a solution to the problem in propositional terms with a solution in doxastic terms, or vice versa, and generate, in some cases, the so-called "*dialogue of the deaf*" between different exponents of the structure of justification. In addition to the aforementioned distinction, there is another in relation to the ambiguity that the very concept of justification entails. According to Alston (2005, 18), in statements of the type "*S's belief is justified*", "*S has a justified belief*" or "*S is justified in believing that P*" the notion of justification can refer to the status or property of a belief, or to the activity of showing the justification of a belief.

From this point of view, the justification status of a belief refers to the epistemic property belonging to the belief itself, while the activity of justifying a belief consists of the action by agent *S* to demonstrate the epistemic status of justification of his belief, which will be a function of a number of reasons of which he is aware and able to quote. We differentiate two hypotheses:

(1) **Hypothesis 1**: Investigating the status of justification, whether from a belief or a proposition, is a function of the activity of justification.
(2) **Hypothesis 2**: Determining the justification status of a belief or proposition is a question independent of the activity of justification.

In hypothesis 1 two possible scenarios are admitted:

(1) In the first scenario, the activity of justification of a belief is performed, citing the appropriate reasons, which effectively confers justification status on that belief, regardless of whether agent S thinks so or not. If this were the case, we would be initiating the *appropriate* activity to establish *justification*.
(2) In the second scenario, the activity of justification of a belief is performed by citing the reasons that seem reasonable to the agent *S*, that is, the reasons that *S* considers that give the belief justification status. In this case we are identifying *justification status*.

Both scenarios are not exclusive. An agent *S* may cite the reasons that he considers reasonable in favor of a belief, and these reasons may coincide with those that effectively confer justification status on the belief. But it can also happen that failed cases of adequate justification are successful cases of reasonable justification (Woods, 2013, p. 104). For hypothesis 1, the problem of epistemic regress is seen in terms of the activity to justify adequately, or reasonably. Indeed, the main difficulty with these kinds of hypotheses is that

they do not give the criteria to determine when the reasons given by the agent are objectively the right reasons, that is, regardless of what the agent considers are the right reasons. In other words, this hypothesis is accused of not providing criteria outside the subjective sphere that allow us to distinguish truth from justification.

Hypothesis 2 considers that the activity of justifying does not reveal or is relevant to the justification status of a belief, but justification is a function of, for example, the correct functioning of certain reliable cognitive processes. The question of what confers the status of justification on a belief motivates the dispute between the internalist and externalist schools of justification. Most internalists commit to the idea that the justification status of a belief is a function of the reasons given by an agent in favor of it, while the externalists consider that the status of justification of a belief is not a function of such activity. In this way, those who analyze the status of justification from the activity of justification, have an internal perspective of the debate on epistemic justification, while those who analyze the debate regardless of the reasons that may be given in the activity of justification, approach the problem from an external point of view.

Thus, although these theories do not deny that the justifying activity exists, they consider that the problem of epistemological concern on justification should not be analyzed in terms of the activity of justifying. Viewed in this way, there is a proclivity for status/activity ambiguity especially when justification status is considered to be a supervening property of a set of potential beliefs-reasons that an agent would be willing to cite. This suggests that the status/activity ambiguity could be relevant to understand the forms of solution that have been devised.

From our point of view, it can be argued that, on the one hand, a proposition P has the status of justification and, on the other, that there is an activity of justifying a proposition P, regardless of whether it is believed. Indeed, an agent *S* can carry out the activity of justifying a proposition P without believing it.

Example 3 When a professor mentions the reasons in favor of an author's (Marx, Hegel or Heidegger) thesis in the exercise of a teaching task, with the purpose of explaining or teaching it to the students, but he does not believe in the truth of that thesis.

Another case is the one in which proposition P still has the status of a hypothesis, in which case carrying out the justifying activity is intended to make that proposition P part of the heritage of our beliefs. In other words, we argue that *the activity of justifying a proposition P can be distinguished from the justifying status of that proposition*. Of course, one might think that the justifying status of a proposition would be supervening from the activity of justifying, but it is also possible to conceive of the justifying status of a proposition P regardless of whether there is any current justifying activity for such a proposition, by some agent *S*.

If it were the case that, for a proposition P, no agent *S* has ever performed the act of justifying it in a sufficiently satisfactory manner, it cannot be said that there is a status of justification for it. But, if for that proposition there already exists or has existed an action on the part of an agent *S* that demonstrated it, that proposition P is justified regardless of whether an activity is currently being carried out by some agent *S* that justifies it.

Example 4 Suppose S utters in 1994 that $P =$ "*Fermat's last theorem is true,*" and imagine that same person *S* uttering the same proposition in late 1995. Before Andrew Wiles carried out his famous proof on the truth of Fermat's theorem, the proposition P could

not be said to be a proposition with justification status. It is only after Wiles' proof that proposition P acquires justification status. Now, that proposition is still justified regardless of whether someone, at this moment, in the whole world, could show why it is. In other words, its justification status, although it depended on a justification activity by Wiles, does not depend on any current activity that proves it.

Of course, it is legitimate to ask what would happen to the status of justification of the proposition P = "*Fermat's last theorem is true*" if humanity disappeared, with all its proof? Does proposition P stop having justification status? Apparently, nothing can have the status of justification if this status is not addressed to someone S in a given time *t*. The plausible answer is that the justification status of the propositions is lost if the activity record that demonstrates it and the agent S for which that proposition P and that demonstration were available are lost. However, the realistic mathematical or Neoplatonic School does not have to accept the latter. Accordingly, even if, after human extinction, a new intelligent race reappeared unaware of the proof, the proposition P = "*Fermat's theorem is true*" would lose its justification status. In any case, the supervenience relationship between activity and the justification status of propositions does not prevent the distinction between activity to justify a proposition and the justification status of a proposition. The same distinction between adequate justification and reasonable justification would apply in this case. Not every reasonable act of justifying a proposition constitutes an act of adequate justification of propositions.

In summary, we can distinguish, on the one hand, the justifying status of a proposition or belief and, on the other, the activity of justifying that belief or proposition. Now, although it could be thought that either the justification status of a belief is a function of the activity of justifying, or that such status is independent of that activity in arguing, for example, that justifying status can be obtained as a function of reliable cognitive processes, we should not confuse justifying status with the activity of justifying itself.

One question we must analyze is how can the propositional justification available to an agent S properly ground his belief? In order to analyze the question, it is convenient to keep in mind certain basic conditions of adequacy that all foundation theory must satisfy. More specifically, we must note the distinction between *reasons for which*, an agent S holds a belief and the *reasons why*, an agent S holds a belief *b*. The *reasons for which* we believe are always *reasons why* we believe, but not vice versa.

Example 5 Suppose agent S receives a severe blow to the head due to an accident and as a consequence S begins to believe that (the accident) is the reason why S believes that Spain is in Europe, but it is not the reason why S believes it, which is instead (suppose) a set of propositions that S learned in geography manuals.

In discussing the founding relationship, we address the reasons for which an agent S believes, that is, those reasons that not only cause belief *b* but motivate it, or, in other words, those reasons that are 'operational' for this belief.

4.1 Regress of Propositional Justification

Let $\Pi = \{P_1, P_2, ..., P_{n-1}, P_n, ...\}$ be an enumerable set of Propositions such that $card\Pi = \aleph_0$. Let $\xleftarrow{Jus}$ be the justification support $X \xleftarrow{Jus} Y$ it reads "*X is justified by Y*" and $\rightarrow_{der}$ be the derivation or inference support $X \rightarrow_{der} Y$ it reads "*X infers Y*".

We can understand these positions as three types of possible response to what is known as Agrippa's trilemma.

The Münchhausen trilemma or Agrippa's trilemma: Whatever the way in which a proposition P is justified, if what is wanted is absolute certainty, it will always be necessary to justify the means of justification, and then the means of that new justification, etc. This simple observation leads without escape to one of the following three alternatives (the three horns of the trilemma):

(1) *An infinite regression or regress of justifications*: P_1 is justified by P_2, P_2 is justified by P_3, P_3 is justified by P_4, etc. $P_1 \underset{Jus}{\longleftarrow} P_2 \underset{Jus}{\longleftarrow} ... \underset{Jus}{\longleftarrow} P_{n-1} \underset{Jus}{\longleftarrow} P_n \underset{Jus}{\longleftarrow} ...$. The need to go back more and more in the search for foundations, an endless process that cannot be carried out, which does not provide any secure foundation.

(2) *An arbitrary cut in reasoning*: P_1 is justified by P_2, P_2 is justified by P_3, and P_n is not justified. $P_1 \underset{Jus}{\longleftarrow} P_2 \underset{Jus}{\longleftarrow} ... \underset{Jus}{\longleftarrow} P_{n-1} \underset{Jus}{\longleftarrow} P_n$. This last proposition can be presented as common sense or as a fundamental principle (postulate or axiom), but in any case it would represent an arbitrary suspension of the principle of sufficient reason resorting to a dogma.

(3) *A circular justification or dialelo*: P_1 is justified by P_2, P_2 is justified by P_3, and P_n is justified by P_1.
$$P_1 \underset{Jus}{\longleftarrow} P_2 \underset{Jus}{\longleftarrow} ... \underset{Jus}{\longleftarrow} P_{n-1} \underset{Jus}{\longleftarrow} P_n$$
$$\underset{Jus}{\downarrow} \rightarrow_{Jus} .... \rightarrow_{Jus} \underset{Jus}{\uparrow}$$
In the process of justification recourse is made to statements that have previously been shown as statements that require justification and therefore without ever coming to a sure justification without being logically flawed.

4.2 Regress of Doxastic Justification

We will remember concepts such as substantive or basic belief and derived or inferential belief.

Definition 22 The more important and basic beliefs of a BS-belief system are the substantive beliefs *(sb)* and constitute the axioms of the system. (Nescolarde-Selva & Usó-Doménech, 2013; Nescolarde-Selva et al., 2015a, 2015b; Usó-Doménech et al., 2022).

Statements such as "*All the power to the people, Black is Beautiful, God exists*", are examples of the content of some BS-belief systems. Substantive beliefs are the focus of interest for believers. Derived beliefs *(df)* are based on substantive beliefs.

Definition 23 The derived or inferential beliefs *db* originate in or are produced or inferred from substantive beliefs *sb*. Derived beliefs may also constitute system theorems.

Over time, some derived beliefs achieve the status of substantive beliefs, giving origin to a more ample body of substantive beliefs that become dogmas. Simultaneously, there is a dynamic involved in a set of beliefs so that the importance of some substantive beliefs increases and others decrease in importance or disappear. Substantive and derived beliefs form sets that may be graphed as a topological structure representing how an individual

organizes concepts and propositions hierarchically in his cognitive structure (Nescolarde-Selva & Usó-Doménech, 2013a, b, 2016; Usó-Doménech et al., 2022).

According to this argument, if a belief is justified, then either it is a substantive belief, or it is justified inferentially from other beliefs. But there are no substantive beliefs, so, if justified, all beliefs would be justified by a chain of inferences. But there is no satisfactory justification chain, because either:

(1) Leads to an infinite regress.

$$sb \rightarrow_{der} d(sb) \rightarrow_{der} d(d(sb)) \rightarrow_{der} d(d(d(sb))) \rightarrow_{der} \ ......$$
$$sb \underset{Jus}{\longleftarrow} d(sb) \underset{Jus}{\longleftarrow} d(d(sb)) \underset{Jus}{\longleftarrow} d(d(d(sb))) \underset{Jus}{\longleftarrow} \ ......$$

(2) There is an arbitrary interruption of the chain of reasons (for example because it is considered that some such reason no longer needs justification).

$$sb \rightarrow_{der} d(sb) \rightarrow_{der} d(d(sb)) \rightarrow_{der} d(d(d(sb))) \rightarrow_{der} \ ...... \rightarrow_{der} d(d(...d(d(sb))))$$
$$sb \underset{Jus}{\longleftarrow} d(sb) \underset{Jus}{\longleftarrow} d(d(sb)) \underset{Jus}{\longleftarrow} d(d(d(sb))) \underset{Jus}{\longleftarrow} \ ...... \underset{Jus}{\longleftarrow} d(d(...d(d(sb))))$$

(3) It incurs a vicious circle.

$$sb \rightarrow_{der} d(sb) \rightarrow_{der} d(d(sb)) \rightarrow_{der} d(d(d(sb)))$$
$$\underset{der}{\uparrow} \overset{\longleftarrow}{}_{der} .... \overset{\longleftarrow}{}_{der} \underset{der}{\downarrow}$$

$$sb \underset{Jus}{\longleftarrow} d(sb) \underset{Jus}{\longleftarrow} d(d(sb)) \underset{Jus}{\longleftarrow} d(d(d(sb)))$$
$$\underset{Jus}{\downarrow} \rightarrow_{Jus} .... \rightarrow_{Jus} \underset{Jus}{\uparrow}$$

Note 4 The derivation chain runs in the opposite direction to the justification chain.

We are going to face the problem, which is typically associated with four traditional hypotheses in relation to the debate on the structure of justification, exposing in a still general and imprecise way and only within the framework of the doxastic and without taking into account the status/distinction of the justification. All these hypotheses constitute attempts to explain the fact that there may be justified beliefs and are consequently conceived as responses to skepticism. We will face the problem of doxastic regress in the following terms:

1. The regress ends in a substantive unwarranted belief.
2. There is an infinite regress of the justification of beliefs.
3. There is a circle of justification for beliefs.
4. The regress ends in a substantive belief justified in a different way than being justified by another derived belief.

Hypothesis 1 is the least popular among justification theorists, as it has been seen as a failed attempt at the problem of regress, and thus as the easiest prey for skepticism. Hypotheses (2), (3) and (4) constitute the traditional theories in the debate about the structure of epistemic justification. Hypothesis (2) is typical of infinite approximations.

Hypothesis (3) is traditionally associated with the coherent nature of justification. Finally, hypothesis (4) is the solution to the regress problem arising from different forms of foundations.

4.3 Chains of Justification

The chains of justification, both doxastic and propositional, can be of two kinds: infinite and finite. Some of the reasons (propositions) of the chain constitute the content of occurring beliefs of agent *S*, some others constitute the content of non-occurring beliefs of *S* (*first order dispositions* to form a belief with said content) and many others constitute the content of *second order dispositions* to form a belief by virtue of certain epistemic practices. Therefore, proposition P may be available to an agent *S* simply because there is a way that agent *S* might come to believe it.

4.3.1 Infinite Chains

To avoid the objection that the mind is finite we must grant that every infinite justification chain contains second order dispositions, because it is impossible for beings like us to harbor an infinite amount of beliefs, whether they are occurring or non-occurring.

Example 6 Consider the proposition P = "*Paris is further north than Barcelona*". In order for this proposition P to be available to an agent *S* and, consequently, to be part of the infinite chain of reasons that justifies any belief for *S*, let's say that in Paris it rains more than in Barcelona, it is not necessary for *S* to believe this to believe proposition P. But if *S* has at home a world map indicating countries that he consults frequently, this is enough to see that Paris is further north than Barcelona, it is an available reason for him, that is, there is a credible way in which *S* could arrive to form this belief, given current epistemic practices. Now the question is, how could proposition P, in a case like this, properly found, even partially, the belief that it rains more in Paris than in Barcelona? It seems clear that *Paris is further north than Barcelona*, cannot be part of the (causal) explanation of why *S* believes that it rains more in Paris than in Barcelona which *S* does not believe nor is willing to believe. It is implausible to suppose that the mere willingness to consult a map to corroborate the latitudes of Paris and Barcelona explains why *S* holds the belief that *it rains more in Paris than in Barcelona*, even before the consultation has taken place.

From the previous example we can infer that second order dispositions cannot function as operative reasons to causally explain why a subject *S* holds a belief. Thus, since the infinite chain of reasons that justifies a proposition P necessarily includes second-order dispositions, it follows that the entire chain cannot possibly contribute to the causal explanation of the belief, and therefore cannot properly ground it. Therefore, we can infer that second-order dispositions cannot function as operational reasons to causally explain why an agent *S* holds a belief.

4.3.2 Finite Chains

Consider an infinite chain, we can assume that what bases a belief is not the entire chain but only a part of it, as long as it is sufficient to satisfy the demands established by the

epistemic standards in force in the context. A belief is never fully justified in the doxastic sense, since, as the finite mind objection points out, it is not possible for beings like us to possess an infinite set of beliefs that function as reasons. However, the doxastic justification of a belief is always partial: it is more or less justified depending on how many reasons in the chain agent S has cited. Suppose that in a certain context S has provided reasons in favor of belief b_1, going as far as link b_4 such that $b_1 \xleftarrow{Jus} b_2 \xleftarrow{Jus} b_3 \xleftarrow{Jus} b_4$ and that this is sufficient to satisfy the standard context. Is b_1 justified doxastically? The problem is that in order to be, the belief must be based on a finite set of reasons. According to Klein (2007b) a belief is doxastically subjectively justified if and only if it is inferred from another belief that is a reason, regardless of whether that reason is propositionally justified, that is, regardless of whether it is part of an infinite chain of reasons available. On the other hand, a belief is doxastically objectively justified only if the reason in question is propositionally justified (Klein, 2007b), that is, if it is part of an infinite chain of available reasons. If a belief is doxastically objectively justified, its content is propositionally justified, and the belief on which it is based is potentially objectively justified, since there will be a reason available for it. And the propositional content of a belief that is merely doxastically subjectively justified is not propositionally justified, and the propositional content on which it is founded is not even potentially doxastically objectively justified. Therefore, a doxastic chain of justification is not infinite, but the beliefs founded on this chain can be objectively or subjectively doxastically justified, depending on whether or not there is an infinite chain of available propositions.

Suppose that agent S_1 provides sufficient reasons for belief b, given the standard context. Likewise, b has a propositional justification, that is, the proposition P that constitutes the content of b belongs to an infinite chain of ratios available for S_1. This means that b is doxastically objectively justified for S_1. Another agent S_2, meanwhile, has provided the same reasons that S_1 in favor of b is in the same context, so the same standards are in effect, but there is no infinite chain of reasons available to it, that is, S_2 has no propositional justification. In that case, S_2's belief is only doxastically subjectively justified. Now, the belief of S_1 enjoys an epistemic status superior to that of the belief of S_2. If the standard context became a little more demanding, S_1 would be able to meet the challenge by providing additional reasons for b, while S_2 would not be able to do so. This means that S_1 is epistemically responsible while S_2 is not. Now, the difference in epistemic status between the beliefs of S_1 and those of S_2 cannot but be due to the fact that the belief of the former has propositional justification while the belief of the latter does not, given that both beliefs are the same in the other aspects. However, if the propositional justification, that is, the property having a proposition belonging to an infinite set of available reasons, makes a difference with respect to the epistemic status of a belief, this must be because it plays a role in its foundation, that is, it explains why the agent has belief. Thus, it is not possible for S_1 to base his belief on only one (finite) part of the chain, since in that case it would not be possible to explain the difference in epistemic status between his belief and that of S_2. In other words, if the superior epistemic status of the belief of S_1 depends on whether its content belongs to an infinite chain of reasons, it must be because that belief is based on that chain in an appropriate way.

Corollary 1 *It is a dilemma*:

(a) *Either the belief of S_1 enjoys an epistemic status superior to that of S_2, in which case it must be founded on the infinite chain of reasons in its entirety.*

(b) *Or the belief of S_1 is founded on a finite set of reasons, in which case it is not possible to explain what gives your belief an epistemic status superior to that of S_2.*

4.4 Epistemic Regress Modes

The propositional/doxastic and status/activity distinctions give rise to four modes of epistemic regress.

(1) *Epistemic regress of doxastic justification of status.* There is a regress of doxastic justification status when, for each belief (or doxastic state) that is justified, or has the property of being justified, there must be another belief (or doxastic state) with this property that supports or guarantees it.

   (a) *Infinitism:* There is infinite regress in determining the justification status of beliefs.
   (b) *Coherentism:* There is a circle in determining the justification status of beliefs.
   (c) *Foundationalism:* The regress ends in a substantive belief whose status is substantive justified belief.

(2) *Epistemic regress of doxastic justification activity.* There is regress of doxastic justification activity when it is affirmed that the reasons that an agent *S* cites to support or guarantee some of his beliefs require, in turn, another set of reasons or guarantees that S believes and must be able to cite.

   (a) *Infinitism:* There is infinite regress in the activity of justifying beliefs.
   (b) *Coherentism:* There is a circle in the activity of justification of beliefs.
   (c) *Foundationalism:* The regress ends in a substantive belief justified by some activity of justification.

(3) *Epistemic regress of propositional justification of status.* There is a regress of propositional justification of status when, for each proposition P_2 that is justified, or has the property of being justified, there must be another proposition P_1, with this property, which supports it.

   (a) *Infinitism:* There is infinite regress in determining the justification status of propositions.
   (b) *Coherentism:* There is a circle in determining the justification status of propositions.
   (c) *Foundationalism:* The regress ends in a basic proposition whose status is justified basic proposition

(4) *Epistemic regress of propositional justification activity.* There is a regress of propositional justifications activity when the propositions that an agent *S* cites to guarantee

each proposition require, in turn, other propositions that *S* could cite, regardless of whether *S* creates them or not.

(a) *Infinitism:* There is infinite regress in the activity of justifying propositions.
(b) *Coherentism:* There is a circle in the activity of justification of the propositions.
(c) *Foundationalism:* The regress ends in a basic proposition justified by some justifying activity.

5 The Three Modes of Epistemic Regress

If the distinction between internalist and externalist conceptions of justification is relative to the nature of justifiers, that is, what kind of entities or phenomena can be justifiers, there is another classification of theories about justification that responds to the different conception that one has regarding its structure. Depending on how the structure of justification is conceived, we can distinguish between its four modes: The regress ends in an unwarranted status/activity, Infinitism, Coherentism and Foundationalism. The main theories regarding the structure of justification are Coherentism and Foundationalism and, to a lesser extent, Infinitism. Foundationalism opposes Agrippa's trilemma by affirming that substantive beliefs do exist or, arguing that the chain of reasons has an end that is not arbitrary. For its part, Coherentism proposes that the reasons form a circle that is not an obstacle to the justification of beliefs. The main difference between the two theories is that, while Foundationalism defends the existence of two types of beliefs according to their justification (substantive and derived or inferential), Coherentism maintains a doxastic monism, that is, that all beliefs are of the same type as regards their justification: all are inferential, and all receive their justification from other beliefs. This is what is called the doxastic conception of justification (only one belief can justify another belief), and according to Davidson (1986) it is the essential feature of Coherentism. Both theories are opposed to justification being able to incur an infinite regress. This is precisely what Infinitism admits, a position, in principle, more counterintuitive, but also supported by Fantl (2003) and Klein (2005, 2012).

5.1 Infinitism

Infinitism maintains that the chain of reasons that justifies a belief must be infinite. Although this position was always recognized, along with Foundationalism and Coherentism as one of the possible solutions to the classic problem of infinite regress, it was not until recently that Infinitism was seriously considered. The main difference between this theory and its rivals lies in the way that Infinitism conceives of justification. If for Foundationalism the reasons merely convey justification of the derived beliefs from the substantive ones, for the Infinitist the reasons increase the justification of the belief they support. Thus, to offer reasons in favor of a belief is not only to transmit the justification from the reasons to the belief, but to improve the justification of that belief. The types of Infinitism that would face the problem of epistemic regress include the following:

5.1.1 I1. Infinity of the Doxastic Justification of Status

It would face the problem of epistemic regress understood in a doxastic version of status. For I1 the justifying status of a belief leads to an infinite (non-circular) regress of justification. For every belief with justifying status, there must always be another belief with this status that supports it, which, in turn, obtained that status by virtue of another belief with justifying status, and the latter by another equal belief, and so on ad infinitum. As every belief is in principle justifiable by another, for I1 the status of the justification of any belief implies a potentially infinite inferential regress of beliefs. Consequently, I1 understands that the justification of a belief is the result of an inferential process that, as it expands in the chain of beliefs that support a belief, widens or increases the justification of that initial or substantive belief. Of course, this idea of increase implies that justification is gradual, which, for the doxastic case, means that there are more or less justified beliefs than others. I1, therefore, must explain the gradual nature of the doxastic justification. But, since a belief cannot be more or less justified only by virtue of the "size" of the chain of beliefs that supports it, Infinitism must distinguish between adequate doxastic justification and complete doxastic justification.

5.1.2 I2. Infinitism of the Doxastic Justification Activity

It would face the problem of epistemic regress understood as a doxastic activity. According to I2, for all beliefs held by an agent *S*, there is a reason that *S* believes and can cite to guarantee his initial belief, and on this last belief there must be another additional belief that supports it, which *S* would also be capable of to quote to support it, and so on ad infinitum.

5.1.3 I3. Infinitism of the Propositional Justification Of Status

It would face the problem of the regress in its propositional version of status. For I3, any proposition P has the status of justification by virtue of another proposition P' from which it receives support or guarantee, and this last proposition P' is justified in function of another proposition P'' with that status, and the latter on the other, and so on ad infinitum.

5.1.4 I.4. Infinitism of the Propositional Justification Activity

This requires understanding the problem of the activities concerning propositions. According to this form of Infinitism, for every proposition P defended by an agent *S*—regardless of whether or not he believes it—there is a proposition P' that *S* can cite to guarantee the initial proposition P, and for this proposition P' there must be another additional proposition P'', which *S* would also be able to cite to defend or guarantee it, and so on ad infinitum.

5.2 Coherentism

If the metaphor that characterized Foundationalism was building, it is networking that defines Coherentism (BonJour, 1985; Lehrer, 1990). According to this conception of justification, our cognitive system is a lattice structure, a network of multiple and

mutual relationships between beliefs, in such a way that it is that bond, and not certain particular unidirectional beliefs, that justifies the system's beliefs. A belief is justified if it is part of a coherent set of beliefs and to the extent that its inclusion increases that coherence. It is a holistic conception of justification; this is a matter of the whole belief system and not of particular linear relationships between certain beliefs. Justification, then, does not maintain a linear and asymmetric structure, but is a relationship of mutual support, in which a belief, which is the conclusion arising from other beliefs that act as premises for its justification, that can in turn serve as a premise for the justification of some of the beliefs that have served as a premise. Thus, all beliefs are of the same type regarding their justification, and since they are all derived beliefs, there are no substantive beliefs. If it is the coherence of the system and the coherent relationships between the different beliefs that form it that determines the justification of any of them, it is important to specify in what the coherence consists. A first, obvious condition is logical consistency: there must be no contradiction between its members. But a coherent system is more than a mere aggregate, it is a structured whole in which its members must keep certain relationships. And the problem consists precisely in determining the type of inferential relations that must occur between its members. Although there is no agreement among the defenders of this school, it seems that these should be explanatory relationships: some beliefs should serve as explanations for others.

Example 7 For example, the set formed by the beliefs "*The car is in the garage*", "*Yesterday was Saturday*", "*Sarah is at home*", being consistent would not form a coherent set (although not inconsistent), because there does not seem to be any relationship between them. On the other hand, if the fact that "*there are clothes hanging out to dry*" could be explained because if "*Sarah is at home*", "*the car is in the garage*" and "*Sarah stays at home resting on Sundays*", then there would be a relationship between them.

However, there are examples that would show that not all inferential relationships between beliefs can be reduced to explanatory relationships (BonJour, 1985, p. 100). This formulation can be read from two perspectives on the justification structure. The first is the *structural-linear perspective*, and the second is the *structural-holistic perspective*.

(1) Under the *structural-linear perspective*, any attempt at circularity is naturally vicious, since it would not be legitimate within the system to support a statement of the structure in the statements that the statement supports. This is the reason why the Foundationalists and Infinitists, who are located in the structural-linear perspective, reject the circular strategy as an attempt to solve the problem of regress.

(2) The Coherentist replaces the linear, unidirectional and asymmetric structure perspective of justification with a holistic, multidirectional and symmetrical conception of the structure of justification, which overcomes the vicious character that a circular structure can have. The reason is that, under this *structural-holistic perspective*, when defining the epistemic status of a belief in terms of the mutual support that it has with other beliefs of the total system that it integrates, no belief enjoys a kind of epistemic priority within the system of which it is part. In other words, in a structural-holistic perspective, no belief is more fundamental than any other, which allows us to think about the structure of justification between the beliefs of a system in terms of reciprocity or mutual support without the circular closure of the system being considered

vicious (Quine, 1969), but being considered virtuous (BonJour, 1985). Therefore, the concept of justification is applicable, more than to isolated beliefs, but to a collection or group of them understood as a symmetrically interrelated system. From the coherentist-holistic perspective, therefore, there is not a stock of substantive beliefs and another stock of derived beliefs.

Coherentism has four classes.

5.2.1 C1. Coherentism of Doxastic Justification of Status

Would face the problem of epistemic regress understood in a doxastic version of status. According to C1, it is possible that a b_1 belief has justification status by virtue of another b_2 belief that supports it, and that this b_2 belief has justification status by virtue of being supported by b_3 belief. For C1 understood in a holistic version, it is the beliefs that keep the relations of mutual support by virtue of which the coherence of the system emerges. Consequently, it must provide a satisfactory explanation of an obvious relationship, namely, the relationship between beliefs about experience and other perceptual states. To give such an explanation, in C1 it is assumed that perceptual states fulfill a double role: one justifiable and the other causal. In their justifying role, these perceptual states have a propositional content that is not involved in their causal function. If the perceptual states are taken in their causal role, that is, independently of their propositional content, it is not possible to establish the logical relationships between the objects of these states and the propositional content of the beliefs that they are supposed to justify. If they are taken in their justifying role, that is, depending on their propositional content, it is necessary to believe in the propositional content of those perceptual states in order for them to be part of the justification process. In the latter case, it would be the beliefs in relation to the propositional contents of the perceptual states that make possible the justification of other (non-perceptual) later beliefs. Under the perspective in which the perceptual states lack propositional content, the only relationship that can exist between them and the empirical beliefs would be, then, of a causal and not justifying nature. For this reason, C1 is committed to the idea that the relation of justification is a relation between propositional contents. C1 assumes that the dynamics of justification remain circumscribed to the doxastic framework of beliefs, even though certain perceptual states are involved in these dynamics, which, in causal roles, do not have justifying implications.

5.2.2 C.2. Coherentism of the Doxastic Justification Activity

This position would face the problem of epistemic regress understood as a doxastic consideration of activity. According to C2, for every belief b_2, the agent S that has it can quote another belief b_1 to support it, and to support the latter, S can quote belief b_1.

5.2.3 C3. Coherentism of the Propositional Justification of Status

This position would face the problem of regress in its propositional version of status. For this type of Coherentism, it is possible for a P_2 proposition to have justification status by virtue of another supporting P_1 proposition, and this P_1 proposition to have justification status by virtue of being supported by proposition P_2. C3 does not have to commit to a

doxastic conception of justification (i.e., only beliefs justify beliefs), but commits to the idea that inferential relations of justification can only occur between propositional contents.

5.2.4 C4. Coherentism of the Propositional Justification Activity

This position would face the problem under activities in relation to propositions. According to this form of Coherentism, regardless of whether agent S believes the propositions he utters or not, for every proposition P_2 uttered, the agent may quote another proposition P_1 to support it, and to support the latter, S may quote the proposition P_2. The advantage that C4 has over its doxastic version is that it avoids any type of objection focused on the restrictive or limited nature of human cognition.

5.3 Foundationalism

Foundationalism is the dominant position and until the nineteenth century it was the only one. This theory maintains an architectural conception of justification. Our cognitive system is like a building whose foundations are the substantive beliefs on which the other derived beliefs are based. Substantive beliefs do not receive their justification from any other belief, either because, given their character, they justify themselves (since they are incorrigible), or because they receive their justification from other types of entities or phenomena, such as, for example, perceptual experience. The first option would correspond to classical Foundationalism, which considers, in its empiricist version, that substantive beliefs are beliefs about sensitive data, about what is given (immediately) in experience (Ayer, 1956, Chapter II). In other versions of this type of Foundationalism, certain intuitions have been proposed as basic beliefs or, in the case of Descartes, indubitable beliefs such as the certainty in my existence as a thinking being (the famous cogito: *I think therefore I am*). Under the second option, substantive beliefs can be about physical objects (and not merely appearances, as was the case in the other type of Foundationalism). Unlike classical Foundationalism, this second version does not consider that substantive beliefs should be incorrigible (Nescolarde-Selva et al., 2015a, 2015b).

The rest of the beliefs of the cognitive system receive their justification inferentially—ultimately—from the substantive beliefs; the latter are its support. The justification is thus an asymmetric and linear structure. Derived beliefs are conditionally justified, justified insofar as those beliefs that have been inferred are justified and, ultimately, can only be justified if, at the end of the chain of justifications, we find some substantive belief. These are intended to avoid an infinite regress or a vicious circle in justification. If, at the end of the chain of justification, there are no beliefs that do not receive their justification from other beliefs, then none of them would be justified, because either an infinite regress or a vicious circle would occur.

Finally, we note that the four perspectives of foundationalism appear in relation to the 4 ways of understanding the problem of regress.

5.3.1 F1. Foundationalism of the Doxastic Justification of Status

This position would face the problem understood in the doxastic version of status. According to F1, regress must end in a substantive belief with a justification status acquired differently from the way derived beliefs have acquired it.

5.3.2 F2. Foundationalism of the Doxastic Justification Activity

This position would face the problem of epistemic regress understood in a doxastic justification activity. For F2, epistemic regress is avoided because, at the end of the chain of reason-beliefs cited by agent *S*, he cites a substantive reason-belief whose justification has been obtained in a different way than the way derived beliefs have acquired it.

5.3.3 F3. Foundationalism of the Propositional Justification of Status

This position would face the problem of the regress in its propositional version of status. For F3 a proposition obtains the status of epistemic justification if the regress ends in a justified basic proposition, the status of which has been obtained differently from the way non-basic or derived propositions have acquired it.

5.3.4 F4. Foundationalism of the Propositional Justification Activity

This position understands the problem in terms of activities concerning propositions. For F4, epistemic regress is avoided because, at the end of the chain of reasons-propositions cited by agent *S*, he cites a basic reason-proposition—regardless of whether he creates it—the justification of which has been obtained differently from that the way non-basic or derived propositions have acquired it.

6 Criticisms

6.1 Criticism of Infinitism

Every infinitist position is inferential. That is to say, it maintains that the beliefs or propositions acquire their justifying status by virtue of the inferential relations that they keep with other beliefs or propositions also justified. From the doxastic point of view this implies that, since all beliefs are inferential or derived, there can be no substantive beliefs justified by something other than other beliefs. In this sense, Infinitism I1 and I2 adopt the intellectualist or doxastical approach, according to which only beliefs can justify beliefs.

Something similar to I1 and I2, happens with I3 and I4, only that instead of adopting a doxastic approach to justification, they adopt an approach according to which only propositions can justify propositions, independently of the agents' doxastic states that sustain them. From this perspective, I3 and I4 would assume the same types of theses and challenges as I2 and I2, but in a propositional key. I3 and I4 hold:

1. The potentially infinite inferences between propositions.
2. The idea that the justification of a proposition is the result of an inferential process that extends to infinity.
3. The justification of a proposition increases as the chain of propositions that supports it expands.
4. By virtue of the above there are degrees of justification for the propositions.

Because of these four ideas, I3 and I4 must also provide a theory for determining the degrees of justification of propositions, as well as the difference between adequate

propositional justification and complete propositional justification. Of course, these explanations must be taken into account once it has been accepted that this theory is a correct theory of the structure of justification.

There are at least two arguments showing that the structure of justification is essentially infinite. The first is known as the *enhancement argument*, (Klein, 2005). This argument is in the form of a regress argument, but in favor of infinity. It starts from the question: under what structure of justification can the justification of a non-evident belief be increased? There are three ways to respond:

(a)   The increase is achieved through a chain of beliefs that at some point turns on itself (circular coherentist structure). It is not correct because any backward belief chain can increase the justification for any belief.

(b)   The increase is achieved by a finite, non-circular chain of beliefs (Foundationalism structure). It is also not correct because any finite chain ends in a substantive belief that, being not supported by other beliefs, is arbitrary. The argument that shows the arbitrariness of the Foundationalism structure is, roughly, the following: suppose that we ask *S* a series of questions in order to request the reasons that he has, and that *S* finally cites a reason *r* that, according to *S*, it is a basic or substantive reason. We could still ask *S* why do you think the basic reason *r* is probably true?

(c)   The increase is achieved by an infinite, non-circular chain of beliefs (infinitist structure). Of course, this argument assumes that skepticism is false. The strategy for the Infinitist is to show that neither (a) nor (b) are correct, after which (c) follows.

The second argument is the *interrogation argument*. It starts from the generally accepted premise that justification is a necessary condition of knowledge. So, if we really want to know, we must aspire to fulfill the condition of justification. One way to fulfill the justification condition is through the questioning exercise. This argument assumes that justification is reached dialectically, which was supported by Plato in *Meno-98a* and, therefore, supposes an internalist position of justification. Faced with a belief b it is always possible to legitimately ask about the bases that support it in order to reach justification about the belief in question. The question is what structure is conducive to that purpose. There are three options: (i) circular, (ii) finite, (iii) infinite and non-circular. Under structure (i) one turns at some point on the reasons given and therefore fails to satisfy the purpose. Under structure (ii) the purpose is not fulfilled because, when the reasoning ends in a reason not supported by another reason, the last reason would be arbitrary. But (iii) does not seem satisfactory either, because if the process of asking continues indefinitely, then, for each reason you have, there will be another, and this does not allow you to meet the objective of achieving justification. Thus, the consequence would be skepticism. However, the Infinitist considers that infinite regress, far from being ineffective with respect to the objective of achieving justification, is the only one of the three possibilities that does not block the interrogative procedure that would allow its realization. After all, human knowledge requires that the possibility of interrogation be available, and this is only possible if a potential series of infinite reasons is available, an arrangement that is only ensured by the structural conception of infinity in any of its versions.

6.2 Criticism of Coherentism

The key point of coherentist perspectives is to provide a definition of coherence that is consistent with the idea of non-vicious mutual support. Now, although there is no standard

definition, this concept can be defined from a variety of aspects and criteria that, according to BonJour (1985, pp. 97–99), encompass its main characteristics. Coherence has to be defined, first, for a system, so that the satisfaction of the concept by the system makes it is possible to apply the concept to some of its parts. Thus, a condition for the coherence of a system is consistency, that is, the non-contradiction between the members that compose it. This is one of the common conditions in the formulations on coherence, but it is not, in any case, a sufficient condition. Indeed, there may be a system maximized in consistency, but whose parts are either not related or related to each other in a superfluous way. In any case, consistency should not be confused with much more demanding criteria such as the mutual derivability between the members of a system, that is, the property that each sub-part of the system is a logical consequence of the inferential combinations of the rest.

One of the most important objections against C1 is known as the criticism of isolation (Olsson, 2017). According to this criticism, C1 does not assign a relevant justifying role to the source of perceptual states, which implies that the coherence of the system does not guarantee any link with reality. In other words, since C1 takes the source of our perceptual states as that which has a causal but not justifying relationship over the rest of our beliefs, that objective source of experience would have no role in the processes of justification, and that implies in turn that there is no reason to think that a coherent belief system reflects what is really happening in the world. Regarding the alleged irrelevance of justification with respect to experience, other objections are formulated such as the one that maintains that a body of propositions like the ones we find in children's stories is, despite its coherence, comprehensively false. These objections show why more than just coherence is needed to ensure the connection of a coherent body of propositions with reality and extend to the possibility that there are alternative and incompatible systems of coherent beliefs. Indeed, it is not possible for two belief systems to be incompatible with each other and equally justified, unless it is accepted that neither of them faithfully represents reality. Therefore, there has to be a way to choose between two incompatible systems, and C1 does not seem to offer a satisfactory decision criterion. Therefore, coherence does not seem to be sufficient to decide the justification of a belief system. Now, to avoid this type of objection, C1 requires assigning a relevant role to experience in justification processes, but any attempt to link experience to such processes runs the risk of bringing the coherentist position closer to some weak or moderate version of Foundationalism. In effect, the attempt to regain the relevance of experience in the processes of justification, sooner or later requires a justifiable asymmetry between beliefs, and that asymmetry goes against the symmetrical and multidirectional proposal of holistic coherentism. A further objection has to do with the fact that the requirement of logical consistency between all the beliefs of the system turns out to be too strong a demand for justification. It does not seem plausible to say that someone is not justified in holding an aggregate of beliefs because there are one or more beliefs in that aggregate that are not consistent with the others. But, arguing a sort of separation by modules or subsets of beliefs in defense of this criticism, so that the inconsistency in one of these modules does not affect the coherence of the others, is not a correct answer, since it is also not clear why the total consistency of a module may be affected by the inclusion of some minimal inconsistency in that subgroup. This type of objection constitutes reason to suspect coherence as a necessary condition for justification. Ultimately, C1's main difficulty lies in its lack of assurance with the truth, for, as consistent as a belief system was, it has no way of ensuring that that system constitutes a correct representation of the world. For C1 in a holistic version, an agent S could have a collection of coherently connected beliefs, regardless of whether or not S can, at some point, show or carry out the activity of justifying if any of those beliefs is coherent with the rest of the beliefs that they

hold. Let's think that each one of us has a great number of beliefs at the moment, many of which we have acquired independently of having shown that those beliefs entered into coherence or mutually supportive relationships with the rest of the beliefs that we already hold. Of course, if we were asked the reasons why we hold any of them, we can give, perhaps with some difficulty, a set of reasons that, in some sense, will be consistent with a wealth of beliefs that are relevant. But that does not mean that a good number of beliefs that we have acquired are justified by reason of some act such as showing that they are coherent with the others. Furthermore, let's think that an agent *S* can have many coherent beliefs without ever having carried out the exercise or activity to justify any of them. If this is so, then at least C1 is consistent with a perspective, according to which the justification status of a belief is independent of the activity of justifying.

Any sequential or linear interpretation of C2 that faces regress in a doxastic version of activity is completely unsatisfactory and obviously incorrect. Of course, if agent *S* believes that P_2, and we ask why he believes it, he can legitimately answer P_1 as the reason for his belief. If we then ask why he believes that P_1 and answers that he believes it because of P_2, it is clear that *S* is making a request for principle, since it is precisely P_2 that was in question from the beginning. The question is how C2 could be holistic in character. Certainly, the formulation of C2 itself does not allow us to clarify clearly how this could be possible, because as it is formulated, it understands the justification of a belief as something that can be determined individually from its connections with other beliefs. However, under the holistic view of justification, an individual belief is derivatively justified in terms of the mutually supportive relationships it has with the total belief system of which it is a part, that is, the justification of a belief can only be established in the measure that it is part of a system. The question that arises, therefore, is how is the activity of justification of a belief carried out based on the total system to which it belongs? Showing that a belief is justified would require showing that it is coherent within the total body of beliefs of which it is a part. But this seems too strong a demand. After all, a belief may be part of a very complex system, and it does not appear that anyone can and need not demonstrate that their belief is justified because it fits coherently into the whole of that complex system. If this were a necessary demand, no real human being, with a vast set of beliefs, could satisfy it. This objection could be answered by proposing a division of the global belief system into compartments or subsets of beliefs that maintain the required coherence with each other and that facilitate agent S in the exercise of showing how a belief enters into coherence with that selected subset of beliefs. However, this strategy is not satisfactory because there does not seem to be a way to explain how the individualization or separation of these belief subsets should be in relation to the evaluated belief, and this could lead to the selected subset being chosen ad-hoc, in such a way that the belief in question does not disturb the coherence of that subset. But, even if a possible answer were given, and there was a criterion to separate certain beliefs by compartments based on the belief to be evaluated, that criterion would sacrifice the global connectivity that is required to guarantee the holistic nature of the system. In other words, a criterion could make the subset of chosen beliefs so specific that that subset would be completely unrelated to other beliefs in the system. As far as the act of showing justification is concerned, C2 seems that if justification status is acquired by consistency, a successful case of adequate justification action with respect to a belief will be one in which agent S succeeds in showing that this belief is consistent with another store of beliefs. But this means that the belief must have the status of justification, via coherence, before S can be successful in its activity of justification.

Under C3, the circular-linear conception is not necessarily vicious. For example, recursive definitions in mathematics can accept that kind of linear circularity. From which it

follows that, under a coherentist position consistent with mathematical recursion, the vicious nature of circularity is blocked. Seen from the point of view of the linear-structure, C3 does not turn out to be, then, easily an objectionable position.

A linear-structure C3 could respond to the argument of the increase in a propositional version of infinitism, arguing that, under a form of recursive circularity, it is possible to increase the degree of justification of the system and its constituent parts depending on the integration of the linearly articulate total system. Seen from a structural-holistic conception, C3 does not have to face the criticism of isolation either, since a completely formal system can be conceived in which the justifying role of experience is not relevant and in which the justification of a proposition P within of a system is defined, solely and exclusively, by the contribution that *P* can make to the coherence of the system of which it is part. For this reason, C3 systems have no obligation to be systems that are connected to or reflect reality. In this same sense, the objection related to the children's story does not hold, nor is the fact that, under this position, there may be incompatible alternative systems. All that C3 requires is the maximization of consistency, a requirement that was considered excessive for C1 and C2 but which, of course, does not apply to C3. Any type of objection that can be made to C3 must be made from an entirely formal level. This is detrimental to the proper epistemological value that this type of coherentism could give us regarding the way we know the world.

In C3 and C4, a set of propositions can only be coherent if someone or something brings them together, and this requires the activity of justifying, that is, of showing how those propositions are coherently connected. Once it has been shown how to put together coherently some propositions with others, we can say that the system that integrates them is coherent. Under the doxastic perspective of coherentism, on the other hand, an agent S acquires beliefs and these are organized—perhaps thanks to some cognitive process—in such a way that they are coherent regardless of the activity of justifying. But that does not happen with propositions. The process that brings them together consistently is the process, carried out by an agent *S*, of showing that they are coherently connected. Thus, it seems that C3 and C4 are at least consistent.

6.3 Criticism of Foundationalism

According to Foundationalists, regress can be avoided by formulating the existence of a justifying basis (doxastic or propositional) whose justification is obtained differently from the way in which that supported by that substantive base obtains justification. Now, for Foundationalists, the question is not only to elaborate an argument that ensures that the epistemic regress can be stopped on the basis of a substantive justification, but also to ensure that any belief supported on that substantive justification will also be justified. The following two questions must be answered:

(1) What is the type of relationship that must exist between the substantive justifying base and what that substantive base justifies? This is the problem of explaining how epistemic (doxastic or propositional) transmission of justification is possible.
(2) What kind of justification is the justifying basic substantive? Or, in other words, what is the source of justification for the substantive basic justification?

Answering both questions is crucial to a Foundationalist theory of the structure of justification. The Foundationalist idea that linear structure preserves justification and that such

structure must having a substantive basis must be supported by a characterization, both of the relationships between the substantive bases with the rest of the structure, and of the substantive bases themselves. Additionally, it must give a substantive explanation of how that substantive base acquires justification. This explanation must be articulated in such a way that it avoids the traditional objections on the arbitrariness of the substantive bases.

Since all forms of Foundationalism resort to non-inferential justification to avoid epistemic regress, the challenge for Foundationalisms is to adequately characterize non-inferential justification for substantive beliefs or underlying propositions. This type of characterization is important, as it is the platform for the structural conception of justification. Stipulating the existence of a justifying basis (substantive beliefs or basic propositions) that is justified differently from the rest of the derived beliefs or derived propositions, allows linear, arboreal or pyramidal structural images of Foundationalism.

6.3.1 Doxastic Justification

When it comes to doxastic justification in relation to the first question, that is to say, the relationship between the substantive bases and what is supported by them, for F1 and F2, the justification must be understood in terms of relations between beliefs and, therefore, they must determine what it is required for one belief to count as an adequate reason for another belief. Because Foundationalism assumes the model that doxastic justification depends on propositional justification (Klein, 2007a), then, when it comes to doxastic justification of status, the property of doxastic justification would be a function of relationship between the propositional contents of the involved doxastic states. This type of relationship can be understood, in turn, in terms of legitimate inferences. In this sense, the doxastic justification of status for F1 would be a function of the legitimate inferential relationships existing between the propositional content of a belief and the propositions that sustain the content of substantive beliefs. For F2 the doxastic justification activity would also respect that inferential condition. An agent *S* appropriately shows the justification for his belief when he is able to demonstrate that his propositional content follows legitimately and inferentially from the propositional content of another, more substantive belief. However, if F1 and F2 accepted that, in both cases, the inferential process is repeated indefinitely, they would have no way to face, in the first case, *the regress of doxastic justification of status*, and in the second, *the regress of doxastic justification activity*. In both cases, not avoiding unlimited inferential regress would entail either I1 (*infinitism of doxastic justification of status*) or I2 (*infinity of doxastic justification activity*).

Thus seen, to avoid the epistemic regress, in regard to the second question—i.e., of what kind is the justification of the justifying basis—F1 must commit to the idea that the justification status of substantive beliefs is obtained not-inferentially, while F2 must commit to the idea that the doxastic justification activity ends by citing a belief that does not require additional inferences to be justified.

6.3.2 Propositional Justification

Regarding the first question—the relationship between the bases and what is supported by them—when it comes to propositional justification of status, F3 will have to argue that

the justification status of non-basic propositions is a function of their relations' legitimate inferential relations with other justified basic propositions. In other words, a non-basic or derived proposition is justified by another basic proposition or one more basic than it, if the justification of the derived one is obtained by virtue of a legitimate inference from the basic justification. For F4, the propositional justification activity carried out by an agent *S* would be successful when the agent shows or quotes the legitimate inferential relationships existing between the basic and derived propositions involved, regardless of whether they are believed by agent *S*. That is, a derived proposition is justified by a more basic one if agent S shows that the derived propositions is obtained by virtue of a legitimate inference from the most basic one, regardless of whether the agent believes in such a proposition or set of propositions. Now, if the justification status of every proposition were a function of its legitimate inference from another proposition, such an inferential process would lead to the regress of propositional justification of status (See Sect. 4.4) and would be sufficient for the adoption of infinity. For its part, the regress of propositional justification activity would be a consequence of carrying out the activity of inferential justification (See Sect. 4.4) for all propositions and would allow some form of infinity.

It is necessary to answer the second question: *what should be the justification of the base to avoid regress?* In order to stop the infinite epistemic regress, the F3 and F4 must affirm that the type of justification of the basic propositions cannot be the product of their inferential relations with other propositions. To avoid *the propositional infinite regress of status* or *propositional activity*, F3 must commit to the idea that the justification status of the basic propositions is acquired non-inferentially, while F4 must argue in favor of the idea according to the which propositional justification activity can be stopped when a proposition is cited that does not require additional inferences to be justified.

6.3.3 Non-inferential Justification of Substantive Beliefs or Basic Propositions

The question is, then, how should the non-inferential justification of substantive beliefs or basic propositions be characterized? According to the distinctions that have been made so far, there would be four characterization challenges of non-inferential justification, all in relation to the four ways of understanding the problem of epistemic regress: two with respect to doxastic/propositional status, and two regarding doxastic/propositional activity. Regarding non-inferential justification, we have four cases:

(a) *Non-inferential justification status of substantive beliefs*: assumes that substantive beliefs acquire justification status regardless of their inferential relationships with other derived beliefs.
(b) *Non-inferential justification activity concerning substantive beliefs*: refers to the activity of showing, regardless of any inference, that a belief has justification.
(c) *Non-inferential justification status of basic propositions*: assumes that basic propositions acquire justification status regardless of their inferential relationships with other derived propositions.
(d) *Non-inferential justification activity concerning basic propositions*: refers to the activity of showing, regardless of any inference, that a proposition is justified.

Now, a characterization of the very activity of non-inferentially justifying, either beliefs b, or propositions P, is problematic. The very idea of non-inferential justification activity, understood

in the sense of activity of non-inferentially establishing justification of a belief/proposition (i.e., showing, independently of other inferences, that a belief has justification) would be self-contradictory or logically inconsistent, since any act of showing that b or P requires appealing to other affirmations with which b or P maintains an inferential relationship. For Alston (1976) the key position of Foundationalism, according to which a basic statement K acquires its justification status independently of the relationship with other cognitions, does not imply that said statement is logically independent of all other possible cognitions, in the sense that although K is justified (status) non-inferentially, it cannot be shown (activity) that K is justified without appealing to other inferences. This observation is important because it answers the objection about arbitrariness that infinity uses in its two key arguments against Foundationalism: the argument of increase and the argument of interrogation. Indeed, Foundationalism could grant Infinitism the fact that every belief or proposition, including substantive or basic ones, requires a potentially unlimited number of questions in order to carry out the activity of showing or establishing the reasons by which it is affirmed that it is substantive or basic. But that does not mean that the justification status of substantive beliefs or basic propositions is acquired based on that infinite inferential process. Foundationalism only needs to explain how the justification status of a belief or proposition is acquired independently of inferential relationships with other beliefs. Now, discarding the logical possibility of carrying out the activity of non-inferentially justifying substantive beliefs or basic propositions does not make sense of the formulations F2 and F4. Indeed, for these perspectives, it is enough that the epistemic regress ends when the agent cites a substantive belief (F2) or basic proposition (F4) that has justification status, a status that it has obtained regardless of its inferential relationship with other derived beliefs or derived propositions. It is in this sense that the F3 and F4 formulations must already have a particular way of determining the non-inferential justification status of substantive beliefs by virtue of which agent S stops when he quotes them. For this reason, F2 or F4 implies the prior adoption of an F1 or F3 formulation respectively.

7 Conclusions

According to Infinitism, a belief can only be justified if, for any legitimate request for justification (and these are potentially unlimited) there is always a new reason to offer. Infinitism maintains that a reason can only be justified by another reason, so there are no substantive beliefs and basic propositions. It denies that a belief can be justified by itself or by a group of which it is a member. The only way out, then, according to Infinitism is that justification consists of a potentially infinite chain.

The most basic and classic objection to Infinitism is that our minds and time are finite and if justification implies an infinite series, there would be no way of knowing whether our beliefs are justified. But if we don't know if our beliefs are justified, can they really be viable?

Both Foundationalism and Coherentism have received criticism and offer problems that are difficult to solve. It seems that Foundationalism is too demanding, since we do not usually keep the justifying trail of our beliefs, much less even for substantive beliefs. For its part, Coherentism seems to leave perceptual experiences without any relevant role in the justification of our empirical knowledge. But these reasons do not have to be repetitive, so Coherentism is also excluded as a solution.

Despite the problems that Foundationalism has, and in particular due to its breadth we do not address all of them in this work. In terms of future work, the authors of this paper are developing the theory that belief systems form an inferential network. This network would have two planes: the plane of beliefs and the plane of associated propositions, and in both, its elements are related bi-directionally, with a relation of inference and another

inverse relation of justification. If we adopt a truth value for each belief, both substantive and derived, that relates to a subjective probability or probability of belief, the belief plane can be transformed into a Bayesian network. Such an approach is consistent with a proposal to base philosophical arguments about knowledge and belief on formal computational models so as to reduce ambiguities that arise with intuitive arguments (Riegler, 2015). The approaches to epistemological regress reviewed here each seek to establish truth via justification. Our distinction between pure and impure systems prioritizes justification by personal experience, acknowledging cognitive limitations and the elusive nature of truth. Because Foundationalism starts from the existence of substantive beliefs and basic propositions, it perfectly fits the objective of our present work, and that is why we have adopted this hypothesis.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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