Modelos y técnicas para el desarrollo de comportamientos de Robots Autónomos

Bayesian Estimation

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Doctorado Depto. CCIA - Robótica y Visión
Contents

1. Uncertainty in Robotics
2. Dynamic State Estimation
3. Bayesian Filters
We are interested in controlling autonomous mobile robots that continuously sense and manoeuvre in a semi-structured environment with a defined goal.

**Interesting**
- Robot guides
- Robot transporters
- Cars
- Vacuum cleaners

**Not interesting**
- Manipulators
- Humanoids
- AGVs
- Game avatars
Uncertainty in Robotics

- Sensors obtain imprecise readings of the environment.
- From them we want to infer (estimate) the most probable state of the world.
- We have to model error and imprecision.
Example problem 1

1. A sonar mounted on a robot gives the distance to an obstacle in front of it: \( z_1 = 3.20 \) m.

2. A infrared sensor towards the same position gives another distance: \( z_2 = 3.00 \) m.

3. After that, a laser gives another distance: \( z_3 = 3.50 \) m.

What is the real distance of the obstacle? What is the error in the measure? How to model it?
Raw sensor data

(a) Sonar data

(b) Laser data

Figure 6.5 Typical data obtained with (a) a sonar sensor and (b) a laser range sensor in an office environment for a "true" range of 300 cm and a maximum range of 500 cm.
Sonar uncertainty

Sonar readings have great uncertainty due to the angular cone and the reflections.

(a) Figure 6.1 (a) Typical ultrasound scan of a robot in its environment. (b) A misreading in ultrasonic sensing. This effect occurs when firing a sonar signal towards a reflective surface at an angle $\alpha$ that exceeds half the opening angle of the sensor.
Probabilistic proximity sensors

The *likelihood* of a given reading \( z \) gives the probability of that reading given that the robot is at position \( x \): \( p(z \mid x) \)
Approximation results

Histogram of readings for two given positions

Sonar
Laser

300cm 400cm
Likelihood functions

Complete histograms and generated likelihood functions
A harder version of the problem

• Given an occupancy grid $m$ and a laser reading $z$

• Compute the likelihood of having obtained the reading from the position $x$
Example problem 2

- A sonar array returns the values \( z = \{\infty, 2, 1, 2, 3, 2, \infty, \infty\} \).

- In what zone of the environment (0,1,2) is the robot placed?

Solution: apply Bayes rule and compute the MAP (Maximum A Posteriori).
Example problem 3

- A robot is moving and the readings of its sonars are:
  
  \[ z_1 = \{\infty, 2, 1, 2, 3, 2, \infty, \infty\} \]
  \[ z_2 = \{4, 2, 1, 2, 3, 3, \infty, \infty\} \]
  \[ z_3 = \{3, 2, 1, 3, 4, \infty, \infty, \infty\} \]

- Where is the robot, given a known (obstacle) map of the environment?

Solution: use a Bayesian filter (Kalman, particle, etc.) to update the robot motion and integrate the sensor readings.
Bayesian Filters

One of the most important techniques in probabilistic robotics is the use of bayesian filters to:

- Estimate the state of the robot
- after integrate the likelihood of the sensor perceptions
- and update it with the robot action model
Probability theory and Robotics

- Random variables and probability functions to model readings.
- Variance and covariance to model error.
- Bayes theorem to infer unknown data.
- Expectations and Bayesian Filters to integrate the readings.
Remembering basic concepts

- Axioms of probability
- Random variables
- Probability Density Functions
- Joint distributions
- Bayes rule
- Expectation, variance, covariance and correlation
Axioms of probability

Let $A, B, \ldots$ be propositions about the outcomes of a given experiment.

The number $P(A)$ represents the probability that the proposition $A$ is true after performing the experiment. It must satisfy the following three axioms:

- $P(A) > 0$
- $P(\text{True}) = 1$
- If $A$ and $B$ are exclusive (there are no outcome in which both are true) then $P(A \lor B) = P(A) + P(B)$
Example

Experiment: our robot is about to enter a given room. Some propositions:

- $A = $ The door is open
- $B = $ There is someone in front of the door
- $C = $ There is a chair in front of the door

$A$ and $C$ are exclusive (there is no event in which both are true)
Frequencies

It’s usual to interpret probabilities as relative frequencies.

We count the number of times that $A$ is true and the number of executions of the experiment. Let’s suppose that our robot tries to enter in the room 30 times:

- A (Open) = |||||||||||||||||||||||| (25)
- B (Someone) = |||||||||||| (12)
- C (Chair) = || (2)

\[
P(A) = \frac{25}{30} = 0.83, \quad P(B) = \frac{12}{30} = 0.4, \quad P(C) = \frac{2}{30} = 0.07
\]
The axioms of Probability

- \( P(A) > 0 \)
- \( P(True) = 1 \)
- \( P(A \lor B) = P(A) + P(B) \)

Derivations:

- \( P(B \lor C') = P(B) + P(C') - P(B \land C') \)
- \( P(\neg A) = 1 - P(A) \)
- \( P(B) = P(B \land C') + P(B \land \neg C') \)
Discrete Random Variables

• $X$ denotes a random variable

• $X$ can take on a countable number of values in
  \[ \{x_1, x_2, \ldots, x_n\} \]

• $P(X = x_i)$ or $P(x_i)$ is the probability that an outcome of
  the experiment produces the value $x_i$ if:

\[
p(x_i) \geq 0
\]

\[
\sum_{i} p(x_i) = 1
\]
Continuous Random Variables

- $X$ takes on values in the continuum

- $p(X = x)$ or $p(x)$ is a probability density function or PDF iff

\[
\forall x \ p(x) \geq 0
\]

\[
\int_{-\infty}^{\infty} p(x) \, dx = 1
\]
PDF and cumulative function

The probability of an event gives an outcome between values $a$ and $b$ is

$$P(a < X \leq b) = \int_{a}^{b} p(x)\,dx$$

The cumulative distribution function is:

$$P(X \leq x) = \int_{-\infty}^{x} p(x)\,dx$$

Relation between the PDF and the cumulative distribution:

$$p(x) = \lim_{h \to 0} \frac{P(x - \frac{h}{2} < X \leq x + \frac{h}{2})}{h} = \frac{\partial P(X \leq x)}{\partial x}$$
Understanding $p(x)$

- What's the gut-feel meaning of $p(x)$?

If

$$\frac{p(a)}{p(b)} = \alpha$$

then

when a value $x$ is sampled from the distribution, you are $\alpha$ times as likely to find that $x$ is “very close to” $a$ than that $x$ is “very close to” $b$. 
Some Probability Density Functions

- **Uniform:**
  
  $$p(x) = \begin{cases} 
  0 & \text{if } x < a \\
  \frac{1}{b-a} & \text{if } a < x < b \\
  0 & \text{if } x > b 
  \end{cases}$$

- **Triangular:**
  
  $$p(x) = \begin{cases} 
  \frac{2(x-a)}{(b-a)(c-a)} & \text{if } a \leq x \leq c \\
  \frac{2(b-x)}{(b-a)(b-c)} & \text{if } c < x \leq b 
  \end{cases}$$
Exponential pdf

\[ p(x; \lambda) = \begin{cases} 
\lambda e^{-\lambda x} & x \geq 0, \\
0 & x < 0 
\end{cases} \]
Normal (Gaussian) pdf

\[ p(x; \mu, \sigma^2) = N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left\{ -\frac{1}{2} \frac{(x - \mu)^2}{\sigma^2} \right\} \]

\[ \mu = \text{mean}, \quad \sigma^2 = \text{variance} \]
Joint distribution

- The joint distribution of two random variables $X$ and $Y$ is given by:

  $$p(x, y) = P(X = x \text{ and } Y = y)$$

- Conditional probability: the probability of $X = x$ given that $Y = y$ is denoted by $p(x \mid y)$.

  $$p(x \mid y) = \frac{p(x, y)}{p(y)}$$

- If $X$ and $Y$ are independents then

  $$p(x, y) = p(x)p(y) \quad p(x \mid y) = p(x)$$
PDF in 2 dimensions

\[ p(x, y) = \text{probability density of random variables} \ (X, Y) \ \text{at location} \ (x, y) \]

\[ P((X, Y) \in R) = \int \int_{(x,y)\in R} p(x, y) \ dy \ dx \]
Multivariate Normal distributions

If $x$ is a multi-dimensional vector (column vector):

$$ x = (x_1, \ldots, x_n)^T $$

the density function of the multivariate Normal distribution is:

$$ p(x; \mu, \Sigma) = \det(2\pi \Sigma)^{\frac{1}{2}} \exp\left\{ -\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu) \right\} $$

Means and covariances:

$$ \mu = (\mu_1, \ldots, \mu_n)^T $$

$$ \Sigma = \begin{bmatrix}
\sigma_1^2 & \cdots & \sigma_{1n} \\
\vdots & \ddots & \vdots \\
\sigma_{n1} & \cdots & \sigma_n^2
\end{bmatrix} $$
Marginals

- **Discrete distributions**
  \[ P(x) = \sum_x P(x, y) \]

- **Continuous distributions**
  \[ p(x) = \int p(x, y) \, dy \]
Graphical insight (1)

**Marginals** $p(x)$ and $p(y)$

\[
p(x) = \int_{-\infty}^{\infty} p(x, y) dy
\]
Graphical insight (2)

Conditional Distributions

\[ p(\text{mpg} \mid \text{weight} = 4600) \]

\[ p(\text{mpg} \mid \text{weight} = 3200) \]

\[ p(\text{mpg} \mid \text{weight} = 2000) \]

\[ p(x \mid y) = \text{p.d.f. of } X \text{ when } Y = y \]
Algebraic manipulation

Based on three rules:

• the **product rule**,  
• the **sum rule** and  
• the **Bayes theorem**
Product rule

Also called the chain rule, it’s derived from the definition of conditional probability

\[ p(x, y \mid H) = p(x \mid y, H) \, p(y \mid H) \]
\[ = p(y \mid x, H) \, p(x \mid H) \]

For generality we include the value \( H \) which represents all other known or assumed quantities.
Sum rule

Also called the total probability theorem, the sum rule follows from the definition of marginalisation

\[
p(x \mid H) = \int p(x, y \mid H) \, dy
\]

\[
= \int p(x \mid y, H) \, p(y \mid H) \, dy
\]
Bayes rule

The Bayes theorem is derived from the product rule

\[
p(x \mid y, H) = \frac{p(y \mid x, H) \ p(x \mid H)}{p(y \mid H)}
\]

\[
= \frac{p(y \mid x, H) \ p(x \mid H)}{\int p(y \mid x, H) \ p(x \mid H) \ dx}
\]

The denominator is a *normalizing constant* that does not depend on \(x\) and such that \(p(x \mid y, H)\) integrates to one. We can then can formulate the rule in this simpler way:

\[
p(x \mid y, H) = \eta \ p(y \mid x, H) \ p(x \mid H)
\]
Using the Bayes rule

\[ P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)} \]

- \( h \) is the ignored variable (hypothesis) that we want to infer
- \( D \) is the data that we know
- \( P(h) \) is the a priori probability of \( h \)
- \( P(D \mid h) \) is the likelihood of obtaining the data \( D \) if the hypothesis \( h \) is satisfied
- \( P(h \mid D) \) is the a posteriori probability of \( h \)
Maximum a posteriori

\[ P(h \mid D) = \frac{P(D \mid h)P(h)}{P(D)} \]

Generally want the most probable hypothesis given the training data

**Maximum a posteriori hypothesis** \( h_{MAP} \):

\[ h_{MAP} = \arg \max_{h \in H} P(h \mid D) \]

\[ = \arg \max_{h \in H} \frac{P(D \mid h)P(h)}{P(D)} \]

\[ = \arg \max_{h \in H} P(D \mid h)P(h) \]
Expectations

μ = E[X] is the expected value of random variable X

\[ E[X] = \int_{x=-\infty}^{\infty} x p(x) \, dx \]

- The average value we’d see if we took a very large number of random samples of X
- The first moment of the shape formed by the axes and the curve
- The best value to choose if you must guess an unknown value drawn from the distribution and you’ll be fined the square of your error
Expectation of a function

\[ E[f(X)] \] is the expected value of \( f(x) \) where \( x \) is drawn from \( X \)'s distribution

\[
E[f(X)] = \int_{x=-\infty}^{\infty} f(x) p(x) \, dx
\]

- The average value we'd see if we took a very large number of random samples of \( f(X) \)
- Note that in general \( E[f(X)] \neq f(E[X]) \)
- If \( f(x) \) is a linear function:

\[
E[aX + b] = aE[X] + b
\]
Variance

The variance of a random variable gives an indication of how the values of a distribution are separated from the mean. We’ll use it as an indication of the quality of an estimation.

\[
V ar[X] = E[(X - \mu)^2] = \\
= \int_{x=-\infty}^{\infty} (x - \mu)^2 p(x) \, dx
\]

\[
V ar[X] = E[X^2] - (E[X])^2
\]

The standard deviation \( \sigma_x \) is defined as \( \sqrt{V ar[X]} \)
Covariance and correlation (1)

The covariance \( \text{Cov}[X, Y] \) of two random variables \( X \) and \( Y \) is defined as

\[
\text{Cov}[X, Y] = E[(X - \mu_x)(Y - \mu_y)] = E[XY] - E[X]E[Y]
\]

The correlation coefficient \( \rho_{xy} \) is defined by the ratio:

\[
\rho_{xy} = \frac{\text{Cov}[X, Y]}{\sigma_x \sigma_y}
\]
Covariance and correlation (2)

Some properties:

\[ |\rho_{xy}| \leq 1 \quad |\text{Cov}[X, Y]| \leq \sigma_x \sigma_y \]

Two variables are called uncorrelated if their covariance is 0, which is equivalent to:

\[ \text{Cov}[X, Y] = 0 \quad \rho_{xy} = 0 \quad E[XY] = E[X]E[Y] \]

Independence: if \( X \) and \( Y \) are independent then they are uncorrelated. The reverse is not in general true. However, if \( X \) and \( Y \) are normal random variables and their correlation coefficient is 0, then they are independent.
Covariance matrix

Given $n$ random variables $\{X_1, \ldots, X_n\}$ their covariance matrix is defined by:

$$\Sigma = \begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \cdots & C_{nn} \end{bmatrix}$$

being

$$C_{ij} = \text{Cov}[X_i, X_j]$$
Example: measurement errors

We measure an object of length $\eta$ with $n$ instruments of varying accuracies. The results of the measurements are $n$ random variables

$$x_i = \eta + \nu_i \quad E[\nu_i] = 0 \quad Var[\nu_i] = E[\nu_i^2] = \sigma_i^2$$

where $\nu_i$ are the measurement errors which we assume independent with zero mean. Our problem is to determine the unbiased, minimum variance, linear estimation of $\eta$.

We wish to find $n$ constants $\alpha_i$ such that the sum

$$\hat{\eta} = \alpha_1 x_1 + \cdots + \alpha_n x_n$$

is a random variable with mean $E[\hat{\eta}] = \alpha_1 E[x_1] + \cdots + \alpha_n E[x_n] = \eta$ and its variance $V = \alpha_1^2 \sigma_1^2 + \cdots + \alpha_n^2 \sigma_n^2$ is minimum, subject to the constraint:

$$\alpha_1 + \cdots + \alpha_n = 1$$
Solution measurement errors

To solve this problem, we note that

\[ V = \alpha_1^2 \sigma_1^2 \ldots \alpha_n^2 \sigma_n^2 - \lambda (\alpha_1 \ldots \alpha_n - 1) \]

for any \( \lambda \) (Lagrange multiplier). Hence \( V \) is minimum if

\[ \frac{\delta V}{\delta \alpha_i} = 2 \alpha_i \sigma_i^2 = 0 \quad \alpha_i = \frac{\lambda}{2 \sigma_i^2} \]

Inserting into the constraint and solving for \( \lambda \), we obtain

\[ \frac{\lambda}{2} = V = \frac{1}{1/\sigma_1^2 + \cdots + 1/\sigma_n^2} \]

Hence

\[ \hat{\eta} = \frac{x_1/\sigma_1^2 + \cdots + x_n/\sigma_n^2}{1/\sigma_1^2 + \cdots + 1/\sigma_n^2} \]
Dynamic State Estimation

- Introduction
- State estimation with a single observation
- Application: laser scan likelihood
- Probabilistic State Dynamics
- Integrating everything: the Bayes Filter
Introduction

- The problem is to track the sequence of states \( \{x_1, x_2, \ldots , x_t\} \) of a dynamical system.

- The states can not be measured directly, but by means of observations. We obtain a set of \( t \) observations \( \{z_1, z_2, \ldots , z_t\} \) observations corresponding to the \( t \) states. The initial state has a distribution \( p(x_1) \).

- Examples:
  - Robot Localization
  - Missile Guidance
  - Tracking of objects in video sequences
State Estimation with Single Observation

- Given the observation \( z = \{z_1, z_2, \ldots, z_n\} \)

- We want to obtain a posterior estimation (named belief) of the current state \( x \) given that observation

\[
Bel(x) \equiv p(x \mid z_1, \ldots, z_n)
\]
Example

- $z_1 = \text{texture descriptor}$
- $z_2 = \text{number of edges}$
- ...

$x \in \{(\text{door-open}, \text{dist}), (\text{door-closed}, \text{dist}), (\text{wall}, \text{dist})\}$
State estimation with single observation

\[ p(x \mid z_1, \ldots, z_n) = \frac{p(z_n \mid x, z_1, \ldots, z_{n-1})p(x \mid z_1, \ldots, z_{n-1})}{p(z_1, \ldots, z_n)} \]

Independence assumption:

\[ z_i \perp \{z_1, \ldots, z_{i-1}, z_{i+1}, \ldots, z_n\} \]
State estimation with single observation

\[ p(\mathbf{x} \mid z_1, \ldots, z_n) = \frac{p(z_n \mid \mathbf{x}, z_1, \ldots, z_{n-1})p(\mathbf{x} \mid z_1, \ldots, z_{n-1})}{p(z_1, \ldots, z_n)} \]

\[ = \eta p(z_n \mid \mathbf{x}, z_1, \ldots, z_{n-1})p(\mathbf{x} \mid z_1, \ldots, z_{n-1}) \]

\[ = \eta_1 \ldots n \prod_{i \in n} p(z_i \mid \mathbf{x})p(\mathbf{x}) \]
Laser Scan Likelihood

Probabilistic State Dynamics

The current state dependend on the previous state:

\[ p(x_t | x_{t-1}) \]

And only on it (Markov assumption):

\[ p(x_t | x_{t-1}, x_{t-2}, \ldots, x_1) = p(x_t | x_{t-1}) \]

Example: \[ p(x_t | x_{t-1}) \propto e^{-\frac{1}{2}(x_t - x_{t-1} - 1)^2} \]

[M. Isard, 1998]
Modeling Actions

- The actions $u$ change the world state according to a distribution

$$p(x_t | x_{t-1}, u),$$

being $x_{t-1}$ the state of the world before executing the action $u$ that leads to the state $x_t$

- Robot motion model:
Integrating everything: Bayes Filter

We obtain a recursive formulation

\[
Bel(x_t) = \eta p(z_t | x_t)p(x_t | u_{t-1}, \ldots, z_1)
\]

\[
\text{Total prob} = \eta p(z_t | x_t) \int_{x_{t-1}} p(x_t | x_{t-1} u_{t-1}, \ldots, z_1)p(x_{t-1} | u_{t-1}, \ldots, z_1) dx_{t-1}
\]

\[
\text{Markov} = \eta p(z_t | x_t) \int_{x_{t-1}} p(x_t | x_{t-1}, u_{t-1})p(x_{t-1} | u_{t-1}, \ldots, z_1) dx_{t-1}
\]

\[
= p(z_t | x_t) \int_{x_{t-1}} p(x_t | x_{t-1}, u_{t-1})Bel(x_{t-1}) dx_{t-1}
\]
The Bayes Filter

Given:

- A robot action $u_t$
- A sensor reading $z_t$
- A a-priori robot state belief estimation

$$ \text{bel}(x_{t-1}) = P(x_{t-1}) $$

Computes:

- The a-posteriori robot state belief estimate

$$ \text{bel}(x_t) = P(x_t \mid x_{t-1}, u_t, z_t) $$
The Bayes Filter Algorithm

\[
Bel(x_t) = p(z_t \mid x_t) \int_{x_{t-1}} p(x_t \mid x_{t-1}, u_{t-1}) Bel(x_{t-1}) dx_{t-1}
\]

**Algorithm Bayes Filter** \((\text{bel}(x_{t-1}), z_t, u_t)\):

*Estimate the new state from the previous and the action*

1. For all \( x_t \in X \):
   \[ \text{bel}^- (x_i) = \sum \text{bel}(x_{t-1}) P(x_t \mid u_t, x_{t-1}) \]

*Incorporate the observations in the belief*

2. \[ \text{bel}(x_t) = \eta P(z_t \mid x_t) \text{bel}^- (x_t) \]

3. return \( \text{bel}(x_t) \)
Bayes Filter Techniques

\[
Bel(x_t) = p(z_t | x_t) \int_{x_{t-1}} p(x_t | x_{t-1}, u_{t-1}) Bel(x_{t-1}) dx_{t-1}
\]

- Kalman Filter
- Particle Filter
- Hidden Markov Models
- Dynamic Bayes Networks
- Partially Observable Markov Decision Process
References


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2. A Taxonomy focused on Coordination

3. Some Strategies for Coordination

4. Conclusions

5. Practical: Player/Stage
Description

MultiRobot Systems (MRS): Set of robots operating in the same environment
MRS vs. Single Robot

**Advantages:**
- Inherent parallelism and redundancy: more efficiency, robustness and flexibility
- Simplify design of robots
- Some tasks require a team

**Disadvantages:**
- More communication
- Harder to test
- N x the trouble
Requerimientos for MRS (1)

- **Robustness**: no single point of failure for the system
- **Optimized**: optimized response to dynamic conditions
- **Speed**: quick response to dynamic conditions
- **Extensibility**: easily extendable to accommodate new functionality
- **Communication**: ability to deal with limited and imperfect communication
- **Resources**: ability to reason about limited resources
- **Allocation**: optimized allocation of tasks
Requerimientos for MRS (2)

- **Heterogeneity**: ability to accommodate heterogeneous teams of robots
- **Roles**: optimized adoption of roles
- **New Input**: ability to dynamically handle new tasks, resources and roles
- **Flexibility**: easily adaptable for different applications
- **Fluidity**: easily able to accommodate the addition/subtraction of robots during operation
- **Learning**: on-line adaptation for especific applications
- **Implementation**: Implemented and proven on physical system
Features to develop MRS

- **Communication:**
  - Direct: dedicated hardware device
  - Indirect: stigmergy

- **Team Composition:**
  - Homogeneous: the same hardware and control software
  - Heterogeneous: differ either in the hardware or in the software

- **System Architecture:**
  - Deliberative Architecture: strategy to reorganize the overall team behaviors
  - Reactive Architecture: reorganize individual tasks

- **Team Size**
Tasks and Domains for MRS

- **Foraging**: rescue and search operations, toxic waste cleaning, mine cleaning, ...
- **Coverage**: snow removal, lawn mowing, car-body painting, ...
- **Multitarget observation**: security, surveillance, recognition, ...
- **Box pushing and object transportation**: stockage, transportation of heavy objects, ...
- **Exploration**: exploration of dangerous environments, ...
- **Flocking**: transhipment operations in harbors and airports, ...
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### Dimensions for MRS Taxonomy focused on Coordination (**)

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<th>Description</th>
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<tr>
<td>Cooperation</td>
<td>Robots that operate together to perform some global task</td>
</tr>
<tr>
<td>Knowledge</td>
<td>That each robot in the team has about its team mates</td>
</tr>
<tr>
<td>Coordination</td>
<td>The actions performed by each robot take into account the actions executed by the other robot</td>
</tr>
<tr>
<td>Organization</td>
<td>The way the decision system is realized within the MRS</td>
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(**) Multirobot Systems: A Classification Focused on Coordination  
*Alessandro Farinelli, Luca Iocchi, Daniele Nardi*  
*IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS, Oct 2004*
A Taxonomy of MRS

- Cooperation

- Cooperative

- Not coordinated

- Strongly centralized
- Weakly centralized
- Distributed

- Knowledge
- Coordination

- ¾ to perform some global task
A Taxonomy of MRS

- Cooperation
  - Cooperative
  - Awareness
    - Aware
    - Unaware

- Without any knowledge about the other robots
- Biologically inspired MRS: stigmergic communication, emergence behavior and self-organization
- Coverage problem: To choose the most promising direction of motion simply evaluating through vision the direction that maximizes the frontal visibility
Unaware Systems

Box-pushing

USC Robotics Research Lab
A Taxonomy of MRS

- **Cooperation**
  - Cooperative

- **Knowledge**
  - Aware
  - Unaware

- The robots have knowledge of the presence of other robots in the environment
A Taxonomy of MRS

- A robot may not take into account the actions performed by other robots in order to accomplish its task.
- It avoids the use of a specific protocol.
- Coverage problem: to choose the direction of motion which is opposite to the average angle subtended by all its neighbors in its visual field.
A Taxonomy of MRS

- **Cooperation**
  - Cooperative

- **Knowledge**
  - Aware
  - Unaware

- **Coordination**
  - Strongly coordinated
  - Weakly coordinated
  - Not coordinated

- It does not rely on an explicit predefined coordination protocol
- To minimize the use of communication
- Coverage problem: to form coalitions between two robots and to calculate the most promising direction of motion as that direction than maximizes the coalition sensorial coverage
Weakly Coordinated Systems

- **Flocking birds** use sensing to monitor the actions of other birds in their vicinity to make local corrections to their own motion.
A Taxonomy of MRS

Based on a system of signals by which the robots exchange information, according to a predefined coordination protocol (set of explicit predefined rules that the robots must follow in order to interact with each other in the environment)
A Taxonomy of MRS

- Leader (or station central) is in charge of organizing the work of the entire team
- Potential to be optimal
- High computation demand required by the leader
- Single point of failure

### Coordination
- Strongly coordinated
- Weakly coordinated
- Not coordinated

### Organization
- Strongly centralized
- Weakly centralized
- Distributed
A Taxonomy of MRS

- The leader is selected dynamically during the mission
- Policies for the allocation of leader
A Taxonomy of MRS

- To take decisions in a completely autonomous fashion
- To entail some kind of communication
- Robust to communication failures and robot malfunctioning
A Taxonomy of MRS focused on Coordination

- Cooperation
  - Cooperative

- Knowledge
  - Aware
  - Unaware

- Coordination
  - Strongly coordinated
  - Weakly coordinated
  - Not coordinated

- Organization
  - Strongly centralized
  - Weakly centralized
  - Distributed
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(3.1) A General Algorithm for Robot Formations Using Local Sensing and Minimal Communication

- Jakob Fredslund, Maja J Matarić
(3.1) A General Algorithm for Robot Formations Using Local Sensing and Minimal Communication

- To establish and maintain formations with some predetermined geometric shape
- Using only local sensing and minimal communication
- Robustness to changes in group size
- Dynamic switching between formations
- Obstacle avoidance
- To keep a single designated neighbor (friend)
- A general analytical measure for evaluating formations
(3.1) A General Algorithm for Robot Formations Using Local Sensing and Minimal Communication

**Assumptions:**
- **Sensors:**
  - Laser scanner (distance to friend)
  - Color camera (identifies the friend using color-blob detection)
  - $\pm 90^\circ$ field of view
  - Camera can be panned (friend in the center of the field view, \textit{camangle})
  - Sonar (collision avoidance)
- Each robot has a unique \textit{ID} detectable by other robots
- Each robot broadcasts its \textit{ID} regularly (every second) as a heartbeat message (to know how many robots $N$ and \textit{lessThanMe})
- The conductor broadcasts indicating the current formation $f$ (every two seconds)
- To avoid obstacle: swerve message with its ID, angle and direction of the turn. Other robots make a swerve of solidarity
- The robots start out in the right order

- Pioneer2 DX with the SICK LMS200 laser, sonars, the Sony PTZ camera, and running Player software
(3.1) A General Algorithm for Robot Formations Using Local Sensing and Minimal Communication

- Chain of friendships
- One robot is the conductor
- Dynamic switching between formations changing the conductor (WEAKLY CENTRALIZED)
(3.1) A General Algorithm for Robot Formations Using Local Sensing and Minimal Communication

\[
n = \lceil N/4 \rceil
\]

if lessThanMe < \lfloor N/2 \rfloor - n
    \[\theta = 45\]
else if lessThanMe < \lfloor N/2 \rfloor
    \[\theta = -45\]
else if lessThanMe > \lfloor N/2 \rfloor + n
    \[\theta = -45\]
else if lessThanMe > \lfloor N/2 \rfloor
    \[\theta = 45\]
(3.1) A General Algorithm for Robot Formations Using Local Sensing and Minimal Communication

- State Data: own ID, N, IDs, lessThanMe, chain of friendships, camangle
- Communications: heart-beat messages, formation messages, swerve messages
- Control Wheels: Translational and Rotational speed
(3.1) A General Algorithm for Robot Formations Using Local Sensing and Minimal Communication

• Evaluation Criteria:

  • Uniform dispersion: the same distance should be kept between all neighboring robots

  • Shape: it should be possible to lay out the desired shape over the positional data and adjust the angles so than all robots are close to this shape

  • Orientation: the stretching from Criterion 2 must not skew the heading more than $\varepsilon_a$
(3.1) A General Algorithm for Robot Formations Using Local Sensing and Minimal Communication
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5. Practical: Player/Stage
(3.2) Coordinated Multi-Robot Exploration

- W. Burgard, M. Moors, C. Stachniss, F. Schneider
- *IEEE Transactions on Robotics* 21(3), 2005
(3.2) Coordinated Multi-Robot Exploration

- To explore an **unknown environment** with a team of robots
- To choose appropriate **target points** (unexplored areas) for the individual robots
- To move every robot to the target point that is closest: two robots choose the **same target**
- To simultaneously take into account the **cost** of reaching a target point and its **utility** (part of the environment will be covered by the robot’s sensor when it reaches its designated target point)
(3.2) Coordinated Multi-Robot Exploration

UnCoordinated Exploration

Coordinated Exploration
(3.2) Coordinated Multi-Robot Exploration

- **Assumptions:**
  - Both the map of the area explored **so far** and the positions of the robots in this map can be communicated between the robots (**STRONGLY CENTRALIZED**).
(3.2) Coordinated Multi-Robot Exploration

- **Occupancy grid**: map to represent the environment
- **Frontier cell** = explored cell that is an immediate neighbor of an unknown
- **Cost** of a cell: proportional to the distance between the robot and that cell
- **Utility** of a frontier cell:
  - number of robots that are moving to that cell or to a place close to that cell
  - part of the environment will be covered by the robot’s sensor when it reaches its designated frontier
(3.2) Coordinated Multi-Robot Exploration

- Computing Costs of Frontiers Cells
  - The minimum-cost path

1) Initialization. The grid cell that contains the robot location is initialized with 0, all others with $\infty$:

$$V_{x,y} \leftarrow \begin{cases} 0, & \text{if } (x, y) \text{ is the robot position} \\ \infty, & \text{otherwise} \end{cases}$$

2) Update loop. For all grid cells $(x, y)$ do:

$$V_{x,y} \leftarrow \min \left\{ V_{x+\Delta x,y+\Delta y} + \sqrt{\Delta x^2 + \Delta y^2} \right\}$$

$$\cdot P(occ_{x+\Delta x,y+\Delta y}) \mid \Delta x, \Delta y \in \{-1, 0, 1\}$$

$$\land P(occ_{x+\Delta x,y+\Delta y}) \in [0, occ_{max}] \right\},$$
(3.2) Coordinated Multi-Robot Exploration

- Computing Utilities of Frontier Cells:
  - If there already is a robot that moves to a particular frontier cell, the utility of that cell can be expected to be lower for other robots

\[ U(t_n | t_1, \ldots, t_{n-1}) = U_{t_n} - \sum_{i=1}^{n-1} P(||t_n - t_i||) \]

- The utility of the adjacent frontier cells in distance \( d \) is reduced according to the probability that the robot’s sensors will cover cells in distance \( d \)

\[ P(d) = \begin{cases} 
1.0 - \frac{d}{\text{max\_range}}, & \text{if } d < \text{max\_range} \\
0, & \text{otherwise}
\end{cases} \]
(3.2) Coordinated Multi-Robot Exploration

Algorithm 1 Goal Assignment for Coordinated Multi-Robot Exploration.

1: Determine the set of frontier cells.
2: Compute for each robot $i$ the cost $V^i_t$ for reaching each frontier cell $t$.
3: Set the utility $U_t$ of all frontier cells to 1.
4: while there is one robot left without a target point do
5: Determine a robot $i$ and a frontier cell $t$ which satisfy:
   $$(i, t) = \arg\max_{(i', t')} \left( U_{t'} - \beta \cdot V^i_{t'} \right).$$
6: Reduce the utility of each target point $t'$ in the visibility area according to $U_{t'} \leftarrow U_{t'} - P(||t - t'||)$. 
7: end while
(3.2) Coordinated Multi-Robot Exploration

To re-compute the target locations:

- **Unlimited Communication:**
  - One robot has reached its designated target location
  - The distance traveled by the robots exceeds a given threshold
  - The time elapsed after computing the latest assignment exceeds a given threshold

- **Limited Communication:**
  - Sub-team of robots: ad-hoc network which forms clusters
  - Each robot stores for each other robot a log of sensor measurement that not been transmitted to the corresponding robot so far
  - The robots maintain a data structure containing the time stamp of the latest sensor measurement of a robot that was transmitted to all other robots
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(3.3) Multirobot Cooperation for Surveillance of Multiple Moving Targets – A New Behavioral Approach

- Andreas Kolling, Stefano Carpin
- *IEEE International Conference on Robotics and Automation, 2006*
(3.3) Multirobot Cooperation for Surveillance of Multiple Moving Targets – A New Behavioral Approach

- Cooperative MultiRobot Observation of Multiple Moving Targets (CMOMMT) task
  - Surveillance and security related tasks

- DISTRIBUTED behavior based control system
  - Assume responsibilities concerning the observation of certain targets
  - Exchange information about observed targets with other robots
(3.3) Multirobot Cooperation for Surveillance of Multiple Moving Targets – A New Behavioral Approach

**Problem Formalization:**

- \( S \): two dimensional, bounded, enclosed region
- \( V \): team of \( m \) mobile robots, \( v_i, i=1,\ldots,m \)
- \( \text{Sensor}\_\text{coverage}(v_i) \): subset of \( S \) observable by \( v_i \)
- \( O(t) \): set of \( n \) targets, \( o_j(t), j=1,\ldots,n \)
- \( B(t) = b_{ij}(t) \) such that
  \[
  b_{ij} = \begin{cases} 
  1 & \text{if } v_i \text{ is observing } o_j(t) \text{ at time } t \\
  0 & \text{otherwise} 
  \end{cases}
  \]
- Maximize
  \[
  A = \sum_{t=1}^{T} \sum_{j=1}^{n} \frac{g(B(t), j)}{T}
  \]

\[
\begin{align*}
 g(B(t), j) &= \begin{cases} 
  1 & \text{if there exist an } i \text{ such that } b_{ij}(t) = 1 \\
  0 & \text{otherwise} 
  \end{cases}
\end{align*}
\]
Assumptions:
- Sensor coverage is much smaller than region $S$
- Speed of targets < speed of robots
- Each robot has a global localization system
- Broadcast communication limited to distance $r_c$
- Sensing range $r_s < r_c$
(3.3) **Multirobot Cooperation for Surveillance of Multiple Moving Targets – A New Behavioral Approach**

- **Local force strategy:**
  - Stay close to targets that are not too far away
  - Stay away from other robots
  - Stay away from targets that get too close
  - Have all surrounding robots and targets influence the movement of the robot

\[
\begin{align*}
  f(v_i, t) &= \sum_{k=1}^{n} t_{ik}(t) + \sum_{k=1}^{m} r_{ik}(t) \\
  f(v_i, t) &= \text{force vector applied to robot } v_i \text{ at time } t \\
  t_{ik}(t) &= \text{force vector from robot } i \text{ to target } k \text{ at time } t \\
  r_{ik}(t) &= \text{force vector from robot } i \text{ to robot } k \ (r_{ii} = 0) \text{ at time } t
\end{align*}
\]
(3.3) Multirobot Cooperation for Surveillance of Multiple Moving Targets – A New Behavioral Approach

- **Force Vectors** depend on distance:
  - Robot-target
  - Robot-robot
- Predictive tracking range
  - Range is predicted linearly
(3.3) Multirobot Cooperation for Surveillance of Multiple Moving Targets – A New Behavioral Approach

- The A-CMOMMT strategy:
  - Improves the local force method

- Share the workload between different robots
  - Each $t_{ik}$ is weighted by a factor $w_{ik}$
  - IF $o_j$ is observed by $v_k$
    THEN $v_i$ should be less attracted to observe $o_i$
    $\Rightarrow$ choice $w_{ij} \leq 1$
Problematic situations with A-CMOMMT

- The robot v1 is subject to differential constraints
- The robot v2’s resources are not being utilized

- Reduce the weight for a target if also observed by another robot ➔ reduced attraction for both robots
(3.3) Multirobot Cooperation for Surveillance of Multiple Moving Targets – A New Behavioral Approach

- The B-CMOMMT strategy:
  - Based on A-CMOMMT

- Incorporate weights for force vectors of robots
- Simple behavioral architecture
  - Follow targets
  - Help
  - Explore
(3.3) *Multirobot Cooperation for Surveillance of Multiple Moving Targets – A New Behavioral Approach*

- **The B-CMOMMT strategy:**
  - Behavioral architecture

  - **Tagging system**
    - one target should only influence the movement of one robot at a particular time
    - Additional assumption: the targets are distinguishable
    - communicate the distance of each tag to the other robots
    - the robot with the larger distance from the tag discards it

  - **Behaviors:**
    - **Follow targets**: direction of movement by considering all and only tagged targets
    - **Help**: move in the direction of the closest robot requesting help
    - **Explore**: movement depend on the position of all robots in communication range
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Conclusions

- A MRS cannot be simply regarded as a generalization of the single robot case
- A MRS can improve either the performance in accomplishing a task, or the robustness and reliability of the system
- The approaches to MRS need to be precisely characterized in terms of:
  - assumptions about the environment
  - internal system organization
- Coordination without Communication ???
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Practical: Player / Stage

1. Let’s move to a practical session

2. We’ll use Player/Stage software

3. A practical implementation of simple MRS
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  *Proceedings of Robotics: Science and Systems, 2005*

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