Finite elements method based on Galerkin's formulation for predicting the sand bars position

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1   **Finite elements method based on Galerkin's formulation for predicting the sand bars**
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22   **Abstract**
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24   The accurate knowledge of the location and movement of the sand bars is important for the
25   coastal manager to establish correctly the coastal defence elements. This work applies a
26   numerical model of finite elements based on Galerkin's formulation for predicting the location of
27   the start, crest and end of the outer bar. The model that shows the best results is the one that
28   combines the variables wave height, period and median sediment size. The mean errors are less
29   than 10m in the distance to the shoreline and 17cm in the depth, while in the formulations
30   proposed by other authors the errors are more than 100m in the distance to the shoreline and 2m
31   in the depth. Thus, the proposed model significantly improves the calculation of the sand bars
32   position. This will allow a better location of the coastal defence elements, and therefore an
33   improvement in their effectiveness and optimisation of the materials to be used in their
34   construction, key to sustainable coastal management. Furthermore, although the model was
35   developed for the area of Valencia (Spain), the model can be used in any area with similar
36   characteristics (open sandy beach), and the methodology (variables and process) could be
37   applied worldwide.
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51   **Keywords:** *sandy beaches; sand bars; numerical modelling; precision profiles; Galerkin*
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1. Introduction

Bars are an important morphological feature of sandy beaches, as they cause the transformation of the wave, and therefore the subsequent modification of the cross-shore beach profile (Holman and Sallenger, 1993). Generally, bars on natural beaches usually move toward the coast when wave energy is low, whereas they move quickly offshore when waves are energetic and the generated currents are strong (Ruessink *et al.*, 2016; Winant *et al.*, 1975). However, although there is huge amount of knowledge of sand bars there are still unknowns. Authors such as Dally and Dean (1984); Sallenger and Howd (1989), among others, have suggested that bar formation and migration are caused by the offshore waves direction, near the bottom, of seasonal flows of breaking waves ("hangover").

Several models have been developed to understand the evolution of the cross-shore beach profile (including the bars): i) the equilibrium profile models (Bruun, 1954; Dean, 1977); ii) descriptive models (Lippmann and Holman, 1990; Splinter *et al.*, 2018); iii) models of empirical evolution of the profile (Kraus and Larson, 1988; Kriebel and Dean, 1985); and iv) process-based models (Bailard, 1982; Thornton *et al.*, 1996). Energy-type models based on sediment transport (based on Bagnold (1966)) have often been used to predict morphological evolution, linking sediment transport to near-bottom flow. Thornton *et al.* (1996) used velocities measured near the bottom to propose an energy transport model. The model adequately predicted the sand bar offshore migration observed during high energy waves and strong medium flows, but not the movement of the bar toward shore and other small changes observed during low energy conditions. Gallagher *et al.* (1998) supported and extended the results of Thornton *et al.* (1996), including in the model the sediment fall velocity by varying it along the profile. This improves predictions of profile change. Again, the model successfully predicted seaward migration and bar erosion observed during storms. However, according to these authors, it was not possible to predict the migration towards the coast that was observed when the waves were moderately strong and the mean currents were weak. Although there are studies that show the migration of the sand bar onshore, as indicated in the following paragraph.

The difficulty of predicting the bar migration to the coast may be due to systematic changes in wave shape that occur over the bar. The waveform is often described as "skewed" or "asymmetric". (Elgar, 1987). Waves with positive asymmetry have high sharp peaks and wide,

shallow trough. [Elgar *et al.* \(2001\)](#) observed that acceleration asymmetry tends to be maximum over the bar (where the break tends to be intense), and suggested that transport proportional to asymmetric acceleration could be responsible for the bar migration towards the coast. [Hoefel and Elgar \(2003\)](#) showed that the addition of wave freestream acceleration skewness leads to accurate predictions of bar migrations both to the coast and to the sea. [Henderson *et al.* \(2004\)](#) presented a model that combined the movement of water and sediment in the boundary layer, and that successfully predicted an event of migration from the bar towards the coast and an event towards the sea. But it failed to predict a second offshore migration event. Moreover, it is shown that, at the wave-resolving level, the onshore migration, largely due to the wave component of nearbed sediment transport ([Falchetti *et al.*, 2010](#); [Hsu *et al.*, 2006](#)), can be properly represented if such sediment transport is described through the third-order moment of the nearbed velocity ([Hsu *et al.*, 2006](#)). The recent work of [Brocchini \(2020\)](#) clarifies how the two perspectives (acceleration skewness for wave-averaging models and third-order moment of the nearbed velocity for wave-resolving models) can be reconciled through the freestream velocity asymmetry.

Among the different models developed for bar prediction, the most commonly used variables are: wave height in deep water (H_o), period (T) or wavelength in deep water (L_o), and median sediment size expressed explicitly (D_{50}) or implicitly (fall velocity; w) ([Demirci and Aköz, 2013](#); [Gallagher *et al.*, 1998](#); [Kömürcü *et al.*, 2007](#)). Some of these authors also incorporate in their models the breaking wave height (H_b) ([Pape *et al.*, 2007](#); [Plant *et al.*, 1999](#)) and/or the slope (m) ([Plant *et al.*, 1999](#); [Różyński, 2003](#)). It is worth mentioning the maximum wave height (H_{max}) used by [Plant *et al.* \(1999\)](#) or the direction of incidence of the waves with respect to the normal of the beach (θ) used by [Pape *et al.* \(2007\)](#). Most of these models also consider the duration of the wave during the test (t).

The accuracy of the models is essential when locating coastal defence works. However, most of the models or numerical formulations obtained for the modelling of the bars have been developed in laboratory tests, where the wave height is generally regular and unique for the same test. This raises the question of what wave height should be used to represent the field observations. Therefore, the aims of this work are: i) determine the wave height (maximum height, mean height, height with a 0.137% probability of being exceeded) that best represents the characteristic points of the bars (crest, start and end); and ii) generate a numerical model that provides the position of

these three characteristic points as precisely as possible, using the least number of variables necessary.

2. Study area

The study area is located to the North and South of the port of Valencia (Valencia, Spain). Nine transects have been analysed in this area, studying a length of 18 km of coastline (Figure 1). To the north of the port of Valencia, 4 transects have been established, divided as follows: i) one at Malvarrosa-Cabanyal beach (P1N); ii) two at Patacona beach (P2N and P3N); and iii) one at Port-Saplaya beach (P4N). In the southern zone the distribution of the 5 transects is: i) two at Pinedo beach (P1S and P2S); ii) two at Saler beach (P3S and P4S); and iii) one at La Dehesa beach (P5S).

The study area is a very dynamic zone that has undergone numerous morphological changes in the studied period (November 1992 to April 2014). In the middle of 2008 the Valencia port underwent an extension of the northern breakwater which was completed in 2010. This caused a greater sheltering of the waves both to the beaches of the North and to those of the South which led to a greater longshore transport towards the south, which implied a significant accretion on the northern beaches, and less obvious erosion on the southern beaches (Aragonés *et al.*, 2019b). The area has also undergone various beach nourishments such as the Pinedo beach in 1999 or the Cabañal-Malvarrosa beach in the period 2004-2006.

Finally, it should be noted that it is a microtidal zone, where astronomical tides (20-30 cm) together with meteorological tides can reach 75 cm (Ecolevente, 2006).

Figure 1. Location of the study area.

3. Methodology

The following sections indicate the procedure used for data collection and the generation of the different models.

3.1. Data collection

Below is a description of the process used to obtain the variables necessary for the subsequent generation of models, such as: position of the bars, waves and sedimentology.

3.1.1. Position of the bars

The position of the bars was analyzed by determining the distance from the shoreline (X) and the depth (Y) of the points of start (s), crest (c) and end (e) of the bar (Figure 2). In those profiles where no bar was found, the distance from each point to the shoreline has been obtained as the average of the position of the previous and posterior profiles, and the corresponding depth in the profile at that distance (López et al., 2017).

Figure 2. Location of the three characteristic points of the bar.

A total of 209 bimonthly collected profiles were analysed, distributed over time as can be seen in the Supplementary data 1: Distribution and use of profiles. The choice of months for sampling was conducted so that representative information was always obtained from the summer (profile after the season of calm swells) and winter profiles (profile after the storm season). The Profiling Bar method (BP) was used to obtain the profiles (Serra Peris and Medina, 1996). Following a methodology similar to terrestrial topography, this method makes it possible to determine the height of the beach profile with an error of less than 2 cm, independently of the mean sea level by astronomical tide and its oscillation by waves.

To establish the position of the bars, the software developed by Pagán *et al.* (2017) was used to load and plot beach profile data. By means of a displacement control the user can obtain, for any entered distance, the elevation in each point of the profile. The start position, the crest, the end and the width of the sandbar are saved, up to a maximum of three sandbars per profile. The accuracy of the results will depend on the precision of the profile survey. Since in this research high-precision beach profile surveys were used, the identification of the sandbars was precise.

Afterwards, an analysis was carried out to determine the outliers to identify possible measurement errors. Those profiles that were considered as outliers in some of the study points were eliminated from the subsequent study (Figure 3). Specifically in this study 7 profiles were eliminated: 3 by outliers in the X_s ; 1 by outlier in the Y_c ; 1 by outlier in the X_c and Y_c ; 1 by outlier in the X_s , X_c and X_e ; and 1 by outlier in Y_e .

Figure 3. Outliers detected in each one of the study points of the bars.

3.1.2. Maritime climate

Data on wave height, period and wave direction were obtained using data from the SIMAR nodes provided by the Public Organization Puertos del Estado (www.puertos.es/en-us/oceanografia/Pages/portus.aspx). These data, recorded every 3 hours since 1958, are based on high-resolution numerical modelling of the atmosphere, sea level and waves of the entire Spanish coastline. Specifically, data from SIMAR node 2081114 was used for transects in the North zone, and SIMAR node 2081113 for transects in the South zone. For each of the profiles the AMEVA software (IHCantabria, 2013) was used to obtain the wave height $H_{s,12}$ (wave height with a probability of 0.137% of being exceeded in the studied period), the mean wave height (H_m), and the maximum wave height (H_{max}) produced between the survey of each profile, as well as their corresponding periods (T) and directions (θ). The breaking wave height (H_b) was also obtained using the formula proposed by Komar and Gaughan (1972) (Equation 1). The use of wave height with probability 0.137% is due to its involvement in sediment transport (Hallermeier, 1978).

$$H_b = H_o \cdot 0.56 (H_o/L_o)^{-1/5} \quad (1)$$

Where H_o is the wave height in deep water (in this case the H_{max} , H_m and $H_{s,12}$), T is the period associated with this wave height, and L_o is the wavelength in deep water obtained as: $L_o = g \cdot T^2 / 2\pi$.

The wave height is obtained in each of the profiles taking into account only the influential wave. In this way, the influence of the enlargement of the port is considered.

3.1.3. Sedimentology

The sedimentology was obtained from the dry beach samples collected during the sampling campaign carried out by the University of Alicante in 2013. For the characterisation of the sedimentology in each of the profiles, the median sediment size (D_{50}) of the sample closest to the transect was used. In the study area, the median sediment size is between 0.167 and 0.332 mm.

3.2. Modellization

The first step in modelling the characteristic points of the bars was to determine the variables to be used. For this purpose, the existing models were studied, analysing the variables used by the different authors (Table 1). The most commonly used variables are wave height and period, or some derived from them such as wavelength. In addition, among the studied models, 58.3% use the profile slope and 33.3% use the median sediment size.

However, the majority of the authors, given that they carry out their studies in flume channel, does not specify the wave height that would correspond to the reality. For this reason, firstly a correlation study is executed to determine which variables most influence the position of the points of the bars. Using the SPSS software, a study of correlations between inputs ($H_{\max}$, $H_{b, \max}$, $T_{\max}$, etc) and outputs (X_s , Y_s , X_c , Y_c , X_e and Y_e) was carried out. This study indicated that the greatest correlations occur with the slope, and in some cases with the median sediment size, but for variables related to wave height the correlations were very low in general, and none of them were outstanding among the rest.

Table 1. Variables used by other authors.

	D_{50}	Slope	Wave height	Period	Wave direction	Others
Horikawa <i>et al.</i> (1973)	No	No	Yes, unspecified	Yes	No	Test duration
Hallermeier (1978)	No	No	Yes, $H_{s,12}$	Yes	No	
Larson and Kraus (1989)	No	No	Yes, breaking	No	No	
Silvester and Hsu (1997)	No	Yes	Yes, unspecified	Yes	No	
Hsu (1998)	No	Yes	Yes, unspecified	Yes	Yes	
Günaydin and Kabdaşlı (2005)	No	Yes	Yes, unspecified	Yes	No	
Kömürcü <i>et al.</i> (2007)	Yes	Yes	Yes, unspecified	Yes	No	
Demirci and Aköz (2013)	Yes	Yes	Yes, unspecified	Yes	No	
Gallagher <i>et al.</i> (1998)	Yes	No	Yes, $H_{s,12}$	No	No	
Pape <i>et al.</i> (2007)	No	No	Yes, H_{rms} and H_b	Yes	Yes	
Plant <i>et al.</i> (1999)	No	Yes	Yes, $H_{\max}$ and H_b	Yes	No	
Różyński (2003)	Yes	Yes	Yes, $H_{s,12}$	Yes	No	

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191 Second, multiple linear regression models were generated using the backward method, which
 192 consists of introducing all possible variables and the SPSS software generates linear models by
 193 eliminating the variables one by one. Thus, once the models had been made for each of the
 194 outputs, it was observed that the slope and the median sediment size remained in practically all
 195 the linear models generated. By contrast, the remaining variables were distributed randomly,
 196 leaving in some models the maximum height, or mean height, or $H_{s,12}$, or a combination of them.
 197 In addition, the generated models did not explain more than 50% of the total sample and did not
 198 solve the problem of choosing between one wave height or another. Therefore, there is no linear
 199 relationship between dependent and independent variables, so it was decided to use non-linear
 200 models, specifically numerical models of finite elements.

201 There are several numerical modelling methodologies that have been applied in the study of
 202 engineering problems. In the last decade, numerical methodologies have been developed based
 203 on the generation of geometric models of finite elements in a hypercube (Navarro-González and
 204 Villacampa, 2013; Navarro-González and Villacampa, 2012; Villacampa *et al.*, 2009). Recently, a
 205 new numerical methodology has been developed that applies in a hybrid way the Galerkin method
 206 and the finite element method (Navarro-González and Villacampa, 2016), which has been
 207 successfully applied in coastal engineering to model the depth of closure (Aragonés *et al.*, 2019a).
 208 This methodology will be applied in this research to generate numerical models, since Galerkin's
 209 formulation of the finite element method (Navarro-González and Villacampa, 2016) improves the
 210 computational efficiency of algorithms from other methodologies based on geometric models of
 211 finite elements previously mentioned (Navarro-González and Villacampa, 2013; Navarro-
 212 González and Villacampa, 2012).

213 The finite element method initially generates a geometric model of finite elements, so that the
 214 model is defined by its value in the nodes of the elements (Villacampa *et al.*, 2009). Specifically,
 215 the geometric model of finite elements is defined dividing Ω in elements, and defining in each
 216 element some points named nodes where the model is obtained. The dimension of the hypercube
 217 is defined by the number of independent or explanatory variables of the relationship to model.
 218 The complexity of the geometric model, c , is the number of subintervals in which each segment
 219 $[0,1]$ is divided. So, the initial domain $[0,1]^n$ is divided into a set of c^n hypercubics elements, and

the number of nodes is $N = (c + 1)^n$. Once the geometric model in a hypercube has been defined, the model is obtained by solving an optimization problem. In this way, for each mesh of finite elements defined in a hypercube, a numerical model is generated.

In the methodology of the Galerking's formulation (Navarro-González and Villacampa, 2016), defined the error function, $e(x)$, the problem of optimization is solved by applying the method of residual weights. So, using the weight functions W_j the optimization problem is equivalent to the fact that the following N-integrals are worth zero: $\int_{\Omega} e(x) w_j = 0$, $j = 1, 2, \dots, N$.

Employing the described methodology different numerical models were generated using different variables in each of them in order to select: i) the wave height that best represents the characteristic points of the bars; and ii) the model that optimizes resources.

First, the first six models in Table 2 (M1-M6) were generated for each of the outputs (X_s , Y_s , X_c , Y_c , X_e and Y_e). Among these six models it was observed that the best results (in all outputs analyzing R^2 , absolute error and absolute mean percentage error) were obtained with the models that included as input the $H_{s,12}$ either in deep water or in break, being slightly better the results with the wave height in deep water. Thus, this variable was selected as the basis for the generation of three more models (M7-M9), in which the rest of the variables were modified to try to optimize the model.

Table 2. Generated numerical models and variables used in each one. The X indicates that the variable has been used in the model.

Models	Variables			
	Wave height (m)	Associated period (s)	D ₅₀ (mm)	Slope
M1	H _{max}	X	X	
M2	H _{b, max}	X	X	
M3	H _m	X	X	
M4	H _{b, m}	X	X	
M5	H _{s,12}	X	X	
M6	H _{b s,12}	X	X	
M7	H _{s,12}		X	
M8	H _{s,12}			X
M9	H _{s,12}	X		X

Once the best model was determined, a validation and stability studies were performed to determine the optimal complexity (mesh density) for the model. The validation was performed using the last profile obtained in each transect, i.e. the last nine available profiles (Supplementary data 1: Distribution and use of profiles).

A perturbation of the input variables was performed to analyze the stability of the chosen model. After the analysis of the variability of the inputs by transects, it was observed that the median sediment size and period are practically constant, while the wave height presents the greatest variation with values of up to 65 cm. For this reason, only the variable wave height is disturbed with different percentages, up to a maximum of 10%. Given that the highest wave height ($H_{s,12}$) used in this work is 5.4 m, applying 10% disturbance implies reaching a maximum variation of 54 cm, very close to the maximum variation in the study area. The disturbance of the input data was randomly performed using Excel's "RANDOM" function.

4. Results

Table 3 shows the correlations obtained between the different inputs and outputs. For all the variables related to waves there are very low correlation values, being the higher value the one that relates the period ($T_{s,12}$) and Ys ($r = 0.128$). The two inputs that present the greatest correlation with outputs are: i) D_{50} with a weak or very weak relationship; and ii) the slope with moderate and strong relationships.

Although the slope has a greater correlation with the study points, obtaining the sediment size in the field is simpler than obtaining the profile to define the slope. Therefore, the first generated models include the D_{50} , and is in the second group of generated models where the slope is included.

Table 3. Correlations (Pearson's coefficient r) between input and output variables.

	Xs	Ys	Xc	Yc	Xe	Ye
$H_{\max}$	0.092	- 0.085	0.069	- 0.101	0.013	- 0.014
$H_{b, \max}$	0.091	- 0.083	0.066	- 0.093	0.018	- 0.013
$T_{\max}$	0.073	- 0.064	0.040	- 0.046	0.035	- 0.017
Direction of $H_{\max}$	0.078	0.076	0.050	0.029	0.091	0.016
Days between $H_{\max}$ and profile survey	0.039	- 0.004	0.040	0.011	0.071	- 0.011
H_m	0.064	- 0.057	0.066	- 0.088	- 0.048	- 0.001
$H_{b, m}$	0.061	- 0.050	0.066	- 0.086	- 0.053	0.004
T_m	0.045	- 0.018	0.057	- 0.068	- 0.057	0.023
Direction of H_m	0.05	- 0.085	0.043	- 0.054	0.026	- 0.040
$H_{s, 12}$	0.039	- 0.011	0.013	- 0.043	- 0.014	0.033
$H_{b, s, 12}$	0.036	0.020	0.008	- 0.030	- 0.018	0.048
$T_{s, 12}$	0.023	0.128	- 0.003	0.028	- 0.026	0.076
Direction of $H_{s, 12}$	- 0.005	0.060	- 0.007	0.008	0.012	0.054
D_{50}	- 0.263	- 0.118	- 0.203	0.051	- 0.181	- 0.084
Slope	- 0.612	0.406	- 0.552	0.407	- 0.514	0.337

After the generation of the numerical models, it was observed that the results for the six study points followed the same trend in terms of R^2 , absolute error and MAPE (Absolute Mean Percentage Error) values. This section only shows the results for the crest (Table 4 - Table 6) it is considered the most relevant factor when locating the sand bars (see the Supplementary data 2: Models results for the results of the start and end points of the bar). The highest R^2 values are obtained for models M1 and M2 (Table 4), and for models M5 and M6 which reach 0.9 and even exceed it in some cases reaching 0.93 in the case of Xe. This indicates that the inclusion of the $H_{s, 12}$ in the model improves the results compared to the use of other possible wave heights such as mean wave height or maximum wave height. Another important variable to include in the model is the period, since if the results of the M7 and M8 models (the two models that do not include the period) are compared with the rest are those that show the lowest R^2 values (< 0.7). Finally, it is observed that using the variable slope (m) instead of the D_{50} also reduces the effectiveness of the model, reaching a maximum value of 0.84 in the distance to the shoreline and 0.78 in the depth.

Table 4. R^2 for X_c and Y_c in the generated numerical models from M1 to M9..

Complexity	M1	M2	M3	M4	M5	M6	M7	M8	M9
Distance to the shoreline (X)									
40	0.718	0.684	0.593	0.566	0.773	0.759	0.471	0.553	0.705
50	0.795	0.768	0.680	0.655	0.841	0.830	0.536	0.601	0.755
60	0.839	0.818	0.748	0.722	0.883	0.877	0.582	0.643	0.794
70	0.867	0.846	0.791	0.771	0.905	0.900	0.620	0.678	0.821
80	0.885	0.866	0.825	0.804	0.916	0.913	0.646	0.709	0.842
Depth (Y)									
40	0.719	0.686	0.616	0.592	0.778	0.767	0.490	0.415	0.643
50	0.802	0.775	0.695	0.672	0.840	0.830	0.566	0.475	0.696
60	0.847	0.828	0.759	0.735	0.876	0.870	0.622	0.525	0.737
70	0.875	0.857	0.797	0.779	0.895	0.891	0.662	0.568	0.763
80	0.892	0.875	0.827	0.807	0.904	0.900	0.690	0.607	0.786

Table 5 and Table 6 show the absolute error and the MAPE, respectively. The trend is the same as observed for R^2 . Thus, in the M5 and M6 models the MAPE error committed is less than 20% in the distance to the shoreline and 15% in the depth. The results of both models are very similar, which was to be expected if it is considered that both use the same inputs with the exception of the wave height. However, since the M6 model includes the breaking wave height ($H_{b, s, 12}$)—which was obtained by using Equation 1, which is a combination of the deepwater wave height (the one used in M5) and the period it could be interpreted that both models are practically the same. For this reason, for the following analyses, the M5 model will be used, which uses wave height in deep water $H_{s, 12}$. Both tables also show that as complexity increases, the error decreases. Thus, in the case of the crest, in the M5 model the absolute error is less than 10 m in the distance to the shoreline from a complexity of 70, and less than 20 cm for a complexity of 60 or more.

Table 5. Absolute error in meters for X_c and Y_c in the generated numerical models from M1 to M9.

Complexity	M1	M2	M3	M4	M5	M6	M7	M8	M9
Distance to the shoreline (X)									
40	20.65	22.26	25.18	26.16	17.40	18.20	29.78	27.55	21.25
50	16.79	18.29	21.80	22.85	13.79	14.50	27.44	25.44	18.31
60	14.14	15.34	19.11	20.23	11.23	11.74	25.57	23.73	16.03
70	12.36	13.44	16.75	17.90	9.65	10.04	23.81	22.19	14.36
80	11.27	12.24	14.93	16.10	8.70	9.05	22.49	20.87	13.11
Depth (Y)									
40	0.35	0.37	0.41	0.43	0.29	0.30	0.50	0.53	0.38
50	0.28	0.30	0.36	0.38	0.23	0.24	0.45	0.49	0.33
60	0.23	0.25	0.31	0.33	0.19	0.20	0.42	0.46	0.30
70	0.20	0.22	0.27	0.29	0.17	0.17	0.39	0.43	0.27
80	0.18	0.20	0.24	0.26	0.15	0.16	0.37	0.40	0.25

Table 6. MAPE for X_c and Y_c in the generated numerical models from M1 to M9.

Complexity	M1	M2	M3	M4	M5	M6	M7	M8	M9
Distance to the shoreline (X)									
40	26.5%	28.7%	33.6%	34.9%	21.8%	22.9%	38.0%	25.2%	18.6%
50	21.1%	23.3%	28.9%	30.2%	17.1%	18.0%	34.9%	23.0%	15.7%
60	17.3%	19.1%	24.7%	26.2%	13.5%	14.2%	32.5%	21.2%	13.6%
70	14.9%	16.5%	21.3%	22.7%	11.4%	12.0%	30.4%	19.8%	12.1%
80	13.3%	14.8%	18.6%	20.2%	10.1%	10.6%	28.7%	18.5%	10.9%
Depth (Y)									
40	18.1%	19.4%	22.0%	22.9%	14.4%	15.0%	26.2%	26.0%	16.8%
50	14.5%	15.8%	18.9%	19.9%	11.4%	11.9%	24.0%	23.8%	14.5%
60	12.0%	13.0%	16.0%	17.0%	9.4%	9.8%	22.3%	22.0%	13.0%
70	10.5%	11.4%	14.0%	14.9%	8.3%	8.5%	20.9%	20.4%	11.9%
80	9.4%	10.3%	12.3%	13.2%	7.6%	7.9%	19.7%	19.0%	11.0%

The goodness of a model must be determined by considering its fit, as well as its behaviour when predicting new data. In this sense, the generated model was validated using data from profiles that were not used in the generation of the model. For these data, the models offer very similar results to those obtained during the generation (Figure 4). For example, for the complexity of 70 the mean absolute error for the distance to the shoreline of the start point (Figure 4a) is less than 10 m, for the crest it is less than 15 m and for the end point it is less than 25 m, while at depths (Figure 4c) they are all less than 30 cm. It should be noted that these errors decrease slightly when using the complexity of 80, but the results between the two models are very similar.

Figure 4. Results of the validations for the M5 model in the complexities of 70 and 80.

To validate a model, input data can be disturbed to see how this affects the results. Figure 5 shows the increase in error caused by a 10% disturbance over the wave height. In the case of the model of complexity 70 the error made over the original model is around 7% in all cases except Xe where it increases to 26%. However, for the 80 complexity model the errors increase by more than 15% in most cases, reaching 38% in the case of Xe. This difference between the perturbations in the 70 and 80 complexity models indicates that the 70 complexity models remain stable because the variations in the results are similar to the perturbations performed (10%), while the 80 complexity models are relatively unstable.

Figure 5. Increased error over the model results for wave height disturbances up to 10%.

Finally, the error made in each of the profiles is compared using some of the formulations proposed by other authors and the M5 model of Comp 70 (Figure 6). In general, the average of the errors for the different authors is quite far from zero. However, there are some formulations, such as [Günaydın and Kabdaşlı \(2005\)](#) or [Silvester and Hsu \(1997\)](#), in which the average error is very close to zero, but the dispersion of the error is much greater than that committed by the M5 model. For example, for the Xc (Figure 6b) the M5 model offers maximum and minimum values of less than 100 m, while the [Günaydın and Kabdaşlı \(2005\)](#) model is around 200 m, and the [Silvester and Hsu \(1997\)](#) model has an error of between -300 m and 250 m. At depth, something similar occurs, although in most models the average errors are less than 1 m, the dispersions are very large. On the crest, the dispersion takes the error to more than 2 m (Figure 6e), while at the start it reaches 4 m of error (Figure 6d) and at the end point to more than 8 m (Figure 6f). The M5 model, however, generally does not exceed 1 m of error in any of the study profiles.

Figure 6. Comparison of the error committed (Real minus Observed) using the formulations of other authors. Where Hor refers to Horikawa et al. (1973), Hall to Hallermeier (1978), L&K to Larson and Kraus (1989), S&H to Silvester and Hsu (1997), H to Hsu (1998), G&K to Günaydın and Kabdaşlı (2005), K to Kömürçü et al. (2007), and D&A to Demirci and Aköz (2013).

5. Discussion

The modelization of the position of sand bars has been extensively studied previously (e.g. Plant et al. (1999); Hsu et al. (2006); Smit et al. (2012); Ruessink et al. (2016)). However, a model that accurately predicts all bar positions has not been achieved. Most models successfully predict offshore movement due to heavy storms, but fail to predict onshore movement (Demirci and Aköz, 2012; Kömürçü et al., 2007; Trowbridge and Young, 1989). This work generates numerical models based on the formulation of Galerkin (Navarro-González and Villacampa, 2016) to determine the position of the start, the crest and the end of the bar in the area of the port of Valencia. The use of Galerkin's formulation of the finite element method and the corresponding improvement in computational efficiency allows analysing relationships in which a greater number of variables are involved without this implying a high time in the execution of the software.

When comparing the results obtained through the generated models and the formulations proposed by other authors, it is observed that there is a great difference in terms of the used variables and the errors committed (Table 1, Table 2 and Figure 6). In the first place, most of the

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3 350 authors deduce the variables to be used from channel tests. Moreover, most tests are performed
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5 351 using a fixed median sediment size and generating a straight profile with a previously established
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7 352 slope, which varies in the different tests, therefore the most used variable (58.3% of the authors
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9 353 studied) to model the bars is the slope (Table 1). However, there are very few who conduct tests
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11 354 modifying the median sediment size (Demirci and Aköz, 2013; Gallagher *et al.*, 1998; Kömürçü
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13 355 *et al.*, 2007). Nevertheless, the results obtained in this work show that when the median sediment
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15 356 size is used, the results are much better (the error is reduced by 54%) than when the bottom
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17 357 slope is used. In addition, the use of the profile slope complicates the data collection in field, since
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19 358 on the one hand the doubt arises up to which depth the profile has to be collected, and on the
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21 359 other hand, if the profile is already taken to determine the slope why not take the complete profile
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23 360 to determine the bar location. In this sense, it is much easier to obtain the median sediment size
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25 361 which, as shown in this work, is sufficient to take just once in the dry beach (provided that the
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27 362 beach does not undergo major sedimentological changes).

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29 363 Another important variable used by all authors is wave height. Most of the tests, mentioned above,
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31 364 are conducted in channels, and generally under regular waves (Demirci and Aköz, 2013; Hsu,
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33 365 1998; Kömürçü *et al.*, 2007). A priori, one could think that this wave height would be similar to the
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35 366 mean wave height that occurs in practice, since it is the most regular. However, this research has
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37 367 shown that it is the maximum wave height and the $H_{s,12}$ which produce results closest to reality
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39 368 (Table 4, Table 5 and Table 6). Specifically, it is the wave height $H_{s,12}$ directly determined in deep
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41 369 water that offers the best results. This result agrees with Splinter *et al.* (2018) according to which
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43 370 outer bars are highly influenced by offshore waves. This may be due to the morphology of the
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45 371 profile and the effect of the waves on the bottom. A real profile is not straight, but has a concave
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47 372 shape (Dean, 1991), with which depth is achieved very quickly in the first meters of profile (from
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49 373 the shoreline), and then a more constant slope is reached. This fast depth reach means that the
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51 374 mean wave (in the study area < 1 m) does not affect the bottom until it is very close to the coast
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53 375 (Hallermeier, 1978), and therefore is not able to produce a large movement of sediment along the
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55 376 profile. In that case, according to the "breakpoint" hypothesis, the bars are formed at the location
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57 377 of the initial wave breaking (Dally, 1987; Dean, 1973) or the existing bars move to that point
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59 378 (Sallenger *et al.*, 1985; Sallenger and Howd, 1989). However, the $H_{s,12}$ with values in the study
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area of between 1.50 m and 5.40 m, is able to generate sediment movement along the profile, and for that reason using this variable the best results are obtained.

When analyzing the results obtained when using the formulations proposed by other authors (Figure 6) large errors are observed, as well as a great dispersion of the data with respect to the proposed model M5. These large errors were also observed by other authors such as [Demirci et al. \(2015\)](#) or [López et al. \(2017\)](#). The possible cause of these errors is explained below. One of the formulations that makes the biggest errors is the one proposed by [Demirci and Aköz \(2013\)](#). This is surprising considering that this is one of the few authors that uses the same variables as the M5 model, and also includes the slope. However, if the data used are analyzed further, it can be seen that although the range of sediment sizes used by [Demirci and Aköz \(2013\)](#) covers the sizes found in the study area, the range of steepness used (0.07-0.263) is much greater than the maximum value occurring in the study area (0.033). This could explain the large errors made when using this formulation, since according to [Demirci and Aköz \(2013\)](#); [Demirci et al. \(2015\)](#) the steepness is one of the variables that most influences the migratory movement of the bars.

[Günaydin and Kabdaşlı \(2005\)](#) obtain relatively good results in comparison with other authors when determining the distance to the shoreline of the different analyzed points, however the errors in the depths increase considerably. In this case the steepness does not seem to be the cause since in their tests they use values between 0.027-0.084, so it includes the values of the study area. One of the possible causes in this case could be, as mentioned above, that these authors do not consider the median sediment size in their formulations. Another possible cause of the errors made by this formulation could be due to the fact that these authors always use the same start slope for the tests, modifying only the wave heights and periods. This could be the cause since, according to several authors, the movement of the bars is not only associated with the incident waves energy, but also with the previous morphological conditions ([Pape et al., 2010](#); [Plant et al., 1999](#)). In other words, the initial characteristics of the beach profile, especially the depth of the water on the bar crest, play an important role in controlling the movement of the bars ([Cheng and Wang, 2018](#)).

Finally, another of the most outstanding formulations regarding the errors committed is the one proposed by [Hsu \(1998\)](#). In this case, the error is not attributable to the wave steepness, as the

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3 408 study steepness (0.0116-0.0619) covers the range of the study area's steepness. However, the
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5 409 median sediment size could be the cause of the error since although this variable is not taken into
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7 410 account in the formulation, a value of 0.15 mm was used for the tests, size too small compared
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9 411 to the usual sediment of sandy beaches. Another peculiarity of the formulation of Hsu (1998) with
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11 412 respect to other authors is the use of the wave incidence angle. The use of this variable may
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13 413 make sense, since according to some authors, the greater the obliquity of the waves, the greater
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15 414 the movement of the offshore bar (Cheng and Wang, 2018; Dubarbier *et al.*, 2015; Walstra *et al.*,
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17 415 2012). However, these same authors indicate that there is no significant correlation between the
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19 416 incident wave angle and changes in bar height.

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21 417 Therefore, to obtain the best model that determines the position of the bars in the study area: i)
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23 418 the independent variables that best describe the characteristics of the study area were analyzed;
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25 419 ii) the model generated was validated with profiles not used during generation; and iii) the stability
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27 420 of the model was studied by disturbing the input variables. The method followed, as well as the
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29 421 accuracy of the obtained model will allow positioning the defense works in an adequate way in
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31 422 the study area or in areas with similar conditions.

32
33 423 **6. Conclusions**

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36 424 The morphodynamics of sand bars remains a difficult research topic due to the complex
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38 425 interaction between breaking waves and sediment transport in the surf zone. In this work a
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40 426 numerical model based on Galerkin's formulation has been developed. From the results obtained
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42 427 it can be stated that:

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44 428 • The use of the median sediment size offers better results than the use of the profile slope.
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46 429 • The wave height that best reflects the changes produced in the sand bars in practice is
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48 430 the wave height with a probability of 0.137% of being exceeded ($H_{s,12}$).
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50 431 • The generated model that includes the wave height $H_{s,12}$, its period and the median
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52 432 sediment size offers very good results, with mean errors lower than 15% (Xs: 23.9%, Xc:
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54 433 11.4%, Xe: 8.2%) in the distances to the shoreline. And in the case of depth, the mean
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56 434 error is below 8% (Ys: 10.2%, Yc: 8.3%, Ye: 5.3%).
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In summary, this research has optimized the existing models, reducing the number of inputs and the calculation time obtaining accurate results.

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