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A Concessions-Based Mechanism for Meta-Bargainig Problems

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ABSTRACT: In 1950, Nash's seminal paper introduced the axiomatic approach to the analysis of bargaining situations. Since then, many bargaining solutions have appeared and been axiomatically analyzed. The fact that agents, when face a bargaining problem, can come up with different solution concepts (that is, different notions of fairness and equity) was first introduced by van Damme (1986) by means of the meta-bargaining model. In this paper we present and axiomatically analyze a mechanism for solving meta-bargaining problems, which we call *Unanimous-Concession*. As an example, we show that the Nash solution is the result of the meta-bargaining process we define, when agents have dual egalitarian criteria. Finally, we compare, from an axiomatic and descriptive point of view, our proposal with other meta-bargaining mechanisms.

Key words: bargaining problem, mechanism, meta-bargaining, axiomatic approach.

JEL Classification: C71, D63, D71

1 Introduction.

The multiplicity of reasonable criteria to determine a solution in a bargaining problem leads to a dilemma. If the agents agree on one of them, this should be applied. But what happens if the agents' criteria are different? Could such a situation fit the Bargaining Model? The following situation, analyzed in experiments by Roth and Murnighan (1982), shows how, in real bargaining, the agents support different criteria.

“Two subjects are given one hundred lottery tickets to divide between them. Each subject's chance of winning a prize is proportional to the number of lottery tickets he receives in the bargaining, but the money values of prizes are different for the two players: \$20 for the first one and \$5 for the second”

The experimental results show that there are two focal proposals: either to split the expected prize equally (20 and 80 tickets respectively, which is the egalitarian solution (Kalai 1977), or to split the lottery tickets equally (50 and 50 tickets, namely the Nash solution (Nash 1950). The final outcome lies within the range between these two options. In Roth and Murnighan's words, *“the observed results suggest that theories which depend only on the feasible set and on the status quo are insufficiently powerful to capture the complexity of this kind of bargaining”*.

Although the traditional bargaining problem has been enriched in different ways (see, for instance, Chun and Thomson 1992), van Damme (1986) was the first author who considered the fact that the agents could support different concepts of fairness and equity by means of a meta-bargaining model. In order to solve these situations, he proposed a mechanism consisting of eliminating those alternatives exceeding their previous demand from the feasible set. By using this mechanism, van Damme provided strategic foundations for the Nash bargaining solution.

From van Damme's work, other mechanisms have arisen: Chun (1985, unpublished), Anbarci and Yi (1992), Marco et al. (1995, unpublished), Naeve-Steinweg (1999, 2002, 2004) and Trockel (2002). These papers show that the formulation of the mechanism is indeed relevant, since, from the strategic point of view, modifications of the procedure considered yield different conclusions. Our work is focused on the axiomatic (cooperative) point of view.

The paper is organized as follows. In Section 2 we introduce the notation and preliminary notions. We present our mechanism, which we

call *Unanimous-Concession*, in Section 3. Then, Section 4 is devoted to characterize it axiomatically. The last two sections are devoted to apply our mechanism to a particular family of bargaining situations and to compare our mechanism with others in the literature.

2 Preliminaries.

A *two-person bargaining problem* is a pair (S, d) , where $S \subseteq \mathbb{R}^2$ and $d \in S$. The points in S represent the feasible utility levels that the individuals can reach if they agree. When this is not the case, they end up at the disagreement point d . We will denote by Σ^2 the class of two-person bargaining problems (S, d) such that:

1. S is convex, closed and bounded from above.
2. S is strictly comprehensive¹, that is if $x \in S$ and $x \geq y$, then $y \in \text{int}(S)$.
3. There exists $x \in S$ such that $x > d$.

A *bargaining solution* is a single-valued function $f : \Sigma^2 \rightarrow \mathbb{R}^2$ such that for all $(S, d) \in \Sigma^2$, $f(S, d) \in S$.

Since Nash's bargaining solution (1950) many others have been introduced in the literature, and most of them have been axiomatically characterized in terms of a set of properties that they satisfy. Some of these properties are shared for most of the solutions, while others are specific for each of these bargaining solutions. We will refer by F , the set of *admissible solutions*, to a set of solutions satisfying a particular group of properties. Obviously, the weaker the properties we consider, the wider the set F of admissible solutions will be.

¹Throughout this paper, the following vectorial notation is used: Let $x, y \in \mathbb{R}^2$,

$$x \geq y \quad \text{means} \quad x_i \geq y_i \quad \text{for all } i = 1, 2.$$

$$x \geq y \quad \text{means} \quad x_i \geq y_i \quad \text{and } x \neq y.$$

$$x > y \quad \text{means} \quad x_i > y_i \quad \text{for all } i = 1, 2.$$

$$\inf\{x, y\} = (\min\{x_1, y_1\}, \min\{x_2, y_2\}).$$

Furthermore, when $S \subseteq \mathbb{R}^2$, $\partial(S)$ and $\text{int}(S)$ represent the boundary and the interior of S , respectively, with the usual topology in $\mathbb{R}^2$; and $PO(S)$ is the set of Pareto optimal points of S ,

$$PO(S) \equiv \{x \in S \mid \forall x' \in \mathbb{R}^2, x' \geq x \Rightarrow x' \notin S\}.$$

A **two-person meta-bargaining problem** is a triple $[(S, d); f, g]$ where $(S, d) \in \Sigma^2$ and $f, g: \Sigma^2 \rightarrow \mathbb{R}^2$ are two admissible bargaining solutions, supported by agents one and two respectively.

Given an admissible family of bargaining solutions F , a **meta-bargaining mechanism** on F is a single valued function

$$\Phi : F \times F \rightarrow F, \quad f, g \in F, \quad \Phi(f, g) = \Phi_{fg} \in F.$$

3 The Unanimous-Concession mechanism.

The *Unanimous-Concession* mechanism U assigns to each meta-bargaining problem $[(S, d); f, g]$ the point in S defined in the following way:

Let $(S, d) \in \Sigma^2$ and let $\{d^k\}$ be the sequence,

$$d^1 = d,$$

and for all $k \in \mathbb{N}, k > 1$,

$$d^k = \begin{cases} \inf \{f(S, d^{k-1}), g(S, d^{k-1})\} & \text{if } d^{k-1} \in \text{int}(S), \\ d^{k-1} & \text{if } d^{k-1} \in \partial(S). \end{cases}$$

Now, let $d^* = \lim_{k \rightarrow \infty} d^k$, and then

$$\mathbf{U}_{fg}(S, d) = d^*.$$

The following figure shows how the *Unanimous-Concession* mechanism is defined.

INSERT FIGURE 1 ABOUT HERE

The interpretation of this mechanism is as follows: each agent has his own proposal, given by the solution functions f and g ; thus, for a bargaining problem (S, d) , the solution outcome is the result of a step-by-step bargaining process, beginning at the disagreement point. In the first step, each agent concedes a certain amount to the other, according to a unanimous criterion: the maximum amount on which both agents agree. So these *concessions* are a natural expression of the idea that what is not actually in dispute should be conceded to the opponent. “*What is not actually in dispute*” is determined by the bargaining solutions proposed by the players. This allows the definition of a new disagreement point in a bargaining problem, in which the same idea can be applied. The point of convergence of this iterative process determines the agreement in the given bargaining problem.

In order to define a mechanism for meta-bargaining problems, it is important to choose the appropriate domain of criteria to be supported by the agents, that is the family F of admissible bargaining solutions. This is important for two reasons: on the one hand, it should contain the solutions which, in general, can be considered fair for bargaining problems; and, on the other hand, this domain must contain only those solutions for which the existence of the mechanism can be guaranteed. For instance, the domain of solutions in which van Damme's mechanism is well defined consists of the family of solutions that satisfy Pareto optimality, scale invariance, symmetry and risk sensitivity. Moreover, he assumes that each agent's proposal gives more utility to himself than his opponent's does. However, for the existence of our mechanism, we only need to restrict the class of admissible solution concepts to be supported by the agents to those that can be considered rational (IR), that is, solutions which give to any agent at least the disagreement utility point. This condition, and the strictly comprehensiveness² of S , ensures the existence and uniqueness of the point d^* in Definition 3 since, if the solutions satisfy (IR), the sequence $\{d^k\}$ is increasing, by construction, and bounded, because the set S is bounded from above, so it converges to some point in S .

In order to ensure efficiency of the meta-bargaining solution outcome, we consider the family of bargaining solutions satisfying (SIR) and (Cont),

(SIR) *Strong individual rationality*:

for all $(S, d) \in \Sigma^2$, $f(S, d) > d$.

(Cont) *Continuity*³:

for any sequence $\{(S^n, d^n)\} \subseteq \Sigma^2$ that converges to $(S, d) \in \Sigma^2$ in

the Hausdorff topology, the sequence $\{f(S^n, d^n)\}$ converges to $f(S, d)$.

From here on, we will consider as *admissible* bargaining solutions only those in the family:

$$F^* = \{\text{bargaining solutions satisfying (SIR) and (Cont)}\},$$

and we will prove that the Unanimous-Concession mechanism is well defined in the class F^* .

Whenever $f, g \in F^*$, then $\mathbf{U}_{fg} \in F^*$.

²Strictly comprehensiveness of the feasible set S is needed to ensure that the bargaining solutions f, g are well defined at each step, since most of the solutions to bargaining problems require the interior of the feasible set S to be non empty.

³All arguments could be made with the weaker condition of *d-Continuity*, in which $S^n = S$, for all n .

By observing that $\mathbf{U}_{fg}(S, d) \geq \inf\{f(S, d), g(S, d)\}$ and being both $f(S, d) > d$ and $g(S, d) > d$, **(SIR)** is satisfied, and only the continuity axiom remains to be proven. However, this property is a direct consequence of the way in which the solution has been defined, since, when $f, g \in F^*$, for all (S, d) , $\mathbf{U}_{fg}(S, d) = \lim_{k \rightarrow \infty} d^k$, and the *infimum* function is continuous.

In the following Proposition, we prove that the *Unanimous-Concession* mechanism, when defined in the class F^* , proposes a Pareto optimal point, for any bargaining situation.

For any pair of solution functions $f, g \in F^*$, the Unanimous-Concession mechanism yields a Pareto optimal point.

Let us show that the point d^* , introduced in Definition 3, belongs to $PO(S)$. If we suppose that $d^* = (d_1^*, d_2^*) \in \text{int}(S)$, given that

$$\begin{aligned} d_1^* &= \lim_{k \rightarrow \infty} \min(f_1(S, d^k), g_1(S, d^k)) \\ d_2^* &= \lim_{k \rightarrow \infty} \min(f_2(S, d^k), g_2(S, d^k)), \end{aligned}$$

there is a subsequence such that, without loss of generality,

$$d_1^* = \lim_{k \rightarrow \infty} f_1(S, d^k).$$

Consider the sequence $\{(S, d^k)\}$, which converges to (S, d^*) , and by applying **(Cont)** to f , $d_1^* = f_1(S, d^*)$, which contradicts the fact that f is **(SIR)**.

The following property provides a complete description of the Unanimous-Concession mechanism. We say that a meta-bargaining mechanism, $\Phi : F \times F \rightarrow F$, satisfies *Independence of Conceded Alternatives* **(ICA)** if:

- for each $f, g \in F$ and each $(S, d) \in \Sigma^2$, if $d' = \inf\{f(S, d), g(S, d)\}$,
- (a) $d' \notin PO(S) \Rightarrow \Phi_{fg}(S, d) = \Phi_{fg}(S, d')$.
- (b) $d' \in PO(S) \Rightarrow \Phi_{fg}(S, d) = d'$.

Part (a) of (ICA) can be seen as a modification of that used in van Damme (1986), in which it is required that a meta-bargaining solution be independent of those alternatives that are not demanded by the agents. Here we require the independence of those alternatives that are simultaneously dominated by the proposals of the agents, so that the problem may be solved in two ways: (i) directly, by applying the mechanism to the initial problem; or (ii) by giving each agent the maximum amount on which both players agree, $\inf\{f(S, d), g(S, d)\}$, which can be viewed as "natural concessions", and applying the mechanism to the problem where the new disagreement point is such unanimous agreement. The fact that both procedures should give the same result is related to the conditions analyzed

in the classical bargaining theory (Kalai 1977; Myerson 1977), called **step by step negotiation** and **composition**. Part (b) says that whenever both agents agree on a Pareto optimal point, such a point must be the solution of the problem.

The following result shows that, in the class F^* , the Unanimous-Concession mechanism is described by the condition **(ICA)**.

The Unanimous-Concession mechanism $\mathbf{U}$ is the only mechanism in the class F^* satisfying **(ICA)**.

It is quite simple to prove that $\mathbf{U}$ satisfies **(ICA)**. Let Φ be a meta-bargaining mechanism, $\Phi : F^* \times F^* \rightarrow F^*$ satisfying **(ICA)**, and let $(S, d) \in \Sigma^2$, $f, g \in F^*$. Then, if $f(S, d) = g(S, d) \in PO(S)$, $d' = \inf\{f(S, d), g(S, d)\} \in PO(S)$, so **(ICA)** implies $\Phi_{fg}(S, d) = d' = \mathbf{U}_{fg}(S, d)$. In other cases, if we name $d' = \inf\{f(S, d), g(S, d)\}$,

$$\Phi_{fg}(S, d) = \Phi_{fg}(S, d') \text{ and } \mathbf{U}_{fg}(S, d) = \mathbf{U}_{fg}(S, d').$$

We can now repeat the above argument to the problem (S, d') and, $\Phi_{fg}(S, d') = \mathbf{U}_{fg}(S, d')$ whenever $f(S, d') = g(S, d') \in PO(S)$, or by denoting $\inf\{f(S, d'), g(S, d')\} = d''$,

$$\Phi_{fg}(S, d') = \Phi_{fg}(S, d'') \text{ and } \mathbf{U}_{fg}(S, d') = \mathbf{U}_{fg}(S, d'').$$

By repeating this process, we obtain the result.

4 Axiomatic characterization.

In order to characterize the *Unanimous-Concession* mechanism, we introduce new properties that analyze the behavior of a meta-bargaining mechanism with regard to the solution concepts used by the agents.

We say that a meta-bargaining mechanism $\Phi : F \times F \rightarrow F$ satisfies *Monotonicity* if neither agent can lose by increasing the outcome he demands for himself.

(Mon) *Monotonicity*:

(a) for all f, h bargaining solutions such that $f_1 \geq h_1$, then

$$(\Phi_{fg})_1 \geq (\Phi_{hg})_1, \text{ for all } g$$

and symmetrically,

(b) for all g, h bargaining solutions such that $g_2 \geq h_2$, then

$$(\Phi_{fg})_2 \geq (\Phi_{fh})_2 \text{ for all } f.$$

On the other hand, we say that a meta-bargaining mechanism $\Phi: F \times F \rightarrow F$ satisfies:

(Un) *Unanimity*:

if $f(S, d) = g(S, d)$ for some $(S, d) \in \Sigma^2$, then

$$\Phi_{fg}(S, d) = f(S, d) = g(S, d).$$

Unanimity implies that whenever the agents agree on a proposed outcome, this will be the solution. But, in general, this axiom is not compatible with *Pareto optimality*, if we allow the agents to propose non Pareto optimal solutions, since $f(S, d) = g(S, d) \in \text{int}(S)$ yields, by Unanimity, $\Phi_{fg}(S, d) = f(S, d) \notin PO(S)$.

The following property is a suitable modification of Unanimity that allows us to make it compatible with efficiency. We say that a meta-bargaining mechanism $\Phi: F \times F \rightarrow F$ satisfies:

(IDS) *Improvement of dominated solutions*:

if for some $(S, d) \in \Sigma^2$,

a) $f(S, d) = g(S, d) \in PO(S)$, then $\Phi_{fg}(S, d) = f(S, d)$, and

b₁) if $f(S, d) \leq g(S, d)$, $\Phi_{fg}(S, d) = \Phi_{fg}(S, f(S, d))$,

b₂) if $g(S, d) \leq f(S, d)$, $\Phi_{fg}(S, d) = \Phi_{fg}(S, g(S, d))$.

This condition means that, when one of the proposals is dominated by the other, the result of the initial problem should be the same if we take the dominated proposal as an initial agreement and then distribute the remainder. When the agents propose Pareto optimal points, this condition coincides with Unanimity.

The following theorem provides a characterization of the Unanimous-Concession mechanism in the class F^* of bargaining solutions.

The Unanimous-Concession mechanism **U** is the only mechanism in the class F^* satisfying **(Mon)** and **(IDS)**.

It is quite simple to prove that **U** satisfies the above mentioned axioms. In order to prove its uniqueness, let Φ be any meta-bargaining mechanism with such properties, and let $(S, d) \in \Sigma^2$ and $f, g \in F^*$.

Now, we consider the auxiliary bargaining solution $h \in F^*$ defined as:

$$h(S, d) = \inf\{f(S, d), g(S, d)\} \quad \text{for all } (S, d) \in \Sigma^2.$$

We can then apply **(Mon)** to the bargaining solutions f, h and the following inequality must be fulfilled:

$$(\Phi_{fg}(S, d))_1 \geq (\Phi_{hg}(S, d))_1. \quad [1]$$

By construction, $h(S, d) \leq g(S, d)$, so **(IDS)** implies

$$\Phi_{hg}(S, d) = \Phi_{hg}(S, h(S, d)).$$

Now, it is possible to apply **(IDS)** to the problem $(S, d^2) \in \Sigma^2$, where

$$d^2 = h(S, d) = \inf\{f(S, d), g(S, d)\},$$

since, by construction, $h(S, d^2) \leq g(S, d^2)$, and we then obtain

$$\Phi_{hg}(S, d) = \Phi_{hg}(S, d^2) = \Phi_{hg}(S, h(S, d^2)).$$

By calling $d^3 = h(S, d^2) = \inf\{f(S, d^2), g(S, d^2)\}$ and successively applying this property, we obtain a sequence of problems (S, d) , (S, d^2) , (S, d^3) , ... such that $\Phi_{hg}(S, d^k)$ converges to a point in the Pareto boundary of S , which coincides, when $f, g \in F^*$, with

$$\mathbf{U}_{fg}(S, d) = \lim_{k \rightarrow \infty} d^k. \quad [2]$$

So we obtain, by joining [1] and [2]:

$$(\Phi_{fg}(S, d))_1 \geq (\Phi_{hg}(S, d))_1 = (\mathbf{U}_{fg}(S, d))_1.$$

With an analogous argument, we obtain that

$$(\Phi_{fg}(S, d))_2 \geq (\Phi_{fh}(S, d))_2 = (\mathbf{U}_{fg}(S, d))_2,$$

and with $\mathbf{U}_{fg}(S, d)$ being a Pareto optimal point,

$$\Phi_{fg}(S, d) = \mathbf{U}_{fg}(S, d).$$

The following examples establish the independence of the properties that characterize the *Unanimous-Concession* mechanism. First, given a bargaining problem (S, d) the *ideal point* $a(S, d)$ is defined by:

$$a_1(S, d) = \max\{x \mid (x, d_2) \in S\}, \quad a_2(S, d) = \max\{x \mid (d_1, x) \in S\}$$

1. The mechanism which provides, for any $f, g \in F^*$, the function, $\Phi_{fg}(S, d) = d + \frac{1}{4}(a(S, d) - d)$, for all (S, d) does not satisfy **(IDS)**, but does satisfy **(Mon)**.

2. The following mechanism satisfies **(IDS)** and does not satisfy **(Mon)**,

$$\Phi_{fg}(S, d) = \begin{cases} \mathbf{U}_{fg}(S, d) & \text{if one proposal is} \\ & \text{dominated by the other} \\ d + \frac{1}{4}(a(S, d) - d) & \text{otherwise} \end{cases}$$

5 An illustration.

In order to illustrate the *Unanimous-Concession* mechanism we are proposing, consider the following problem concerning the optimal provision and cost-sharing of a public good.

Two agents, indexed by $i = 1, 2$, consume one public good x , and share its cost. The technology for producing the public good is owned by both agents and uses as input the private good y . Agent i 's consumption set is $\mathbb{R}_+ \times Y_i$, $Y_i \subset \mathbb{R}_+$, with elements (x, y_i) . Here x is a level of public good and $y_i \in Y_i$ is agent i 's net loss of the private good, i.e. $y_i \geq 0$ means that agent i contributes y_i units of private good to the production of the public good, so that no private transfers are allowed.

The production function is $x = f(y)$, where $y = y_1 + y_2$, and the cost function is $c(y) = f^{-1}(x)$. The agents' utility functions are assumed to take the form $u_i(x, y_i) = h_i(x) - a_i y_i$, $i = 1, 2$. Individual decisions for the provision of the public good, z_i^* , are defined by

$$\arg \max_{x \geq 0} \{u_i(x, c(x))\} = \arg \max_{x \geq 0} \{h_i(x) - a_i c(x)\}, \quad i = 1, 2.$$

Under standard assumptions, once the *optimal* level of public good x^* is determined, the cooperative game associated with this situation is the following bargaining problem (S, d)

$$d_i = u_i(z_i^*, c(z_i^*)), \quad i = 1, 2, \text{ and}$$

$$S = \{(u_1, u_2) \mid \frac{a_2}{a_1} u_1 + u_2 \leq h_2(x^*) + \frac{a_2}{a_1} h_1(x^*) - a_2 c(x^*)\},$$

therefore agents face a "*linear*" bargaining problem. In particular, if we fix

$$f(y) = \frac{1}{2}y,$$

$$u_1(x, y_1) = x - \frac{2}{3}y_1 \quad u_2(x, y_2) = 2\sqrt{x} - y_2,$$

the bargaining problem is given by

$$S = \{(u_1, u_2) \mid \frac{3}{2}u_1 + u_2 \leq 2\} \quad d = (0, \frac{1}{2}).$$

Consider now that two agents, both supporting egalitarian criteria, do not agree on what is relevant: either gains from the disagreement point, or losses from the ideal point (that is, one supports the egalitarian bargaining solution (Kalai 1977), and the other proposes the rational equal-

loss bargaining solution (Herrero and Marco 1993)). In such a case, what we have is in fact a meta-bargaining problem. Let us see how the *Unanimous-Concession* mechanism solves this problem.

Agent one proposes the utility distribution $(\frac{3}{5}, \frac{11}{10})$, which implies gains of $\frac{3}{5}$ from the disagreement point, $(0, \frac{1}{2})$, for both agents. Agent two proposes the utility distribution $(\frac{2}{5}, \frac{7}{5})$, which implies equal losses of $\frac{3}{5}$ from the ideal point $(1, 2)$ and contributions of 5.4 and 2.6 units of private good for agent one and agent two respectively. The maximum amount of utility distribution on which both agents agree is $\inf\{(\frac{3}{5}, \frac{11}{10}), (\frac{2}{5}, \frac{7}{5})\} = (\frac{2}{5}, \frac{11}{10})$. This point has been denoted by d^2 in the sequence $\{d^k\}$ used to define the Unanimous-Concession mechanism. The next term of this sequence is obtained by setting the infimum of the solution outcomes of the bargaining problem (S, d^2) , and so on. The utility distribution provided by the Unanimous-Concession mechanism is then $(\frac{1}{2}, \frac{5}{4})$, which coincides with the Nash bargaining solution to the problem (S, d) (Nash 1950).

The result in the above example (agents supporting egalitarian and rational equal-losses arrive at the Nash solution when the Unanimous-Concession mechanism is applied) is a general property of our mechanism for “linear” bargaining problems. If we consider the subclass of bargaining problems Σ_L^2 in which the feasible sets S are linear half-spaces of $\mathbb{R}^2$,

$$S = \{(x, y) \in \mathbb{R}^2 \mid ax + by \leq m; a > 0, b > 0\},$$

then we have the following result.

For any $(S, d) \in \Sigma_L^2$, if f, g are the egalitarian and the rational equal-losses bargaining solutions, then

$$\mathbf{U}_{fg}(S, d) = N(S, d),$$

where $N(S, d)$ stands for the Nash bargaining solution.

It is clear that, in the class Σ_L^2 , the Nash bargaining solution is given by

$$N(S, d) = d + \frac{1}{2}(a - d),$$

where $a = a(S, d)$ denotes the *ideal point* of the problem (S, d) . In order to prove that the Unanimous-Concession mechanism gives the same proposal, we observe that (see Figure 2) the sequence $\{d^k\}$ that defines $\mathbf{U}_{fg}$ satisfies:

$$d^2 = d + v^1 \Rightarrow a^2 = a(S, d^2) = a - v^1,$$

$$d^3 = d^2 + v^2 = d + v^1 + v^2 \Rightarrow a^3 = a(S, d^3) = a - v^1 - v^2,$$

and so on. By construction, both sequences $\{d^k\}$ and $\{a^k\}$ converge to the same point $d^* = \mathbf{U}_{fg}(S, d)$, so that

$$d^* = d + \sum_i v^i = a - \sum_i v^i$$

and then, $\sum_i v^i = \frac{1}{2}(a - d)$, so $\mathbf{U}_{fg}(S, d) = d + \frac{1}{2}(a - d)$.

INSERT FIGURE 2 ABOUT HERE

6 Concluding remarks: a comparison with other meta-bargaining mechanisms.

From the initial meta-bargaining mechanism introduced by van Damme (1986), and its modification (Naeve-Steinweg 1999), other different mechanisms have been discussed in the literature: *Chun's mechanism* (Chun 1985; modified in Naeve-Steinweg 2002), the *Minimal Agreement procedure*⁴ (Anbarci and Yi 1992), the *Meta-bargaining Game* (Trockel 2002), and the *Averaging mechanism* (Naeve-Steinweg 2004).

The mechanisms of van Damme and Chun were introduced in order to support, from an strategic point of view (non-cooperative) Nash and Kalai-Smorodinsky solutions, respectively. The Meta-bargaining Game also supports Nash solution. The first work that considers the axiomatic (cooperative) approach to meta-bargaining problems is Marco et al. (1995). From then, this methodology has been applied to the analysis of other mechanisms (see Naeve-Steinweg 1999, 2002 and 2004). As Naeve-Steinweg (2002) points out, from a non-cooperative approach, “we can interpret meta-bargaining theory as a means to give a justification for a certain bargaining solution. [...] Based on van Damme’s paper it seems that the meta-bargaining approach singles out the Nash solution. However, it has to be stressed that the optimality strongly depends on mechanism we use. In particular, we have seen that also the Kalai-Smorodinsky (1975) solution is optimal for the appropriate mechanism.”⁵ That is why we are interested in the meta-bargaining process from a normative point of view, in which the agents have different equity criteria (represented by the bargaining solutions they support) and, as mentioned in Naeve-Steinweg (2004), “this leads to the interpretation of a mechanism as an arbitration scheme”.

In order to compare the *UC*-mechanism with the other ones, the first thing we must mention is that the *Minimal Agreement procedure* is, under

⁴This procedures was first suggested in van Damme (1986).

⁵As mentioned in Naeve-Steinweg (2002), by defining an appropriate mechanism, or by changing the framework, many other bargaining solutions are optimal.

some conditions, very similar to our proposal.⁶ To a better understanding the differences among them, in the following Figure we show how the different meta-bargaining mechanisms work (graphically):

INSERT FIGURE 3 ABOUT HERE

To analyze the axiomatic differences among these mechanisms, we will compare which properties they fulfil. The properties we analyze, additionally to the ones we have previously introduced in Sections 3 and 4, are the following:

We say that a meta-bargaining mechanism $\Phi : F^* \times F^* \rightarrow F^*$ satisfies *Impartiality* (Marco et al. 1995) if all opinions should be taken into account equally, regardless of whose opinion it is.

(Im) *Impartiality*:

for each $f, g \in F^*$, and all $(S, d) \in \Sigma^2$, then $\Phi_{fg}(S, d) = \Phi_{gf}(S, d)$.

A meta-bargaining mechanism $\Phi : F^* \times F^* \rightarrow F^*$ satisfies *Generalized Midpoint Domination* (Naeve-Steinweg 2004) when a minimal amount of cooperation should enable the agents to reach at least the average of their preferred outcomes.

(GMD) *Generalized Midpoint Domination*:

for each $f, g \in F^*$, and all $(S, d) \in \Sigma^2$, then $\Phi_{fg}(S, d) \geq \frac{1}{2}f(S, d) + \frac{1}{2}g(S, d)$.

A meta-bargaining mechanism $\Phi : F^* \times F^* \rightarrow F^*$ satisfies *Step-by-step Bargaining* (Naeve-Steinweg 2004) if the mechanism is invariant under a certain decomposition.

(STEP) *Step-by-step Bargaining*:

for each $f, g \in F^*$, and all $(S, d) \in \Sigma^2$, such that there is $T \subseteq S$ satisfying $(T, d) \in \Sigma^2$, $f(S, d) = f(T, d)$, $g(S, d) = g(T, d)$ and the segment joining $f(T, d)$ and $g(T, d)$, belongs to $PO(T)$,

$$[f(T, d), g(T, d)] \subseteq PO(T),$$

then $\Phi_{fg}(S, d) = \Phi_{fg}(S, \Phi_{fg}(T, d))$.

⁶The *Minimal Agreement procedure* has some shortcomings, since it requires the proposals to be in a determinate order ($f_2 \leq g_2$; $f_1 \geq g_1$) and, in some sets, it may not converge. But, whenever exists and converges to a Pareto optimal point, the outcome coincides with the one provided by our mechanism (see Marco et al, 1995, for additional details in comparing both mechanisms).

Finally, next table shows which properties are fulfilled by each meta-bargaining mechanism.⁷

	<i>UC</i>	<i>Min Agr</i>	<i>van Damme</i>	<i>Chun</i>	<i>M-Game</i>	<i>Av</i>
<i>PO</i>	✓	× (*, **)	× (*)	× (*)	×	✓
<i>Im</i>	✓	×	✓	✓	×	✓
<i>Un</i>	× (*)	✓	✓	✓	✓	× (*)
<i>IDS</i>	✓	×	×	×	×	×
<i>Mon</i>	✓	✓ (**)	×	×	×	✓
<i>GMD</i>	×	×	×	×	×	✓
<i>STEP</i>	×	×	×	×	(***)	✓
<i>ICA</i>	✓	×	×	×	×	×

(*) If the bargaining solutions f, g are required to be *PO*, this property is fulfilled by the mechanism.

(**) Whenever this mechanism is well defined.

(***) Not applicable.

UC: Unanimous Concession mechanism (Marco, Peris and Subiza 1995).

Min Agr: Minimal Agreement procedure (Anbarci and Yi 1992).

van Damme: van Damme's mechanism, as modified in Naeve-Steinweg (1999).

Chun: Chun's mechanism, as modified in Naeve-Steinweg (2002).

M-Game: Meta-bargaining Game (Trockel 2002).

Av: Averaging mechanism (Naeve-Steinweg 2004).

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⁷As we have mentioned in Section 3, an important question is to select the appropriate domain of admissible bargaining solutions. We have compared the different mechanisms in the class F^* , where the *UC* mechanism has been axiomatically characterized.

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Figure 1

Figure 2

Panel a: Minimal Agreement/Unanimous Concession Mechanisms

Panel b: Modified van Damme's Mechanism

Panel c: Modified Chun's Mechanism

Panel d: Trockel's Meta-Bargaining Game

Panel e: Averaging Mechanism

In **panel a**, $d_2 = \inf\{f(S,d), g(S,d)\}$.

In **panel b**, $S^2 = \{x \in S / x \leq \sup\{f(S,d), g(S,d)\}\}$ where for $x, y \in \mathbb{R}^2$, $\sup\{x, y\} = (\max\{x_1, y_1\}, \max\{x_2, y_2\})$.

In **panel c**, for $j \in \{1, 2\}$, e_{1j} (resp. e_{2j}) is the point belonging to the line joining d and $a(S,d)$ whose coordinate j is such a coordinate of the proposal of agent 1 (resp. agent 2). Then,

$$S^2 = \{x \in S / x \leq \min\{\max\{e_{11}, e_{12}\}, \max\{e_{21}, e_{22}\}\}\}.$$

In **panel d**, S is normalized in such a way $a(S,d) = (1, 1)$. The tangent line to δS at point $f(S,d)$ is denoted by $t(f)$, $p(t(f))$ is the normal vector to $t(f)$ such that the inner product $p(t(f)) \cdot f(S,d) = 1$. Analogously for $g(S,d)$, $t(g)$ and $p(t(g))$. Then,

$$\phi_{fg}(S,d) = (\min\{f_1(S,d), 1/2p_1(t(f))\}, \min\{g_2(S,d), 1/2p_2(t(g))\}).$$

In **panel e**, $d_2 = [f(S,d) + g(S,d)]/2$.

Figure 3

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