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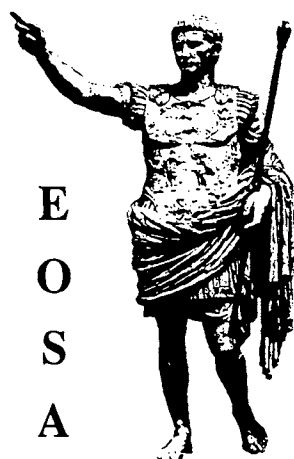
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## PSF CALCULATION FOR HOLOGRAPHIC OPTICAL ELEMENTS WITH ANNULAR APERTURES

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### Abstract

The imaging characteristics of systems with annular pupils is studied. The aberration-free Point-Spread Function (PSF) is discussed for increasing values of the obscuration of the pupil using a computer program. The results are compared with the corresponding results for systems with circular pupils.

### 1.- Theoretical analysis

Consider a holographic optical element (HOE) with an annular pupil [1] having inner and outer radii of  $\epsilon a$  and  $a$ , where  $\epsilon$  is called its obscuration ratio. The PSF of the system is given by:

$$I(x', y') = \frac{1}{N} \left| \iint_{\Sigma} f(r, \theta) \exp(i \Delta(r, \theta; x', y')) r dr d\theta \right|^2$$

where  $N$  is a normalized factor that makes  $I$  equal to 1 in the centre of the aberration-free image,  $\Sigma$  is the illuminated surface of the hologram,  $f(r, \theta)$  is the function that takes into account the amplitude variation on the surface of the hologram.  $\Delta(r, \theta; x', y')$  is the wavefront aberration and is a function of points  $(x', y')$  on the image plane and of point  $(r, \theta)$  on the hologram plane. If we assume that cartesian coordinates of the gaussian point are  $(x_g, 0, z_g)$ ,  $R_g \gg a$  and, that the HOE does not have any aberrations,  $\Delta$  function can be written approximately as:

$$\Delta(r, \theta; x', y') = \frac{2\pi}{\lambda_c R_g} r (x' \cos \alpha_g \cos \theta + y' \sin \theta)$$

where  $R_g$  is the distance from the gaussian point to the center of the hologram and  $\lambda_c$  is the reconstruction wavelength and  $\cos \alpha_g = z_g/R_g$ . In order to solve this integral, we have used a numerical integration procedure similar to those proposed by Chung and Hopkins [2] for conventional optical elements.

### 2.- Numerical example

As a numerical example we will consider two holographic lenses with a diameter of  $D = 4$  cm recorded with a plane wave that forms a  $30^\circ$  angle with the optical axis and a convergent wave with the source point separated 40 cm from the hologram centre and situated on the axis normal to the holographic plate in the first case, and the source point is forming a  $-45^\circ$  angle with the optical axis in the second. The hologram is reconstructed with the same recording geometry using a plane wave with the same wavelength used in the recording step (633 nm). We also assumed that  $f(r, \theta) = 1$ .

Figures 1-2 shows the aberration-free PSF for the on-axis convergent beam with two different values for the obscuration ratio of the pupil  $\epsilon = 0.5$  and  $0.75$ . Figure 3 and 4 show the aberration-free PSF for the out-axis convergent beam with the same values for  $\epsilon$ . These figures shows that the values of the secondary maxima of a distribution relative to the principal maximum rise as  $\epsilon$  increases. In Figures 1 and 2, the PSF has radial symmetry, as is shown by the kurtosis' values  $k_{x1} = k_{y1} = 4.05$  and  $k_{x2} = k_{y2} = 3.03$ . Figures 3 and 4, does not have radial symmetry and the PSF has an elliptical distribution. This can be shown

with the kurtosis values  $k_{x3} = 4.00$ ,  $k_{y3} = 5.26$  and  $k_{x4} = 3.29$ ,  $k_{y4} = 4.18$ . To obtain a diffraction figure with radial symmetry for an "out axis" holographic lens, we must use an elliptical pupil with axis values of:  $a$  and  $a/\cos \alpha_g$ . On the other hand, we have analyzed other annular filters with different functions  $f(r, \theta)$  and the influence of aberrations on PSF. Finally, we have also found that the defocus tolerance for a given Strehl ratio is larger for a HOE with an annular pupil than for the same HOE with a circular pupil.

### 3- Conclusions

We have obtained numerical results on the PSF for aberration-free holographic optical elements with uniform illumination and annular pupils. These results indicate that it is possible to improve the imaging quality of HOE's by using apodization filters, such as annular filters, on the HOE surface.

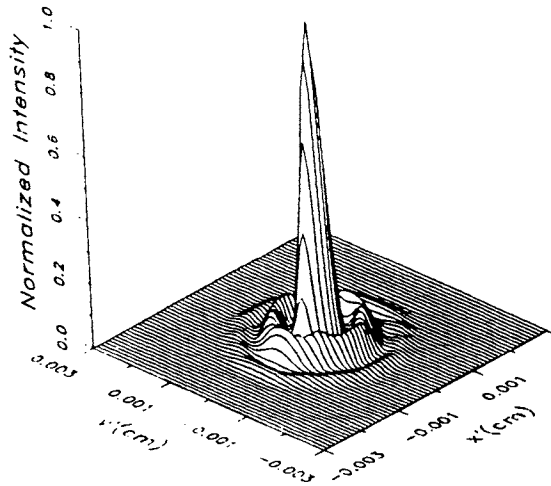


Figure 1

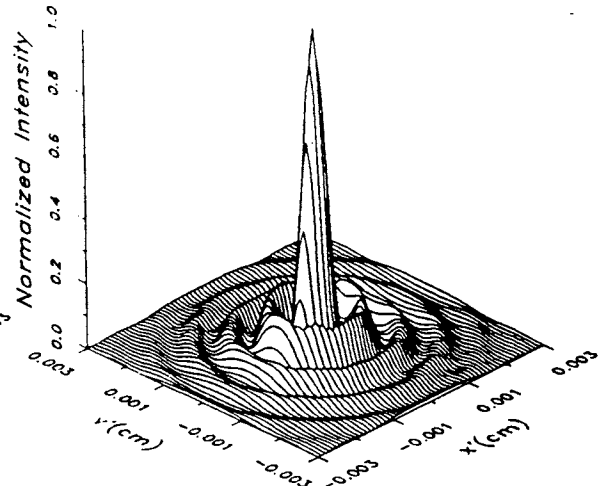


Figure 2

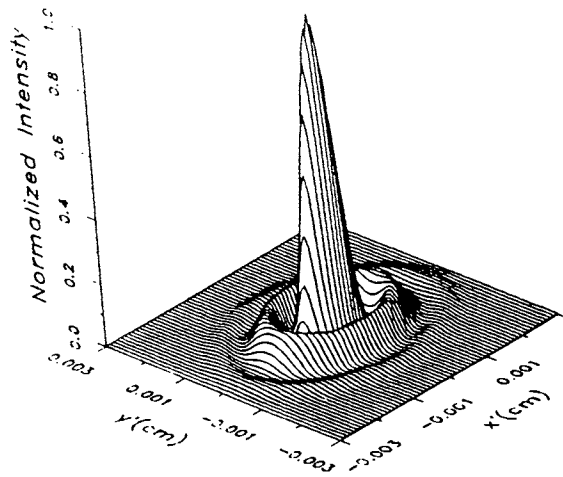


Figure 3

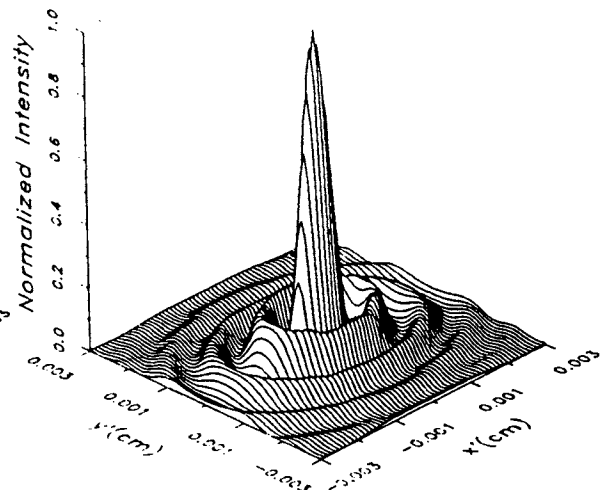


Figure 4

### 4-References

- [1] V. N. Mahajan. *J. Opt. Soc. Am. A.* **72**, 1258 (1982).
- [2] C. S. Chung and H. H. Hopkins. *J. Modern Opt.*, **35**, 1485 (1988).