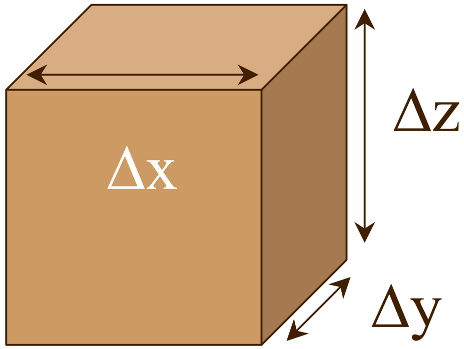
BALANCE DE ENERGÍA: TERMINOS A CONSIDERAR

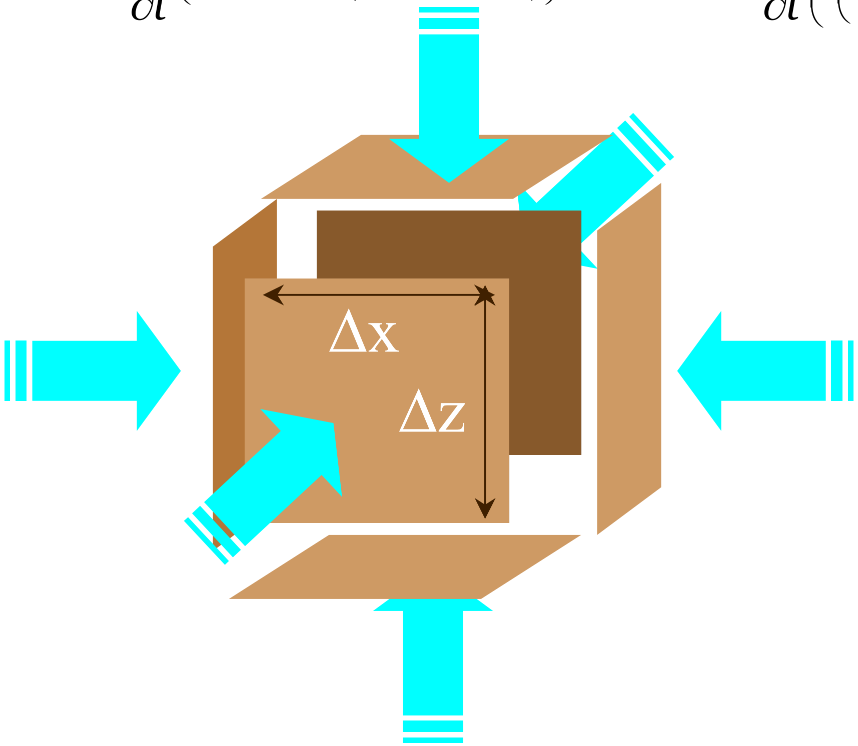


$$ACUMULACIÓN = (ENTRADA - SALIDA) + GENERACIÓN$$

$$\left\{\begin{matrix}\text{velocidad de}\\ \text{acumulación}\\ \text{de energía}\\ \text{cinética e interna}\end{matrix}\right\} = \left\{\begin{matrix}\text{velocidad de}\\ \text{entrada de energía}\\ \text{cinética e interna}\\ \text{por movimiento}\\ \text{convectivo}\end{matrix}\right\} - \left\{\begin{matrix}\text{velocidad de}\\ \text{salida de energía}\\ \text{cinética e interna}\\ \text{por movimiento}\\ \text{convectivo}\end{matrix}\right\} +$$

$$+\left\{\begin{matrix}\text{velocidad neta}\\ \text{de adición de calor}\\ \text{por conducción}\end{matrix}\right\} - \left\{\begin{matrix}\text{velocidad neta}\\ \text{de trabajo comunicado}\\ \text{por el sistema a los}\\ \text{alrededores}\end{matrix}\right\} + \left\{\begin{matrix}\text{velocidad neta}\\ \text{de trabajo comunicado}\\ \text{por los alrededores}\\ \text{sobre el sistema}\end{matrix}\right\}$$

$$\left\{\begin{array}{c}\text{velocidad de}\\ \text{acumulación}\\ \text{de energía}\\ \text{cinética e interna}\end{array}\right\} \approx \frac{\partial}{\partial t}(U+K) = \frac{\partial}{\partial t}\left(\Delta x \Delta y \Delta z\left(\rho\hat{U} + \rho\hat{K}\right)\right) = \Delta x \Delta y \Delta z \frac{\partial}{\partial t}\left(\left(\rho\hat{U} + \frac{1}{2}\rho v^2\right)\right)$$



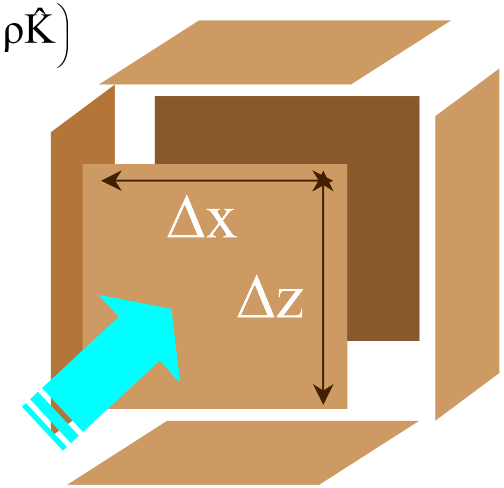
Velocidad neta de entrada de calor por conducción

$$dQ = \vec{n} \bullet \vec{q}\ dS$$

$$:= \Delta y \Delta z \left(q_x\big|_x - q_x\big|_{x+\Delta x}\right) + \Delta x \Delta z \left(q_y\big|_y - q_y\big|_{y+\Delta y}\right) + $$
$$+ \Delta x \Delta y \left(q_z\big|_z - q_z\big|_{z+\Delta z}\right)$$

BALANCE DE ENERGÍA: ENTRADA-SALIDA POR CONVECCIÓN*

$$\left\{\begin{array}{l}\text{velocidad de entrada o}\\ \text{salida de energía cinética e}\\ \text{interna por convección por}\\ \text{una cara del elemento}\end{array}\right\} = \text{Caudal entra} \bullet \text{Concentración} = Q \bullet \left(\rho\hat{U} + \rho\hat{K}\right)$$



EJEMPLO: Cara situada en posición y de dimensiones Δx -Δz

$$dQ = \vec{n} \bullet \vec{v}\, dS \Rightarrow Q = \vec{n} \bullet \vec{v}\, \Delta x \Delta z = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \Delta x \Delta z = v_y \Delta x \Delta z$$

$$\left\{\begin{array}{l}\text{velocidad de entrada de}\\ \text{energía cinética e interna}\\ \text{por movimiento convección}\\ \text{por la cara considerada}\end{array}\right\} = v_y \Delta x \Delta z \bullet \left(\rho\hat{U} + \frac{1}{2}\rho v^2\right)\Big|_y$$

$$\begin{array}{l}\text{velocidad de}\\ \text{entrada de energía}\\ \text{cinética e interna}\\ \text{por movimiento}\\ \text{convección}\end{array} = \begin{array}{l} \Delta y \Delta z \left(v_x(\rho\hat{U} + \frac{1}{2}\rho v^2)\Big|_x - v_x(\rho\hat{U} + \frac{1}{2}\rho v^2)\Big|_{x+\Delta x} \right) + \\ + \Delta x \Delta z \left(v_y(\rho\hat{U} + \frac{1}{2}\rho v^2)\Big|_y - v_y(\rho\hat{U} + \frac{1}{2}\rho v^2)\Big|_{y+\Delta y} \right) + \\ + \Delta x \Delta y \left(v_z(\rho\hat{U} + \frac{1}{2}\rho v^2)\Big|_z - \rho v_z(\rho\hat{U} + \frac{1}{2}\rho v^2)\Big|_{z+\Delta z} \right) \end{array}$$

- CONSIDERACIONES PREVIAS:

$$dW = \vec{F} \bullet \vec{\mathrm{dr}} \Rightarrow \frac{dW}{dt} = \vec{F} \bullet \frac{\vec{\mathrm{dr}}}{\mathrm{dt}} = \vec{F} \bullet \vec{\mathrm{v}}$$

- FUERZAS QUE ACTÚAN:

  - La fuerza de la gravedad (de los alrededores sobre el sistema):

$$\vec{F} = \rho\,\vec{g} = \rho\begin{pmatrix} g_x & g_y & g_z \end{pmatrix}$$

  - Esfuerzos viscosos (del sistema hacia los alrededores): $d\vec{F} = \vec{n} \bullet \vec{\vec{\pi}}\ \mathrm{dS}$

$$\vec{\vec{\pi}} = \begin{pmatrix} \pi_{xx} & \pi_{xy} & \pi_{xz} \\ \pi_{yx} & \pi_{yy} & \pi_{yz} \\ \pi_{zx} & \pi_{zy} & \pi_{zz} \end{pmatrix} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} + \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix}$$

EJEMPLO: Cara situada en posición y + Δ y de dimensiones Δx -Δz $\vec{n} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$

$$\vec{F} = \vec{n} \bullet \vec{\vec{\pi}}\ \Delta\mathrm{S} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \tau_{xx} + p & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} + p & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} + p \end{pmatrix} \Delta x \Delta z = \begin{pmatrix} \tau_{yx} \Delta x \Delta z \\ \left(\tau_{yy} + p\right) \Delta x \Delta z \\ \tau_{yz} \Delta x \Delta z \end{pmatrix}$$

VELOCIDAD DE TRABAJO COMUNICADO POR LOS ALREDEDORES SOBRE SISTEMA

- La fuerza de la gravedad:

$$\frac{dW}{dt} = \rho\, \vec{g} \bullet \vec{v} = \rho \begin{pmatrix} g_x & g_y & g_z \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \rho \left(g_x v_x + g_y v_y + g_z v_z \right)$$

- Esfuerzos viscosos:

EJEMPLO: Cara situada en posición y + Δ y de dimensiones Δx -Δz $\vec{n} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$

$$\left.\frac{dW}{dt}\right|_y = \begin{pmatrix} \tau_{yx} \Delta x \Delta z & \left(\tau_{yy} + p\right) \Delta x \Delta z & \tau_{yz} \Delta x \Delta z \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = \left[\tau_{yx} \Delta x \Delta z v_x \quad \left(\tau_{yy} + p\right) \Delta x \Delta z v_y \quad \tau_{yz} \Delta x \Delta z v_z \right]_{y+\Delta y}$$

La contribución total de todas las caras:

$$\frac{dW}{dt} = - \left[\begin{aligned} &\Delta y \Delta z \left(\left(\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z\right)\Big|_{x+\Delta x} - \left(\tau_{xx} v_x + \tau_{xy} v_y + \tau_{xz} v_z\right)\Big|_{x} \right) + \\ &+ \Delta x \Delta z \left(\left(\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z\right)\Big|_{y+\Delta y} - \left(\tau_{yx} v_x + \tau_{yy} v_y + \tau_{yz} v_z\right)\Big|_{y} \right) + \\ &+ \Delta x \Delta y \left(\left(\tau_{zx} v_x + \tau_{zy} v_y + \tau_{zz} v_z\right)\Big|_{z+\Delta z} - \left(\tau_{zx} v_x + \tau_{zy} v_y + \tau_{zz} v_z\right)\Big|_{z} \right) \end{aligned} \right]$$

BALANCE DE ENERGÍA

$$ACUMULACIÓN = (ENTRADA - SALIDA) + GENERACIÓN$$

Dividiendo por el volumen del elemento considerado($\Delta x \Delta y \Delta z$) y haciendo $\Delta x, \Delta y, \Delta z \rightarrow 0$

$\rho(\vec{v} \cdot \vec{g})$

$-\left[\vec{\nabla} \cdot \left((\frac{1}{2}\rho v^2 + \rho \hat{U})\vec{v} \right)\right]$

$$\frac{\partial}{\partial t}(\rho \hat{U} + \frac{1}{2}\rho v^2) = -\left(\frac{\partial}{\partial x} v_x(\rho \hat{U} + \frac{1}{2}\rho v^2) + \frac{\partial}{\partial y} v_y(\rho \hat{U} + \frac{1}{2}\rho v^2) + \frac{\partial}{\partial z} v_z(\rho \hat{U} + \frac{1}{2}\rho v^2) \right) -$$

$-(\vec{\nabla} \cdot p\vec{v})$

$-\vec{\nabla} \cdot \vec{q}$

$$-\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right) + \rho(v_x g_x + v_y g_y + v_z g_z) - \left(\frac{\partial}{\partial x} p v_x + \frac{\partial}{\partial y} p v_y + \frac{\partial}{\partial z} p v_z \right) -$$

$$-\left(\frac{\partial}{\partial x}(\tau_{xx}v_x + \tau_{xy}v_y + \tau_{xz}v_z) + \frac{\partial}{\partial y}(\tau_{yx}v_x + \tau_{yy}v_y + \tau_{yz}v_z) + \frac{\partial}{\partial z}(\tau_{zx}v_x + \tau_{zy}v_y + \tau_{zz}v_z) \right)$$

$-\vec{\nabla} \bullet [\vec{\vec{\tau}} \bullet \vec{v}]$

Ecuación de continuidad

$$\rho\left[\frac{\partial}{\partial t}(\hat{U} + \frac{1}{2}v^2) + (\vec{v} \cdot \vec{\nabla}(\hat{U} + \frac{1}{2}\rho v^2)) \right] + (\hat{U} + \frac{1}{2}v^2)\left[\frac{\partial \rho}{\partial t} + (\vec{\nabla} \cdot \rho\vec{v}) \right] = -(\vec{\nabla} \cdot \vec{q}) + \rho(\vec{v} \cdot \vec{g}) - (\vec{\nabla} \cdot p\vec{v}) - (\vec{\nabla} \bullet [\vec{\vec{\tau}} \bullet \vec{v}])$$

$$\rho \frac{D}{Dt}(\hat{U} + \frac{1}{2}v^2) = -(\vec{\nabla} \cdot \vec{q}) + \rho(\vec{v} \cdot \vec{g}) - (\vec{\nabla} \cdot p\vec{v}) - (\vec{\nabla} \bullet [\vec{\vec{\tau}} \bullet \vec{v}])$$

BALANCE DE ENERGÍA

$$\rho \frac{D}{Dt}\left(\hat{U} + \frac{1}{2}v^2\right) = -(\vec{\nabla}\cdot\vec{q}) + \rho(\vec{v}\cdot\vec{g}) - (\vec{\nabla}\cdot p\vec{v}) - \left(\vec{\nabla}\bullet\left[\vec{\vec{\tau}}\bullet\vec{v}\right]\right)$$

Ecuación energía mecánica

$$\rho \frac{D}{Dt}\left(\frac{1}{2}v^2\right) = p(\vec{\nabla}\cdot\vec{v}) - (\vec{\nabla}\cdot p\vec{v}) + \rho(\vec{v}\cdot\vec{g}) - \left(\vec{\nabla}\cdot\left[\vec{\vec{\tau}}\cdot\vec{v}\right]\right) + (\vec{\vec{\tau}}:\vec{\nabla}\vec{v})$$

$$\rho \frac{D\hat{U}}{Dt} = -(\vec{\nabla}\bullet\vec{q}) \; -(\vec{\vec{\tau}}:\vec{\nabla}\vec{v}) - p\left(\vec{\nabla}\bullet\vec{v}\right)$$

$$d\hat{U} = \left(\frac{\partial\hat{U}}{\partial\hat{V}}\right)_T d\hat{V} + \left(\frac{\partial\hat{U}}{\partial T}\right)_{\hat{V}} dT = \left[-p + T\left(\frac{dp}{dT}\right)_{\hat{V}}\right] d\hat{V} + \hat{C}_V dT$$

$$\rho \frac{D\hat{U}}{Dt} = \left[-p + T\left(\frac{dp}{dT}\right)_{\hat{V}}\right]\rho\frac{D\hat{V}}{Dt} + \rho\hat{C}_V\frac{DT}{Dt}$$

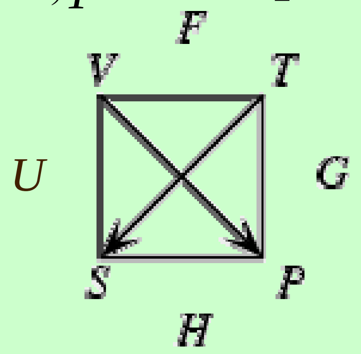
$$\rho\frac{D\hat{V}}{Dt} = \rho\frac{D}{Dt}\left(\frac{1}{\rho}\right) = -\frac{1}{\rho}\frac{D\rho}{Dt} = (\vec{\nabla}\cdot\vec{v})$$

$$\rho\hat{C}_V\frac{DT}{Dt} = -(\vec{\nabla}\cdot\vec{q}) - T\left(\frac{\partial p}{\partial T}\right)_{\hat{V}}(\vec{\nabla}\cdot\vec{v}) - (\vec{\vec{\tau}}:\vec{\nabla}\vec{v})$$

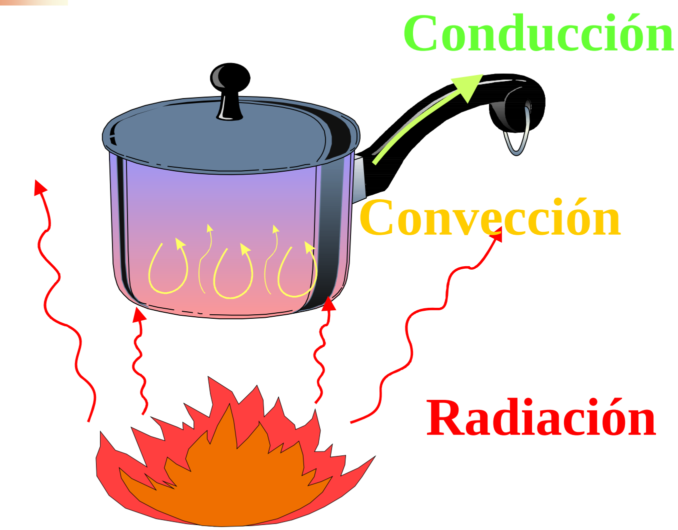
TERMODINÁMICA

$$d\hat{U} = TdS - pdV$$

$$\left(\frac{\partial\hat{U}}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - p$$



$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

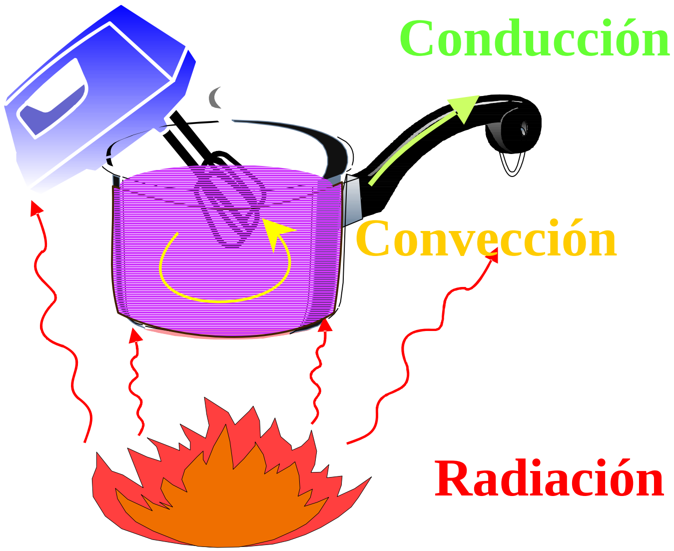


Convección LIBRE

1. Las características de flujo están determinadas por el efecto de flotación del fluido caliente.

2. Los perfiles de velocidad y temperatura están íntimamente relacionados.

3. El número de Nusselt depende de los números de Grashof y Prandtl.



Convección FORZADA

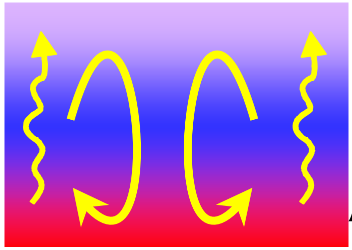
1. Las características de flujo estás determinadas fundamentalmente por una fuerza externa.

2. Primeramente se hallan los perfiles de velocidad, que se utilizan después para calcular los perfiles de temperatura (procedimiento general para fluidos cuyas propiedades físicas son constantes).

3. El número de Nusselt depende de los números de Reynolds y Prandtl.

$$\rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} p - \left[\vec{\nabla} \cdot \vec{\vec{\tau}}\right] + \rho \vec{g}$$

⇒ **Convección forzada**



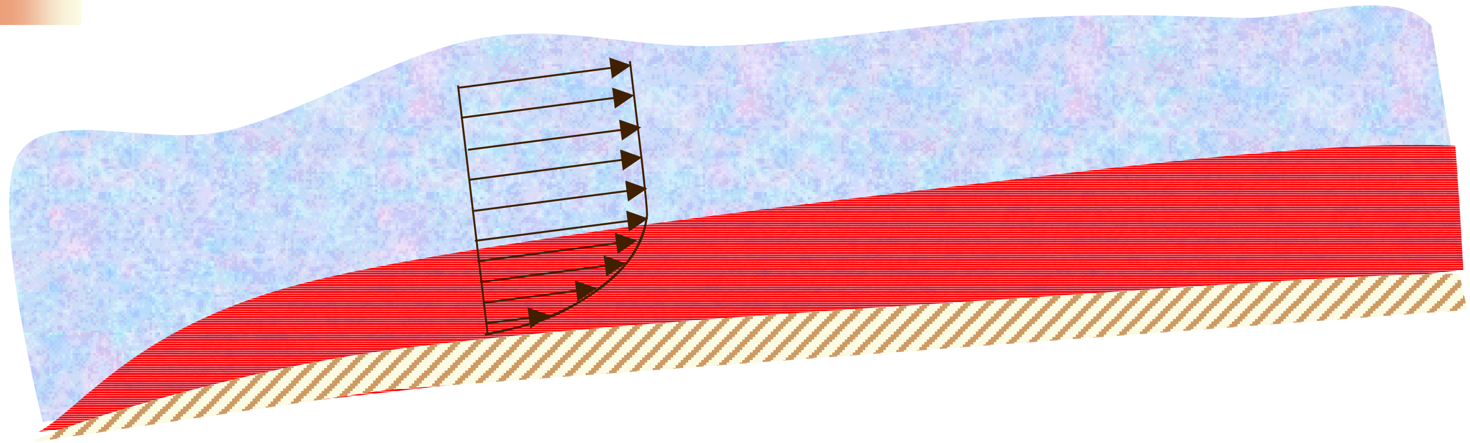
Convección natural: $\vec{\nabla} p = \bar{\rho} \vec{g}$

$$\bar{\beta} = \frac{1}{\bar{V}} \left(\frac{\partial \bar{V}}{\partial T} \right)$$

$$\rho = \bar{\rho} + \left(\frac{\partial \rho}{\partial T} \right) \left(T - \bar{T} \right) = \bar{\rho} + \left(\frac{\partial \left(1/\bar{V} \right)}{\partial T} \right) \left(T - \bar{T} \right) = \bar{\rho} - \bar{\rho} \left(\frac{1}{\bar{V}} \left(\frac{\partial \left(\bar{V} \right)}{\partial T} \right) \right) \left(T - \bar{T} \right) = \bar{\rho} - \bar{\rho} \bar{\beta} \left(T - \bar{T} \right)$$

$$\rho \frac{D\vec{v}}{Dt} = -\rho \vec{g} - \left[\vec{\nabla} \cdot \vec{\vec{\tau}}\right] + \bar{\rho} \vec{g} + \bar{\rho} \bar{\beta} g \left(T - \bar{T} \right) = -\left[\vec{\nabla} \cdot \vec{\vec{\tau}}\right] + \bar{\rho} \bar{\beta} g \left(T - \bar{T} \right)$$

Ec.	Forma especial	Ecuación en forma simbólica		Sistema coordenado: Rect.	Cil.	Esf.
Cont.	—	$(\nabla \cdot v) = 0$	(A)	Tabla 2.1 Ec. A	Tabla 2.1 Ec. B	Tabla 2.1 Ec. C
Movimiento	Convección forzada	$\rho \frac{Dv}{Dt} = -\nabla p + \mu \nabla^2 v + \rho g$	(B)	Tabla 2.2 b Ecs. D, E, F	Tabla 2.3 b Ecs. D, E, F	Tabla 2.4 b Ecs. D, E, F
Movimiento	Convección libre	$\rho \frac{Dv}{Dt} = \mu \nabla^2 v - \bar{\rho} \bar{\beta} g (T - \bar{T})$	(C)	—	—	—
Energía	En función de $\hat{U}$	$\rho \frac{D\hat{U}}{Dt} = k \nabla^2 T + \mu \Phi_v$	(D)	—	—	—
Energía	En función de $\hat{C}_v$ y T	$\rho \hat{C}_v \frac{DT}{Dt} = k \nabla^2 T + \mu \Phi_v$	(E)	Tabla 8.2 Ec. A	Tabla 8.2 Ec. B	Tabla 8.2 Ec. C



Balances materia $\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$

Balance cantidad de movimiento $v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \upsilon \frac{\partial^2 v_x}{\partial y^2}$ $\frac{v_x}{v_\infty} = \frac{v_y}{v_\infty} = 0$ $\frac{v_x}{v_\infty} = 1$

Balance energía $v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$

$y = 0, T = Ts$

$y \to \infty, T = T_\infty$

$y = 0, \frac{T - Ts}{T_\infty - Ts} = 0$

$y \to \infty, \frac{T - Ts}{T_\infty - Ts} = 1$

SOLUCIÓN AL ANÁLISIS EXACTO DE LA CAPA LIMITE

$$\eta = \frac{y}{2}\sqrt{\frac{v_\infty}{\upsilon x}} = \frac{y}{2x}\sqrt{\frac{x v_\infty}{\upsilon}} = \frac{y}{2x}\sqrt{Re_x} \qquad f' = 2\frac{v_x}{v_\infty} = 2\frac{T - T_s}{T_\infty - T_s}$$

$$\left.\frac{df'}{d\eta}\right|_{y=0} = f''(0) = \left.\frac{d\left[2(v_x / v_\infty)\right]}{d\left[(y/2x)\sqrt{Re_x}\right]}\right|_{y=0} = \left.\frac{d\left[2\left[(T - T_s)/(T_\infty - T_s)\right]\right]}{d\left[(y/2x)\sqrt{Re_x}\right]}\right|_{y=0} = 1.328$$

$$\left.\frac{d\left[\left[(T - T_s)\right]\right]}{d(y)}\right|_{y=0} = \left.\frac{d[T]}{d(y)}\right|_{y=0} = \frac{1.328}{4x}\sqrt{Re_x}\left(T_\infty - T_s\right) = \frac{0.332}{x}\sqrt{Re_x}\left(T_\infty - T_s\right)$$

$$q_y = \frac{Q_y}{A} = h_x(T_s - T_\infty) = -k\left.\frac{\partial T}{\partial y}\right|_{y=0} \quad \Rightarrow \quad h_x = -\frac{k}{(T_s - T_\infty)}\left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{0.332k}{x}Re_x^{1/2}$$

$$\frac{h_x x}{k} = Nu_x = 0.332\, Re_x^{1/2}$$

Pr=1

SOLUCIÓN AL ANÁLISIS EXACTO DE LA CAPA LIMITE

$$\frac{\delta}{\delta_t} = \Pr^{1/3}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left[\frac{0.332}{x} \mathrm{Re}_x^{1/2} \Pr^{1/3}\right]$$

$$h_x = 0.332 \frac{k}{x} \mathrm{Re}_x^{1/2} \Pr^{1/3}$$

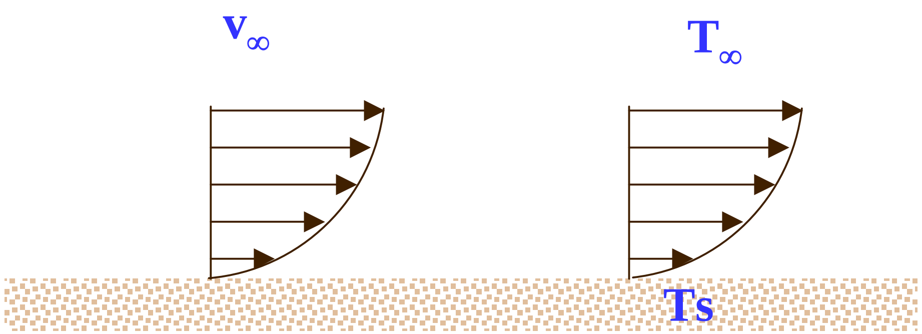
$$\frac{h_x x}{k} = \mathrm{Nu}_x = 0.332 \mathrm{Re}_x^{1/2} \Pr^{1/3}$$

$$Q_y = h_L A(T_s - T_\infty) = \int_A h_x (T_s - T_\infty) dA$$

$$h_L (WL)(T_s - T_\infty) = 0.332 kW \Pr^{1/3} (T_s - T_\infty) \int_0^L \frac{\mathrm{Re}_x^{1/2}}{x} dx$$

$$h_L L = 0.332 k \Pr^{1/3} \left(\frac{v_\infty \rho}{\mu}\right)^{1/2} \int_0^L x^{-1/2} dx = 0.664 k \Pr^{1/3} \left(\frac{v_\infty \rho}{\mu}\right)^{1/2} L^{1/2} = 0.664 k \Pr^{1/3} \mathrm{Re}_L^{1/2}$$

$$\mathrm{Nu}_L = \frac{h_L L}{k} = 0.664 \Pr^{1/3} \mathrm{Re}_L^{1/2}$$



$$\frac{d}{dy}\left(\frac{v_x}{v_\infty}\right)\bigg|_{y=0} = \frac{d}{dy}\left(\frac{T - T_s}{T_\infty - T_s}\right)\bigg|_{y=0}$$

$$Pr = 1 \Rightarrow \mu Cp = k \qquad \mu Cp\,\frac{d}{dy}\left(\frac{v_x}{v_\infty}\right)\bigg|_{y=0} = k\,\frac{d}{dy}\left(\frac{T - T_s}{T_\infty - T_s}\right)\bigg|_{y=0}$$

$$\frac{\mu Cp}{v_\infty}\frac{d}{dy}(v_x)\Big|_{y=0} = \frac{k}{(T_\infty - T_s)}\frac{d}{dy}(T - T_s)\Big|_{y=0}$$

$$-\frac{d(T - T_s)}{dy}\bigg|_{y=0} = \frac{h}{k_f}(T_s - T_\infty)$$

$$\frac{\mu Cp}{v_\infty}\frac{d}{dy}(v_x)\Big|_{y=0} = \frac{k_f}{(T_\infty - T_s)}\frac{h}{k_f}(T_\infty - T_s) \;\Rightarrow\; \frac{\mu Cp}{v_\infty}\frac{d}{dy}(v_x)\Big|_{y=0} = h$$

$$\frac{\mu Cp}{v_\infty}\frac{d}{dy}\left(v_x\right)\Big|_{y=0} = h$$

$$C_D = \frac{\tau_0}{\rho v_\infty^2/2} = \frac{2\mu}{\rho v_\infty^2}\frac{dv_x}{dy}\Big|_{y=0}$$

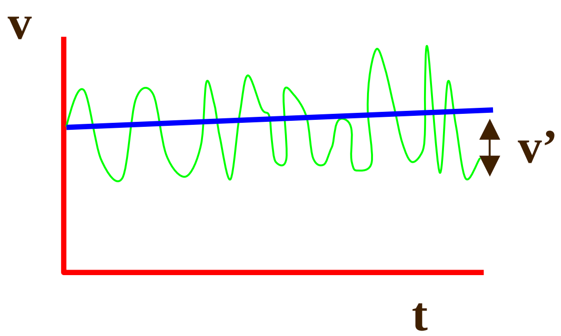
$$\frac{Cp}{v_\infty}\tau_0 = h = \frac{Cp}{v_\infty}\left(\frac{\rho v_\infty^2}{2}C_D\right) = Cp\left(\frac{\rho v_\infty}{2}C_D\right)$$

$$\frac{h}{Cp\,\rho\,v_\infty} = \frac{C_D}{2}$$

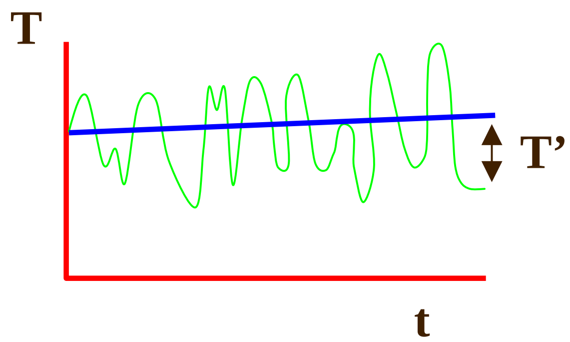
Pr=1

$$\frac{h_x x}{k} = Nu_x = 0.332\,Re_x^{1/2}\,Pr^{1/3} \qquad C_{Dx} = \frac{0.664}{Re_x^{1/2}}$$

$$\frac{Nu_x}{\sqrt{Re}\,Pr^{1/3}} = \frac{C_D}{2}\sqrt{Re} \Rightarrow \frac{Nu_x}{Re\,Pr^{1/3}} = \frac{C_D}{2} = St\,Pr^{2/3}$$



$$v = \bar{v} + v'$$



$$T = \bar{\bar{T}} + T'$$

$$\rho \hat{C}_p \frac{\partial T}{\partial t} = -\left(\frac{\partial}{\partial x} \rho \hat{C}_p v_x T + \frac{\partial}{\partial y} \rho \hat{C}_p v_y T + \frac{\partial}{\partial z} \rho \hat{C}_p v_z T \right) + k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial x^2} \right) +$$
$$+ \mu \left[2 \left(\frac{\partial v_x}{\partial x} \right)^2 + \left(\frac{\partial v_x}{\partial y} \right)^2 + 2 \left(\frac{\partial v_x}{\partial y} \frac{\partial v_y}{\partial x} \right) + \dots \right]$$

$$\frac{\partial}{\partial x} \rho \hat{C}_p v_x T = \frac{\partial}{\partial x} \rho \hat{C}_p (v_x + v_x')(\bar{T} + T') = \frac{\partial}{\partial x} \rho \hat{C}_p (v_x \bar{T} + v_x' \bar{T} + v_x' \bar{T} + v_x' T')$$

Promedio con el tiempo:

$$\frac{\partial}{\partial x} \overline{\rho \hat{C}_p v_x T} = \frac{\partial}{\partial x} \rho \hat{C}_p \left(\overline{\bar{v}_x \bar{T}} + \overline{v_x' \bar{T}} + \overline{\bar{v}_x T'} + \overline{v_x' T'} \right) = \frac{\partial}{\partial x} \rho \hat{C}_p \left(\bar{v}_x \bar{T} + 0 + 0 + \overline{v_x' T'} \right)$$

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial x^2}\right)=k\left(\frac{\partial^2\left(\bar{T}+T'\right)}{\partial x^2}+\frac{\partial^2\left(\bar{T}+T'\right)}{\partial y^2}+\frac{\partial^2\left(\bar{T}+T'\right)}{\partial x^2}\right)$$

Promedio con el tiempo:

$$k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial x^2}\right)=k\left(\frac{\partial^2\left(\bar{T}\right)}{\partial x^2}+\frac{\partial^2\left(\bar{T}\right)}{\partial y^2}+\frac{\partial^2\left(\bar{T}\right)}{\partial x^2}\right)$$

$$\Phi_v=2\left[\left(\frac{\partial v_x}{\partial x}\right)^2+\left(\frac{\partial v_y}{\partial y}\right)^2+\left(\frac{\partial v_z}{\partial z}\right)^2\right]+\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)^2+\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial z}\right)^2+\left(\frac{\partial v_x}{\partial z}+\frac{\partial v_z}{\partial x}\right)^2-\frac{2}{3}\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)$$

$$\left[\overline{\left(\frac{\partial v_x}{\partial x}\right)^2}\right]=\left[\overline{\left(\frac{\partial\left(\bar{v}_x+v_x'\right)}{\partial x}\right)^2}\right]=\left(\frac{\partial v_x}{\partial x}\right)^2+\overline{\left(\frac{\partial v_x'}{\partial x}\right)^2}+0$$

$$\overline{\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)^2}=\overline{\left(\frac{\partial\left(\bar{v}_y+v_y'\right)}{\partial x}+\frac{\partial\left(\bar{v}_x+v_x'\right)}{\partial y}\right)^2}=\left(\overline{\left(\frac{\partial\left(\bar{v}_y+v_y'\right)}{\partial x}\right)^2+\left(\frac{\partial\left(\bar{v}_x+v_x'\right)}{\partial y}\right)^2}+2\overline{\left(\frac{\partial\left(\bar{v}_x+v_x'\right)}{\partial y}\right)\left(\frac{\partial\left(\bar{v}_y+v_y'\right)}{\partial x}\right)}\right)$$

$$\overline{\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)^2}=\left(\overline{\left(\frac{\partial(\bar{v}_y ')}{\partial x}\right)^2}+\overline{\left(\frac{\partial(v_y ')}{\partial x}\right)^2}+\overline{\left(\frac{\partial(\bar{v}_x)}{\partial y}\right)^2}+\overline{\left(\frac{\partial(v_x ')}{\partial y}\right)^2}+2\overline{\left(\frac{\partial(\bar{v}_x\bar{v}_y+\bar{v}_x v_y '+v_x '\bar{v}_y+v_x ' v_y ')}{\partial y}\right)}\right)$$

$$\overline{\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)^2}=\left(\overline{\left(\frac{\partial(\bar{v}_y ')}{\partial x}\right)^2}+\overline{\left(\frac{\partial(v_y ')}{\partial x}\right)^2}+\overline{\left(\frac{\partial(\bar{v}_x)}{\partial y}\right)^2}+\overline{\left(\frac{\partial(v_x ')}{\partial y}\right)^2}+2\left(\frac{\partial(\overline{\bar{v}_x\bar{v}_y}+0+0+\overline{v_x ' v_y '})}{\partial y}\right)\right)$$

$$\overline{\left(\frac{\partial v_y}{\partial x}+\frac{\partial v_x}{\partial y}\right)^2}=\left(\frac{\partial \overline{v_y}}{\partial x}+\frac{\partial \overline{v_x}}{\partial y}\right)^2+\overline{\left(\frac{\partial v_y '}{\partial x}+\frac{\partial v_x '}{\partial y}\right)^2}$$

$$\overline{\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)}=\overline{\left(\frac{\partial(v_x+v_x ')}{\partial x}+\frac{\partial(v_y+v_y ')}{\partial y}+\frac{\partial(v_z+v_z ')}{\partial z}\right)}=\left(\frac{\partial(\overline{v_x})}{\partial x}+\frac{\partial(\overline{v_y})}{\partial y}+\frac{\partial(\overline{v_z})}{\partial z}\right)$$

$$\Phi_v=\Phi^l{}_v+\Phi^t{}_v$$

LA ECUACIÓN DE ENERGÍA EN RÉGIMEN TURBULENTO

$$\rho\hat{C}_p\frac{\partial\overline{T}}{\partial t}=-\left(\frac{\partial}{\partial x}\rho\hat{C}_p\overline{v}_x\overline{T}+\frac{\partial}{\partial y}\rho\hat{C}_p\overline{v}_y\overline{T}+\frac{\partial}{\partial z}\rho\hat{C}_p\overline{v}_z\overline{T}\right)-\left(\frac{\partial}{\partial x}\rho\hat{C}_p\overline{v_x'T'}+\frac{\partial}{\partial y}\rho\hat{C}_p\overline{v_y'T'}+\frac{\partial}{\partial z}\rho\hat{C}_p\overline{v_z'T'}\right)+$$

$$-\left(\frac{\partial q^{(l)}{}_x}{\partial x}+\frac{\partial q^{(l)}{}_y}{\partial y}+\frac{\partial q^{(l)}{}_z}{\partial z}\right)+\mu\left(\Phi_v^{(l)}+\Phi_v^{(t)}\right)$$

$$\overline{q}_x^{(t)}=\rho\hat{C}_p\overline{v_x'T'};\quad \overline{q}_y^{(t)}=\rho\hat{C}_p\overline{v_y'T'};\quad \overline{q}_z^{(t)}=\rho\hat{C}_p\overline{v_z'T'}$$

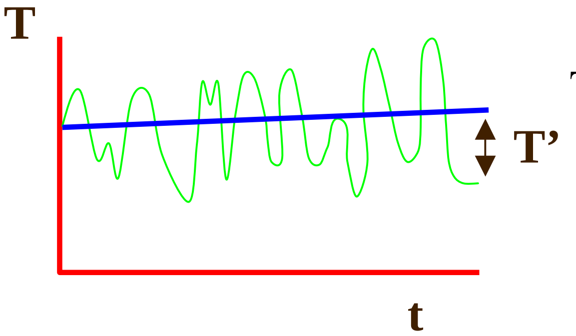
$$\rho\hat{C}_p\frac{\partial\overline{T}}{\partial t}=-\left(\frac{\partial}{\partial x}\rho\hat{C}_p\overline{v}_x\overline{T}+\frac{\partial}{\partial y}\rho\hat{C}_p\overline{v}_y\overline{T}+\frac{\partial}{\partial z}\rho\hat{C}_p\overline{v}_z\overline{T}\right)-\left(\frac{\partial q^{(l)}{}_x}{\partial x}+\frac{\partial q^{(l)}{}_y}{\partial y}+\frac{\partial q^{(l)}{}_z}{\partial z}\right)+$$

$$-\left(\frac{\partial q^{(t)}{}_x}{\partial x}+\frac{\partial q^{tl)}{}_y}{\partial y}+\frac{\partial q^{tl)}{}_z}{\partial z}\right)+\mu\left(\Phi_v^{(l)}+\Phi_v^{(t)}\right)$$

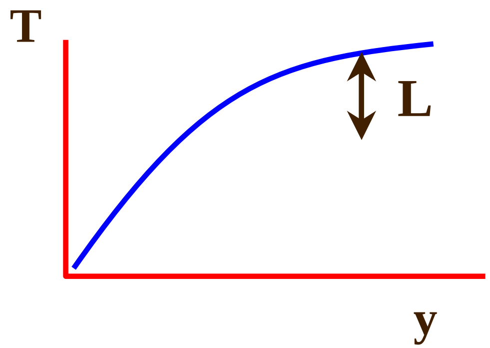
$$\rho\hat{C}_p\frac{D\overline{T}}{Dt}=-(\vec{\nabla}\cdot\vec{\overline{q}}^{(l)})-(\vec{\nabla}\cdot\vec{\overline{q}}^{(t)})+\mu\Phi_v^{(l)}+\mu\Phi_v^{(t)}$$

$$(\vec{\nabla}\cdot\vec{\overline{v}})=0 \qquad \rho\frac{D\vec{\overline{v}}}{Dt}=-\vec{\nabla}\overline{p}-\left[\vec{\nabla}\cdot\vec{\overline{\tau}}^{(l)}\right]-\left[\vec{\nabla}\cdot\vec{\overline{\tau}}^{(t)}\right]+\rho\vec{g}$$

DISTRIBUCIÓN DE TEMPERATURA A PARTIR DE LA TEORÍA DE LONGITUD DE MEZCLA



$$T = \bar{\bar{T}} + T'$$



$$T\big|_{y\pm L} = +T\big|_{y} \pm L\left.\frac{d\bar{T}}{dy}\right|_{y}$$

$$T\big|_{y\pm L} - T\big|_{y} = \pm L\left.\frac{d\bar{T}}{dy}\right|_{y} \quad \Rightarrow \quad T' = \pm L\frac{d\bar{T}}{dy}$$

$$q_y^{(t)} = \left.\frac{Q_y}{A}\right|_{turb} = \rho\hat{C}_p\overline{(v_y'T')} = \rho\hat{C}_p\overline{v_y'L\frac{d\bar{T}}{dy}} = \rho\hat{C}_p\,\overline{v_y'L}\,\frac{d\bar{T}}{dy}$$

$$\overline{v_y'L} \rightarrow \alpha^{(t)}$$

$$Pr_{turb} = \frac{\upsilon^{(t)}}{\alpha^{(t)}} = \frac{L^2\left|d\bar{v}_x/dy\right|}{\overline{\left|Lv_y'\right|}} = \frac{L^2\left|d\bar{v}_x/dy\right|}{L^2\left|d\bar{v}_x/dy\right|} = 1$$

DISTRIBUCIÓN DE TEMPERATURA A PARTIR DE LA TEORÍA DE LONGITUD DE MEZCLA

$$q_y = \left.\frac{Q_y}{A}\right| = -\rho \hat{C}_p (\alpha + \alpha^{(t)}) \frac{d\overline{T}}{dy}$$

SUBCAPA LAMINAR

$$-\tau = \rho \upsilon \frac{dv_x}{dy} = \tau_0 \qquad \int_0^{v_x|_\xi} dv_x = \frac{\tau_0}{\rho \upsilon} \int_0^\xi dy \qquad v_x|_\xi = \frac{\tau_0 \xi}{\rho \upsilon}$$

$$\frac{\rho \upsilon v_x|_\xi}{\tau_0} = \frac{\rho \hat{C}_p \alpha}{q_y} (T_s - T_\xi)$$

$$q_y = \frac{Q_y}{A} = -\rho \hat{C}_p \alpha \frac{dT}{dy} \qquad \int_{T_s}^{T_\xi} dT = -\frac{q_y}{\rho \hat{C}_p \alpha} \int_0^\xi dy \qquad T_s - T_\xi = \frac{q_y \xi}{\rho \hat{C}_p \alpha}$$

NÚCLEO TURBULENTO

$$\frac{h}{\rho \hat{C}_p (v_\infty - v_x|_\xi)} = \frac{C_D}{2} \qquad \frac{q_y}{\rho \hat{C}_p (v_\infty - v_x|_\xi)(T_\xi - T_\infty)} = \frac{\tau_0}{\rho (v_\infty - v_x|_\xi)^2}$$

$$\frac{\rho (v_\infty - v_x|_\xi)}{\tau_0} = \rho \hat{C}_p \frac{(T_\xi - T_\infty)}{q_y}$$

$$\frac{\rho \upsilon v_x|_\xi}{\tau_0} = \frac{\rho \hat{C}_p \alpha}{q_y}(T_s - T_\xi)$$

$$\frac{\rho (v_\infty - v_x|_\xi)}{\tau_0} = \rho \hat{C}_p \frac{(T_\xi - T_\infty)}{q_y}$$

$$\frac{\rho}{\tau_0}\left[v_\infty + v_x|_\xi \left(\frac{\upsilon}{\alpha} - 1\right)\right] = \rho \hat{C}_p \frac{(T_s - T_\infty)}{q_y}$$

$$h = \frac{q_y}{(T_s - T_\infty)} \qquad C_D = \frac{\tau_0}{\rho v_\infty^2 / 2}$$

$$\frac{v_\infty + v_x|_\xi (\upsilon / \alpha - 1)}{v_\infty^2 C_D / 2} = \frac{\rho \hat{C}_p}{h}$$

$$\frac{h}{\rho \hat{C}_p v_\infty} = \frac{C_D / 2}{1 + (v_x|_\xi / v_\infty)(\upsilon / \alpha - 1)}$$

$$\frac{h}{\rho \hat{C}_p v_\infty} = \frac{C_D/2}{1 + (v_x|_\xi / v_\infty)(\upsilon/\alpha - 1)}$$

$$v^+ = y^+ = 5$$

$$v^+ = \frac{v_x|_\xi}{(\sqrt{\tau_0/\rho})} = \frac{v_x|_\xi}{(v_\infty \sqrt{C_D/2})} 5 \Rightarrow \frac{v_x|_\xi}{(v_\infty)} = 5\sqrt{\frac{C_D}{2}}$$

Analogía de Prandtl

$$St = \frac{C_D/2}{1 + 5\sqrt{C_D/2}(Pr - 1)}$$

Subcapa laminar
Capa de transición
Núcleo turbulento

Analogía de Von Karman

$$St = \frac{C_D/2}{1 + 5\sqrt{C_D/2}\left\{Pr - 1 + \ln\left[1 + \frac{5}{6}(Pr - 1)\right]\right\}}$$