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OPTIMIZATION OF RECONSTRUCTION GEOMETRY FOR MAXIMUM DIFFRACTION EFFICIENCY IN HOE: THE INFLUENCE OF RECORDING MATERIAL

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ABSTRACT

In this paper the relation between recording and reconstruction geometries for maximum diffraction efficiency in thick holographic lenses is analyzed. Theoretical expressions are presented when variations in recording material due to processing are taken into account. A particular holographic lens is studied in both theoretical as well as experiment terms, finding the optimized reconstruction geometry for maximum diffraction efficiency and the aberration that appear.

1.- INTRODUCTION

Holographic Optical Elements (HOEs) are Diffractive Optical Elements (DOEs), and therefore work by diffracting light from a generalized grating structure with nonuniform groove spacing[1]. It is possible to consider two important questions associated with the making of thick HOEs[2]. The first is the design and analysis of the HOE; the second the recording material used to record the HOE. In order to quantify the performance of a HOE one can evaluate aberrations and diffraction efficiency.

As a result of the variations in recording material introduced by the photochemical process of the HOE, the reconstruction geometry corresponding to maximum diffraction efficiency has to be changed with respect to the obtention geometry, which evidently gives rise to the appearance of aberrations in the HOE during the reconstruction stage[3,4].

In this paper, we present matricial relations between the reconstruction and recording coordinates of the sources in a Holographic Lens (HL), when these changes in recording material due to chemical process appear, and when recording and readout wavelengths are different and when Bragg's Law is complied with. According to this, and upon introducing the experimental values of recording material variations due to processing into the equation, it is possible to evaluate the aberrations in the reconstruction stage of HLs when diffraction efficiency is maximum. We present theoretical and experimental results in the optimization of reconstruction geometry of an HL for maximum diffraction efficiency and when recording and reconstruction wavelengths are identical. Also, we evaluate the aberrations introduced due to the changes between recording and reconstruction geometries, using a root mean square of wave front aberration. In such a case it is possible to find geometries in which the combinations of wavelengths and recording material changes give way to situations in which the aberrations are minimized and, at the same time, high diffraction efficiencies are achieved.

2.- INFLUENCE OF RECORDING MATERIAL AND ITS PROCESSING IN THE DESIRED HOE

In the photochemical processing of holographic recording material used for making thick phase holograms we use liquid solutions and as consequence of this there is a change in the average refractive index and a deformation of the recording material. As a result of all these variations, the reconstruction geometry for maximum diffraction efficiency will be different from the recording geometry. For a holographic grating, the grating vector can be written as[5]:

$$\mathbf{K} = \mathbf{k'}_R - \mathbf{k'}_O \tag{1}$$

where $\mathbf{k'}_R$ and $\mathbf{k'}_O$ are the propagation vector of the reference (R) and object (O) beams in the medium of refractive index n_R, respectively. Using the Effective Holographic Grating Model[6,7], the relationship that exists between the grating vectors before, $\mathbf{K}$, and after processing, $\mathbf{K}^*$, are expressed as:

$$K_x^* = K_x \tag{2}$$

$$K_z^* = \frac{1}{T_e} K_z \tag{3}$$

where $T_e = t_e/t_R$, t_e is the "effective thickness" and t_R is the initial thickness of the medium, respectively. T_e is expressed as follows[7]:

$$T_e = \frac{\operatorname{tg}\phi}{\operatorname{tg}\delta + \operatorname{tg}\phi} T \qquad (4)$$

where $T = t_C/t_R$, t_C is the thickness of the medium after processing, ϕ is the angle of Bragg plane inclination with respect to the Z axis and δ is the "cut" or "shear" angle[7]. This shear effect is due to the inclination of the Bragg planes and these δ angles will be small (only a few degrees). After processing, Bragg's Law can be written as:

$$\mathbf{K}^* = \mathbf{k}'_C - \mathbf{k}'_I \qquad (5)$$

where $\mathbf{k}'_C$ and $\mathbf{k}'_I$ are the propagation vectors of the reconstruction (C) and image (I) beams in the medium of refractive index n_C after processing.

The effective thickness is directly related to the reconstruction angle which complies with Bragg's Law. If this angle is α_C, the following equation can be obtained for transmission gratings by developing equation (5):

$$\sin\alpha_C = \frac{1}{2}\left[\frac{N}{T_e} + \mu\right]\sin\alpha_R + \frac{1}{2}\left[\frac{N}{T_e} - \mu\right]\sin\alpha_O \qquad (6)$$

where α_R and α_O are the recording angles of the grating, $N = n_C/n_R$ and $\mu = \lambda_C/\lambda_R$, λ_R and λ_C being the recording and the reconstruction wavelengths, respectively. It is possible to determine the T_e/N quotient through experimentation by using:

$$\frac{T_e}{N} = \frac{\sin\alpha_R + \sin\alpha_O}{2\sin\alpha_C + \mu(\sin\alpha_O - \sin\alpha_R)} \qquad (7)$$

3.- HIGH DIFFRACTION EFFICIENCY CONDITION IN THE HOE

It is possible to apply the relationship presented in the previous section to holographic lenses by using an approximation of the local grating[8], which produces a set of relationships between the coordinates of the recording and reconstruction sources when spherical and collimated wave fronts are used and Bragg's Law is complied with in the reconstruction stage.

The interference pattern, locally, looks like a plane grating having a grating vector $\mathbf{K}(x, y)$:

$$\mathbf{K}(x, y) = \mathbf{k}'_R(x, y) - \mathbf{k}'_O(x, y) \qquad (8)$$

The z component of the propagation vector $\mathbf{k}'_q(x, y)$, k'_{qz}, can be derived from the x and y components, k'_{qx} and k'_{qy}, yielding:

$$k'_{qz} = \sqrt{\left(\frac{2\pi n_q}{\lambda_q}\right)^2 - k'^2_{qx} - k'^2_{qy}} \qquad (9)$$

After processing of the HL, and in order to obtain high diffraction efficiency, it is necessary to fulfill the Bragg condition given by:

$$\mathbf{K}^*(x, y) = \pm[\mathbf{k}'_C(x,y) - \mathbf{k}'_I(x,y)] \qquad (10)$$

In this equation, the $\pm$ sign refers to the use of the +1 diffracted order (+) or -1 diffracted order (-) of the holographic lens.

Taking into account equations (2)-(3), the components of the grating vectors before and after processing, $\mathbf{K}$ and $\mathbf{K}^*$, are expressed with the following equations:

$$K_x^*(x, y) = K_x(x, y) \quad (11)$$
$$K_y^*(x, y) = K_y(x, y) \quad (12)$$
$$K_z^*(x, y) = [1/T_e(x, y)]\cdot K_z(x, y) \quad (13)$$

Now, the T_e parameter will be different for any local grating and it will be a function of any (x,y) point of the HL surface. Substituting equations (8) and (10) into equations (11)-(13), we obtain the equations:

$$k_{Cx}'(x, y) - k_{Ix}'(x, y) = \pm [k_{Rx}'(x, y) - k_{Ox}'(x, y)] \quad (14)$$
$$k_{Cy}'(x, y) - k_{Iy}'(x, y) = \pm [k_{Ry}'(x, y) - k_{Oy}'(x, y)] \quad (15)$$
$$k_{Cz}'(x, y) - k_{Iz}'(x, y) = \pm [1/T_e(x,y)]\cdot[k_{Rz}'(x, y) - k_{Oz}'(x, y)] \quad (16)$$

Equations (14) and (15) are always sastisfied, because of the image equations[10].

Generally, aspheric recording or reconstruction wave fronts will be necessary to satisfy equations (14)-(15) simultaneously, due to the (x, y) dependence. If we want to use spheric and collimated wavefronts, the previous equations are only satisfyied approximately. This situation requires that $T_e(x,y)$ remain constant, as we will see immediately. In section 5 with a holographic lens made in the laboratory, we will verify that this approximation is acceptable.

Firstly, we will consider that the $T_e(x,y)$ function is constant, taking the mean value of the $T_e(x,y)$ function over the entire surface of the HL, $<T_e(x,y)>$:

$$T_e(x,y) \approx < T_e(x,y) > \equiv T_e \quad (17)$$

Using the equation for the propagation vectors of the light in air at the point of the HL with coordinates $(x,y, z = 0^-)$, $\mathbf{k}_q$:

$$\mathbf{k}_q(x, y) = (2\pi/\lambda_q)\cdot(\mathbf{r}_q / r_q) \quad (18)$$

and using equation (9), it is possible to obtain the expression for the propagation vectors in the medium, $\mathbf{k}_q'$. Expanding the expression obtained for $\mathbf{k}_q'$ by series of powers, we retain terms not higher than the second power, and by substituing the final expression of $\mathbf{k}_q'$ into equations (14)-(16) with $T_e(x,y) \approx T_e$ (Equation (17)), we obtain, for a specified recording geometry, the expressions for the coordinates of the reconstruction sources needed to fulfill the Bragg condition:

$$\begin{bmatrix} \dfrac{x_C}{R_C} \\ \dfrac{x_I}{R_I} \end{bmatrix} = \pm \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} \dfrac{x_R}{R_R} \\ \dfrac{x_O}{R_O} \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} \dfrac{y_C}{R_C} \\ \dfrac{y_I}{R_I} \end{bmatrix} = \pm \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} \dfrac{y_R}{R_R} \\ \dfrac{y_O}{R_O} \end{bmatrix} \quad (20)$$

$$\begin{bmatrix} \dfrac{1}{R_C} \\ \dfrac{1}{R_I} \end{bmatrix} = \pm \begin{bmatrix} a & b \\ b & a \end{bmatrix} \begin{bmatrix} \dfrac{1}{R_R} \\ \dfrac{1}{R_O} \end{bmatrix} \quad (21)$$

Similary, for a specified reconstruction geometry, the required expressions for the coordinates of the recording sources which satisfy Bragg's Law are:

$$\begin{bmatrix} \frac{x_R}{R_R} \\ \frac{x_O}{R_O} \end{bmatrix} = \pm \begin{bmatrix} c & d \\ d & c \end{bmatrix} \begin{bmatrix} \frac{x_C}{R_C} \\ \frac{x_I}{R_I} \end{bmatrix} \qquad (22)$$

$$\begin{bmatrix} \frac{y_R}{R_R} \\ \frac{y_O}{R_O} \end{bmatrix} = \pm \begin{bmatrix} c & d \\ d & c \end{bmatrix} \begin{bmatrix} \frac{y_C}{R_C} \\ \frac{y_I}{R_I} \end{bmatrix} \qquad (23)$$

$$\begin{bmatrix} \frac{1}{R_R} \\ \frac{1}{R_O} \end{bmatrix} = \pm \begin{bmatrix} c & d \\ d & c \end{bmatrix} \begin{bmatrix} \frac{1}{R_C} \\ \frac{1}{R_I} \end{bmatrix} \qquad (24)$$

In the above matricial relations the parameters a, b, c and d are defined as:

$$a = \frac{1}{2}\left[\frac{N}{T_e} + \mu\right] \qquad b = \frac{1}{2}\left[\frac{N}{T_e} - \mu\right] \qquad (25)$$

$$c = \frac{1}{2}\left[\frac{T_e}{N} + \frac{1}{\mu}\right] \qquad d = \frac{1}{2}\left[\frac{T_e}{N} - \frac{1}{\mu}\right] \qquad (26)$$

Expressions (25) and (26) show that if $\mu = T_e/N$, the coordinates of the reconstruction beam depends only on the reference beam coordinates and viceversa, and the same occurs with the image beam and object beam coordinates.

4.- ANALYSIS OF ABERRATIONS

In order to improve the performance of the final holographic lens, it is helpful to know the aberrations that it contains. The relationship between the ϕ_C phase of the reconstruction wave front of wavelength λ_C at the HL, and the ϕ_I phase of the image wave front at the HL, is given by [11]:

$$\phi_C = \phi_I \pm (\phi_O - \phi_R) \qquad (27)$$

where ϕ_R and ϕ_O are the phases of the object and reference waves, respectively, and the ± refers to the positive and negative first diffracted orders from the HL. When the desired Gaussian phase $\phi_I{}^D$ differs from the actual image phase ϕ_I, aberrations appear. The wave front aberration, Δ, is the difference between the desired image phase $\phi_I{}^D$ and the actual image phase ϕ_I:

$$\Delta = \phi_I - \phi_I{}^D \qquad (28)$$

Taking into account equation (27), aberrations can be written as:

$$\Delta = \phi_C - \phi_I{}^D \pm (\phi_O - \phi_R) \qquad (29)$$

When the phases ϕ_O, ϕ_R, ϕ_C and $\phi_I{}^D$ are the phases of spherical waves, their values in the plane of the hologram are:

$$\phi_q(x, y) = (2\pi/\lambda_q)\cdot(r_q - R_q) \qquad (30)$$

According to equation (30), the wave front aberration Δ will be expressed in radians. Usually, the wave front aberration is expressed in unities of reconstruction wavelengths[10]. In this case, we define the wave front aberration as W(x,y), where the relation between Δ and W is given by:

$$\Delta(x, y) = (2\pi/\lambda_C)\cdot W(x, y) \qquad (31)$$

and wave front aberration in wavelengths is $W(x, y)/\lambda_C$. Using equations (29), (30) and (31), W (x, y) is given by:

$$W = r_C - r_I \pm \mu(r_O - r_R) - [R_C - R_I \pm \mu(R_O - R_R)] \qquad (32)$$

We take as the measure of the aberrations the root mean square value[12], A of the wave front aberration $(1/\lambda_C)\cdot W(x, y)$ over the entire pupil of the HL, i.e.,

$$A = \frac{1}{\lambda_C}\left[\frac{\iint W^2(x, y)\,dx\,dy}{\iint dx\,dy}\right]^{1/2} \cong \frac{1}{\lambda_C}\left[\sum_{i=0}^{N}\sum_{j=0}^{N}\frac{W_{ij}^2}{(N+1)^2}\right]^{1/2} \qquad (33)$$

In order to approximate the integral, we use a set of $(N + 1)^2$ sample points where $W_{ij} \equiv W(x_i, y_j)$ is the sample value of W (x, y) in an incremental area $\Delta x\Delta y$ located at point (x_i, y_j) on the HL, with $\Delta x = D_x/N$ and $\Delta y = D_y/N$, and D_x and D_y being the dimensions of the HL, and:

$$x_i = \left[i - \frac{N}{2}\right]\Delta x \qquad\qquad y_j = \left[j - \frac{N}{2}\right]\Delta y \qquad (34)$$

5.- NUMERICAL EXAMPLE AND EXPERIMENTAL RESULTS

The optimization procedure for recording or reconstruction geometry for maximum diffraction efficiency described above is illustrated here for a holographic lens having the following parameters (fig.1): $R_R = \infty$, α_R=40°, β_R = 0°, R_O = - 325 mm, α_O = 0°, β_O = 0°, λ_R = 633 nm, D = 80 mm; where D is the diameter of the HL. The HL was fabricated in bleached photographic emulsion Agfa -Gevaert 8E75 HD, using AAC developer and R-9 solvent bleaching[13]. Due to the symmetry of the system respect to the XZ plane, it is possible to reduce the analysis at that plane.

Once the lens is processed, the next step is to find the value of T_e/N. Since at each point of the lens the exposure values, the spatial frequency and the slant angle of the interference fringes are different, the T_e/N parameter will be different as well. Given the symmetrical nature of the HL, we will only determine T_e/N on the X axis points of the lens. We will show that these measurements will suffice in obtaining acceptable diffraction efficiencies. The same wavelength is used in reconstruction as in recording (λ_C = 633 nm).

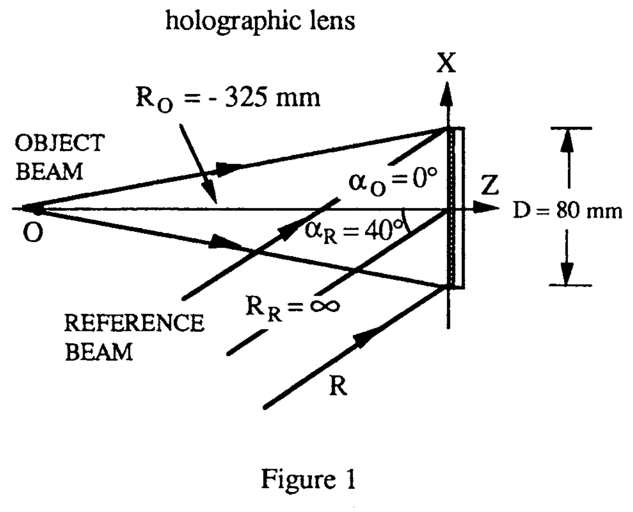

Figure 1

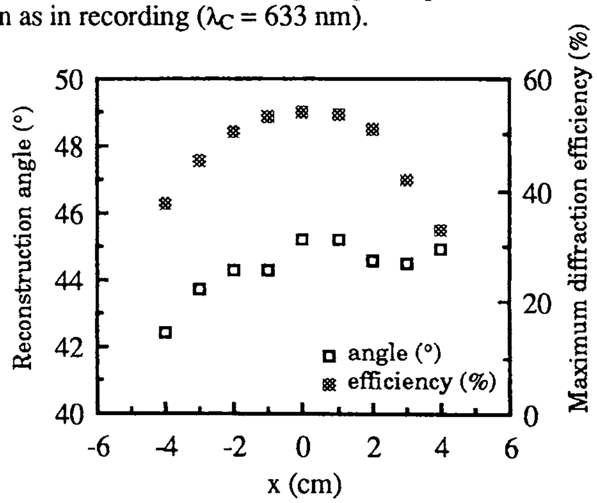

Figure 2

Equation (7) is used to determine $(T_e/N)(x)$. It is modified as necessary according to the local grating of each point of the X axis with μ being 1:

$$\frac{T_e}{N}(x) = \frac{\sin \alpha_R(x) + \sin \alpha_O(x)}{2 \sin \alpha_C(x) + \sin \alpha_O(x) - \sin \alpha_R(x)} \tag{35}$$

We have chosen to use the points that go from -4 cm to + 4 cm as values of x, at 1 cm intervals, and we identify the value of the reconstruction angle that produces maximum diffraction efficiency in each case. We call this angle $\alpha_C{}^B(x)$.

Fig.2 shows the experimental values for maximum diffraction efficiency and their corresponding reconstruction angles for each point x on the lens.

By knowing the reconstruction angle for maximum diffraction efficiency, it is possible to calculate T_e/N by using equation (35). In fig.3 we see the values of $T_e/N(x)$ as well as the average value obtained $<T_e/N(x)>=0.85$.

According to equations (19)-(21), and using $T_e/N = 0.85$ and $\mu = 1$, the desired reconstruction parameters for achieving high diffraction efficiency are $R_C = -3683$ mm, $\alpha_C = 44.4°$, $\beta_C = 0°$.

Fig.4 shows diffraction efficiency as a function of coordinate x for $T_e/N = 1.00$ (identical recording and reconstruction geometries), for $T_e/N = 0.85$ and also shows maximum diffraction efficiency when measured experimentally. We can see that by using the $T_e/N = 0.85$ reconstruction geometry, approximatly maximum diffraction efficiencies can be obtained.

In this case, the root mean square value of the wavefront aberration over the entire surface of the holographic lens is $A = 54\,\lambda_C$.

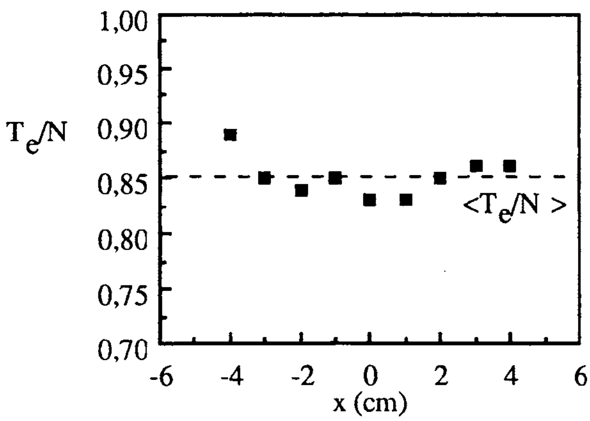

Figure 3

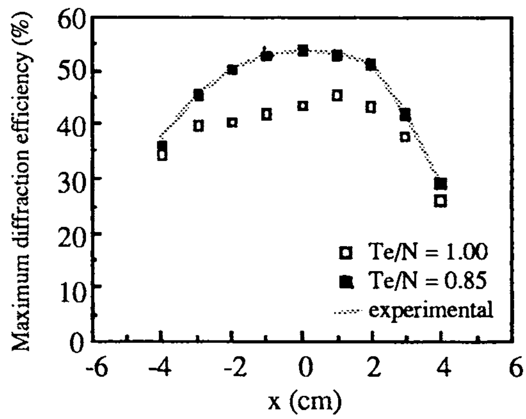

Figure 4

Once the T_e/N value of the lens being analyzed is known, it is possible to use the equation in the opposite direction. In other words, for any given reconstruction geometry (which does not differ greatly from the recording geometry used in the asymmetric lens studied), we want to be able to find the recording geometry needed if $T_e/N = 0.85$ and $\mu = 1$.

If the desired reconstruction geometry is $R_C = \infty$, $\alpha_C = 40°$, $\beta_C = 0°$, $R_I = -325$ mm, $\alpha_I = 0°$, $\beta_I = 0°$ and $T_e/N = 0.85$, by using equations (22)-(24), we obtain $R_R = 4333$ mm, $\alpha_R = 36.5°$, $\beta_R = 0°$, $R_O = -351$ mm, $\alpha_O = -2.8°$, $\beta_O = 0°$. By choosing these recording sources, and in spite of the variations that take place in the recording material during processing, we can ensure that during reconstruction with a collimated beam at a 40° angle to the Z axis, the image beam will be divergent, well have the desired characteristics and will have maximum diffraction efficiency. Nevertheless, aberrations will appear in this beam, just as has been pointed out earlier. The root mean square value of the wavefront aberration over the entire surface of the holographic lens is $A = 39\,\lambda_C$.

6.- DISCUSSION AND FINAL REMARKS

Equations (11)-(13) are obtained by applying the hypothesis on local gratings which includes the introduction of the recording material and its processing into the fabrication of holographic lenses. They show the relationship between the vector of each local grating at one point (x, y) of the lens before and after processing. Included in the equations is the effective thickness and its relation to the deformations in the recording material. First of all these equations and the maximum diffraction efficiency equations in the reconstruction stage of the lens, require that relationships presented in (14)-(16) must be met. These link wave vectors to the material and are nothing more than Bragg's Law applied to each and every local grating on the lens.

As a consequence of these expressions -and when all of the beams used in recording and reconstruction are collimated or spheric wavefronts- new equations can be proposed which relate the different coordinates of the reconstruction and image sources to the object and reference sources and vice-versa if Bragg's Law is satisfied. Here the most important issue is that in all of these equations (19)-(26) the effective thickness (expressed by T_e), the index quotients (expressed by N) and the wavelength quotients (expressed by μ) are all present. Expressions (19)-(26) offer a new set of possibilities in the area of recording and reconstruction geometries of HLs as a function of the values that can be assigned to the three aforementioned parameters -T_e, N and μ- which were not possible in other geometric models that did not include the variations in recording material due to processing. All of this can lead to a study of the influence that these equations can have on the pre-design of Holographic Lenses.

However, we must consider the fact that the recording and reconstruction wavelengths are different. In our case, we add the important influence of the recording material and equations (25) and (26), which put parameters T_e, N and μ on equal footing. Thus, if the right choices are made as regards recording material and processing (T_e and N), and recording and reconstruction wavelengths (μ), it is possible to find situations in which some aberrations are compensated for. In the case we analyzed experimentally, it is possible to cancel coma and astigmatism and ensure maximum diffraction efficiency [4] if $\mu = N/T_e$, that is, if the reconstruction wavelength is 745 nm (in this case A is 1.5 λ_C, much lower than those obtained with a reconstruction wavelength of 633 nm). Finally, it will be possible to obtain techniques for minimizing aberrations by using aspheric wavefronts during recording that are obtained from holograms which are computer generated[9] or by using recursive techniques, that is using as recording beams wavefronts which come from other holographic lenses[2]. And now there are greater possibilities for succes when using these techniques because there are more parameters to combine (T_e, N and μ).

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