PROSPECTIVE TEACHERS' COMPETENCE OF FOSTERING STUDENTS' UNDERSTANDING IN SCRIPT WRITING TASK

Jennifer Dröse & Lena Wessel

Paderborn University

In different research traditions teachers' diagnostic competence has always been characterised as being interwoven with fostering students in order to enhance their understanding. While the first one has been investigated thoroughly there is only a limited empirical access to the second one so far, especially in a content-specific way meaning focussing on the specificity of mathematical content areas. As prospective teachers have been shown to struggle with formulating adequate diagnostic judgements and fostering students, we especially investigate their practices of fostering students' understanding identified in script writing tasks analysed with the epistemic matrix. The results indicate that there are three typical impulse pathways in the matrix.

THEORETICAL BACKGROUND

Prospective teachers' competence of fostering students' understanding

For teaching that is centred around students' understanding, teachers' diagnostic competence has been found to be important (Empson & Jacobs, 2008). This competence has already been defined and conceptualized in different frameworks, which often do not focus on the mathematical content specifically. Therefore, the authors of this paper follow a content-specific approach on conceptual and procedural knowledge elements of the current learning content (here: conditional probabilities) or the prior mathematical content as described and explained in Dröse, Griese & Wessel (accepted). Due to space limitations this explanation cannot be presented in detail here.

While diagnostic competence is well conceptualized, there is not yet a definition of teachers' competence of fostering students' understanding applicable. Ball, Thames & Phelps (2010) describe 'knowledge of content and students' as well as 'knowledge of content and teaching' as important facets of teachers' knowledge. 'Knowledge of content and teaching' particularly comprises that teachers need to make instructional decisions, meaning to know "when to pause for more clarification, (…) when to ask a new question or pose a new task" (p. 401). But, the competence of fostering students' understanding can be seen as relating more aspects than knowledge facets alone:

(1) Teacher student interaction is often based on a task that is linked to the learning trajectories intended by the teacher, and the learning goal(s) set for the individual student. These aspects fall within the teacher's subject matter and pedagogical content knowledge (Ball et al. 2010).

(2) When students solve the given task, their individual thinking and learning pro-

2022. In C. Fernández, S. Llinares, A. Gutiérrez, & N. Planas (Eds.), *Proceedings of the 45th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 243-250). PME.

cesses need to be perceived and interpreted by the teacher in order to guide the teacher's decision-making within the learning process. These aspects are described as cognitive diagnostic thinking processes by Loibl et al. (2020).

(3) What is more in order to react adequately and enhance students' understanding are communicational skills building up on the made decisions, taking the students' thinking processes as well as the task or the learning goal into consideration and guiding the students' learning process through the teacher.

Current studies, e.g. Prediger & Buró (2021) identified teachers' competence of 'Enhancing students understanding' to be a central job for teachers' expertise when working with students at-risk next to the jobs of 'Specifying learning contents' and 'Monitoring students' learning process'. As prospective teachers have been shown to struggle with identifying content-specific aspects in their diagnostic judgements (Jansen & Spitzer, 2009) and therefore maybe also in fostering students' understanding, the authors limit their research to prospective teachers. For them, an important practice to become proficient in is, amongst others, the practice of explaining. It is needed in discursive situations of fostering students in one-on-one or classroom discussions. For this paper, the authors rely on earlier research on explaining practices because of its identified potential when extending ideas to the competence of fostering students in discursive settings.

The epistemic matrix for characterising explaining practices

For capturing the different modes of the practice of explaining, Erath and Prediger (2014) developed further the epistemic matrix (cf. Fig. 1). The epistemic matrix distinguishes epistemic modes and logical levels of an explaining practice: Logical levels referring to the content of explaining (explanandum) are unfolded in conceptual and procedural levels and their epistemic modes are characterised as follows:

- *Labelling & naming*: mode that expresses names and labels, e.g. in one word
- *Explicit formulation*: more elaborated mode, including e.g. definitions of concepts or formulation of procedures
- *Exemplification*: mode of expressing examples and counterexamples
- *Meaning & connection*: mode of expressing connections between conceptual and procedural knowledge or to other concepts, pre-knowledge or graphical representation
- *Purpose*: mode of describing the "inner mathematical or everyday functions" (Erath & Prediger, 2014, p. 18) of concepts and procedures
- *Evaluation*: mode that "appears in the context of presenting solutions in class" (Erath & Prediger, 2014, p. 18)

We can rely on the matrix with its epistemic modes as well as logical levels for systematizing epistemic fields of an explanation in a content-specific approach. Since our study focuses on the current mathematical content of conditional probabilities, we exemplify the epistemic fields for this content in the next Section. As the epistemic modes of “purpose” and “evaluation” are connected to rather general classroom discussions, we focus only on four modes for the content of conditional probabilities.

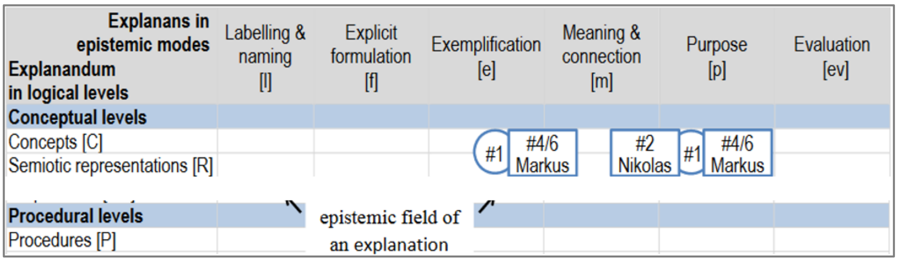


Figure 1: Excerpt of an example of epistemic matrix (Erath & Prediger, 2014)

The epistemic matrix for explaining conditional probabilities

In order to follow the content-specific focus on conceptual and procedural knowledge elements, the epistemic matrix is now exemplarily filled out for the current mathematical content of conditional probabilities (Fig. 2). We differentiate current mathematical content from prior mathematical content because focussing on the content - and with it on students’ prior knowledge in contrast to the current content - as separate and constituent element of diagnostic judgments and fostering students’ understanding has only rarely been investigated (Dröse, Griese & Wessel, accepted). The following explanations regard Fig. 2 for the conceptual and the procedural level:

On the *conceptual level*, concepts concerning stochastic (in)dependence are knowledge elements of the current mathematical content (cf. row --CC--) (Hoffrage et al., 2015), while they build upon conceptual knowledge elements from prior mathematical content (cf. row --CP--), e.g. the part-whole or part-of-part relationship as concepts of fractions and the multiplication of fractions (Post & Prediger, 2020; Prediger & Schink, 2009). The epistemic modes can be described as follows. Here examples are given:

- *Naming:* “conditional probability”, --CC--, |L|
- *Explicit formulation:* “the definition of conditional probabilities”, -CC--, |F|
- *Exemplification:* “distinguish joint and conditional probabilities”, --CC--, |E|
- *Meaning & connection:* “visualize conditional probabilities”, --CC--, |M|

On the *procedural level* different procedures can be focused either in the current mathematical content of conditional probabilities (cf. row --PC--) (see overview in Binder et al., 2020) or in prior mathematical contexts as routine calculations on fractions (cf. row --PP--) (Prediger & Schink, 2009). Again, different epistemic modes for these procedures can be distinguished (examples given):

- *Naming:* “rule of Bayes”, row --PC--, column |L|
- *Explicit formulation:* “formulate the formula of Bayes”, --PC--, |F|
- *Exemplification:* “express conditions of applying the formula”, --PC--, |E|
- *Meaning & connection:* “explaining the formula”, --PC--, |M|

Epistemic modes / Logical levels	Labeling & naming |L|	Explicit formulation |F|	Exemplification |E|	Meaning & connection |M|
Conceptual levels				
Current mathe-matical content --CC--	Term: Conditional probability	Definition of conditional probabilities	Distinguishing joint and conditional probabilities	Concept or visuali-zation of conditional probabilities
Prior mathe-matical content --CP--	Term: fraction	Definition of fractions, parts etc.	Examples of fractions and parts	Unit square as visualization
Procedural levels				
Current mathe-matical content --PC--	Rule of Bayes	Formulating the formula of Bayes	Expressing conditions of application of the formula	Explaining the formula
Prior mathe-matical content --PP--	Formula of multiplying fraction	Procedure of multiplying fractions	Expressing conditions of application of the formula	Explaining the formula part-of-part relation

Figure 2: Epistemic matrix exemplarily specified for conditional probabilities (adapted from Erath & Prediger, 2014)

Research questions

For pursuing the described research interest, we investigate this research questions:

(RQ1) How can teachers' competence of fostering students' understanding be investigated with the epistemic matrix?

(RQ2) Which pathways through the epistemic matrix can be identified in prospective teachers' moves for fostering a student (hereafter abbreviated as PTMF)?

METHODS

Data collection

The data was collected in a university mathematics education course with n=26 prospective secondary school teachers in Germany. The sample can be characterized as follows: 81% of the prospective teachers' study for upper secondary school and 19% study for vocational schools as well as 69% of the prospective teachers attend the course in their sixth semester and 31% attend the course in their eighth semester and all in the last year of their bachelor programme. The course covers among other topics content knowledge and pedagogical content knowledge on conditional probabilities.

For assessing the prospective teachers' competences, a vignette displayed in Fig. 3 is used as an already established instrument in mathematics education research (cf. overview in Buchbinder & Kuntze, 2018). The vignette, consisting of a written student solution and a following transcript, based on a real dialogue (in Post & Prediger, 2020), that give insights into the student's understanding and obstacles concerning conditional probabilities and fractions as the underlying prior mathematical content. For assessing the prospective teachers' competence of fostering students' understanding a script wri-

ting task inspired by lesson plays (Zazki, Liljedahl, & Sinclair, 2009) is integrated.

Transcript vignette

For the following task you should put yourself in the teachers' position and react appropriately within the situation.

Background information for the following scene: The class in a German upper secondary school has covered conditional probabilities and their calculation.

In the following you will read two transcripts displaying excerpts from conversations between two students (Ole and Nazan) and their teacher, regarding the task displayed on the right.

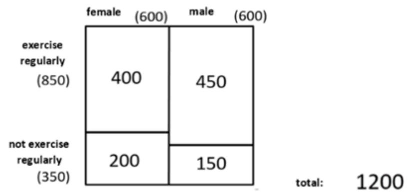
Task: Exercising Teenagers

In a survey, 1200 teenagers were asked if they exercise regularly. 600 out of the 1200 teenagers are female.

$\frac{1}{8}$ of the female teenagers do not exercise regularly. $\frac{3}{8}$ of the teenagers are male and exercise on a regular basis.

What is the probability that a random male person exercises regularly?

(JIM study 2018)



Part 1: Ole solves the task. He writes down the following solution.

$\frac{3}{8}$ = 0.375= 0.375%, the probability is 0.375%.

The following interaction with the teacher evolves [not printed here].

Task: Analyze the two transcripts:

Formulate an impulse for continuing the dialogue. Express how you would react to foster Ole. Write a continuation for the transcript.

Figure 3: Transcript vignette with task for prospective teachers
(completely printed in Dröse, Griese & Wessel, accepted)

Data analysis

The 26 written documents containing PTMF were coded in two steps.

(1) Two raters coded the written documents containing PTMF for logical levels with the codes: --conceptual level of current mathematical content--, --conceptual level of prior mathematical content--, --procedural level of current mathematical content-- and --procedural level of prior mathematical content-- with an interrater reliability of Cohen's $\kappa = 0.89$.

(2) In a second step, for each knowledge element the epistemic mode has been coded and double-checked by the second rater.

(3) The codes have been displayed in the epistemic matrix and the pathways through the epistemic matrix have been categorized into different types.

EMPIRICAL FINDINGS ON PROSPECTIVE TEACHERS' COMPETENCE OF FOSTERING STUDENTS' UNDERSTANDING

The application of the data analysis method to the written documents enabled us to access which epistemic fields (epistemic mode + logical level) have been addressed in the dialogue containing PTMF by the prospective teachers in which order. With this identification of addressed epistemic fields, we suggest a tool for investigating a facet of teachers' competence of fostering students' understanding (RQ1). These analyses provide us with three main types of moves for fostering Ole in the given data set. For assessing RQ2, the three main types are now presented and illustrated by examples.

The first type is characterised by starting with an |exemplification| (|E|) on the --procedural level of the prior mathematical content-- (--PP--) of shortening fractions. After that, the moves continue addressing the |meaning and connections| (|M|) also in the --prior mathematical content but on a conceptual level-- (--CP--) of fractions. This type was found among 5 out of 17 dialogues containing PTMF. The following extract shows only the teachers' moves, as they would have been set by the prospective teachers. The example pathways through the epistemic matrix as displayed in Fig. 4.

1 T: So then try out, if it is the same.

3 T: Okay. And what is the meaning of 3/8? Perhaps it is helpful if you read the text again carefully and include the unit square.

5 T: Right. And the 3/4? What is this part? And what has been searched for in the task?

The second type found among 4 out of 17 dialogues containing PTMF has the same starting point but continues by addressing different epistemic modes in the --conceptual level of the current mathematical content--(--CC--). The example pathway that addresses the |exemplification| (|E|) of the current mathematical content is displayed in Fig. 4.

1 T: Have a look at the two fractions again, if the two numbers can be the same.

3 T: 3/8 is the probability, that out of all teenagers a random chosen person is male and exercise. Can you see the difference [to 450/600]?

The third type found by 8 out of 17 prospective teachers also has the same starting point and continues on the --conceptual level of the prior mathematical content-- (--CP--) as well as afterwards on the --conceptual level of the current mathematical content-- (--CC--). The pathway through the epistemic matrix for the following teacher moves is again displayed in Fig. 4.

1 T: We can calculate, if it is the same. With which number can 450/600 be shortened?

3 T: Very good. And is this the same part as the one you have had before?

5 T: Ok, lets have a look at the task again. Which part had to be calculated?

7 T: Very good, so we have calculated a conditional probability. What has been the condition in this task?

Fig. 4 displays all three types found for the continuation of the dialogue with teachers' move in order to foster Ole. T1-T7 describe the turns of the teacher moves.

Contrasting the three types, we see that all prospective teachers start with addressing the same epistemic field. After that, the types take different routes through the epistemic matrix. The first type addresses also the --conceptual level of the prior mathematical content-- while it does not reach the current mathematical content of the task. The second type reaches the current mathematical content, but might miss potential students' obstacles in the --conceptual level of the prior mathematical content-- belonging to the --procedural level of the prior mathematical content--. The third type addresses the --conceptual level of the prior mathematical content-- as well as the --conceptual level of the current mathematical content-- and therefore unites impulses of type one and two.

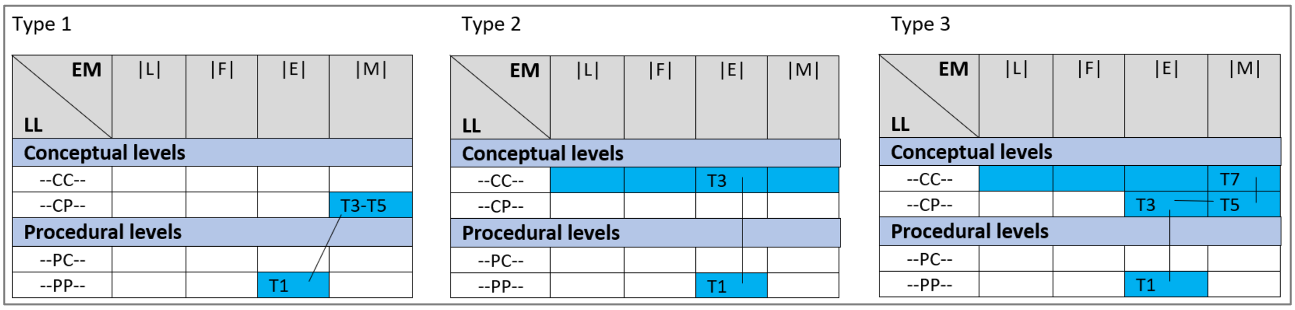


Figure 4: Three types of pathways through the epistemic matrix identified for Ole

DISCUSSION AND OUTLOOK

Concerning the first research question, we see that applying the epistemic matrix for explaining (Erath & Prediger, 2014) to the written prospective teachers' moves for fostering Ole provides different types of teacher moves and therefore might give deeper insights into what is more in teachers' competence for fostering students under-standing than the already identified facets (Ball et al. 2008; Loibl et al., 2020).

The second research question aims at investigating which pathways through the epistemic matrix prospective teachers take in their written moves. This led to the afore mentioned three types of teacher moves for fostering Ole. Those types cover different logical levels and epistemic modes that are of theoretical importance as they enrich the already existing research on prospective teachers' obstacles (Jansen & Spitzer, 2009) with deeper knowledge on prospective teachers' moves for a specific vignette. In addition, this is especially important for teacher educators as knowing prospective teachers' moves makes it possible to adjust teaching-learning arrangements to the prospective teachers' competencies.

Meanwhile, our research is limited due to the small sample size of only 26 prospective teachers and the specific content and given transcript vignette. Future research has to extend on the one hand the sample size and provide further insights for other mathematical content areas and vignette formats (Buchbinder & Kuntze, 2018).

Acknowledgements.

We thank Birgit Griese for her support and collaboration.

References

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.

Buchbinder, O., & Kuntze, S. (2018). Representations of Practice in Teacher Education and Research – Spotlight on Different Approaches. In S. Kuntze & O. Buchbinder (Eds.), *Mathematics teachers engaging with representations of practice. A dynamically evolving field* (pp. 1–8). Springer.

Dröse, J., Griese, B., & Wessel, L. (accepted). Prospective teachers' diagnostic judgements on students' understanding of conditional probabilities. *Presentation at CERME 12.*

Empson, S. B., & Jacobs, V. J. (2008). Learning to Listen to Children's Mathematics. In T. Wood & P. Sullivan (Eds.), *International Handbook of Mathematics Teacher Education.* (Vol. 1, pp. 257–281). Sense.

Erath, K., & Prediger, S. (2014). Mathematical Practices as Under-determined Learning Goals: The Case of Explaining Diagrams in Different Classroom Microcultures. In Oesterle, S., Liljedahl, P., Nicol, C., & Allan, D. (Eds.*) Proceedings of the Joint Meeting3-17of PME 38 and PME-NA 36*, Vol. 3, pp. 17-24. Vancouver, Canada: PME.

Binder, K., Krauss, S., & Wiesner, P. (2020). A new visualization for probabilistic situations containing two binary events: The frequency net. *Frontiers in Psychology*, *11*(750), 1–21.

Hoffrage, U., Krauss, S., Martignon, L., & Gigerenzer, G. (2015). Natural frequencies improve Bayesian reasoning in simple and complex inference tasks. *Frontiers in Psychology 6*(1473), 9–22.

Jansen, A., & Spitzer, S. M. (2009). Prospective middle school mathematics teachers' reflective thinking skills: Descriptions of their students' thinking and interpretations of their teaching. *Journal of Mathematics Teacher Education*, *12*(2), 133–151.

Loibl, K., Leuders, T., & Dörfler, T. (2020). A Framework for Explaining Teachers' Diagnostic Judgements by Cognitive Modeling (DiaCoM). *Teaching and Teacher Education*, 91, 103059.

Post, M. & Prediger, S. (2020). Decoding and discussing part-whole relationships in probability area models: The role of meaning-related language. In J. Ingram, K. Erath, F. Rønning & A. Schüler-Meyer (Eds.), *Proceedings of the Seventh ERME Topic Conference on Language in the Mathematics Classroom* (pp. 105–113). Montpellier, France: ERME.

Prediger, S., & Buró, R. (2021, in press). Fifty ways to work with students' diverse abilities? A video study on inclusive teaching practices in secondary mathematics classrooms *International Journal of Inclusive Education*.

Prediger, S. & Schink, A. (2009). 'Three eights of which whole?' - Dealing with changing referent wholes as a key to the part-of-part-model for the multiplication of fractions. In M. Tzekaki, M. Kaldrimidou & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education.* (pp. 4409–4416) Thessaloniki, Greece: PME.

Zazkis, R., Liljedahl, P., & Sinclair, N. (2009). Lesson plays: Planning teaching vs. teaching planning. *For the learning of mathematics, 29*(1), 40-47.