Contests with Dominant Strategies*

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Abstract

In this paper we first characterize the class of contest success functions (CSFs) that yield contests with equilibria in dominant strategies. Then we study the optimal contest as a decision problem under uncertainty. We consider the classical criteria of Wald, Laplace, Hurwicz and Savage. We find that the CSF that maximizes aggregate effort under these criteria has the form of an additively separable cut-off CSF.

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1. Introduction

A contest is a game in which contestants make efforts in order to win a prize. The core of this game is the Contest Success Function (CSF) that maps the contestants efforts into the probabilities that they win the prize. The standard approach to solve these games is to assume that agents play Nash or Bayesian equilibrium strategies. This approach has produced important insights into the structure of contests, but it is not free from problems. In particular, Bayes-Nash equilibrium is a demanding concept both from the informational and the computational points of view.¹ Thus, it would be desirable to have an equilibrium concept that does not require much computation or information. These concerns have a long history in game theory and mechanism design going back, at least, to the Wilson doctrine: "I foresee the progress of game theory as depending on successive reduction in the base of common knowledge required to conduct useful analysis of practical problems" (Wilson 1987).² In a series of papers, Bergemann and Morris look for mechanisms in which the planner knows nothing about the agents' beliefs and higher order beliefs about other agents' types (see their 2012 book for a summary of their work).³ In this paper, instead of looking for solution concepts for which robust implementation is possible (as they do) we take a direct route and look for Dominant Strategies directly.

We characterize contests in which there are dominant strategies and find the contest in this class for which total effort (revenue) is maximized according to some criterion. These contests are called optimal. In line with the informational paucity required on contestants, we assume that the contest organizer has very little information. Thus, instead of the usual Bayesian approach, we use decision criteria under uncertainty in order to find the optimal contest.

We first focus our attention on a class of Contest Success Functions (CSFs) which are additively separable in the efforts of all contestants. Such CSFs have been used, among others by Skaperdas and Vaidya (2012), Polishchuk and Tonis (2013), Cubel and Sanchez-Pages (2016) and Skaperdas, Toukanb and Vaidya (2016). With these CSFs, payoffs of each contestant are additively separable in the efforts of all other contestants and thus, these efforts, are irrelevant in order to determine the payoff maximizing effort of each contestant. We prove that given a CSF yielding equilibrium in dominant strategies, there is a CSF which yields the same equilibrium in dominant strategies and the same payoffs which is additively separable. Thus, if the set of additively separable CSFs does

¹For the complexity of computing Nash equilibrium see e.g. Daskalakis, Goldberg and Papadimitriou (2009). These problems might explain why Nash equilibrium predictions do not always fare well in experiments, see e.g. Dechenaux, Kovenock and Sheremeta (2015).

²The assumption of a common prior is specially problematic in contests like trials, awarding scholarships or job applications.

³Other notions of robustness have appeared in the literature. Corchon and Ortuño-Ortín (1994) assume that agents have complete information about a part of the environment and look for a mechanism yielding socially optimal allocations in Uniform Nash Equilibrium and Bayesian Equilibrium for any possible prior.

not exhaust the set of all CSFs yielding contests with equilibria in dominant strategies, there is no loss of generality in considering only this class of CSFs.

Next, we consider the optimal contest. We assume that the planner can choose the CSF so in the sequel, abusing language, we identify the optimal mechanism with the CSF which is optimal for the planner. We assume that the planner only cares about total effort. In line with our previous discussion on the information in the hands of agents, we assume that the planner (either an external agent or a surrogate of contestants) has little information about the environment. In fact, we assume that she only knows the upper bound of valuations. Thus, the planner faces a decision problem under uncertainty.⁴ We explore the optimal contests according to four classical criteria:

1. Equal Probability (Laplace). All valuations are equally likely, so the planner chooses the mechanism that maximizes expected revenue in accordance with the uniform distribution.
2. Maximin (Wald). If the planner chooses a mechanism her payoff will be the minimum for this mechanism. The planner chooses a mechanism that maximizes this minimum payoff.
3. Optimism-Pessimism (Hurwicz). Select a constant α . For each possible mechanism choose the smallest (denoted by r) and the largest (denoted by R) revenues. Given $\alpha \in [0, 1]$, choose the mechanism for which $\alpha r + (1 - \alpha)R$ is maximized.
4. Minimax Regret (Savage). Define regret to be the difference between the revenue that could have been obtained if true valuations were known and the revenue that is actually obtained. Apply the minimax criterion to regret across all possible valuation profiles.

We present a parametric CSF family that, by a judicious choice of the relevant cutoffs, implements the optimal contest in each of the four criteria explained above.⁵ We call this family the $\tilde{V}$ -CSF and it can be informally described as follows: given a valuation $\tilde{V}$ and a vector of efforts, we divide the agents into two groups: those who make at least an effort equal to $\tilde{V}/n$ and those who do not. The first group has a higher probability of obtaining the prize than the second group. Inside each group, all agents have the same probability of obtaining the prize. The difference in probability between the two groups is such that it makes agents with value $\tilde{V}$ indifferent to either making no effort or making an effort exactly equal to $\tilde{V}/n$. If no one makes an effort, everybody qualifies for a lottery.

The optimal mechanism under Maximin and Optimism-Pessimism criteria turns out not to be very interesting: in the first case, given the extremely pessimistic expectation of valuations

⁴The only papers that we are aware of that take a similar approach to ours is by Bergemann and Schlag (2011) on the design of monopoly pricing and by Rezai and van der Ploeg (2017) on the design of climate policy.

⁵In the Minimax Regret case we assume that the optimal CSF is of a cutoff type, which is not the case in the other three criteria where cutoff mechanism emerge as the optimal ones.

embedded in this criterion, an optimal mechanism is to give equal shares of the prize for everybody irrespective of whether they make an effort or not (i.e. $\tilde{V} = 0$) (in fact, any mechanism is optimal!) and in the second case the optimal mechanism is to set $\tilde{V}$ equal to the maximal valuation. Under the Laplace criterion, the optimal mechanisms set $\tilde{V}$ equal to the maximal valuation divided by two. Under the Minimax regret criterion, the optimal mechanisms set $\tilde{V}$ equal to the maximal valuation multiplied by $n/(n+1)$ where n is the number of contestants.

From a positive point of view, our paper studies contests where equilibrium is not a consequence of assumptions about information, coordination or the capabilities of the agents to forecast each other's actions. Under dominant strategies the outcome of the contest is, assuming that agents are maximizers, unavoidable and thus, contests with an equilibrium in dominant strategies and socially undesirable outcomes like war or lobbying may be more difficult to stop than outcomes resulting from the coordination of agents in certain strategies.

From a normative point of view we provide mechanisms that maximize total effort and do not rely on the planner having statistical information and on agents coordinating in some strategies whose individual optimality may depend on fine details about the information structure. Thus, when designing job applications, public procurement, or assigning scholarships, it would be a plus to know the optimal contests in which both the planner and the agents are required to know little about the problem they are facing.

2. The Model and Preliminary Results

There are n players and, each player, say i , can take an action $g_i \in \mathbb{R}_+$ which is interpreted as an effort or a monetary expense. Vectors are denoted by bold letters. Let $\mathbf{g} = (g_1, g_2, \dots, g_n)$. A prize is awarded with probabilities $\mathbf{p}(\mathbf{g}) = (p_1(\mathbf{g}), p_2(\mathbf{g}), \dots, p_n(\mathbf{g}))$. The probability vector $\mathbf{p}(\mathbf{g})$ belongs to the $n-1$ dimensional simplex S^{n-1} and can also be interpreted as the share in the prize won by the players. The function $p_i(\mathbf{g})$ is called the contest success function (CSF). We assume that the probability of success of any player is determined by the same function, i.e. the CSF is anonymous. Formally, $p_i(\mathbf{g}) = p(g_i, \mathbf{g}_{-i})$ for all $i = 1, 2, \dots, n$. We also assume that $p(g_i, \mathbf{g}_{-i})$ is increasing in g_i ($p(g_i, \mathbf{g}_{-i}) \geq p(g'_i, \mathbf{g}_{-i})$ whenever $g_i > g'_i$) and decreasing in g_j for all $j \neq i$ ($p(g_i, g_j, \mathbf{g}_{-i-j}) \leq p(g_i, g'_j, \mathbf{g}_{-i-j})$ whenever $g_j > g'_j$). Each player i values the prize at $v_i \in [0, \bar{V}]$. Let $\mathbf{v} = (v_1, v_2, \dots, v_n)$. The vector $\mathbf{z}_{-i}$ is a $n-1$ dimensional vector whose i th component has been deleted.

All individuals are risk neutral and have linear payoff functions on the cost of the action, so that the payoff function of player i can be written as

$$u_i(\mathbf{g}) = v_i p(g_i, \mathbf{g}_{-i}) - g_i. \quad (2.1)$$

We refer to $(\mathbf{v}, \mathbf{p})$ as the contest game.

In this paper we analyze contests in which agents do not have access to statistical information about the types of other agents. We only assume that each agent knows her own valuation and the domain $[0, \bar{V}]$.⁶ Thus, provided that dominant strategies exist, the most natural equilibrium concept is an equilibrium in dominant strategies.

Our first objective is to identify the class of CSFs with the properties described above that yield contests with equilibria in dominant strategies⁷. We start by exploring what we can get assuming that the CSF is twice continuously differentiable⁸. For simplicity, we assume interiority of the maximization problem.⁹ From the first order condition of payoff maximization,

$$\frac{\partial p(g_i, \mathbf{g}_{-i})}{\partial g_i} v_i = 1.$$

Assuming dominant strategies, it must be that

$$\frac{dg_i}{dg_k} = -\frac{\frac{\partial^2 p(g_i, \mathbf{g}_{-i})}{\partial g_i \partial g_k}}{\frac{\partial^2 p(g_i, \mathbf{g}_{-i})}{\partial g_i^2}} = 0 \Leftrightarrow \frac{\partial^2 p(g_i, \mathbf{g}_{-i})}{\partial g_i \partial g_k} = 0,$$

which implies that $p(g_i, \mathbf{g}_{-i})$ is additively separable in g_i and $\mathbf{g}_{-i}$ for all i , and since we have assumed an anonymous CSF,

$$p(g_i, \mathbf{g}_{-i}) = F(g_i) + H(\mathbf{g}_{-i}) + B.$$

Define $\varphi(g_i) = F(g_i) - F(0)$, and $\Psi(\mathbf{g}_{-i}) = H(\mathbf{g}_{-i}) - H(\mathbf{0})$. Note that $\varphi(0) = 0$ and $\Psi(\mathbf{0}) = 0$. Let $A = B + F(0) + H(\mathbf{0})$. Then,

$$p(g_i, \mathbf{g}_{-i}) = \varphi(g_i) + \Psi(\mathbf{g}_{-i}) + A.$$

When $g_i = 0$ and $\mathbf{g}_{-i} = \mathbf{0}$, $p(g_i, \mathbf{g}_{-i}) = 1/n$. Thus, $A = 1/n$. Since $\sum_{i=1}^n p(g_i, \mathbf{g}_{-i}) = 1$, then $\sum_{i=1}^n \varphi(g_i) = -\sum_{i=1}^n \Psi(\mathbf{g}_{-i})$. Let us see that this relation implies that $\Psi(\mathbf{g}_{-i}) = -\frac{1}{n-1} \sum_{j \neq i} \varphi(g_j)$. Let $g_i \neq 0$ and $g_j = 0$ for all $j \neq i$, then

$$\sum_{k=1}^n \varphi(g_k) = \varphi(g_i) = -\sum_{j=1, j \neq i}^n \Psi(g_i, \mathbf{0}_{-i-j}) = -(n-1)\Psi(g_i, 0, 0, \dots, 0).$$

Let $g_i \neq 0$, g_j and $g_j \neq 0$ and $g_k = 0$ for all $k \notin \{i, j\}$, then

$$\sum_{l=1}^n \varphi(g_l) = \varphi(g_i) + \varphi(g_j) = -\Psi(g_i, \mathbf{0}_{-i-j}) - \Psi(g_j, \mathbf{0}_{-i-j}) - \sum_{k=1, k \neq i, j}^n \Psi(g_i, g_j, \mathbf{0}_{-i-j-k}).$$

⁶In the standard Bayesian setting agents know the domain of preferences, a vector of prior probabilities of occurrence of the valuations of other agents and the statistical distribution of other agents' valuations. In our setting, there is no knowledge of both priors and the probability distribution.

⁷We refer to dominant strategies in the weak sense.

⁸As far as we know, the first paper using differentiability to find mechanisms with dominant strategies is Laffont and Maskin (1980).

⁹Given that $u_i(0, \mathbf{g}_{-i}) \geq 0$ and $u_i(g_i, \mathbf{g}_{-i}) \leq 0$ for all $g_i \geq v_i$, interiority is a valid assumption as long as $\frac{\partial p(0, \mathbf{g}_{-i})}{\partial g_i} v_i > 1$.

Since $-\Psi(g_i, \mathbf{0}_{-i-j}) = \frac{1}{n-1}\varphi(g_i)$, and $-\Psi(g_j, \mathbf{0}_{-i-j}) = \frac{1}{n-1}\varphi(g_j)$,

$$\varphi(g_i) + \varphi(g_j) = \frac{1}{n-1}\varphi(g_i) + \frac{1}{n-1}\varphi(g_j) - (n-2)\Psi(g_i, g_j, 0, 0, \dots, 0).$$

Thus, $\frac{1}{n-1}(\varphi(g_i) + \varphi(g_j)) = -\Psi(g_i, g_j, 0, 0, \dots, 0)$. Applying an induction argument, we get that $\frac{1}{n-1} \sum_{j \neq i} \varphi(g_j) = -\Psi(\mathbf{g}_{-i})$. Clearly $\varphi(\cdot)$ must be increasing for $p(g_i, \mathbf{g}_{-i})$ to be increasing in g_i and decreasing in g_j , and in order to guarantee that probabilities are non negative we must have that $0 \leq \varphi(g_i) \leq 1/n$ for all $g_i \in \mathbb{R}_+$. Thus,

$$\begin{aligned} p(g_i, \mathbf{g}_{-i}) &= \varphi(g_i) - \frac{1}{n-1} \sum_{j \neq i} \varphi(g_j) + \frac{1}{n} \text{ with} \\ 0 &\leq \varphi(g_i) \leq 1/n \text{ for all } g_i \in \mathbb{R}_+ \text{ and } \varphi \text{ increasing} \end{aligned} \tag{2.2}$$

These CSFs are additively separable in the strategies of all agents. This implies that the payoff function of each contestant is additively separable in the strategies chosen by the other contestants. Therefore, the payoff maximizing strategy do not depend on the other contestants' strategies.

The following proposition shows that, when looking for CSFs yielding dominant strategies, there is no loss of generality in restricting ourselves to additively separable CSFs of the form (2.2).

Proposition 1. *Let $(\mathbf{v}, \mathbf{p})$ be a contest game that has a dominant strategy equilibrium. Then, there is a contest game $(\mathbf{v}, \hat{\mathbf{p}})$ with the same dominant strategy equilibrium where the CSF has a separable representation of the form (2.2).*

The proof of Proposition 1 is based on the revelation principle applied to our contest game.¹⁰ Formally, a CSF that yields contests with an equilibrium in dominant strategies defines a mechanism Γ where the strategy space for each agent is $\mathbb{R}_+$ and the outcome function gives, for each $\mathbf{g} \in \mathbb{R}_+^n$, an allocation $(p(g_i, \mathbf{g}_{-i}), g_i)_{i=1}^n$. The game induced by Γ is the contest game. Let $\mathbf{g}^*$ be the dominant strategy equilibrium of the contest game. The existence of dominant strategies plus the anonymity assumption on the CSF imply that $g_i^* = G(v_i)$, i.e. the equilibrium effort of contestant i is a function of his own valuation and this function is the same for all players. Let $\mathbf{G}(\mathbf{v}_{-i}) = (G(v_j))_{j=1, j \neq i}^n$. The probability of winning in equilibrium can be written as $p(g_i^*, \mathbf{g}_{-i}^*) = p(G(v_i), \mathbf{G}(\mathbf{v}_{-i})) = P(v_i, \mathbf{v}_{-i})$. Therefore, for each CSF that yields an equilibrium in dominant strategies, we can associate a direct mechanism $([0, \bar{V}]^n, f)$ such that, for each

¹⁰The revelation principle has been applied to crowdsourcing contests by Chawla, Hartline and Sivan (forthcoming in GEB). In these contests, firms use inputs obtained from Internet users to build projects. There are similarities and differences of this paper with our approach. Among the former they consider anonymous mechanisms and study the optimal prior independent contest. Among the latter they assume that the planner is only interested in the submission with the highest quality. They use Bayesian equilibrium as equilibrium concept and their setting is an all pay auction.

$(V_1, \dots, V_n) \in [0, \bar{V}]^n$, $f(V_1, \dots, V_n) = (P(V_i, \mathbf{V}_{-i}), G(V_i))_{i=1}^n = (p(G(V_i), \mathbf{G}(\mathbf{V}_{-i})), G(V_i))_{i=1}^n$ in which truth telling is a dominant strategy, that is, the mechanism is incentive compatible (IC)

$$v_i P(v_i, \mathbf{V}_{-i}) - G(v_i) \geq v_i P(V_i, \mathbf{V}_{-i}) - G(V_i), \quad \forall (V_i, \mathbf{V}_{-i}), \quad V_i \neq v_i. \quad (2.3)$$

and individually rational (IR)

$$V_i P_i(V_i, \mathbf{V}_{-i}) - G_i(V_i) \geq 0, \quad \text{for all } (V_i, \mathbf{V}_{-i}) \quad (2.4)$$

From now on, we denote a direct revelation mechanism associated with a contest game, in short, a **revelation contest**, by $(\mathbf{P}, \mathbf{G})$ where the payoff of an agent of type v_i given $(V_i, \mathbf{V}_{-i})$ is

$$u(V_i, \mathbf{V}_{-i} \mid v_i) = v_i P(V_i, \mathbf{V}_{-i}) - G(V_i).$$

In order to prove Proposition 1 we first need the characterization of the revelation contests that are incentive compatible and individually rational. This characterization is given in the following lemma. The proof follows from Lemma 2 in Myerson (1981) applied to revelation contests. All the formal details are given in the Appendix.

Lemma 1. *The revelation contest $(\mathbf{P}, \mathbf{G})$ is IC and IR if and only if*

$$(1) P(V_i, \mathbf{V}_{-i}) = \phi(V_i) - \frac{1}{n-1} \sum_{j \neq i} \phi(V_j) + \frac{1}{n}$$

where $\phi : [0, \bar{V}] \rightarrow \mathbb{R}_+$ is an increasing function that satisfies $\phi(0) = 0$ and $\phi(\bar{V}) \leq \frac{1}{n}$.

$$(2) G(V_i) = V_i \phi(V_i) - \int_0^{V_i} \phi(s) ds, \text{ and } G(\cdot) \text{ is increasing.}$$

The formal proof of Proposition 1 uses Lemma 1 and is given in the Appendix. We present here the main ideas behind it. If a CSF yields contests with an equilibrium in dominant strategies, is associated with a revelation contest, $(\mathbf{P}, \mathbf{G})$, which is incentive compatible and individually rational. This revelation contest should be in the class described in Lemma 1. From the revelation contest, we want to construct a CSF in the class described in (2.2) defined for all $g_i \in \mathbb{R}_+$. Since $G(\cdot)$ is increasing, its range is in $[0, G(\bar{V})]$. If $G(\cdot)$ were strictly increasing and a continuous function, its range would be exactly this. Suppose for the time being that this is the case. Thus, if $g_i \in [0, G(\bar{V})]$, there is a $V_i \in [0, \bar{V}]$ such that $G(V_i) = g_i$ and we can define $\varphi(g_i) = \phi(V_i)$. But the CSF should be defined for all $g_i \in \mathbb{R}_+$, so the domain of g_i also includes $(G(\bar{V}), \infty)$. We have to extend $\varphi(\cdot)$ in such a way that $g_i = G(v_i)$ is a dominant strategy for each agent i . We only need to define $\varphi(g_i) = \phi(\bar{V})$ for all $g_i \in (G(\bar{V}), \infty)$. Clearly, with this extension no agent will make more of an effort than $G(\bar{V})$. Thus, the CSF of the form (2.2) defined in this way produces the same dominant strategy profile as the initial game, and so the proof would be complete if $G(\cdot)$ were continuous. In the Appendix, we provide a construction that deals with the problem of a discontinuous $G(\cdot)$.

To close this section, we relate some CSFs already present in the literature with the class defined in (2.2). For example, Cubel and Sanchez-Pages (2016) in a framework of group contests, provide an axiomatic characterization of the following family where p_k represents the probability that group k obtains the prize:

$$p_k(\mathbf{g}) = \max\{\min\{\frac{1}{k} + h_k(g_k) - \frac{1}{k} \sum_{l \in \mathbf{k}} h_l(g_l), 1\}, 0\}.$$

Under anonymity, if each group has one agent, this CSF can be written as

$$\begin{aligned} \frac{1}{k} + h(g_k) - \frac{1}{k} \sum_{l \in \mathbf{k}} h(g_l) &= \frac{1}{k} + \varphi(g_k) - \frac{1}{k-1} \sum_{l \in \mathbf{k}, l \neq k} \varphi(g_l) \\ \text{where } \varphi(g_k) &= \frac{k-1}{k} h(g_k) \end{aligned}$$

and, if $h(0) = 0$ and $h(g_k) \leq \frac{1}{k-1}$, we have an example of a CSF of the form (2.2).

The symmetric version of the difference-form contest function derived by Skaperdas and Vaidya (2012) takes the form:

$p(g_1, g_2) = \frac{1}{2} + \frac{\alpha}{2}(h(g_1) - h(g_2))$ with $0 \leq h(g_i) \leq 1$, and $\alpha \leq 1$ (this one is also a particular case of the ones we described where $\varphi(g_k) = \frac{\alpha}{2}h(g_k)$). They endogenously derive this form in a model of persuasion. In a subsequent paper, Skaperdas, Toukanb and Vaidya (2016) prove that the contest game has an equilibrium in dominant strategies. Finally, Polishchuk and Tonis (2013) derive optimal CSFs through Bayesian implementation. They show that, in some cases, the optimal CSF is of the form described in (2.2).

3. The Optimal CSF under Uncertainty

In this section we introduce optimality criteria that will allow us to find the optimal CSF in the class of CSFs that yields equilibrium in dominant strategies (in short, the optimal CSF or, abusing language, the optimal mechanism in what follows). These criteria must reflect the informational constraints faced by the planner. A common assumption in the literature on mechanism design is that the planner does not have better information than agents.¹¹ This is because if the planner is an external agent (as a regulator), it does not seem natural to assume that she has access to information that is unknown to the agents. And if the planner is a surrogate of agents (i.e. "agents might choose the mechanism themselves", see Maskin and Sjoström (2002)) this assumption also holds. This implies that in our case, the planner should not have any statistical information about the interval of valuations. Thus, the planner faces a decision problem under uncertainty (see Milnor (1954)).

¹¹There is a literature evolving from the paper by Myerson (1983) which assumes that the principal has private information. This literature is important but it does not form the backbone of mechanism design. More references on this topic can be found in <https://sites.google.com/site/tmylovanov/informed-principal>

We now briefly describe the classical decision criteria under uncertainty:

1. Equal Probability (Laplace). All valuations are equally likely, so the planner chooses the CSF that yields dominant strategies that maximizes expected revenue in accordance with the uniform distribution.
2. Maximin (Wald). If the planner chooses a CSF her payoff will be the minimum over all states of nature. The planner chooses the CSF that yields equilibrium in dominant strategies that maximizes this minimum payoff.
3. Optimism-Pessimism (Hurwicz). Select a constant α . For each possible CSF choose the smallest (denoted by r) and the largest (denoted by R) revenues. Given $\alpha \in [0, 1]$, the planner chooses the CSF that yields equilibrium in dominant strategies that maximizes $\alpha r + (1 - \alpha)R$.
4. Minimax Regret (Savage). Define regret to be the difference between the revenue that could have been obtained if true valuations were known and the revenue that is actually obtained. Apply the minimax criterion to regret across all possible valuation profiles.

The Laplace criterion is like a Bayesian agnostic criterion in which, since there is no information that makes any of the states different from others it yields the same probability to all of them. The Wald criterion assumes that the state of the world that minimizes the payoffs of the contest organizer is the one chosen. The Hurwicz criterion corrects this by putting some weight on the most optimistic outcome. Finally, the Savage criterion chooses the decision that minimizes the maximum regret. These four criteria are termed classical because they have been around for some years. More recently, other writers have proposed other criteria, see for example Barberá and Pattanaik (1986) and Bossert, Pattanaik and Xu (2000).

Before proceeding with the application of these criteria, recall that, because of Proposition 1, there is no loss of generality to search for the optimal CSF in the class of separable CSFs defined in (2.2).

The optimal CSF that we obtain under these criteria belongs to a particular subclass of the separable ones. Each CSF in this subclass is defined by a threshold $\tilde{V}$. We refer to this CSF as the $\tilde{V}$ -CSF. Formally, the $\tilde{V}$ -CSF is defined as

$$\begin{aligned} p_i(\mathbf{g}) &= \varphi(g_i) - \frac{1}{n-1} \sum_{j \neq i} \varphi(g_j) + \frac{1}{n}, \text{ with} \\ \varphi(g_i) &= 1/n \text{ if } g_i \geq \tilde{V}/n, \text{ and } \varphi(g_i) = 0 \text{ otherwise.} \end{aligned} \tag{3.1}$$

In an additively separable CSF the probability that, say, agent i obtains the prize has three sources: A constant one ($1/n$), another part that depends on her effort and another part that depends on everybody else's efforts. In a $\tilde{V}$ -CSF, the second part is a fair lottery if i makes a minimum effort

of $\tilde{V}/n$ (which we will call the cut-off effort in the sequel) and zero otherwise. And the third part depends on how many contestants other than i have made the cut-off effort. In the worst possible case this is $-1/n$ and thus the probability that i obtains the prize is either 0 (if she does not make the cut-off effort) or $1/n$ (if she makes this effort). In the best possible case, i.e. when no other agent has made the cut-off effort, i obtains the prize with probability $2/n$ if she makes the cut-off effort or $1/n$ otherwise. In a $\tilde{V} - CSF$, contrarily to an auction with a reservation price, if you post a bid less than the cut-off, you have some probability to win the contest as long as somebody else has made an effort less than the cut-off too. In this case, if $m < n$ contestants (apart from i) bid less than the cut-off, i obtains the prize with probability $-(n-1-m)/(n-1)n + 1/n = m/n(n-1)$.

In the following lemma we show the dominant strategy obtained in the contest game defined by a CSF in this subclass.

Lemma 2. *Let $(\mathbf{v}, \mathbf{p})$ be the contest game defined by a $\tilde{V} - CSF$. Then, $g_i = \tilde{V}/n$ is a strictly dominant strategy for all agents with $v_i > \tilde{V}$ and $g_i = 0$ is a strictly dominant strategy for agents with $v_i < \tilde{V}$. For an agent with $v_i = \tilde{V}$, both $g_i = 0$ and $g_i = \tilde{V}/n$ are weakly dominant strategies.*

Proof. Let i be such that $v_i > \tilde{V}$. Clearly, an effort strictly above $\tilde{V}/n$ is dominated by $\tilde{V}/n$ because both give the same probability of winning but the first one is more costly. If $g_i < \tilde{V}/n$, the difference between the probability of winning when effort is equal to $\tilde{V}/n$ and effort is g_i is $1/n$. The difference in efforts is $\tilde{V}/n - g_i$. Given that $v_i > \tilde{V}$, $v_i/n > \tilde{V}/n > \tilde{V}/n - g_i$. Thus, $g_i = \tilde{V}/n$ is a dominant strategy for all i such that $v_i > \tilde{V}$. Let i be such that $v_i < \tilde{V}$. Any effort $g_i < \tilde{V}/n$ is dominated by $g_i = 0$, because the probability of winning does not change but $g_i > 0$. Let us see finally that the payoff associated with an effort $\tilde{V}/n$ is smaller than the one at $g_i = 0$. Again, the difference in the probability of winning is $1/n$, and the difference in efforts is $\tilde{V}/n$, since $v_i < \tilde{V}$, $v_i/n < \tilde{V}/n$. Thus, $g_i = 0$ is a dominant strategy for all $v_i < \tilde{V}$. The agent with $v_i = \tilde{V}$ is indifferent between $g_i = 0$ and $g_i = \tilde{V}/n$. The proof that for $v_i = \tilde{V}$, both $g_i = 0$ and $g_i = \tilde{V}/n$ are weakly dominant strategies, follows directly from the previous arguments. ■

3.1. The Laplace Criterion

Suppose valuations for each agent are randomly and independently drawn from a continuous distribution with density function $f : [0, \bar{V}] \rightarrow \mathbb{R}_+$. Let F be the distribution function. Under the Laplace criterion F is assumed to be the Uniform distribution. The following proposition characterizes the optimal CSF (the one that maximizes expected revenue) under the Laplace criterion.

Proposition 2. *Under the Laplace criterion the optimal CSF is the $\bar{V}/2 - CSF$.*

Proof. If a CSF yields contests with an equilibrium in dominant strategies, the contest game has associated an incentive compatible and individually rational revelation contest, $(\mathbf{P}, \mathbf{G})$. Let us

see first what the optimal revelation contest is under the Laplace criterion. Once we have this optimal revelation contest, we derive the CSF.

By Lemma 1, if $(\mathbf{P}, \mathbf{G})$ is an incentive compatible and individually rational revelation contest, $G(V_i) = V_i \phi(V_i) - \int_0^{V_i} \phi(s) ds$. Thus, the expected revenue is:

$$\begin{aligned}
\int_0^{\bar{V}} G(V_i) f(V_i) dV_i &= \int_0^{\bar{V}} V_i \phi(V_i) f(V_i) dV_i - \int_0^{\bar{V}} \left(\int_0^{V_i} \phi(s) ds \right) f(V_i) dV_i \\
&= \int_0^{\bar{V}} V_i \phi(V_i) f(V_i) dV_i - \int_0^{\bar{V}} \left(\int_s^{\bar{V}} f(V_i) \phi(s) dV_i \right) ds \\
&= \int_0^{\bar{V}} V_i \phi(V_i) f(V_i) dV_i - \int_0^{\bar{V}} (1 - F(s)) \phi(s) ds \\
&= \int_0^{\bar{V}} \phi(V_i) \left(V_i - \frac{(1 - F(V_i))}{f(V_i)} \right) f(V_i) dV_i.
\end{aligned}$$

For the uniform distribution

$$\int_0^{\bar{V}} G(V_i) f(V_i) dV_i = \int_0^{\bar{V}} \phi(V_i) (2V_i - \bar{V}) \frac{1}{\bar{V}} dV_i. \quad (3.2)$$

We can decompose (3.2) as:

$$\int_0^{\bar{V}/2} \phi(V_i) (2V_i - \bar{V}) \frac{1}{\bar{V}} dV_i + \int_{\bar{V}/2}^{\bar{V}} \phi(V_i) (2V_i - \bar{V}) \frac{1}{\bar{V}} dV_i,$$

for all $V_i \in [0, \bar{V}/2]$, $(2V_i - \bar{V}) \leq 0$, thus, since ϕ has to be increasing, the first term is maximized when $\phi(V_i) = 0$ for all $V_i \in [0, \bar{V}/2]$. The second term is bounded above by $\phi(\bar{V}) \frac{\bar{V}}{4}$. Thus, given that $\phi(\bar{V}) \leq 1/n$, the term $\phi(\bar{V}) \frac{\bar{V}}{4}$ is maximized at $\phi(\bar{V}) = 1/n$. Therefore, for all $V_i > \bar{V}/2$, $\phi(V_i) = 1/n$. By (2) in Lemma 1, $G(V_i) = 0$ for all $V_i \in [0, \bar{V}/2]$, and $G(V_i) = \bar{V}/2n$ for all $V_i \in [\bar{V}/2, \bar{V}]$. Therefore, the optimal CSF should be such that, at the dominant strategy $\mathbf{g}^*$ of the contest game, $g_i^* = 0$ for all agent i such that $v_i \in [0, \bar{V}/2]$, and $g_i^* = \bar{V}/2n$ for all i such that $v_i \in [\bar{V}/2, \bar{V}]$. As Lemma 2 shows, the $\bar{V}/2 - CSF$ satisfies this requirement. ■

3.2. The Maximin Criterion

This criterion reflects pessimism about the state of nature: it assumes that the planner always expects the state of nature to be the worst possible. Given these expectations, the planner chooses the CSF that maximizes efforts. Given these extremely pessimistic expectations, the planner can only expect that agents make zero effort which is the only best reply when the valuation is zero. Given this, the planner is indifferent to a large class of mechanisms because they all raise the same effort. For reasons of symmetry with the other criteria we choose the $0 - CSF$. Note that the $0 - CSF$ can be interpreted as a pure lottery, since it gives all agents an equal chance to obtain the prize.

Proposition 3. *Under the Maximin criterion, any separable CSF is optimal. In particular, the 0 – CSF is optimal.*¹²

The proof is trivial and is omitted.

The previous mechanism can be considered as a formalization of the practice that some publicly provided goods are offered for free. The informal argument behind this is that certain goods must be offered free of charge in order to make sure that everybody has a chance to consume them. In this interpretation, p_i is the proportion of the prize obtained by i .

3.3. The Optimism-Pessimism Criterion

The Optimism-Pessimism criterion reflects a compromise (measured by α) between being pessimistic -expecting all agents to have the lowest valuation- and optimistic -expecting all agents to have the greatest valuation. Thus, given α , for each possible CSF choose the smallest (denoted by r) and the largest (denoted by R) revenues. Choose the CSF for which $\alpha r + (1 - \alpha)R$ is maximized.¹³ Given the result in the Maximin case, it is obvious that under the Optimism-Pessimism criterion the optimal mechanism is chosen with emphasis on the maximal valuation. Our next proposition formalizes this idea.

Proposition 4. *Under the Optimism-Pessimism criterion, if $\alpha < 1$, the optimal CSF is the $\bar{V}$ – CSF*

Proof. If a CSF yields contests with an equilibrium in dominant strategies, the contest game has associated an incentive compatible and individually rational revelation contest, $(\mathbf{P}, \mathbf{G})$. Let $G(\mathbf{V}) = \sum_{i \in N} G(V_i)$ be the revenue raised by the revelation contest. The smallest revenue is obtained at $\mathbf{0}$ and the largest at $\bar{\mathbf{V}}$. Then, according to the Hurwicz criterion the payoff from this revelation contest is

$$\alpha G(\mathbf{0}) + (1 - \alpha) G(\bar{\mathbf{V}}).$$

Since for any incentive compatible and individually rational revelation contest $G(\mathbf{0}) = 0$, the revelation contest that maximizes $\alpha G(\mathbf{0}) + (1 - \alpha) G(\bar{\mathbf{V}})$ is the revelation contest that maximizes revenue at $\bar{\mathbf{V}}$. By part (2) in Lemma 1,

$$G(\bar{V}) = \bar{V} \phi(\bar{V}) - \int_0^{\bar{V}} \phi(s) ds$$

which is maximized when the integral is zero and $\phi(\bar{V})$ has the maximal value. Since $0 \leq \phi(V_i) \leq 1/n$, $G(\bar{V})$ is maximal when $\phi(\bar{V}) = 1/n$, and $\phi(V_i) = 0$ for all $V_i \in [0, \bar{V})$. Thus, $G(V_i) = 0$ for

¹²We note that our result here is driven by the maximin criterion which forces the planner to focus on the worst possible event and not by the solution concept, i.e. equilibrium in dominant strategies.

¹³For $\alpha = 1$ this reduces to the Maximin criterion.

all $V_i \in [0, \bar{V})$, $G(\bar{V}) = \bar{V}/n$, and $\alpha G(\mathbf{0}) + (1 - \alpha) G(\bar{\mathbf{V}}) = (1 - \alpha) \bar{V}$. The optimal CSF should be such that, at the dominant strategy $\mathbf{g}^*$ of the contest game, $g_i^* = 0$ for each agent i such that $v_i \in [0, \bar{V})$, and $g_i^* = \bar{V}/n$ for each agent i such that $v_i = \bar{V}$. As Lemma 2 shows, the $\bar{V} - CSF$ satisfies this requirement. ■

In our case, the Optimism-Pessimism criterion breaks the indifference of the designer over all mechanisms that occurred in the Maximin criterion. It does it by considering a best case scenario.

3.4. Minimax Regret Criterion

Finally, we look for the optimal mechanism under the Minimax Regret criterion. This approach may be motivated by assuming that the planner might discover ex post the true state so she protects herself against this possibility by trying to minimize the maximum regret, see Schlag (2006) for additional discussion and motivations. For the analysis of this criterion, we restrict the attention to the class of $\tilde{V} - CSF$ defined in (3.1).

Proposition 5. *Under the Minimax regret criterion, among the $\tilde{V} - CSF$, the optimal CSF is the $\frac{n}{n+1} \bar{V} - CSF$*

Proof. A contest game defined by a $\tilde{V} - CSF$ has an associated incentive compatible and individually rational revelation contest, $(\mathbf{P}, \mathbf{G})$, such that $\phi(V_i) = \varphi(g_i^*)$ where g_i^* is the dominant strategy of the contest game. Since for a $\tilde{V} - CSF$, $g_i^* = 0$ if $V_i \leq \tilde{V}$, and $g_i^* = \tilde{V}/n$ if $V_i > \tilde{V}$, then $\phi(V_i) = 0$ for all $V_i \leq \tilde{V}$ and $\phi(V_i) = 1/n$ if $V_i > \tilde{V}$. Let us call the revelation contest with these characteristics a $\tilde{V}$ -cut-off mechanism. If true valuations are known, the maximal revenue that we can get will coincide with the maximal valuation. That is, for each $\mathbf{V}$, the maximal revenue will be $\max_i V_i$. Given a $\tilde{V}$ -cut-off mechanism with $\tilde{V} \in [0, \bar{V}]$ the revenue will be $\frac{k}{n} \tilde{V}$ where k is the number of agents with $V_i > \tilde{V}$. If at $\mathbf{V}$ $\max_i V_i \leq \tilde{V}$, the revenue given by the mechanism will be zero. Given $\tilde{V} \in [0, \bar{V}]$, among all states of the world where $\max V_i > \tilde{V}$, the maximal regret is obtained at the state where $\max V_i = \bar{V}$ and there is only one agent with valuation above $\tilde{V}$. Thus, the regret is $\bar{V} - \frac{1}{n} \tilde{V}$. Among all states of the world where $\max V_i \leq \tilde{V}$, the maximal regret is obtained at the state where $\max V_i = \tilde{V}$, and the regret is $\tilde{V}$.

Thus, given a $\tilde{V}$ -cut-off mechanism, if $\tilde{V} < \bar{V}$, the maximal regret is given by $\bar{V} - \frac{1}{n} \tilde{V}$ if $\bar{V} - \frac{1}{n} \tilde{V} > \tilde{V}$, otherwise the maximal regret is $\tilde{V}$. Therefore, the maximal regret among all the $\tilde{V}$ -cut-off mechanism such that $\tilde{V} < \bar{V}$ is minimized if $\tilde{V} = \frac{n}{n+1} \bar{V}$. Finally, if $\tilde{V} = \bar{V}$, the maximal regret is $\bar{V}$. Since $\bar{V} > \frac{n}{n+1} \bar{V}$, among all $\tilde{V}$ -cut-off mechanism, the maximal regret is minimized if $\tilde{V} = \frac{n}{n+1} \bar{V}$. Thus, the optimal CSF should be such that, at the dominant strategy $\mathbf{g}^*$ of the contest game, $g_i^* = 0$ for each agent i such that $v_i \in [0, \frac{n}{n+1} \bar{V}]$, and $g_i^* = \bar{V}/(n+1)$ for each agent i such that $v_i \in [\frac{n}{n+1} \bar{V}, \bar{V}]$. As Lemma 2 shows, the $\frac{n}{n+1} \bar{V} - CSF$ satisfies this requirement. ■

The previous result is partial because it only considers the subclass of $\tilde{V} - CSF$. In the previous criteria, the optimal CSF was in this subclass but we do not know if this is true here. Still the result allows a comparison with the optimal mechanisms found in the previous criteria. In particular, the cut-off used here is larger than the one in the Laplace criterion and, when the number of contestants tends to infinity, it converges to the cut-off in the optimism-pessimism criterion.

4. Conclusions

In this paper we have characterized CSFs that yield contests with equilibria in dominant strategies and we have found total effort maximizing mechanisms according to four well known decision criteria under uncertainty. In this final section we consider the possible use of our results.

1. Our characterization of CSFs can be used to single out contests in which there is equilibrium in dominant strategies, provided that we can obtain empirical estimates of those CSFs (see Jia, Skaperdas and Vaidya (2013) and the references therein). Given the robustness of dominant strategies, we would expect that in these contests, efforts made by agents should match very closely those predicted by the theory.
2. The CSFs characterized in this paper can be tested in the lab to see if the behavior there matches the theoretical prescriptions. If this were the case, then it could be argued that the failure of the theoretical prediction of Nash equilibrium in many lab contests is due to coordination problems and not to the fact that agents value winning per se or they are spiteful or inequality-averse, etc. See Dechenaux, Kovenock and Sheremeta (2015) for a list of possible explanations of overbidding in contests with complete information. Another explanation of the failure of game theoretical predictions in experiments is provided by Gill and Prowse (2012). They report that, in a sequential-move real-effort competition, payoffs are affected by "Disappointment Aversion" in such a way that the dominant strategy predicted when payoffs are standard is not chosen by subjects. In particular there is a discouragement effect for second movers when, according to standard payoffs, there should be none.
3. We consider optimality (total effort maximization) by assuming that the planner or designer faces a decision problem under uncertainty. We showed that a special class of CSFs, the $\tilde{V} - CSF$, where $\tilde{V}$ stands for a given cut-off, is optimal in three out of four of the decision criteria used in this paper. Roughly speaking, the $\tilde{V} - CSF$ gives each participant an exogenous share of the prize but allows this share to increase with one's effort and to decrease with the efforts of competitors. As far as we can tell, this CSF does not resemble any known CSF. In a $\tilde{V} - CSF$, contrarily to an auction with a reserve price, if you post a bid less than the cut-off, you have some probability to win the contest as long as somebody else has made an effort

less than the cut-off too. And when there are more than two contestants, the probability of winning the contest is always less than one, no matter how different efforts might possibly be.

4. A matter of concern is that three of our optimal mechanisms depend on the planner knowing the upper bound of valuations. This characteristic, which runs counter to our motivation that mechanisms should depend as little as possible on information is, in fact, directly imported from the decision criteria under uncertainty that we use. In all of them, it is assumed that the decider knows all possible states of nature. As far as we know, the assumption that the planner knows the domain of economies she is facing (restricted or unrestricted), is universally made in the literature of mechanism design. This points to a deep difficulty in our understanding of uncertainty. After all, one of the uses of mechanism design is to be a testbed of our abstract theories and to point out its limits.
5. In some contests, in which only the project submitted by the winner counts, a reasonable optimality criterion would be to maximize expected highest effort, see Serena (2017). This new goal may produce very different results than the goal used in this paper of maximizing aggregate effort. "Results are in sharp contrast with the standard sum of efforts maximization" (Serena, 2017). Thus we leave this extension to a different paper.
6. Our tools can be applied to quantity setting oligopoly in which market revenue is written as $R_i(x_i, x_{-i})K_i$ where x_i is the output of firm i , x_{-i} is the vector of outputs of its competitors and K_i is a positive number that measures the strength of demand. Our results here can be used to characterize the class of inverse demand functions that yield dominant strategies (assuming that costs are linear).

5. APPENDIX

The proof of Lemma 1 uses two more lemmas. The first one is a direct adaptation to our framework of Lemma 2 in Myerson (1981). We present here the statement of this lemma without proof. The second Lemma is an auxiliary technical result.

Lemma 3. (Myerson, 1981) *A revelation contest $(\mathbf{P}, \mathbf{G})$ is IC and IR if and only if*

- (1) *For all $\mathbf{V}_{-i} \in [0, \bar{V}]^{n-1}$, if $s_i \leq t_i$, $s_i, t_i \in [0, \bar{V}]$ then $P(s_i, \mathbf{V}_{-i}) \leq P(t_i, \mathbf{V}_{-i})$.*
- (2) *$u(s_i, \mathbf{V}_{-i} \mid s_i) = u(0, \mathbf{V}_{-i} \mid 0) + \int_0^{s_i} P(s, \mathbf{V}_{-i}) ds$.*
- (3) *$u(0, \mathbf{V}_{-i} \mid 0) \geq 0$ for all $\mathbf{V}_{-i} \in [0, \bar{V}]^{n-1}$.*

Lemma 4. *Let $\phi : [0, \bar{V}] \rightarrow \mathbb{R}$ and $\psi : [0, \bar{V}]^{n-1} \rightarrow \mathbb{R}$ be such that $\phi(0) = 0$, $\psi(\mathbf{0}_{-i}) = 0$ for all i , and $\sum_{i=1}^n \phi(V_i) = -\sum_{i=1}^n \psi(\mathbf{V}_{-i})$. Then, $\psi(\mathbf{V}_{-i}) = -\frac{1}{n-1} \sum_{j \neq i} \phi(V_j)$.*

Proof. Let $(V_i, \mathbf{0}_{-i})$, then

$$\sum_{i=1}^n \phi(V_i) = \phi(V_i) = - \sum_{j=1, j \neq i}^n (V_i, \mathbf{0}_{-i-j}) = -(n-1)\psi(V_i, 0, 0, \dots, 0).$$

Let $(V_i, V_j, \mathbf{0}_{-i-j})$, then

$$\sum_{i=1}^n \phi(V_i) = \phi(V_i) + \phi(V_j) = -\psi(V_i, \mathbf{0}_{-i-j}) - \psi(V_j, \mathbf{0}_{-i-j}) - \sum_{k=1, k \neq i, j}^n (V_i, V_j, \mathbf{0}_{-i-j-k}).$$

Since $-\psi(V_i, \mathbf{0}_{-i-j}) = \frac{1}{n-1}\phi(V_i)$, and $-\psi(V_j, \mathbf{0}_{-i-j}) = \frac{1}{n-1}\phi(V_j)$,

$$\phi(V_i) + \phi(V_j) = \frac{1}{n-1}\phi(V_i) + \frac{1}{n-1}\phi(V_j) - (n-2)\psi(V_i, V_j, 0, 0, \dots, 0).$$

Thus, $\frac{1}{n-1}(\phi(V_i) + \phi(V_j)) = -\psi(V_i, V_j, 0, 0, \dots, 0)$. Applying an induction argument, we get that $\frac{1}{n-1} \sum_{j \neq i} \phi(V_j) = -\psi(\mathbf{V}_{-i})$. ■

Proof of Lemma 1

If the revelation contest $(\mathbf{P}, \mathbf{G})$ is incentive compatible and individually rational, it satisfies (1), (2) and (3) in Lemma 3. First of all, note that condition (2) in Lemma 3 implies that

$$G(V_i) = V_i P(V_i, \mathbf{V}_{-i}) + G(0) - \int_0^{V_i} P(s, \mathbf{V}_{-i}) ds. \quad (5.1)$$

Let us see that $G(\cdot)$ is increasing. By (1) in Lemma 3, $P(V_i, \mathbf{V}_{-i})$ is increasing in V_i . Suppose that there is a pair V_i, V'_i such that $V'_i > V_i$ but $G(V_i) > G(V'_i)$. Thus $P(V'_i, \mathbf{V}_{-i}) \geq P(V_i, \mathbf{V}_{-i})$ and by incentive compatibility,

$$\begin{aligned} P(V_i, \mathbf{V}_{-i}) V_i - G(V_i) &\geq P(V'_i, \mathbf{V}_{-i}) V_i - G(V'_i) \text{ or equivalently,} \\ (P(V_i, \mathbf{V}_{-i}) - P(V'_i, \mathbf{V}_{-i})) V_i &\geq G(V_i) - G(V'_i) > 0 \end{aligned}$$

which is a contradiction. Thus if $P(V_i, \mathbf{V}_{-i})$ is increasing in V_i , $G(V_i)$ is increasing in V_i too. Given (5.1),

$$V_i (P(V_i, \mathbf{V}_{-i}) - P(V_i, \mathbf{V}'_{-i})) = \int_0^{V_i} (P(s, \mathbf{V}_{-i}) - P(s, \mathbf{V}'_{-i})) ds,$$

which implies that P is separable, that is:

$$P(V_i, \mathbf{V}_{-i}) = F(V_i) + H(\mathbf{V}_{-i}).$$

Let $\phi(V_i) = F(V_i) - F(0)$, and $\psi(\mathbf{V}_{-i}) = H(\mathbf{V}_{-i}) - H(\mathbf{0}_{-i})$ and let $A = F(0) + H(\mathbf{0}_{-i})$. Thus, $P(V_i, \mathbf{V}_{-i}) = \phi(V_i) + \psi(\mathbf{V}_{-i}) + A$. Note that $\phi(0) = 0$ and $\psi(\mathbf{0}_{-i}) = 0$, therefore $P(\mathbf{0}) = A$ for all i . Thus, $A = 1/n$. Given that $\sum_{i=1}^n P(V_i, \mathbf{V}_{-i}) = 1$, $\sum_{i=1}^n \phi(V_i) = -\sum_{i=1}^n \psi(\mathbf{V}_{-i})$. By Lemma 4 using this equality and playing with vectors of valuations of the form $(V_i, \mathbf{0}_{-i})$, $(V_i, V_j, \mathbf{0}_{-i-j})$ and

so on, $\psi(\mathbf{V}_{-i}) = -\frac{1}{n-1} \sum_{j \neq i} \phi(V_j)$. Thus, $P(V_i, \mathbf{V}_{-i}) = \phi(V_i) - \frac{1}{n-1} \sum_{j \neq i} \phi(V_j) + \frac{1}{n}$. By Lemma 3, $P(\cdot)$ is increasing in V_i , thus $\phi(\cdot)$ is increasing, and since $P(\cdot)$ is a probability distribution, $0 \leq \phi(V_j) \leq 1/n$. By point (3) in Lemma 3, $u(0, \mathbf{V}_{-i} \mid 0) \geq 0$ which implies that $G(0) = 0$. By (5.1) and point (1)

$$G(V_i) = V_i \phi(V_i) - \int_0^{V_i} \phi(s) ds.$$

Finally, let us see that a revelation contest $(\mathbf{P}, \mathbf{G})$ that satisfies (1) and (2) in the statement of this Lemma, is incentive compatible and individually rational. Let us see first that $(\mathbf{P}, \mathbf{G})$ is individually rational, that is, $P(V_i, \mathbf{V}_{-i}) V_i - G(V_i) \geq 0$. Given that $0 \leq \phi(V_i) \leq 1/n$ for all V_i , $P(V_i, \mathbf{V}_{-i}) V_i - G(V_i) \geq \int_0^{V_i} \phi(s) ds \geq 0$. In order to prove that $(\mathbf{P}, \mathbf{G})$ is incentive compatible, we have to show that for each individual with valuation v_i ,

$$v_i P_i(v_i, \mathbf{V}_{-i}) - G_i(v_i) \geq v_i P_i(V_i, \mathbf{V}_{-i}) - G_i(V_i), \quad \forall (V_i, \mathbf{V}_{-i}),$$

which is equivalent to proving that

$$\phi(V_i) (V_i - v_i) \geq \int_0^{V_i} \phi(s) ds - \int_0^{v_i} \phi(s) ds. \quad (5.2)$$

If $V_i > v_i$,

$$\int_0^{V_i} \phi(s) ds - \int_0^{v_i} \phi(s) ds = \int_{v_i}^{V_i} \phi(s) ds,$$

and since ϕ is increasing, for all $s \in [v_i, V_i]$, $\phi(s) \leq \phi(V_i)$. Therefore,

$$\int_{v_i}^{V_i} \phi(s) ds \leq \phi(V_i) (V_i - v_i).$$

If $V_i < v_i$,

$$\int_0^{V_i} \phi(s) ds - \int_0^{v_i} \phi(s) ds = - \int_{V_i}^{v_i} \phi(s) ds$$

and since ϕ is increasing, for all $s \in [V_i, v_i]$, $\phi(s) \geq \phi(V_i)$. Therefore,

$$- \int_{V_i}^{v_i} \phi(s) ds \leq -\phi(V_i) (v_i - V_i) = \phi(V_i) (V_i - v_i).$$

Thus, in both cases (5.2) holds and therefore the revelation contest $(\mathbf{P}, \mathbf{G})$ is incentive compatible. ■

Proof of Proposition 1. If a CSF yields contests with an equilibrium in dominant strategies, it has an associated revelation contest, $(\mathbf{P}, \mathbf{G})$, which is incentive compatible and individually rational. By Lemma 1, $(\mathbf{P}, \mathbf{G})$ is such that $P(V_i, \mathbf{V}_{-i}) = \phi(V_i) - \frac{1}{n-1} \sum_{j \neq i} \phi(V_j) + \frac{1}{n}$, where $\phi : [0, \bar{V}] \rightarrow \mathbb{R}_+$ is an increasing function that satisfies $\phi(0) = 0$ and $\phi(\bar{V}) \leq \frac{1}{n}$. Consider a partition of $[0, \bar{V}]$ in subintervals, such that, in each of the intervals of the partition, either $\phi(\cdot)$ is strictly

increasing and continuous, or it is constant¹⁴. Since $G(\cdot)$ is given by (2) in Lemma 1, in each of the intervals of the partition, $G(\cdot)$ is strictly increasing and continuous, or it is constant.

If the range of $G(\cdot)$ is $[0, G(\bar{V})]$, the definition of $\varphi(\cdot)$ is an easy task immediate from $\phi(\cdot)$. That is, for all $g_i \in [0, G(\bar{V})]$, there is a $V_i \in [0, \bar{V}]$ such that $G(V_i) = g_i$ and we can define $\varphi(g_i) = \phi(V_i)$. But the CSF should be defined for all $g_i \in \mathbb{R}_+$, so the domain of g_i also includes $(G(\bar{V}), \infty)$. We have to extend $\varphi(\cdot)$ in such a way that $g_i = G(v_i)$ is a dominant strategy for each agent i . We only need to define $\varphi(g_i) = \phi(\bar{V})$ for all $g_i \in (G(\bar{V}), \infty)$. Clearly, with this extension no agent will make more effort than $G(\bar{V})$. Thus, the CSF of the form (2.2) defined in this way produces the same dominant strategy profile as the initial game. If the range is not $[0, G(\bar{V})]$, is because $\phi(\cdot)$ is constant in all the domain, or $\phi(\cdot)$ is not a continuous function. In the first case, $\phi(V_i) = k$ for all $V_i \in [0, \bar{V}]$, define $\varphi(g_i) = k$ for all g_i . In the second case, the discontinuities of $\phi(\cdot)$ imply that $G(\cdot)$ is not a continuous function either. The range is included in $[0, G(\bar{V})]$ but it has holes. For all g_i in the range and in $(G(\bar{V}), \infty)$ we define $\varphi(\cdot)$ as before. It only remains to define $\varphi(\cdot)$ at all the possible efforts, g_i , in the holes of $[0, G(\bar{V})]$ produced by the discontinuities. The holes which are not in the range, are of the form $[G^m, G^{m+1})$ or $(G^l, G^{l+1}]$. Consider first a hole of the form $[G^m, G^{m+1})$. We can associate this hole with two intervals in the domain: $[V^{m-1}, V^m)$ and $[V^m, V^{m+1}]$ with the following properties: $G(V^m) = G^{m+1}$; and $G^m = \lim_{k \rightarrow m} G(V^k)$ with $V^k \in [V^{m-1}, V^m)$ (at V^{m-1} and at V^{m+1} the interval could be open or closed, but this is irrelevant now for the present analysis). For all $g_i \in [G^m, G^{m+1})$, $\varphi(g_i) = \lim_{k \rightarrow m} \phi(V^k)$ with $V^k \in [V^{m-1}, V^m)$. For the hole of the form $(G^l, G^{l+1}]$ we can associate two intervals in the domain: $[V^{l-1}, V^l]$ and $(V^l, V^{l+1}]$ with the following properties $G(V^l) = G^l$, and $G^{l+1} = \lim_{k \rightarrow l} G(V^k)$ with $V^k \in (V^l, V^{l+1}]$ (at V^{l-1} and at V^{l+1} the interval could be open or closed, but it is irrelevant now for this analysis).

We already know that an agent with valuation v_i has a payoff at $g_i = G(v_i)$ that is greater than or equal to the payoff at any other possible effort in the range of $G(\cdot)$ or at any effort in $(G(\bar{V}), \infty)$. It remains to prove that the payoff is also greater than or equal to any possible effort in the holes. For that, it is sufficient to prove first that an agent with valuation V^m prefers to choose the effort $G(V^m) = G^{m+1}$ rather than any other effort $g \in [G^m, G^{m+1})$. Since in all this interval the probability of winning is the same (given the definition of $\varphi(\cdot)$), the greater payoff in this interval is obtained at $g = G^m$. Note that $[G(V^{m-1}), G^m)$ is in the range of $G(\cdot)$. Thus, for any given vector of effort for the other agents (g_{-i}), and for all $g \in [G(V^{m-1}), G^m)$,

$$(\varphi(G^{m+1}) - \frac{1}{n-1} \sum_{j \neq i} \varphi(g_j) + \frac{1}{n}) V^m - G^{m+1} \geq (\varphi(g) - \frac{1}{n-1} \sum_{j \neq i} \varphi(g_j) + \frac{1}{n}) V^m - g;$$

or equivalently

$$\varphi(G^{m+1}) V^m - G^{m+1} \geq \varphi(g) V^m - g$$

¹⁴We can partition the space of valuations in this way because a monotone function has at most a countable number of discontinuities.

Since for all $g \in [G(V^{m-1}), G^m]$ and this interval is in the range, there is $V^k \in [V^{m-1}, V^m]$ such that $G(V^k) = g$. Thus,

$$\phi(V^m) V^m - G(V^m) \geq \phi(V^k) V^m - G(V^k),$$

and therefore

$$\phi(V^m) V^m - G(V^m) \geq \lim_{k \rightarrow m} \phi(V^k) V^m - \lim_{k \rightarrow m} G(V^k) = \lim_{k \rightarrow m} \phi(V^k) V^m - G^m.$$

Thus, it is a dominant strategy for an agent with valuation V^m to choose effort $g = G(V^m)$.

Finally, it remains to prove that an agent with valuation V^l prefers to choose the effort $G(V^l) = G^l$ to any other effort $g \in (G^l, G^{l+1}]$. We omit this part since it is basically a replica of the above argument. ■

6. References

References

- [1] Baik, K. H. (1998). "Difference-Form Contest Success Functions and Effort Level in Contests". *European Journal of Political Economy*, 14, 685-701.
- [2] Barberá, S., and P. Pattanaik (1986). "Falmagne and the Rationalizability of Stochastic Choices in terms of Random Orderings". *Econometrica* 54, 3, 707—715.
- [3] Bergemann, D. and Schlag, K. (2011). "Robust monopoly pricing," *Journal of Economic Theory*, 146, 6, 2527-2543.
- [4] Bergemann, D. and S. Morris (2012). *Robust Mechanism Design: The Role of Private Information and Higher Order Beliefs*, World Scientific, Singapore.
- [5] Bossert, W., P. K. Pattanaik and Y. Xu (2000) "Choice under complete uncertainty: axiomatic characterizations of some decision rules". *Economic Theory* 16, 295–312.
- [6] Chawla, S., J. D. Hartline and B. Sivan (forthcoming). "Optimal crowdsourcing contests". *Games and Economic Behavior* <https://doi.org/10.1016/j.geb.2015.09.001>.
- [7] Che, Y.-K. and I. Gale (2000), "Difference-Form Contests and the Robustness of All-Pay Auctions," *Games and Economic Behavior* **30**, 22–43.
- [8] Corchon, L. and I. Ortuno-Ortin (1994). "Robust implementation under alternative information structures". *Economic Design*, 1, 1, 159–171.

- [9] Cubel, M. and S. Sanchez-Pages (2016). "An axiomatization of difference-form contest success functions". *Journal of Economic Behavior and Organization* (forthcoming).
- [10] Daskalakis, C., P. W. Goldberg and C. H. Papadimitriou (2009). "The Complexity of Computing a Nash Equilibrium". *SIAM J. Comput.*, 39, 1, 195–259.
- [11] Dechenaux, E., D. Kovenock and R. Sheremeta (2015). "A Survey of Experimental Research on Contests, All-Pay Auctions and Tournaments," *Experimental Economics*, 18, 609-669.
- [12] Gill, D. and V. Prowse (2012). "A structural analysis of disappointment aversion in real effort competition". *American Economic Review*, 102, 1, 469-503.
- [13] Jia, H. S. Skaperdas, S. Vaidya (2013). "Contest functions: Theoretical foundations and issues in estimation". *International Journal of Industrial Organization* 31, 211–22.
- [14] Laffont, J. J. and E. Maskin (1980). "A Differential Approach to Dominant Strategy Mechanisms". *Econometrica*, 48, 6, 1507-1520.
- [15] Maskin E, Sjöström T (2002). "Implementation Theory". In: Arrow K, Sen A, Suzumura K *Handbook of Social Choice Theory Vol. I*. Amsterdam: North Holland, 237-288.
- [16] Milnor, J. (1954), "Games against nature," in: R. Thrall, C. Coombs and R. Davis (eds.), *Decision Processes*, Wiley, New York, 49.
- [17] Myerson, R. (1981). "Optimal Auction Design". *Mathematics of Operations Research* 6, 1, 58-73.
- [18] Myerson, R (1983). "Mechanism Design by an Informed Principal". *Econometrica*, 51, 6, 1767-1797.
- [19] Polishchuk, L and A. Tonis (2013). "Endogenous contest success functions: a mechanism design approach". *Economic Theory*, 52, 1, 271-297.
- [20] Rezai, A. and F. van der Ploeg (2017). "Climate Policies Under Climate Model Uncertainty: Max-Min and Min-Max Regret Uncertainty". OxCarre, Oxford Centre for the Analysis of Resource Rich Economies.
- [21] Schlag, K. (2006). "Why Minimax Regret?". Mimeo, October 23.
- [22] Serena, M. (2017). "Quality Contests" *European Journal of Political Economy*, 46, 15-25.
- [23] Skaperdas, S. and S. Vaidya (2012). "Persuasion as a contest". *Economic Theory* 51, 2, 465-486.

- [24] Skaperdas, S., A. Toukanb and S. Vaidyac (2016). "Difference-Form Persuasion Contests". *Journal of Public Economic Theory* 1-28.
- [25] Sonnenschein, H. (1972). "Market excess-demand functions". *Econometrica*. 40 (3): 549–563.
- [26] Wilson, R. (1987): "Game-Theoretic Approaches to Trading Processes," in *Advances in Economic Theory: Fifth World Congress*, ed. by T. Bewley, chap. 2, 33–77. Cambridge, University Press, Cambridge.