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2 **USE OF ARTIFICIAL NEURAL NETWORKS TO PREDICT 3-D ELASTIC**

3 **SETTLEMENT OF FOUNDATIONS ON SOILS WITH AN INCLINED BEDROCK**

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11

12 **Abstract**

13 Application of the theory of elasticity for the calculation of foundation settlements yields

14 equations that are well-established and consolidated in geotechnical standards and/or

15 recommendations. These equations are corrected by an influence factor to increase precision and

16 encompass the existing complex geotechnical casuistry. The study presented herein utilizes neural

17 networks to improve the determination of the influence factor (I_a), which considers the effect of

18 a finite elastic half-space limited by an inclined bedrock under the foundation. The results

19 obtained with the utilization of artificial neural networks demonstrate a notable improvement in

20 the predicted value of the influence factor in comparison with existing analytical equations.

21

22 **Keywords:** artificial neural networks (ANN); foundations; soil/structure interaction; settlement;

23 elasticity; finite-element modelling.

24

25 **1. Introduction**

26

27 The main objectives of a geotechnical engineer, when completing a foundation design, are to
28 determine the safety factor, while guaranteeing adequate functionality and economics. The safety
29 factor is determined by calculating the allowable load. Settlement calculation leads to adequate
30 functionality of the foundation for the actual working pressures, which are usually one-third of
31 the ultimate bearing capacity of the soil. Finally, the economic aspect of the design is very
32 important, obviously always within the safety limits recommended by the standards.

33 Therefore, proper foundation design must ensure that the structure does not suffer excessive
34 displacements. In this sense, the current standards (e.g. CEN 2004) consider the Serviceability
35 Limit State (SLS), which delimits the conditions beyond which the structure no longer fulfils the
36 requirements of functionality, comfort, durability or appearance. Generally, SLS refers to
37 situations that are solvable, repairable or that can admit remedial measures without serious
38 inconveniences to the users. The verification of SLS in a shallow foundation consists of
39 quantifying the total settlement of the foundation and the angular distortion between two adjacent
40 columns and verify whether these values exceed the maximum allowable limits.

41 Therefore, it is very important to evaluate, as accurately as possible, the deformations of the
42 supporting soil for the adopted working pressure levels.

43 Nowadays there are very sophisticated methods for the calculation of shallow foundation
44 settlements, but analytical methods based on the theory of elasticity ("elastic methods") are still
45 widely used in geotechnical practice (mainly during early design phases), and present in all
46 geotechnical standards and recommendations (e.g. CEN 2004; AASHTO 2017). Elastic methods
47 offer versatile solutions that can be easily obtained through laboratory and/or field tests.
48 Therefore, more accurate prediction of settlements depends on the improvement and
49 complementation of existing equations. Although elastic methods are not the best predictors of
50 soil behaviour, when considering working loads far from failure (e.g., shallow foundations where

51 a safety factor of 3 is accepted), these methods provide a more than acceptable prediction, as
52 demonstrated by the more than 200 real cases studied by Burland *et al.* (1985).

53 There are three main categories of methods for the computation of the elastic settlement in a
54 shallow foundation:

- 55 • Empirical methods, which are based on the compilation and correlation of settlement
56 measured in structures and load tests with the results from in situ data (e.g., SPT, CPT,
57 pressuremeter, etc.). The procedures developed by Terzaghi *et al.* (1967), Meyerhof
58 (1965) and Burland *et al.* (1985) are included in this category.
- 59 • Semiempirical methods, which combine field observations and theoretical studies. This
60 category includes, among others, the methods proposed by Schmertmann *et al.* (1978),
61 Briaud (2007) and Akbas and Kulhawy (2009).
- 62 • Methods based on theoretical solutions supported by the theory of elasticity, such as those
63 developed by Bowles (1987) and Mayne and Poulos (1999).

64

65 All the equations based on the elasticity theory present a similar structure. The general expression
66 for calculating the elastic settlement of a foundation that transmits uniform pressure distribution
67 (q_{net}) resting on an elastic, homogeneous and isotropic medium is (Mayne and Poulos 1999):

68

$$69 \quad s = q_{net} B \frac{(1-\nu^2)}{E} I \quad (1)$$

70

71 where s is the settlement of the foundation, B is the foundation width, E is Young's modulus for
72 the soil, ν is Poisson's ratio of the soil and I is the displacement influence factor. Displacement
73 influence factors are coefficients that modify the general equation and adapt it to specific cases
74 not covered by the general equation, improving its accuracy. This, when employing elastic
75 methods, the use of I is absolutely necessary to improve the prediction of settlements. Therefore
76 it is very important to develop new displacement influence factors to broaden the application of

77 elastic methods. A comprehensive explanation on most of the existing displacement influence
78 factors can be found in Milovic (1992) and Mayne and Poulos (1999).

79 There is limited scientific literature focused on the proposal of influence factors that consider a
80 shallow foundation, with specific stiffness, that rests on an elastic finite half-space with inclined
81 bedrock (i.e. two-layer model, with a deformable layer over a rigid inclined layer). Han *et al.*
82 (2007) carried out the most detailed study of the problem up to date, using the finite difference
83 method. The authors applied a two-dimensional plane strain model to investigate the influence of
84 an inclined incompressible layer (bedrock) on the settlement of a purely flexible strip load on a
85 compressible soil layer. The study highlighted the importance of considering the actual inclination
86 (i.e. the dip) of the rigid layer in settlement calculations. Unacceptable results, from a settlement
87 viewpoint, could be obtained if the dip is not taken into account (resulting in tilting or inadequate
88 differential settlements in the foundation). However, the study applied a load directly to the
89 ground surface (i.e. without considering any element of the foundation with a specific stiffness)
90 and therefore is limited to this specific situation. Nowadays, most foundations present a specific
91 stiffness, being perfectly flexible in very few cases, and therefore it is necessary to study the
92 problem including the consideration of foundation stiffness. Foundation stiffness is evaluated by
93 the foundation flexibility factor (K_f) proposed by Brown (1969). K_f is one of the most widely used
94 parameters to define the stiffness of a shallow foundation and it is defined as follows:

95

$$96 \quad K_f = \left(\frac{E_c}{E_s}\right) \left(\frac{t}{a}\right)^3 \quad (2)$$

97

98 where E_c refers to the elastic modulus of the foundation material (i.e. concrete), E_s is the
99 representative elastic modulus of the soil beneath foundation base (i.e. value at a depth $z=a$), t is
100 foundation thickness, and a is the equivalent radius of the foundation.

101

102 According to the value of the foundation flexibility factor (K_f) the foundations can be considered
103 perfectly rigid when $K_f > 10$, perfectly flexible when $K_f < 0.01$ and intermediately flexible for K_f
104 values between 0.01 and 10.

105 Díaz and Tomás (2016) analysed the influence of an inclined rigid layer (i.e. bedrock) on the
106 elastic settlements of a shallow foundation. Two-hundred and seventy-three 3D non-linear finite
107 element models were developed considering the foundation stiffness (K_f), inclination (α), and the
108 depth of the rigid layer (z) as variables. Statistical analysis of the results enabled the proposition
109 of an analytical equation that can be applied to the calculation of settlements.

110 Artificial Neural Networks (ANN) employ artificial intelligence to simulate the biological
111 structure of the human brain and nervous system through their architecture. This concept was
112 firstly introduced in 1943 by McCulloch and Pitts (1943), and expanded by Werbos (1974)
113 through the development of the backpropagation algorithm, becoming a practical tool in the field
114 of forecasting and prediction.

115 ANN have been successfully applied to several geotechnical engineering problems during the last
116 decades (e.g. Zounemat-Kermani *et al.* 2009; Tarawneh 2013; Mozumder and Laskar 2015;
117 Ochmański *et al.* 2015; Benali *et al.* 2017). More specifically, ANN have also been used for the
118 prediction of foundation settlements, with highly satisfactory results (e.g. Shahin *et al.* 2002;
119 Zhang *et al.* 2011; Shahin 2014; Shahin 2014; Baziar *et al.* 2015; Harikumar *et al.* 2016).

120 The objective of the study presented herein is to apply ANN to predict 3-D elastic settlements of
121 shallow foundations on soils with a rigid inclined layer. From a database containing 273 registries
122 derived from Finite Element Method (FEM) models, 212 (77.4%) were used to train the neural
123 network and the remaining 61 (22.6%) were used to test the network. ANN predictions were then
124 compared with the predictions obtained from the application of the equation recently proposed by
125 Díaz and Tomás (2016), based on traditional analytical data-fitting derived from the FEM 3D
126 models.

127

128 **2. Methodology**

129

130 **2.1. FEM model**

131

132 FEM software ANSYS V.11 (Ansys 2007a; Ansys 2007b; Ansys 2007c) was used to model the
133 case in which a shallow foundation rests on an elastic finite half-space (i.e., a compressible layer
134 over a rigid layer). A 3D nonlinear model with contact elements between the foundation and the
135 soil was developed to simulate the foundation-soil interface through the Mohr-Coulomb law.
136 These contact elements enable the consideration of the friction between materials. Herein an
137 interface friction angle equal to 2/3 of the friction angle of the soil was considered. This value,
138 recommended by Potyondy (1961), is commonly accepted for the computation of soil-concrete
139 friction.

140 A refinement of the mesh in the zone below the foundation (higher concentration of stress) was
141 implemented, and elements with a size of 1/10 of the foundation width were used in this zone.
142 The size of the elements was progressively increased to 1/2 of the foundation width in the limits
143 of the model.

144 The model presents a deformable top layer over rigid soil that can be considered incompressible
145 (i.e. rigid) for the normal range of pressures present in shallow foundations. This layer, located at
146 depth z , presents a specific inclination (α) with respect to the horizontal. Figure 1 depicts a scheme
147 of the model adopted.

148 The finite element models were solved by varying the key variables of the problem. Table 1 shows
149 the specific parameters for which the models were solved for. Combination of these values yielded
150 273 finite element models.

151

Variable	Values								
z/B	0.25	0.5	0.75	1	1.5	2	3	5	10
α (°)	0	15	30	45	60				
K_f	0.001	0.01	0.1	1	10	30	100		

152 *Table 1. Values of the variables used in the FE models.*

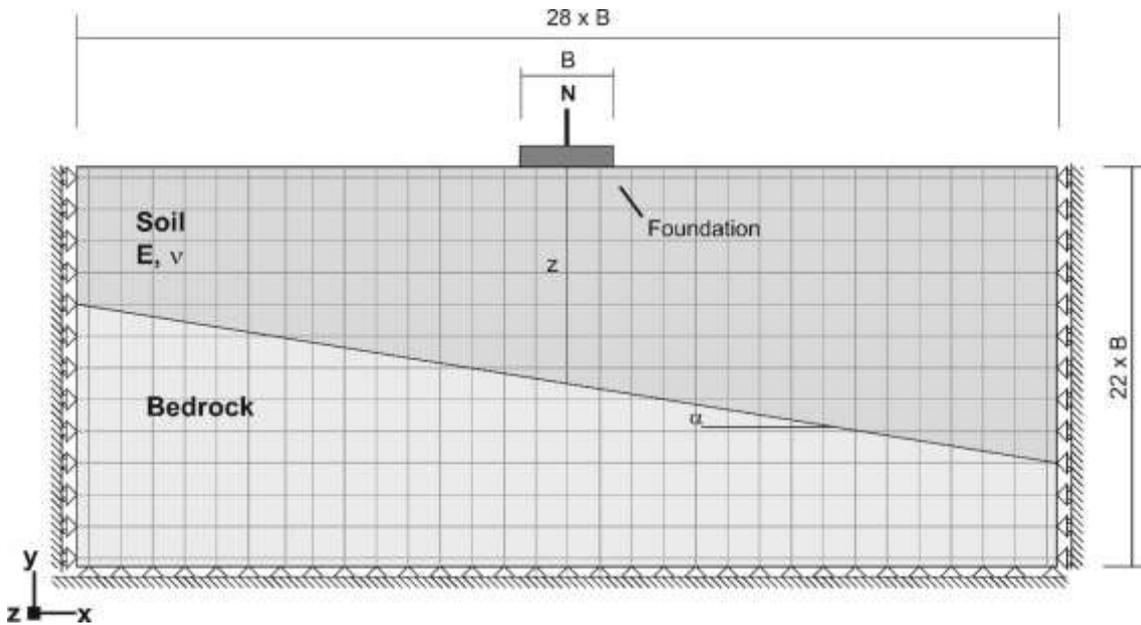
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154 Further details on the model and procedure carried out can be found in Díaz and Tomás (2016).

155

156

157



158

159 *Fig. 1. Geometry of the 3D FEM model adopted for modelling the settlement of a foundation resting on an elastic*
160 *finite half-space with an inclined bedrock (rigid layer) at specific depth.*

161

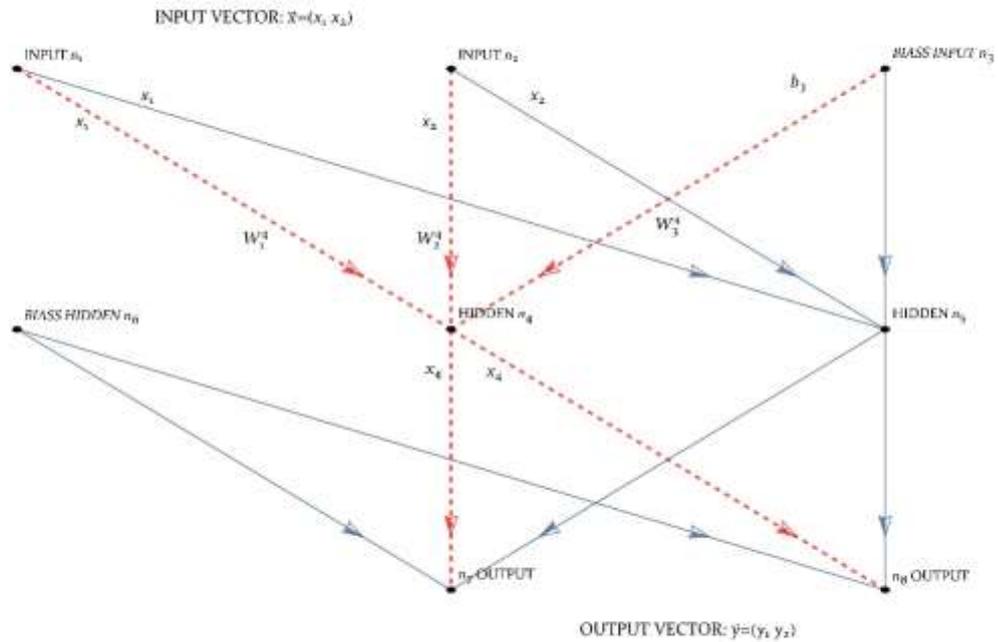
162

163 **2.2. Neural networks**

164 ANN systems mimic the behaviour of the human brain, in the sense that they require a learning
165 process, accomplished from input datasets and known outputs, to predict results (output data)
166 associated with cases (input data) not used in the training. Utilization of a higher number of cases
167 for training enables more reliable predictions of the network. Another analogy with the brain can
168 be made by considering the internal structure or network architecture. There are many types of
169 network architectures, but basically all are constituted of nodes or perceptrons (neurons in the
170 brain) with connections (synapses) between neurons. Each neuron processes the input signals
171 received from other neurons and transmits the result to their output neurons. The concept of neural
172 networks is very old, but its massive application is much more recent.

173

174 One of the network architectures most commonly used is the feedforward neural network. Figure
 175 2 shows a scheme of the structure of a very simple feedforward neural network, consisting of
 176 three layers of neurons.



177
 178 Fig. 2. A feedforward neural network with one hidden layer. n_1, n_2 : input-neurons; n_3, n_6 : Bias-neurons; n_4, n_5 : hidden
 179 layer neurons; n_7, n_8 : output-neurons.

180
 181 Two of these layers, input and output, are always present, with any number of hidden layers (one
 182 in this case) in between. The connection structure is as follows: any neuron is fully connected to
 183 all neurons in the previous layer for receiving data, and with all the neurons in the next layer for
 184 transmitting the result processed. The exceptions are BIAS neurons that do not receive input data
 185 and are used to correct the bias. Neural network learning is accomplished by automatic fitting,
 186 through an iterative process of the synaptic weights. While neurons transmit resulting processed
 187 data to all outbound connections, these data are weighted independently at each connection by
 188 the corresponding synaptic weight, so that each neuron connected to the output receives a different

189 value. The processing of the sum of input data by each neuron is performed by a nonlinear
190 activation function, which can take several forms and response settings within each concrete form.
191 The advantage of using neural networks rather than analytical function fits (such as least-squares
192 methods), becomes clear when considering that in this case there are 12 synaptic weights or fitting
193 parameters. These fit parameters can be easily expanded by increasing the number of neurons and
194 hidden layers, and are covered by the nonlinear input-output dependencies through the activation
195 function. During the learning process, synaptic weights are automatically adjusted through an
196 iterative process that seeks the minimum MSE between the target data (actual) and the network
197 output data for the same inputs. In compact form, the response of an active neuron (not BIAS)
198 can be written as Equation (3).

199

$$200 \quad x_j = \sigma \left(\sum_{i=m}^{n-1} x_i w_i^j + b_n^j \right) \quad (3)$$

201

202 where x_j is the result of neuron j of layer k , $\sigma(x)$ is the activation function, m is the number of
203 the first neuron in the previous layer, n is the number of the first neuron in the previous layer
204 (BIAS), x_i is the result of neuron i of layer $k-1$, w_i^j is the synaptic weight of i, j connection, and
205 b_n^j is the BIAS weight connection.

206

207 It is worth noting that ANN have been considered as an alternative to traditional analytical fitting
208 in detriment of other techniques of supervised machine learning (e.g. support vector machines),
209 because ANNs achieved prediction errors that were considered adequate for the purposes of the
210 present work.

211

212 **3. Prediction of 3-D elastic settlements by means of neural networks**

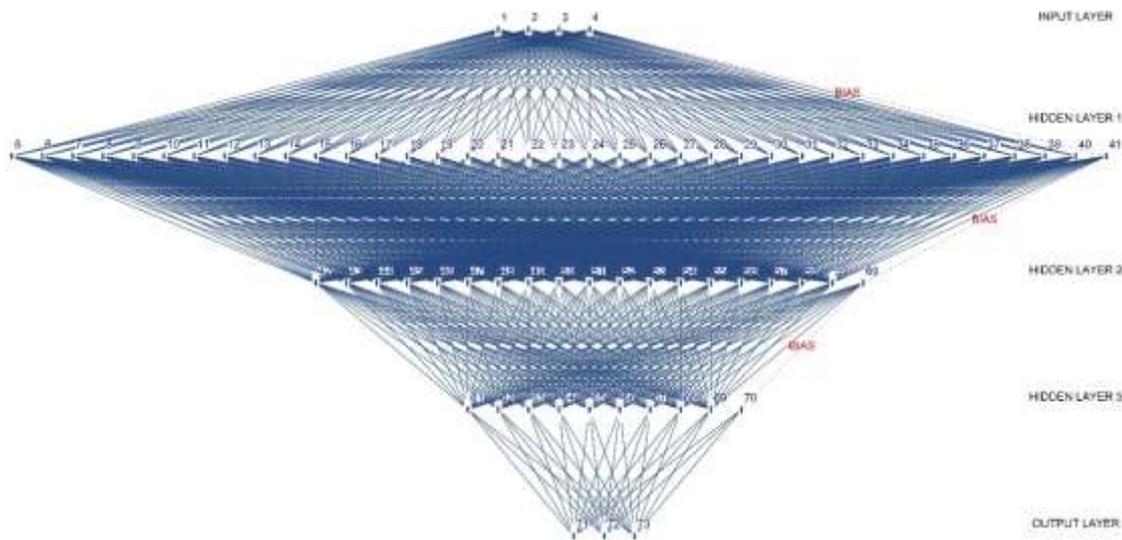
213

214 A total of 273 non-linear finite element models were solved to analyze the relationship (I_a)
215 between the settlement of a shallow foundation on an infinite elastic half-space (s_∞) and the

216 highest settlement obtained for the same foundation resting on a deformable layer over an inclined
217 rigid layer (s_a). The results were used to implement an ANN to obtain an approximation function
218 that enables the prediction of settlements.

219

220 The network used herein consists of an input layer with three neurons (plus one BIAS neuron),
221 hidden layer 1 with 36 neurons, hidden layer 2 with 18 neurons and hidden layer 3 with nine
222 neurons. Each hidden layer has one additional BIAS neuron to correct the bias. The output layer
223 contains three neurons. The total number of neurons is 73 and, with the used network topology,
224 1011 synaptic connections were created (Figure 3).



225

226 *Fig. 3. Overview of the neural network used.*

227

228 In Figure 3, the numbers in the boxes indicate the number of the neuron. Of the five layers of
229 neurons, the topmost and the bottommost constitute the input and output layers, respectively. The
230 three intermediate layers correspond to hidden layers.

231

232 Although there are studies about the determination of the optimal size and architecture of the
233 network (e.g. Hunter *et al.* 2012), the process carried out in herein focused on fixing the number
234 of layers and neurons, through a previous study where different network configurations were
235 analysed. In this analysis, the values of MSE (mean squared error), MRE (Mean Relative

236 Estimation Error between the predicted and the target data) and R^2 (coefficient of determination)
237 obtained for each configuration were compared, followed by the selection of the configuration
238 with better values. In addition, the network designed herein can be used with a higher volume of
239 data in the future.

240 The high number of connections and neurons generated a neural network with high fitting
241 capability (1011 synaptic weights). Input vectors present three components, coinciding with the
242 number of neurons in the data input layer (without the BIAS neuron), representing the input
243 variables of each case (K_f , α , z/B) previously defined. The output vectors also present three
244 components, for topology requirements of this type of neural network, although the desired result
245 (I_α) is unidimensional. In each case, I_α is obtained as the mean value of the three components of
246 the output vector.

247

248 On the other hand, in this work, the sigmoid symmetric activation function $\sigma(x)$ has been used in
249 the interval $[-1, +1]$, as is shown in Equation (4).

250

$$251 \quad \sigma(x) = \frac{1}{1+e^{-sx}} \quad (4)$$

252

253 where s is the parameter that adjusts the smoothness of the response function.

254

255 Although the activation function selected was the sigmoid symmetric, several activation functions
256 were previously tested. The function with the lowest MSE value and successful convergence was
257 selected.

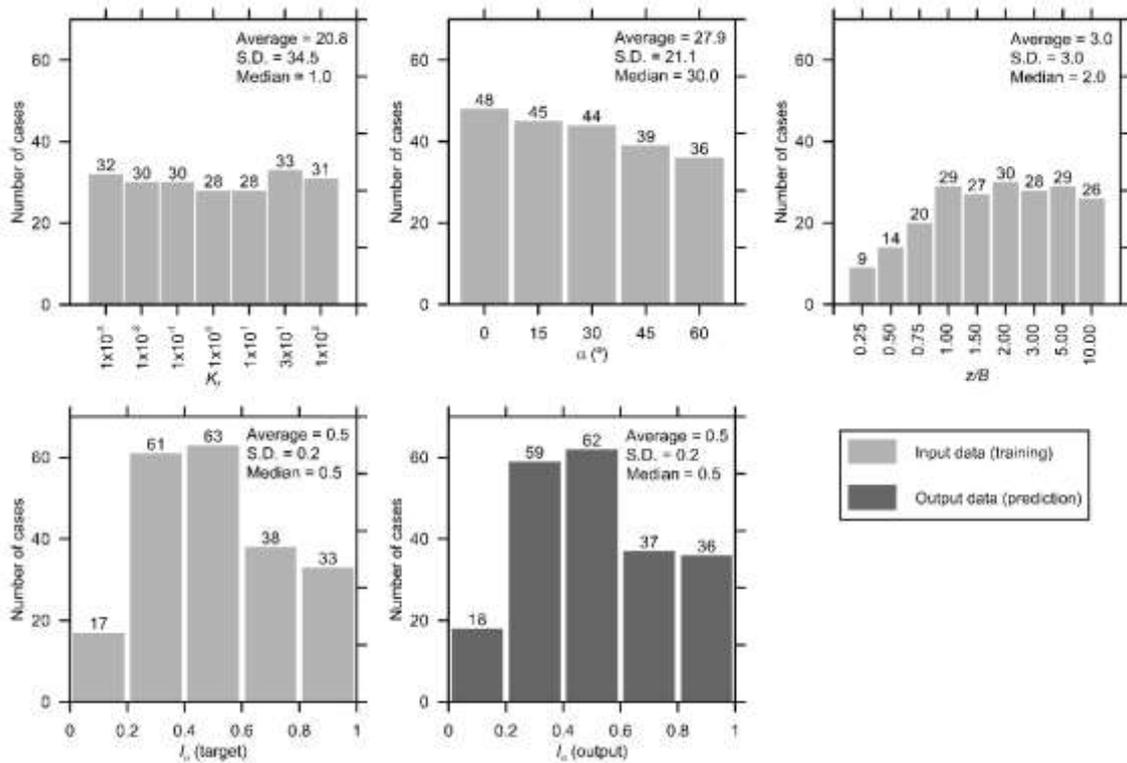
258

259 **4. Analysis of results**

260

261 A dataset with 273 input-output vectors corresponding to the relationship between the settlement
262 of a shallow foundation resting on an infinite elastic half-space and the highest settlement

263 obtained for the same foundation on a deformable layer over an inclined rigid layer was used in
 264 the implementation of ANN. ANN was trained with 212 randomly selected data vectors, followed
 265 by an accuracy test for the trained network, carried out with the remaining 61 vectors. These two
 266 steps correspond to the two mandatory phases required before using a neural network: training
 267 and validation.
 268 Regarding the input database used to train the network, a brief characterization with its main
 269 statistics is presented in Figure 4.



270

271

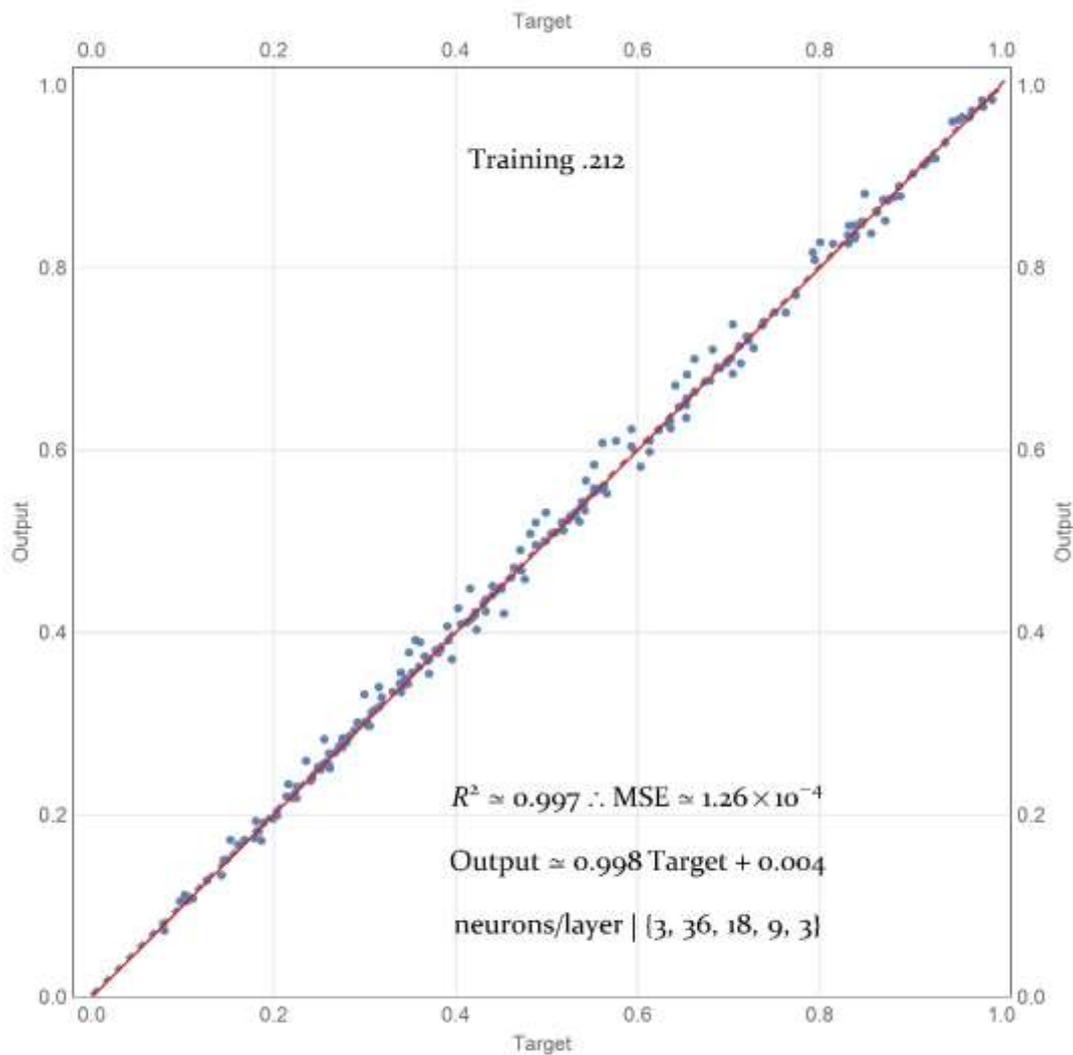
Fig. 4. Histograms of the input and output variables. S.D. refers to standard deviation.

272

273 It is worth noting that, to avoid overfitting, the neural network was trained with 20 sets of
 274 randomly chosen input data. For each series, five re-trainings were carried out. The synaptic
 275 weights selected for this process were those that presented the lowest MSE values. However, it
 276 should be noted that the differences obtained were negligible and after $4 \cdot 10^5$ iterations in all re-
 277 trainings, MSE was always under $1.5 \cdot 10^{-4}$.

278

279 Figure 5 shows the results of the network training, providing MSE and MRE values equal to
280 $1.26 \cdot 10^{-4}$ and 2.04%, respectively. Figure 5 represents the linear regression between the target
281 data (used in the training) and the output data network, obtained from the trained network using
282 the corresponding input parameters.
283 The fitting equation provided an R^2 value of 0.997.
284



285

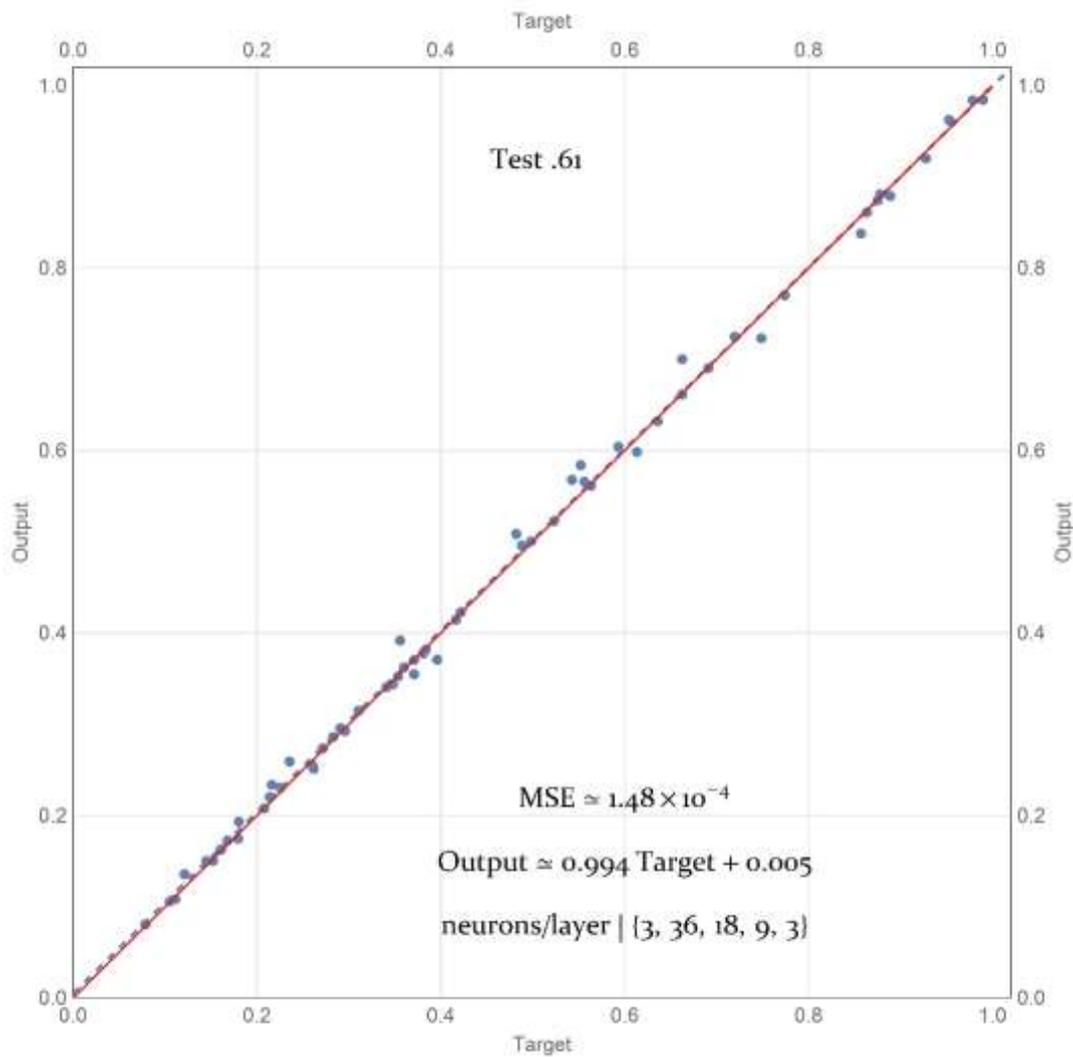
286 *Fig. 5. Comparison and regression of actual settlements (Target) and artificial neural network predicted settlements*
287 *(Output) using training data for 212 study cases.*

288

289 Figure 5 also shows that the points line up very well with a straight line that crosses the origin
290 with a slope near 1 (i.e. 45 degrees). This indicates high quality parameter fitting of the neural
291 network during the training.

292 Once the artificial neural network was trained, the next step consisted of the application of a test
293 to the 61 vectors not used for training, with the results shown in Figure 6.

294 The values of the foundation settlement predicted by ANN provided MSE value equal to $1.48 \cdot 10^{-4}$
295 ⁴, R^2 equal to 0.998 and a fitting line slope of 1.000 (i.e. 45°).



296

297 *Fig. 6. Comparison of actual settlements (Target) and the settlements predicted by means of the artificial neural*
298 *network (Output) for 61 study cases (Test).*

299

300 An application was created with the trained network to obtain the output values (I_α) from any set
 301 of input values ($K_f, \alpha, z/B$). K_f is the foundation flexibility factor, according to Brown (1969), α
 302 and z are the dip and the depth of the rigid layer under the central point of the foundation,
 303 respectively, and B is the foundation width, as previously described. Figure 7 shows the
 304 appearance of the application generated from the specific training used herein.
 305

Neural Network			
{3, 36, 18, 9, 3}.: MSE $\approx 1.26 \times 10^{-4}$			
K_f	α	z/B	I_α
30.	0.	3.	"0.5086"

306

307

Fig. 7. Application with the trained network.

308

309 The equation obtained in the original analytical fitting (Díaz and Tomás 2016) was:

310

$$311 \quad I_\alpha = 0.1261 \cdot e^{-(8.7510 \cdot K_f + 0.0949)} + \left(\frac{1}{1529} \cdot \alpha + 0.7690\right)^2 - 1.1715 \cdot e^{-(0.4892 \cdot \frac{z}{B} + 0.7061)} - 0.0002 \cdot K_f + \frac{z}{B} \cdot 0.0249 \quad (5)$$

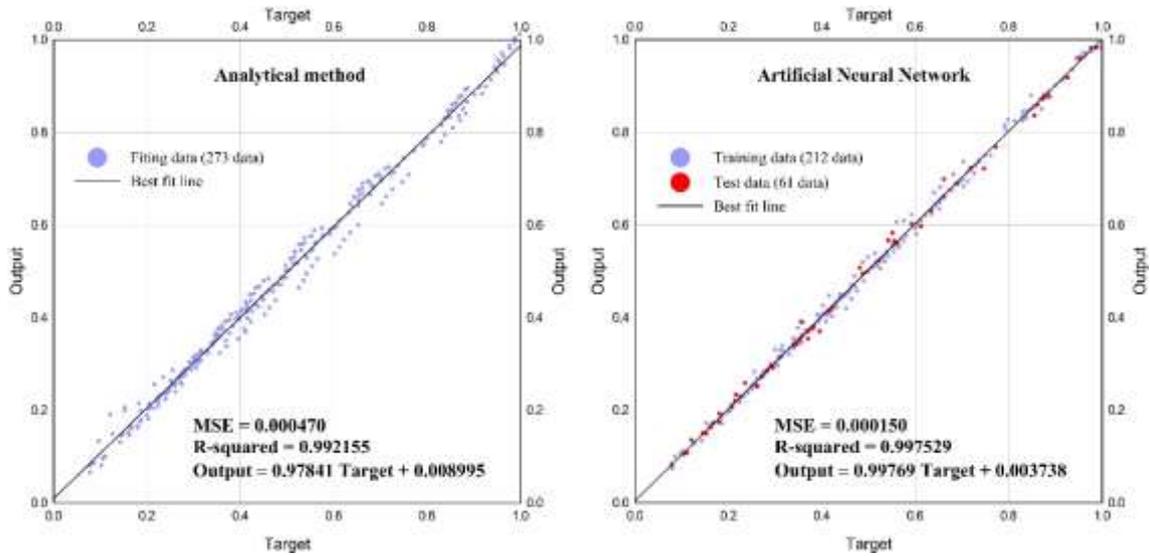
312

313 Using data from the trained network, the target-prediction regressions of both fits (analytical and

314 ANN) were computed, followed by a comparison of accuracies. Figure 8 shows the point clouds,

315 and the MSE and MRE values. In both cases, the errors obtained with the trained ANN were lower

316 than the errors obtained with the analytical fit.



317

318 *Fig. 8. Comparison of the fitting performed over the results provided by: (left) the analytical method proposed by*
 319 *Díaz and Tomás (2016); and (right) trained ANN.*

320

321 Table 2 summarizes the main results of the fitting. Prediction of foundation settlement by means
 322 of ANN provided better results for all values and evaluated statistical parameters.

323

Parameter	Analytical fit	ANN
Coefficient of determination (R^2)	0.992	0.997
Mean quadratic error (MSE)	$4.70 \cdot 10^{-4}$	$1.26 \cdot 10^{-4}$
Mean relative error (MRE)	4.19%	2.04%

324

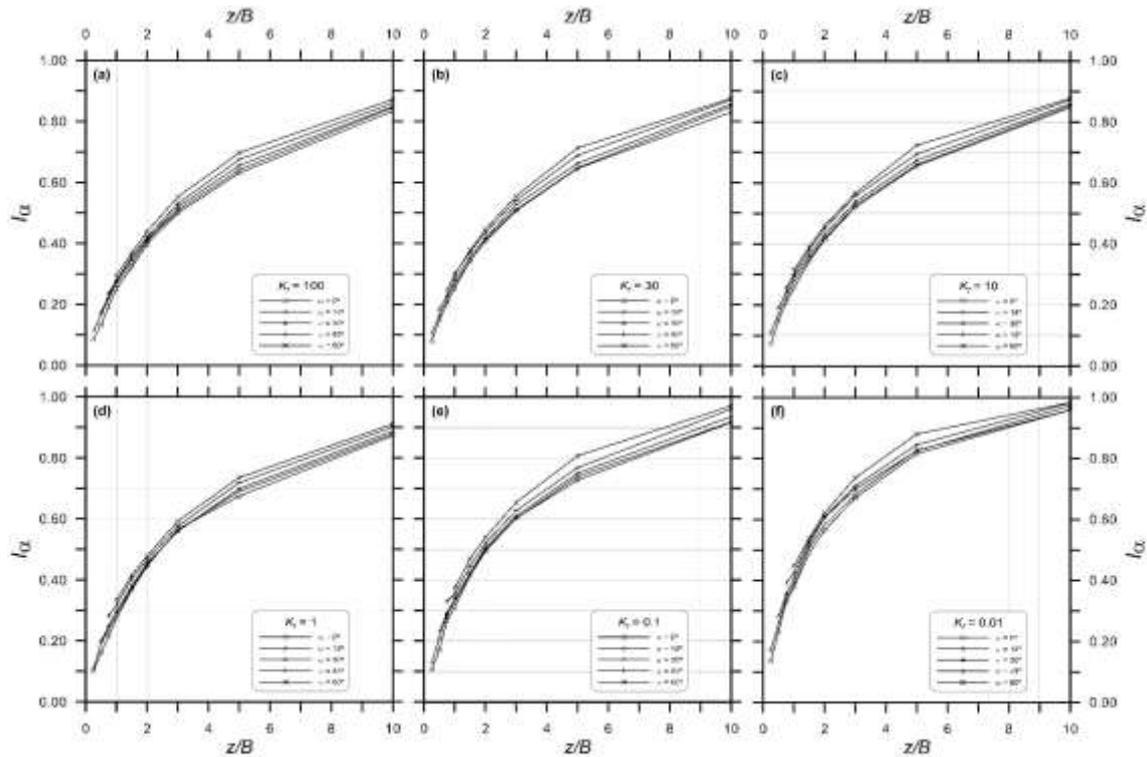
Table 2. Comparison of the statistical parameters obtained for the analytical fit and the ANN (training).

325

326 5. Design charts

327

328 The results obtained from the trained ANN were conveniently organized and represented in six
 329 design charts to quickly estimate I_α for different values of the input variables (K_f , α , z/B). Figure
 330 9 shows these design charts.



331

332 *Fig. 9. Design charts for the calculation of I_α from different z/B ratios and α values. The figure covers the following*
 333 *K_f values: a) 100, b) 30, c) 10, d) 1, e) 0.1 and f) 0.01.*

334

335

336

337 6. Conclusions

338

339 The study presented herein predicted the settlement of foundations resting on a finite half-space
 340 with an inclined rigid layer, by means of artificial neural networks. This study demonstrated the
 341 feasibility of ANN to predict the settlement of shallow foundations under these conditions. The
 342 ANN was developed with 273 results of 3D non-linear FEM models, of which 212 corresponded
 343 to training and 61 to testing. Subsequently, an application was developed with the trained network
 344 to obtain I_α from any set of input values (K_f , α , z/B). These I_α predicted values were compared
 345 with those obtained from the analytical fitting of the results of FEM models. It was verified that
 346 ANN was capable of accurately predicting the settlement of foundations resting on a finite half-
 347 space with an inclined rigid layer.

348 The results also established that the ANN method provides better results than traditional analytical
349 regression methods (squared coefficient of determination equal to 0.997, mean quadratic error
350 equal to $1.26 \cdot 10^{-4}$ and mean relative error equal to 2.04%).

351 Furthermore, six synthetic design charts were built using the input and output parameters derived
352 from the ANN. These charts relate I_α with other key parameters (K_f , α , z/B), enabling I_α estimations
353 that can be utilized at design stages.

354 Finally, it must be highlighted that artificial neural networks present another advantage over
355 traditional regression methods: once the model has been trained and tested, it can be utilized as
356 an accurate and quick tool for the estimation of settlement under the conditions studied herein.

357

358

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365

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