Efficient Coupling Integrals Computation of Waveguide Step Discontinuities using BI-RME and Nyström methods

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ABSTRACT

This paper describes a novel technique for the very efficient and accurate computation of the coupling integrals of waveguide step discontinuities between arbitrary cross section waveguides. This new technique relies on solving the Integral Equation (IE) that provides the well-known Boundary Integral – Resonant Mode Expansion (BI-RME) method by the Nyström approach, instead of using the traditional Galerkin version of the Method of Moments (MoM), thus providing large savings on computational costs. Comparative benchmarks between the results provided by the new technique and the original BI-RME method are successfully presented.

Keywords: Nyström, BI-RME, Modal Analysis.

1. INTRODUCTION

Filters, diplexers and many passive waveguide devices used in real applications can be considered as the cascade of step discontinuities between two waveguides with arbitrary cross-sections [1]. The electrical behaviour of waveguide step discontinuities can be characterized through multimode equivalent circuits, such as the generalized scattering, impedance or admittance matrices. The computation of these matrices requires the knowledge of the modal spectrum of the waveguides, as well as a large number of coupling integrals involving the modal fields of both waveguides. The method followed in this paper for computing such coupling integrals is based on the use of a fast algorithm for the modal characterization. Once the modal chart is obtained, the evaluation of the coupling coefficients can be performed in a very simple and efficient way by post-processing some of the matrices already used in the mode determination with a minimum additional numerical effort. The BI-RME method [2] is chosen for the calculation of the modal spectrum of an arbitrary cross-section waveguide because it presents high speed and accuracy, which are mainly due to the use of rapidly convergent expressions for the Green functions. The BI-RME method is based on the solution of an integral equation, which can be solved numerically by two basic approaches. The first technique, which is traditionally employed, is based on the Method of Moments (Galerkin version). The second one is the Nyström technique, that has been revealed as a very efficient new way for solving IEs. In this paper, a fast and simple implementation of the BI-RME method based on the Nyström technique is presented and applied to the computation of the required coupling integrals with a few additional numerical operations. Comparisons between the numerical results obtained by the new technique and the results obtained by the classical BI-RME approach fully validate the new implementation. The gain in efficiency is successfully proven through computation cost benchmarks, and the accuracy is measured by comparing results provided by the new technique and the original BI-RME implementation.

2. THEORY

The BI-RME method computes the cut-off frequencies and the field patterns of arbitrarily shaped waveguide modes. In this work, the combination of the BI-RME method with the Nyström technique is applied to the solution of the TE and TM modes of the arbitrary waveguides. The development of this method starts with an integral equation that, after applying the Nyström method, leads to a linear matrix eigenvalue problem. Accuracy aspects of this procedure are directly related to the rigorous treatment of the singular behaviour of the integral equation kernel, which must be carefully considered.

Besides, the BI-RME method implemented with the Nyström technique also allows to calculate the coupling integrals between arbitrary waveguide modes and the modes of the standard rectangular waveguide. The computation of the coupling integrals follows the same approach as the original method [3] but obtaining a different formulation, which is related to the new segmentation technique.

2.1 TM Modes

The original BI-RME method is followed in order to compute the TM modes. First, the boundary condition to the axial component of the electric field is imposed, thus leading to an integral equation (see equations (9a) and (9b) in [2]) that is solved employing a Nyström quadrature. The following algebraic equations are obtained

$$\sum_{\substack{j=1\\ j\neq i}}^{N} \omega'_{ij} g(\mathbf{s}_i,\mathbf{s}_j) b'_j + \omega'_{ii} b'_i + \sum_{m=1}^{M'} \frac{\psi_m(\mathbf{s}_i)}{h'^2_m} a'_m = 0 \qquad i = 1,\ldots,N \tag{1a}$$

$$\left(1 - \frac{k'^2}{h'^2_m}\right) \cdot a'_m = k'^2 \sum_{j=1}^{N} \gamma'_j \frac{\psi_m(\mathbf{s}_j)}{h'^2_m} b'_j \qquad m = 1,\ldots,M' \tag{1b}$$

where N is the number of points used for the segmentation of the perturbing contour, $\mathbf{s}_i$ and $\mathbf{s}_j$ are the discrete observation and source points, and g is the rapidly convergent scalar two-dimensional Green function. On the other hand, M' is the number of TM modes of the surrounding standard rectangular waveguide, and h'_m and ψ_m are, respectively, the cut-off wavenumber and normalized scalar potential of the m-th TM mode of such rectangular waveguide.

The values ω'_{ji} and γ'_j are Nyström quadrature weights. In this approach, a simple one point quadrature is chosen. However, as can be seen in (1a), the Nyström method requires the evaluation of the kernel with coincident source and observation points. In such cases, the singular contribution of the kernel is approximated by its Taylor expansion, which gives place to a regular and a singular term. To compute the value ω'_{ii} , the regular term is extracted, whereas the singular one is analytically solved.

The equations (1a) and (1b) form an eigenvalue problem, whose solution provides as eigenvectors the modal expansion coefficients (a'_m) and the amplitudes of the longitudinal current density in the discrete points of the contour (b'_j), and as eigenvalues the cut-off wavenumbers of the perturbed waveguide (k').

2.2 TE Modes

For solving the TE case, the BI-RME method is used again. After applying Nyström to the integral equation that outcomes from imposing the corresponding boundary condition, the following set of algebraic equations is obtained

$$\frac{1}{k^2} \sum_{\substack{j=1\\ j\neq i}}^{N} \omega_{ij} \frac{\partial^2 g(\mathbf{s}_i,\mathbf{s}_j)}{\partial l \partial l'} b_j + \frac{\omega_{ii} b_i}{k^2} - \sum_{\substack{j=1\\ j\neq i}}^{N} \nu_{ij} \mathbf{t}(\mathbf{s}_i) \mathbf{G}_{st}(\mathbf{s}_i,\mathbf{s}_j) \mathbf{t}(\mathbf{s}_j) b_j - \nu_{ii} b_i - \sum_{m=1}^{M} \frac{\mathbf{t}(\mathbf{s}_j) \mathbf{e}_m(\mathbf{s}_i)}{h_m^2} a_m = 0 \qquad i = 1,\ldots,N \tag{2a}$$

$$\left(1 - \frac{k^2}{h_m^2}\right) \cdot a_m = k^2 \sum_{j=1}^{N} \gamma_j \frac{\mathbf{t}(\mathbf{s}_j) \cdot \mathbf{e}_m(\mathbf{s}_i)}{h_m^2} b_j \qquad m = 1,\ldots,M \tag{2b}$$

where $\mathbf{G}_{st}$ is the rapidly convergent solenoidal dyadic Green function and $\mathbf{t}$ is the unitary tangent vector on the arbitrary contour. In this case, M represents the number of TE modes of the surrounding standard rectangular waveguide, and h_m and $\mathbf{e}_m$ are, respectively, the cut-off wavenumber and the transverse electric field of the m-th TE mode of the rectangular waveguide. In this problem, the unknowns are again the modal expansion coefficients (a_m), the amplitudes of the transverse current density in the discrete points of the contour (b_j), and the cut-off wavenumbers of the perturbed waveguide (k).

The quadrature weights ω_{ji}, ν_{ii}, ν_{ji} and γ_j of the Nyström method are obtained following the same procedure as the TM case. However, as it can be seen in (2a), the computation of ω_{ii} is related to the evaluation of the double partial derivative of g with respect to the observation and source contour parameters l and l', which is not integrable even in the

Cauchy principal value definition. Therefore, the treatment of this hipersingularity is different with regard to the TM case. In this case, ω_{ii} is computed via the traditional method of the subtraction of the singularity.

2.3 Coupling integrals

The computation of the coupling integral coefficients between the modes of a rectangular waveguide and an arbitrarily shaped waveguide is defined by the following expression

$$I_{pq} = \int_{S_2} \mathbf{e}_{1p} \cdot \mathbf{e}_{2q} \, ds \tag{3}$$

where $\mathbf{e}_{1p}$ is the p-th mode transverse electric field of the rectangular waveguide and $\mathbf{e}_{2q}$ is the q-th mode transverse electric field of the arbitrary waveguide. Once the corresponding currents, modal expansion coefficients and mode cut-off wavenumbers are obtained for the TE and TM cases applying BI-RME and Nyström methods, then the coupling integrals are easily obtained through the following expressions

$$\mathrm{I}_{pq}^{\text{TM-TM}} = h'_p \sum_{j=1}^{N} \omega'_j \frac{\psi_p(\mathbf{s}_j)}{h'^2_p} b'^q_j + \frac{a'^q_p}{h'_p} \tag{4a}$$

$$\mathrm{I}_{pq}^{\text{TM-TE}} = \frac{1}{k_q} \sum_{j=1}^{N} \omega_j \mathbf{e}'_p(\mathbf{s}_j) \cdot \mathbf{t}(\mathbf{s}_j) b^q_j \tag{4b}$$

$$\mathrm{I}_{pq}^{\text{TE-TE}} = k_q \left[\sum_{j=1}^{N} \omega_j \frac{\mathbf{t}(\mathbf{s}_j) \cdot \mathbf{e}_p(\mathbf{s}_j)}{h^2_p} b^q_j + \frac{a^q_p}{h^2_p} \right] \tag{4c}$$

$$\mathrm{I}_{pq}^{\text{TE-TM}} = 0 \tag{4d}$$

where $\mathbf{e}'_p$ is the transverse electric field of the p-th TM mode of the rectangular waveguide, ω_j and ω'_j are quadrature weights that result from applying again the Nyström approach, and the rest of variables are previously defined. It must be pointed out that all values in (4a) and (4c) have been already evaluated in the modal spectral analysis. Therefore, the scalar product in (4b) is the only expression that needs a new numerical evaluation.

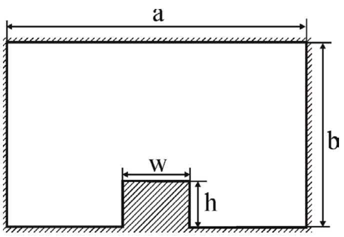
3. RESULTS

The accuracy and efficiency improvement provided by this new method is proved through successful comparisons with numerical data obtained with a revisited implementation of the classical BI-RME approach. Accuracy aspects of this simple and fast procedure are considered, setting the maximum relative error to 0.5% in the cut-off frequencies computation. Obviously, the Nyström method with one-point quadrature should need more discrete points than Galerkin technique in order to obtain the same accuracy. The followed approach is to take first the same number of points N as the classical Galerkin case, and then to increase N in order to satisfy the imposed bound. Comparative benchmarks between the new technique and the original BI-RME method are successfully presented confirming the improvement in efficiency issues. In order to validate the new technique, one example of practical interest is given.

The cut-off frequencies and the coupling integrals are evaluated for a ridge waveguide. All tests have been performed on a PC Pentium II @ 400 MHz.

3.1 Ridged waveguide

The geometry to be tested is a WR-75 standard waveguide perturbed by a central single ridge shown in Fig. 1. As can be seen in Table I, the cut-off frequencies for the TE and TM modes are obtained with computation time savings and a small loss of accuracy with respect to the classical BI-RME Galerkin implementation using the same discretization order (without extra N points). In this comparison, the number of rectangular waveguide modes is 600 and 140 single ridge valid cut-off frequencies are obtained. The mean and maximum error of these frequencies and the gain of efficiency achieved are shown in Table I.



a = 19.05 mm
b = 9.525 mm
w = 3 mm
h = 3 mm

Figure 1: Single ridged waveguide geometry.

	N extra points	Time reduction	Time	Max. error	Mean error
TM	0 %	38.9 %	1.98 s	0.05 %	0.02 %
TE	0 %	32 %	6.12 s	0.19 %	0.05 %

Table I: Single ridge cut-off frequencies comparison between classical Galerkin BI-RME and Nyström BI-RME.

The coupling integrals for the first 21 modes of the single ridge and 30 modes of a WR-75 waveguide have been also computed. A maximum absolute difference with Galerkin BI-RME equal to $4\cdot10^{-3}$ has been observed, thus confirming the accuracy of the new technique. Table II shows some of these coefficients and their associate absolute errors.

Modes (A)	**Modes (B)** $\mathbf{TE_{10}}$	$\mathbf{TE_{20}}$	$\mathbf{TE_{01}}$	$\mathbf{TE_{11}}$	$\mathbf{TM_{11}}$	$\mathbf{TE_{30}}$	$\mathbf{TE_{21}}$	$\mathbf{TM_{21}}$
$\mathbf{TE_{10}}$	-0.851 *$1.7\cdot10^{-3}$*	0	-0.214 *$2.3\cdot10^{-3}$*	0	--	0.228 *$1.5\cdot10^{-3}$*	0	--
$\mathbf{TE_{01}}$	0	-0.7619 *$2\cdot10^{-4}$*	0	-0.593 *$1\cdot10^{-3}$*	--	0	-0.058 *$1.9\cdot10^{-3}$*	--
$\mathbf{TE_{20}}$	0	-0.602 *$1.4\cdot10^{-3}$*	0	0.7649 *$6\cdot10^{-4}$*	--	0	0.158 *$1.1\cdot10^{-3}$*	--
$\mathbf{TE_{11}}$	-0.0269 *$2\cdot10^{-4}$*	0	0.918 *$1.2\cdot10^{-3}$*	0	--	0.299 $2.6\cdot10^{-3}$	0	--
$\mathbf{TM_{11}}$	0.329 *$4\cdot10^{-3}$*	0	-0.186 *$2\cdot10^{-3}$*	0	-0.769 *$1\cdot10^{-3}$*	0.218 *$1.4\cdot10^{-3}$*	0	0
$\mathbf{TE_{21}}$	0	0.0084 *$3\cdot10^{-4}$*	0	-0.2134 *$3\cdot10^{-4}$*	--	0	0.9276 *$9\cdot10^{-4}$*	--
$\mathbf{TM_{21}}$	0	0.067 *$2.1\cdot10^{-3}$*	0	0.0108 *$2\cdot10^{-4}$*	0	0	0.046 *$1.6\cdot10^{-3}$*	-0.9586 *$3\cdot10^{-4}$*
$\mathbf{TE_{30}}$	0.0203 *$2\cdot10^{-4}$*	0	-0.129 *$1.1\cdot10^{-3}$*	0	--	0.832 *$1.4\cdot10^{-3}$*	0	--

Table II: Coupling integral coefficients between the WR-75 waveguide modes **(A)** and the single ridged waveguide modes **(B)**. The absolute difference between classical Galerkin BI-RME and Nyström Bi-RME coupling integrals is also shown in italic.

4. CONCLUSIONS

A fast and accurate algorithm has been presented for the determination of the coupling coefficients between a rectangular waveguide and an arbitrarily shaped waveguide. The complete modal characterization of the arbitrary waveguide has been performed by a new BI-RME version based on the Nyström approach. This new method offers some advantages compared to standard Galerkin BI-RME method: the first one is the simplicity of the implementation and the second improvement is the reduction of the computation time. Obviously, the expected accuracy drops with respect to the classical case, but the difference has been bounded in this work at very low values for both the cut-off frequencies and the coupling integrals. The novel technique has been successfully applied to the case of a single ridge waveguide, which fully validates the proposed method.

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