Learning to “notice” the mathematics teaching. Adopting a socio-cultural perspective on student teachers’ learning

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Abstract

There are several different approaches used by mathematics teacher educators to help pre-service primary teachers develop the useful skills and knowledge required to teach mathematics. One of these approaches is concerned with learning to notice relevant aspects in mathematics teaching. But making mathematics teaching meaningful is a challenge to pre-service primary teachers. In addition, the instructional tasks that they must carry out in their teacher education programs influence their learning and ways of knowing. Nowadays communication and information technologies are providing new conditions and tools to assist pre-service primary teachers in learning to notice relevant aspects of mathematics teaching and to develop their relationships to pupils’ ways of thinking. It is assumed that online discussions encourage the collaborative process in which meaning is negotiated and that knowledge is constructed when videotaped extracts from mathematics lessons are analysed. From here, the question of how forms of participation operate to mediate meanings in conversation is central to our understanding of the contribution of socio-cultural perspectives on mathematics teacher learning. This situation establishes three levels with their relationships. Here, I describe the relationships between these three levels and how it is possible to identify an emergent research agenda that reflects aspects of socio-cultural perspectives in order to study the pre-service primary teachers’ learning processes.

INTRODUCTION

In this paper I propose to describe three levels which should govern our thoughts on the praxis of teacher education and on research into student teachers’ learning processes. The first level concerns mathematics-teaching situations in Primary Education. These situations comprise a mathematical activity, a teacher

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and a group of learners in a school-type institutional context. The second level concerns a group of student teachers who try to understand the different variables which affect the mathematics-teaching situations described in Level 1 above. Finally, the third level concerns the researcher/teacher-educator who attempts to understand student teachers’ learning processes in the situations described in Level 2 above.

The first aim of this work is to characterise a procedure for observing student teachers’ learning processes by adopting sociocultural references regarding these processes (Goss, 2008). Secondly, the references adopted in order to understand student teachers’ learning processes will enable us to generate certain reflections on the characteristics of learning environments which incorporate technological resources to facilitate, within teacher-education programmes, interaction and communication among the students involved (Andriessen, Erkens, van de Laan, Peters, Coirier, 2003; Llinares, 1998; Ponte, Oliveira, Varandas, Oliveira, Fonseca, 2007).

**Figure 1.** Three levels: N1-primary mathematics teaching; N2- primary teacher students learning to “notice” in mathematics teaching, N3-teacher educators/researchers learning to “notice” the primary teachers’ learning processes.
Frame 1: A PRIMARY MATHEMATICS TEACHING SITUATION

Mari Cruz is a primary-school teacher in a 3rd-year primary class. There are 22 pupils in her group, all 8 or 9 years old. Mari Cruz usually uses a textbook as the basis of her classes, but also likes to use other resources when presenting activities to the children. The aim of today’s class is to present multiplicative-structure problems. Mari Cruz hopes that the children will learn to analyse the situations and to identify the quantities and the relationships between them as a first and essential step in their initial comprehension of the problem. In particular, one of the objectives of this year’s programme is that the children should learn to identify the data in situations, as well as the relationships they have with unknown quantities. This objective is part of a more general objective running throughout the programme, namely that the children should develop their problem-solving skills. For today’s class Mari Cruz has decided to present a situation to the pupils and then get them to state a problem connected to the situation. Stating a problem is a skill that she considers important in the development of the pupils’ mathematical competence. To this end, Mari Cruz shows two boxes of chocolates to the children and asks them to think of a problem in which the two boxes play a role. At the outset Mari Cruz intends the pupils to concentrate on relevant quantities as a way of linking problem-solving situations with reality. She says:

*I have two chocolate-boxes. What do I need to know in order to find out how many chocolates there are in the two boxes?*

In this situation, Mari Cruz believes that the important thing is that the pupils begin to realise that in each situation there are certain quantities that can be considered to be data, that relationships can be established between them, and that questions can be raised concerning them. By presenting this open-ended situation, Mari Cruz is creating an opportunity to discuss with the pupils the relevant quantities proposed. Once the situation has been presented, some of the pupils raise their hands to ask permission to suggest certain quantities. After allowing the pupils to suggest possible quantities for a few moments, Mari Cruz centres the discussion on the appropriateness of the quantities suggested and on why certain quantities might be irrelevant in the situation presented.

In the second part of the class Mari Cruz tries to get the pupils to pose a question in this situation. Posing a question is a relevant mathematical activity for children of this age, since it obliges them to identify the relationships between the quantities they consider to be data and an unknown quantity which is to be discovered. Primary-school children are not accustomed to carrying out this kind of activity, which is therefore a mathematical challenge for them. After a brief exchange of proposals, Mari Cruz selects some of the pupils’ formulations and writes them on the blackboard, thus reaching a statement of the problem.
In this situation Mari Cruz has offered her pupils the opportunity to think and communicate their ideas about a situation whose solution is not an algorithm or a previously-established procedure. In this situation, the pupils are expected to argue about the appropriateness of the quantities suggested as possible data for the problem (the number of chocolates in each box), and also to discuss among themselves why certain quantities might be appropriate in some situations, but not in others where other conditions prevail. From this point of view, the task proposed by Mari Cruz is in itself a problem for the pupils, in the sense that they might perceive it as being interesting and as something that motivates them to “carry out mathematical research”. Furthermore, by stating the problem in this manner she is able to connect with their previous knowledge and in particular with knowledge obtained outside school, so that they can use this knowledge to create mathematical ways of discussing the relevance of the quantities in a given situation. The situation thus enables the pupils to think and prompts them to communicate their mathematical ideas to others.

In this situation Mari Cruz has presented an activity which is appropriate to her pupils. She has also facilitated an exchange of information among the pupils at appropriate moments, and has helped them to develop their arguments. She has furthermore encouraged the pupils to work both individually and collectively in order to refine their arguments and to establish a balance between the information supplied and the pupils’ own autonomy. These references around which Mari Cruz’s classroom teaching is organised reveal a sociocultural aspect of classroom praxis involving respect for everyone’s ideas and that the appropriateness or otherwise of a solution must depend on the arguments generated.

In this teaching situation we can discern certain aspects (Fennema, Romberg, 1999; Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, Human, 1997) consider relevant in the development of mathematical competence in primary-school children:

The nature of the activities
- The role of the teacher
- The characteristics of the social culture of the classroom
- The mathematical resources supporting the learning process, and in particular the characteristics of the mathematical discourse generated in the classroom
- Manifestations of equity and equality.

Student teachers should be able to identify these aspects of mathematics teaching, and determining to what extent they are present in particular situations is a skill they should develop. A teacher should be able to “observe” what happens in a classroom in order to “analyse” it and “interpret” it. The skills of “observing” and “interpreting” what occurs in mathematics lessons develop over
time and depend on knowledge of teaching and learning processes in mathematics. From this point of view, teacher education programmes should help student teachers to start “noticing” in mathematics teaching (Llinares, 2004). They should learn to “notice in mathematics teaching” in the sense of learning to observe and interpret things that occur in mathematics lessons. Learning to observe involves examining what happens in classrooms where mathematics is taught and learned, and identifying relevant aspects of both (Hiebert, Morris, Berk, Jansen, 2007; Lin, 2005; Llinares, 2006; Morris, 2006; Star, Strickland, 2008; van Es, Sherin, 2002).

Frame 2: LEARNING TO NOTICE IN MATHEMATICS TEACHING

Virginia, Patricia, Estefania, and Judith are student teachers who, together with a group of fellow-students, are participating in an activity which consists of analysing a videoed extract of Mari Cruz’s mathematics lesson, described in the previous frame. The mathematics-teaching analysis activity is part of a learning environment in the Mathematics Teaching subject in our teacher education programme. This subject is organised following a b-learning methodology, and brings together face-to-face instruction and internet-based activities (Llinares, Valls, Roig, 2008; Valls, Callejo, Llinares, 2008). The activity these students are carrying out at the moment belongs to a learning environment whose objective is to develop the skill of observing mathematics teaching and making it meaningful. To this end, the on-line activity to be carried out consists of watching the video of Mari Cruz’s class, reading documents on the characteristics of lessons which favour the learning of mathematics through understanding (Hiebert et al., 1997), and participation in an online debate on how to identify and relate those aspects of the teaching process which are relevant in furthering the learning process (Llinares, 2002). At the end of the online debate, the student teachers are asked to write a joint report on the characteristics of Mari Cruz’s mathematics teaching as observed in the video, and forward it to their tutor. Figure 2 shows the online platform which supports this activity.

In this learning environment, the teacher educator provides documents with theoretical information on teaching and learning mathematics and about critical characteristics of classes that promote mathematical competence, the nature of the mathematical tasks, the cognitive demand of an activity, the role of the primary teacher, the social culture of the classroom, and the notion of equity. Video-clip (children of 8-9 years of age) begins with the Mari Cruz (the primary teacher) showing the primary pupils two boxes of chocolates and asking what data will be necessary in order to formulate a problem (How many chocolates could there be in these boxes?).
In one of the conversational chains generated in the debate on the analysis of the situation, the contributions concentrate on the appropriateness of the problem presented by Mari Cruz as a mathematically relevant activity for 3rd-year primary school children, and on the objectives that could be achieved, taking into account the way in which Mari Cruz manages the interaction with her pupils and the mathematical discourse generated in the classroom. The following is an example of the contributions made, in an exchange between Virginia, Cristina and Judith.

- **Reply to Cristina (VIRGINIA, 10:40:29; 11/01/2006)**

  Cristina, I think you say that the important thing is that the children should know what algorithm to apply. But in my opinion I think you are wrong because the aim of the task is to get them to use the algorithm as a tool in order to solve the problem, but it's not the aim of the task as the teacher presents it to them. Do you agree?

  - **Reply to Virginia (JUDITH, 12:03:00; 11/01/2006)**

    I agree with you when you say that the question is not whether they know what algorithm to apply, but rather in seeing whether the pupils, using the knowledge they already have, can think about the problem given and develop strategies to solve it.


  What I really meant was that the important thing is that the pupils should realise what question they have to ask themselves and then apply the best algorithm to achieve a quick, economical, reliable solution to the problem. Besides, as the teacher says, the objective is to get the pupils to see the problem as a process in which the algorithm used will be the endpoint.

- **Reply to Patricia (VIRGINIA, 10:22:38; 11/01/2006)**

  I think the most important thing is that they should realise what question they have to ask themselves, which is how many chocolates there are in the box. The number of chocolates is secondary, because it doesn't matter what it is. It could be 8, 14 or 50.

These exchanges between Virginia and Judith, prompted in the first place by Cristina's previous contribution to the online debate, show how they identify what they consider to be relevant in the mathematical activity presented to the pupils by Mari Cruz, and what the objectives are. In the opinion of these student teachers, making the task presented by Mari Cruz meaningful has become
the topic of the discussion, which in turn reveals how difficult it is for them to assign meaning to a primary-education mathematical activity that is not simply the application of algorithms.

On the other hand, we can interpret these exchanges as attempts to gradually improve their interpretations. However, the interchanges also provide the pre-service primary teachers with the opportunity to identify and refine the interpretations of aspects of mathematics teaching that are relevant in order to understand the primary teacher's decisions. Another aspect in the process of negotiation of meaning in the online debate is when pre-service teachers analyse mathematics lessons is the way in which they refine their arguments, identify and describe specific events, disagree and propose new ideas. This characteristic of the interaction shows how arguments can be constructed in interaction with their peers, as an activity directed towards an objective (Callejo, Valls, Llinares, 2007; Llinares, Valls, 2007; Rey, Penalva, Llinares, 2006).

The interaction between Virginia and Judith reveals that learning to “identify” those aspects which might be important in a mathematics-teaching situation is a complex activity. For instance, the student teachers need to learn to “analyse” the roles played by the mathematical problem and by the teacher in the interaction between the pupils and the problem itself, and to determine the characteristics which make it a suitable medium for learning mathematics. The student
teachers also need to learn to see how a certain way of managing mathematical communication in the classroom facilitates or hinders the learning process, and how the teacher's management of whole-group or small-group work makes it possible to generate different opportunities for learning mathematics in the classroom. Mari Cruz's approach to mathematical ideas is an aspect which the student teachers need to learn to identify when they observe mathematics lessons.

On the other hand, the fact that the student teachers do actually develop a more complex view of mathematics teaching is revealed in their integration of ideas from different perspectives (of mathematical problems, of learning, of classroom communication, of assessment procedures ...). It is also revealed when the student teachers are seen to be able to provide empirical evidence to support their more theoretical thoughts. In this sense, the opportunity to participate in virtual debates (online discussion sites) enables Judith and Virginia and their fellow-students to overcome time/pace limitations in the collaborative process, and gives them a chance to write about what they identify as being relevant in the mathematics lessons observed and to relate this evidence to theoretical principles. Providing student teachers with the opportunity to express their ideas in public thus makes it easier for them to negotiate meanings and therefore to learn. These learning opportunities may help student teachers to identify certain aspects of mathematics teaching, interpret them and integrate theoretical principles into their interpretations.

In this context, we can assume that the text in the online discussions provides the focus for progressive discourse and simultaneously embodies the progress made. Writing as a way of participating in online discussions might allow pre-service mathematics teachers to become acquainted with and understand the topic they are writing about. We assume that participation in online discussions on a specific issue might encourage pre-service primary teachers to move beyond descriptions of events in the classroom to begin to endow them with meaning. This process might support the way in which they actively engage in testing the relevance of theoretical information in relation to their beliefs and its practical application in the analysis of mathematics teaching (e.g. features of the task, the role of the teacher, the social culture of the mathematics classroom, mathematical tools to support learning, and equity and accessibility).

The virtual learning environment was designed taking into consideration Wells' (2002) model that relates different opportunities for sense-making that prospective teachers might encounter. Firstly, their experience is taken into account since the pre-service teachers have the chance to observe aspects of mathematics teaching from different perspectives, starting from their own initial conceptions. Secondly, the preservice teachers are encouraged to use some elements of theoretical information. And finally, opportunities for knowledge building are created when they engage in discussions leading to meaning-making with others to extend and transform their collective understanding with respect to some aspect of the jointly undertaken activity.
Frame 3. LEARNING TO NOTICE IN PRE-SERVICE MATHEMATICS TEACHER’S LEARNING

In order to understand the student teachers’ learning processes as generated in Frame 2, it is necessary to clearly identify what is being learnt and to adopt a theoretical perspective which will make it possible to assign meaning to the processes generated.

One of the professional skills that pre-service primary teachers should develop as a component of becoming a teacher is learning to notice significant events as a way of structuring their attention. The ability to notice what is happening in the mathematics classroom and to endow it with meaning from the perspective of mathematics learning is critical in enabling pre-service primary teachers to conceptualize a contemporary view of mathematics teaching. We can consider three key components of pre-service teachers’ ability to notice: first, identifying what is important for mathematics learning in a teaching situation; secondly, making connections between specific classroom interactions and the broader concepts and principles; and finally, using what is known about a context to reason about a given situation.

Some studies point out that the analysis of mathematics lessons recorded as videoclips provides pre-service teachers with experience in observing and interpreting, and goes beyond the usual concerns with classroom management issues (Cochrane-Smith, Lytle, 1999). Further, it allows them to focus on more complex issues and use their reflections to provide a more complex view of mathematics teaching. Recently, we have been adopting socio-cultural perspectives on learning to teach in order to explain how pre-service teachers assign meaning to mathematics teaching (Callejo, Linares & Valls, 2008; Linares & Valls, 2007). Learning from a sociocultural point of view has to do with how people appropriate and master tools for thinking and acting in a community of practices (Wenger, 1998; Wertsch, 1991). From these perspectives, knowledge construction in collaborative settings is based on the assumption that learners engage in specific discourse activities and that the nature of the participation and content of this discourse is related to the knowledge thereby constructed. From these perspectives, “knowledge building” has to do with how the pre-service primary teacher is engaged in meaning-making with others in an attempt to extend and transform his or her collective understanding (Linares, Olivero, 2008). Knowledge building typically involves constructing, using and progressively improving representational artifacts. Here, the question of how modes of participation operate to mediate meanings in conversation is central to our understanding of the contribution of the design of learning environments in teacher education. From here, we assume that pre-service teachers construct arguments in interaction with their peers in order to construct knowledge about mathematics teaching as well as to develop the skills needed to learn from practice. At least
three conditions need to be fulfilled in order for collaborative learning processes
to be knowledge-productive (Wenger, 1998):

i) a focus on shared interests,
ii) engagement in joint activities, and
iii) development of a shared repertoire of resources (experience, stories,
tools, ways of addressing recurrent problems)

Such a situated, constructed view of cognition locates knowledge con-
struction processes in the actions of persons, provoked by their activities in a
context. In the interaction, pre-service teachers create focal points around which
the negotiation of meaning and reciprocal understanding organize processes such
as noticing, representing, naming, describing, interpreting, using,... Weng-
er (1998) called this process reification. The process of reification shapes the pre-
service teachers' experience of creating "objects" about mathematics teaching.
From this perspective, meaning arises out of a process of negotiation that com-
bines both participation and reification. This process, leading to the construc-
tion of the knowledge needed for noticing the teaching of mathematics, begins
with the possibility of making the student teachers' initial conceptions public and
debatable regarding the nature of mathematics, the learning and the teaching
of mathematics and the role of the teacher. The progressive use of theoretical ideas
as conceptual tools in analysing and interpreting the teaching-learning situations,
and modification in the way of participating in the spaces set up for social inter-
action are manifestations of the knowledge construction process.

Another characteristic is that throughout the interactive learning environ-
ments the pre-service primary teachers have the opportunity to generate a focus
on shared interests when they engage in joint activities, and develop a shared
repertoire of resources. One characteristic of this socio-cultural perspective is the
important role played by artifacts like the subjects' written contributions, which
mediate in knowing. Writing about a topic is considered to be a powerful way
of knowing. Here, the important thing is that individuals come to understand a
topic better when they have to write to communicate to others – write for others.
Writing is understood from this perspective to be a tool for collaborative reflec-
tion and, the same time, problem solving (Wells, 2002). The way the pre-service
primary teachers participate in the online discussions illustrates how they inter-
pret events in mathematics teaching. Discrepancies in their exchanges are indica-
tors of their engagement in meaning-making with others. Their behaviour in these
debates shows how they create focal points to generate a process of noticing and
interpreting that produces "objects" referring to mathematics. Writing enables
the pre-service primary teachers to work through ideas such as considering how
to make ideas about characteristics of tasks in the video-taped lesson and the role
of the primary teacher comprehensible to others. In this context, writing is seen
as a resource for thinking and communicating knowledge in which pre-service teachers are attempting to build ideas and connect them with the evidence negotiating their interpretation.

We have also been able to identify another characteristics in our analysis of pre-service mathematics teachers’ learning in these interactive learning environments: what the pre-service primary teachers observed in the video contrasted strongly with their beliefs, and thus determined what they found relevant (Cos, Valls, 2006). This might explain why, when they were asked to design a mathematical task for primary pupils, they did not consider, for example in the cases of pre-service primary teachers, the possibility of defining the task as “formulating a problem”, which was what they had seen in the video-clips.

SOME FINAL OBSERVATIONS

A line of research has recently taken shape at an international level, in an attempt to provide information related to the processes by which student teachers learn to identify and interpret aspects of mathematics teaching, and thus to obtain information as to how these processes operate. This research agenda on learning to notice in preservice mathematics teachers’ learning processes is aimed at describing and explaining these processes (as we have seen in Frame 3) (García, Sánchez, Escudero, Llinares, 2006). The agenda poses the question of how the use of case studies and videos which reflect teaching situations can provoke fruitful discussion on mathematics teaching among student teachers and how to characterise the way in which the discussion evolves. One way of carrying out research within this agenda is through cycles of “teaching experiments” (Design-based Researcher Collective, 2003). Teaching experiments consist of the design and implementation of learning environments (García, 2001) and their subsequent analysis (cycles of design-based research). The learning environments designed to this end involve problem-solving tasks in which student teachers can use ideas derived from The Didactics of Mathematics in order to identify and interpret relevant aspects of a mathematics teaching/learning situation (D’Amore, 2006). Furthermore, in the learning environments social interaction spaces are set up to enable the student teachers to negotiate and discuss the meanings of different ideas such as a teacher’s management of classroom interaction or the way in which ideas are linked to the events observed in the videoed praxis (as described in Frame 2).

These learning environments use mathematics teaching records as learning resources, considering them as a medium through which student teachers can reflect on different aspects of teaching, the learning of mathematics and the role of the teacher. In other words, recorded teaching situations are seen as media through which student teachers can reflect on pupils’ mathematical cognition,
the demands made by the mathematical tasks to be carried out, and the teacher's management of interaction among the pupils. Recorded teaching events thus become empirical evidence to which student teachers can link their more general thoughts (Contreras, Blanco, 2002; Linares, 2009; Penalva, Escudero, Barba, 2006).

The second idea relates to the role played by interaction in student teachers’ learning processes. The importance given to interaction in professional knowledge-building situations is justified by theoretical principles which consider that the development of thought processes depends to a great extent on social discourse. These theoretical principles assume that the creation of knowledge is related to the way in which people interact and to the way in which they negotiate alternative interpretations and integrate different views of the situation in which they find themselves (Borga, Gadamidis, 2008). This point of view stresses the importance in student teachers’ learning processes of opportunities to discuss teaching situations from a research perspective, generating questions, offering alternative interpretations and linking general reflections to the empirical evidence provided (Cochran-Smith, 1999).

We should finally point out that adopting sociocultural viewpoints in order to analyse student teachers’ learning processes also imposes conditions on the way in which learning environments are designed and the design of the research itself. The link between the teacher-educator and the researcher into student teachers’ learning processes is a characteristic of this research agenda.

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References


