

# Measurement of the absorption coefficient of the glass substrate of holographic plates

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## **ABSTRACT**

We present a simple method of evaluating the absorption of the glass substrate of holographic plates which makes use of transmission data. The method is based on the analysis of the transmission of a plane parallel glass plate as a function of the angle of incidence assuming that multiple interferences occur at the plate when it is illuminated with a coherent beam of light from a laser. It is necessary to know the absorption of the glass substrate in order to be able to correct the data for the transmission and diffraction efficiencies of holographic diffraction gratings. The agreement between calculated and measured data confirms the applicability of this method.

**Subject terms:** holographic recording materials, glass; optical constants; optical materials; absorption coefficient; multiple interferences

In order to calculate the intrinsic efficiencies of the transmitted and diffracted beams corresponding to a holographic diffraction grating, the values of the intensities of the incident, diffracted and transmitted beams measured experimentally are corrected using Fresnel's equations [1-3]. This correction is done to take into account reflections which occur at the various interfaces - air-material, material-glass, glass-air - of the holographic plate. However, in order to obtain a more accurate measurement of these intrinsic efficiencies, it is necessary to take into account the absorption of the substrate on which the recording material -whether a photographic emulsion, dichromated gelatin, photoresin, photopolymer or any other type of material- will be deposited. Usually this substrate is a plane parallel glass plate, and therefore it will not have a high absorption. However, it is sometimes essential to know the value of this absorption in order to obtain accurate values of the various holographic parameters that characterize a diffraction grating. For example, it is necessary to know the exact value of the absorption of the glass substrate in order to determine the absorption and scattering,  $A \& S$ , of holographic diffraction gratings [3, 4]. This may be calculated (as a percentage) using the equation  $A \& S = 100 - \eta(\%) - \tau(\%)$ , where  $\eta$  is the diffraction efficiency of the diffracted beam and  $\tau$  is the diffraction efficiency of the transmitted beam.

In this paper we present a simple method of evaluating the absorption of the glass substrate of a holographic plate. Our experimental study was carried out on the plane parallel glass plate of the type of photographic plate used in holography. However, the same procedure may be used with any other type of holographic material deposited on a glass plate or simply with any glass plate of similar characteristics to those used as substrates in holography.

If a collimated beam of light of intensity  $I_i$  incides on a plane parallel glass

plate of dielectric material, assumed to be lineal, homogeneous and isotropic, at an angle of incidence  $\theta$  to the perpendicular, the intensity  $I_t$  on leaving the material will be less than on entering due to absorption. Therefore:

$$I_t = I_i \exp(-\alpha d / \cos\theta) \quad (1)$$

where  $\alpha$  is the coefficient of absorption,  $d$  is the thickness of the medium and  $\theta'$  is the angle of refraction inside the medium. Then the transmittance  $T$  of the plate is:

$$T = I_t / I_i = \exp(-\alpha d / \cos\theta) \quad (2)$$

Equation (2) may be used to calculate the coefficient of absorption  $\alpha$  of a plane parallel glass plate by simply making a collimated beam of monochromatic light (since  $\alpha$  is a function of the wavelength of the light) incide on a glass sheet and measuring the incident intensity  $I_i$  and transmitted intensity  $I_t$ . However, this procedure does not give an accurate result since due to the refraction which takes place at the air-glass interface of the first side of the plate, the light which actually incides is less than that measured. This does not pose a problem since by applying Fresnel's equations it is possible to calculate the true intensity of light inciding on the plate once the measured intensity is corrected taking into account the light reflected at the air-glass interface. The same applies to the transmitted light, since the intensity of the light measured by the detector is less than that of the light which is actually transmitted due, in this case, to the light reflected by the glass-air interface. After correcting in the same way as for the incident light, the true intensity transmitted by the plate may be calculated, before the glass-air interface. Taking this into account, the corrected equation for the transmittance is:

$$T = (t_{ag}t_{ga})^2 \exp(-\alpha d / \cos\theta') \quad (3)$$

Where  $t_{ag}$  and  $t_{ga}$  are the amplitude transmission coefficients for the air-glass and glass-air interfaces, respectively, and  $T$  is the transmittance measured experimentally.

However, in spite of having corrected the transmitted and incident intensities, their quotient does not give the correct value of transmittance, since it is necessary to take into account the effect of the multiple reflections that occur at the glass plate, as can be seen in Figure 1. On illuminating with a coherent beam of light, a diagram is obtained of the interference arising from reflections taking place at both sides of the plate. In this situation the superposition of an infinite number of waves with the same phase difference  $\varphi$  is produced [5]:

$$\varphi = \frac{4\pi}{\lambda} n d \cos\theta' \quad (4)$$

with amplitudes that decrease at a geometrical rate as can be seen in Figure 1, where  $r_{ag}$  and  $r_{ga}$  are the amplitude reflection coefficients for the air-glass and glass-air interfaces, respectively. The remaining parameters are fully defined in Figure 1. In equation (4)  $n$  is the refractive index of the plate,  $\lambda$  is the wavelength in air of the radiation used and  $\theta'$  is the angle of refraction in the medium.

The amplitude of the  $m$  wave is less than that of the  $m-1$  wave due to the factor  $r_{va}^2 \exp(-\alpha s)$ , where:

$$s = d / \cos\theta' \quad (5)$$

and the phase differs in  $\varphi$ . The wave superposition has a complex amplitude:

$$U = U_1 + U_2 + U_3 + \dots \quad (6)$$

and taking Figure 1 into account:

$$U = U_1(1 + h + h^2 + \dots) = \frac{U_1}{1 - h} \quad (7)$$

where  $h = r_{ga}^2 \exp(-\alpha s) \exp(j\varphi)$ . Equation (7) may be written as:

$$U = \frac{\exp(-\alpha s) t_{ag} t_{ga} U_0}{1 - \exp(-\alpha s) \exp(j\varphi) r_{ag}^2} \quad (8)$$

where  $U_0$  is the amplitude of the incident wave. Taking into account the relationship between the transmission and reflection coefficients [6]:

$$t_{ag} t_{ga} = 1 - r_{ag}^2 = 1 - r^2 \quad \text{and} \quad r = |r_{ag}| = |r_{ga}| \quad (9)$$

we arrive at the result that the transmittance  $T$  measured experimentally is:

$$T = \frac{|U|^2}{|U_0|^2} \quad (10)$$

and may be expressed as:

$$T = \frac{T_{max}}{1 + \left(\frac{2f}{\pi}\right)^2 \sin^2\left(\frac{2\pi}{\lambda} d \sqrt{n^2 - \sin^2\theta}\right)} \quad (11)$$

where:

$$T_{max} = \frac{(1 - r^2)^2 \exp(-\alpha s)}{[1 - r^2 \exp(-\alpha s)]^2} \quad (12)$$

and the quantity:

$$\mathcal{F} = \frac{\pi r \exp(-\alpha s)}{1 - r^2 \exp(-\alpha s)} \quad (13)$$

is a parameter known as *finese* [7].

From equation (11) it can be deduced that  $T$  oscillates as  $\theta$  increases about a mean line  $T_{med}$  which decreases with  $\theta$ . The value of  $T_{med}$  may be calculated using the equation:

$$T_{med} = \frac{1}{2} (T_M + T_m) \quad (14)$$

where  $T_M$  and  $T_m$  correspond to the transmittance values, for which  $\sin^2(\varphi/2)$  takes the values 0 and 1, respectively. Using equations (11), (12) and (14) and after carrying out a series of operations, we arrive at the equation:

$$T_{med} = \frac{T_{max}}{2 [1 + (2\mathcal{F}/\pi)^2]} \quad (15)$$

As can be seen  $T_{med}$  is a function of  $\alpha$ ,  $n$  and the product  $\alpha d$ . As we have pointed out, the transmittance oscillates as the angle of incidence increases and the frequency of these oscillations is greater at high values of  $d$ , as occurs in the case of the glass substrate of holographic plates.

In the particular case in which light polarized perpendicular to the plane of incidence is used, the factor  $r^2$  which appears in equations (12) and (13) is obtained as follows [6]:

$$r^2 = \left[ \frac{n \cos\theta' - \cos\theta}{n \cos\theta' + \cos\theta} \right]^2 \quad (16)$$

In order to analyze the absorption of the glass substrate of a holographic plate, we used the glass backing of a BB-640 holographic plate from Holographic Recording Technologies (Steinau, Germany) [8], measuring 2.5" x 2.5" and with a thickness of  $1.83 \pm 0.01$  mm. The refractive index of the glass plate is  $n = 1.527 \pm 0.002$  for a wavelength of 632.8 nm [9]. The emulsion of the plate was completely removed and the glass plate was immersed in a chromic mixture solution for 24 hours. The plate was then rinsed in running water and dried. The plane parallel glass plate was then mounted on a motorized rotation stage which was controlled electronically using a DC point-to-point motion controller connected to a personal computer using an IEEE-488 interface. The rotating device had a resolution of  $0.001^\circ$ . Figure 2 shows a schematic optical arrangement of the experimental setup. The plate was illuminated using a collimated beam from an He-Ne laser polarized perpendicular to the plane of incidence. The incident,  $I_i$ , and transmitted,  $I_t$ , light intensities were measured by means of an optical power meter. The transmittance  $T$  was calculated as the quotient  $I_t/I_i$  varying the angle of incidence  $\theta$  between  $0^\circ$  and  $35^\circ$  with intervals of  $0.1^\circ$ . Adjusting the equation for  $T_{med}$  to the experimental data for  $T$  by means of minimum squares, and since the values of  $T$  oscillate with a high frequency about the value of  $T_{med}$ , it is possible to obtain an acceptable value for the product  $\alpha d$ , where  $\alpha$  is the absorption coefficient of the glass and  $d$  its thickness, provided that a large number of values of the transmittance is taken. Taking into account that in our experiments the value of the incident intensity was  $I_i = 500 \pm 1 \mu\text{W}/\text{cm}^2$  and that the absolute errors of  $I_t$  and  $\theta$  were  $1 \mu\text{W}/\text{cm}^2$  and  $0.001^\circ$ , respectively, the values



obtained of  $\alpha d$  and  $\alpha$  with this adjustment are  $\alpha d = 0.027 \pm 0.001$  and  $\alpha = 0.0148 \pm 0.0006 \text{ mm}^{-1}$  for a wavelength of 632.8 nm, taking into account that for the glass substrate of the BB-640 plate,  $d = 1.83 \pm 0.01 \text{ mm}$ .

Figure 3 shows the experimental results obtained for the transmittance  $T$  of the glass backing on which the BB-640 emulsion is deposited. The continuous line in this figure corresponds to the theoretical curve for  $T_{med}$  obtained taking the value of  $\alpha d$  calculated prior to adjustment of the experimental data for the transmittance. As can be seen, the curve calculated for  $T_{med}$  actually corresponds to the line which is half way between the values of maximum and minimum transmittance. Adjustment of the equation corresponding to  $T$  without taking into account any type of interference but considering Fresnel's reflections at the two interfaces, expressed as  $(1-r^2)^2 \exp(-\alpha s)$  (see Equations (3) and (9)), gives a value of 0.020 for  $\alpha d$ , which is slightly lower than that obtained when multiple interferences are taken into account.

The agreement between theoretical and experimental results and the ease with which the experiment can be performed makes the technique proposed an attractive method to evaluate the absorption of the plane parallel glass substrate of a holographic plate. The results obtained are satisfactory, while the experimental setup and technique used are straightforward in terms of their practical application. The method may be applied to any plane parallel glass plate whose thickness is similar to that of holographic plates.

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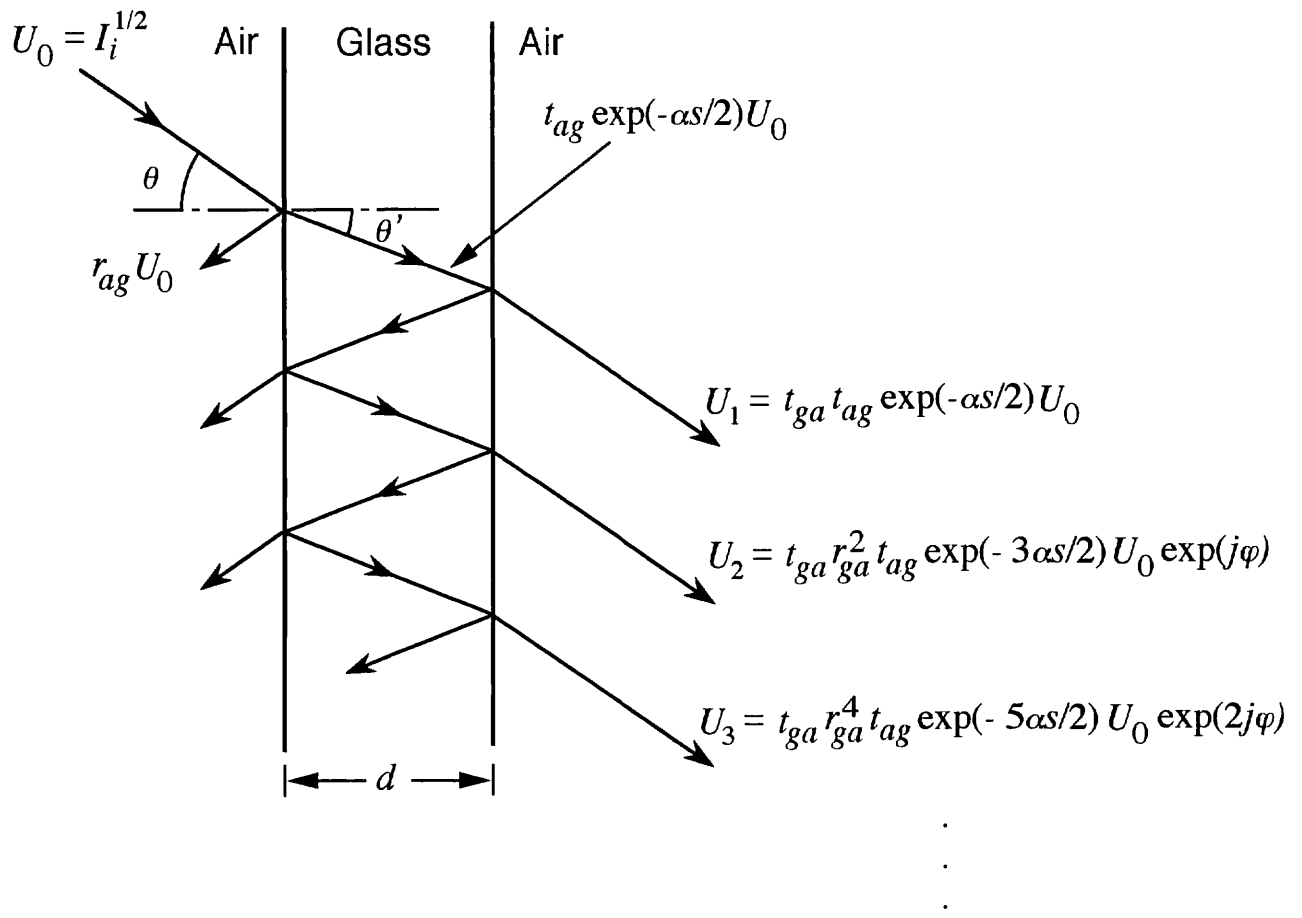
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## FIGURE CAPTIONS

Figure 1: Multiple reflections at a plane parallel glass plate illuminated by a coherent beam of light with a wavelength of  $\lambda$ .

Figure 2: Experimental arrangement used to measure the transmittance of the glass substrate of holographic plates.

Figure 3: Experimental values of the transmittance of the glass substrate of BB-640 holographic plates. The continuous line corresponds to the theoretical curve for  $T_{med}$  obtained by substituting the values of  $ad = 0.027 \pm 0.001$  and  $n = 1.527$  in equation (15).



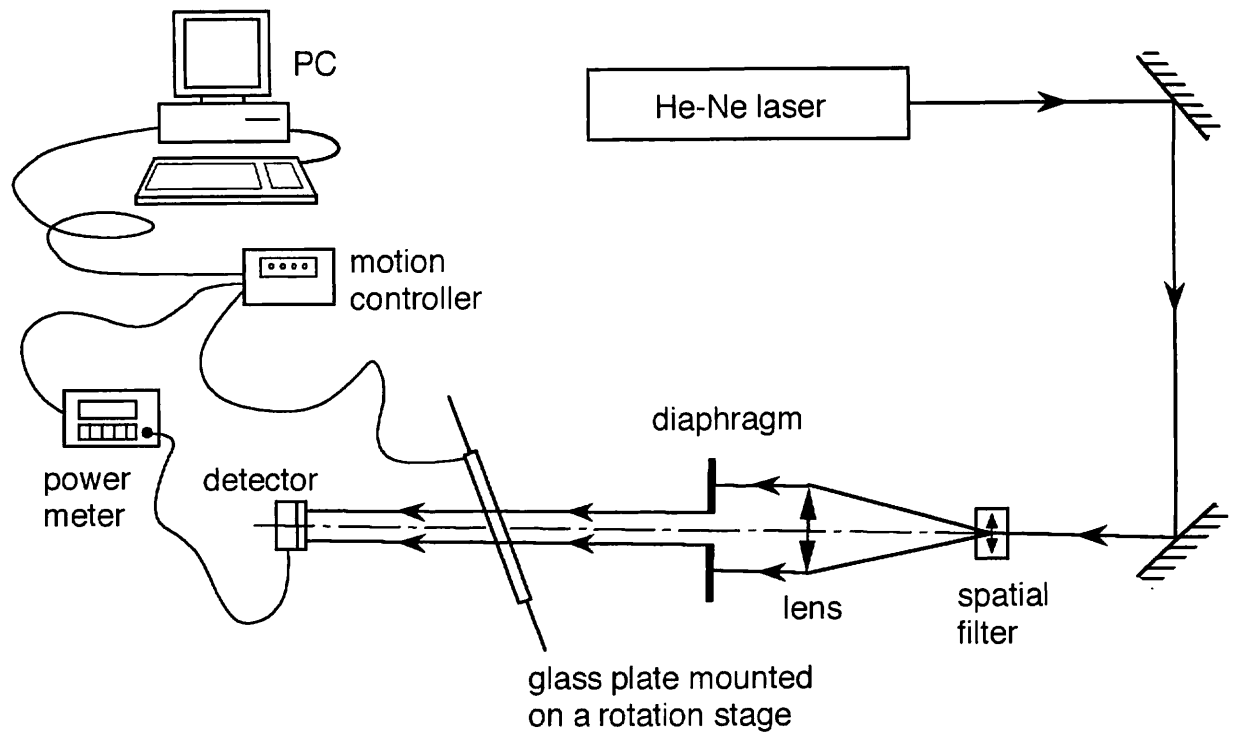


FIGURE 2

