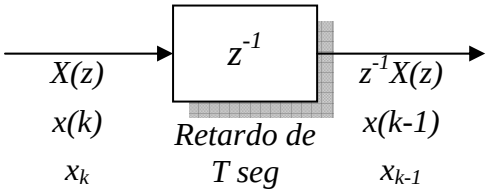
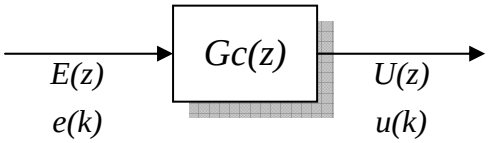


Relación entre z^{-1} y un retraso de una muestra



$$G_c(z)=\frac{U(z)}{E(z)}=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\cdots+b_mz^{-m}}{1+a_1z^{-1}+a_2z^{-2}+\cdots+a_nz^{-n}}$$

$$U(z)\left(1+a_1z^{-1}+\cdots+a_nz^{-n}\right)=E(z)\left(b_0+b_1z^{-1}+\cdots+b_mz^{-m}\right)$$

$$U(z)=-a_1z^{-1}U(z)-\cdots-a_nz^{-n}U(z)+b_0E(z)+b_1z^{-1}E(z)+\cdots+b_mz^{-m}E(z)$$

$$u_k=-a_1u_{k-1}-\cdots-a_nu_{k-n}+b_0e_k+b_1e_{k-1}+\cdots+b_me_{k-m}$$

Expresión genérica de un controlador

$$G_c(z)=K_d\,\frac{z+z_{cd}}{z+z_{pd}}\,\frac{z+z_{ci}}{z+z_{pi}}=\frac{K_dz^2+K_d\left(z_{cd}+z_{ci}\right)z+\left(z_{cd}\cdot z_{ci}\right)}{z^2+\left(z_{pd}+z_{pi}\right)z+\left(z_{pd}\cdot z_{pi}\right)}=$$

$$=\frac{K_d+K_d\left(z_{cd}+z_{ci}\right)z^{-1}+\left(z_{cd}\cdot z_{ci}\right)z^{-2}}{1+\left(z_{pd}+z_{pi}\right)z^{-1}+\left(z_{pd}\cdot z_{pi}\right)z^{-2}}$$

$$G_c(z)=K_p+\frac{K_d}{T}\,\frac{z-1}{z}+\frac{K_iT}{2}\,\frac{z+1}{z-1}=K_p+\frac{K_d}{T}\left(1-z^{-1}\right)+\frac{K_iT}{2}\,\frac{1+z^{-1}}{1-z^{-1}}=$$

$$=\frac{K_p\left(1-z^{-1}\right)+\dfrac{K_d}{T}\left(1-2z^{-1}+z^{-2}\right)+\dfrac{K_iT}{2}\left(1+z^{-1}\right)}{1-z^{-1}}=$$

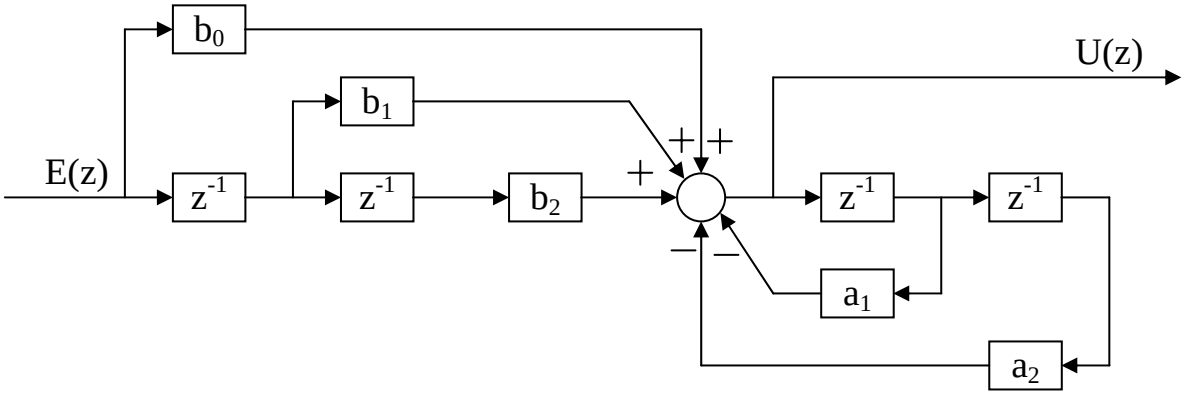
$$=\frac{\left(K_p+\dfrac{K_d}{T}+\dfrac{K_iT}{2}\right)+\left(-K_p-\dfrac{2K_d}{T}+\dfrac{K_iT}{2}\right)z^{-1}+\dfrac{K_d}{T}z^{-2}}{1-z^{-1}}$$

Controladores de segundo orden PID vistos

$$G_c(z) = \frac{U(z)}{E(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$u_k = -a_1 u_{k-1} - a_2 u_{k-2} + b_0 e_k + b_1 e_{k-1} + b_2 e_{k-2}$$

Expresión genérica de un controlador de segundo orden



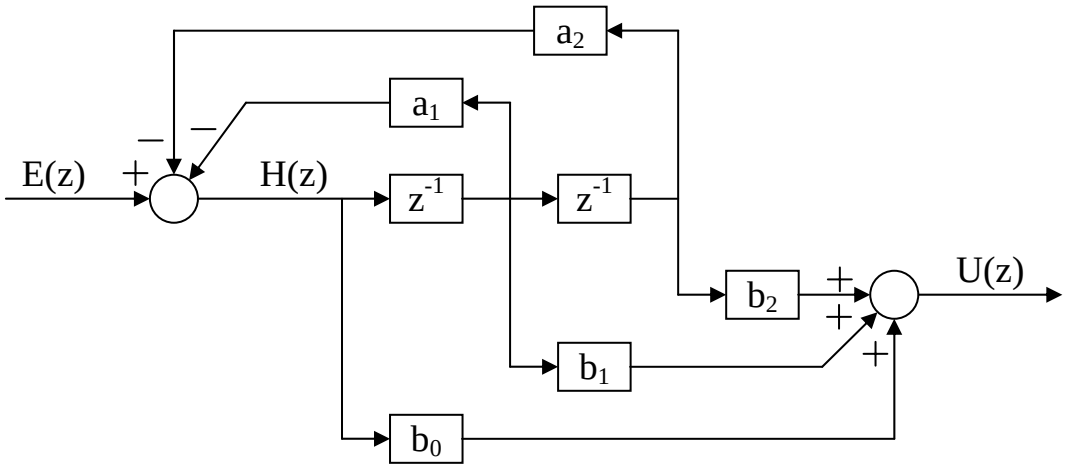

```
ek1 = 0; ek2 = 0; uk1 = 0; uk2 = 0
Cada T segundos hacer:
    ek = LeerEntrada()-LeerSalida()
    uk = b0·ek + b1·ek1 + b2·ek2 – a1·uk1 – a2·uk2
    EscribirControl(uk)
    ek2 = ek1
    ek1 = ek
    uk2 = uk1
    uk1 = uk
```

Método de programación directa

$$Gc(z) = \frac{U(z)}{E(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{H(z)}{E(z)} \frac{U(z)}{H(z)}$$

$$\frac{H(z)}{E(z)} = \frac{1}{1 + a_1 z^{-1} + a_2 z^{-2}} \rightarrow H(z) = E(z) - a_1 z^{-1} H(z) - a_2 z^{-2} H(z)$$

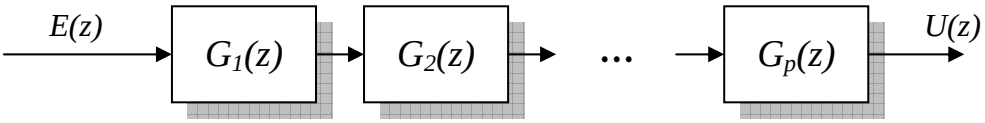
$$\frac{U(z)}{H(z)} = b_0 + b_1 z^{-1} + b_2 z^{-2} \rightarrow U(z) = b_0 H(z) + b_1 z^{-1} H(z) + b_2 z^{-2} H(z)$$



Método de programación estándar

$$Gc(z) = G_1(z)G_2(z)\cdots G_p(z)$$

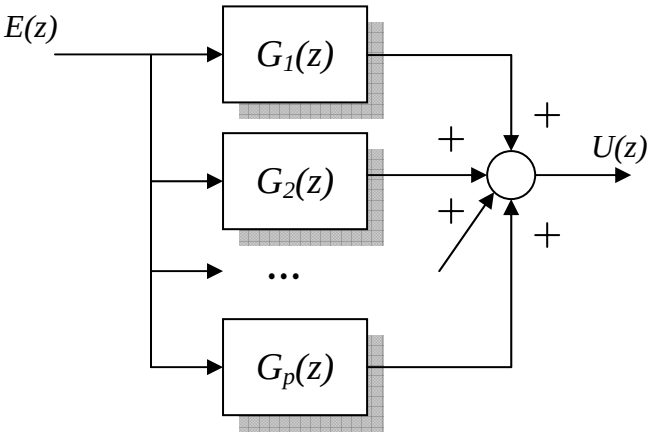
$$G_i(z) = \begin{cases} \dfrac{1+b_i z^{-1}}{1+a_i z^{-1}} \\ \dfrac{1+e_i z^{-1}+f_i z^{-2}}{1+c_i z^{-1}+d_i z^{-2}} \end{cases}$$



Método de programación serie

$$Gc(z) = A + G_1(z) + G_2(z) + \cdots + G_p(z)$$

$$G_i(z) = \begin{cases} \dfrac{b_i}{1+a_i z^{-1}} \\ \dfrac{e_i + f_i z^{-1}}{1+c_i z^{-1}+d_i z^{-2}} \end{cases}$$



Método de programación serie