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Mathematics Teacher's Knowledge, knowledge-based reasoning, and contexts

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Research on mathematics teacher is multi-faceted, as reflected in the three articles in this issue of the Journal of Mathematics Teacher Education. These articles address different aspects of research on mathematics teacher education and raise empirical and conceptual issues. The first study focuses on how *audible conversational* affordances and constraints the development of noticing during a meeting on a lesson study analysis, as part of a field experience in a primary teacher education program. The other two studies address different aspects of mathematics teacher knowledge from a highly methodological and conceptual perspective.
In all three articles, presence of teacher knowledge is more or less explicit. In the Amador and Carter study, noticing is considered as a knowledge-based-reasoning process because attending to students' thinking can increase pre-service mathematics teachers' pedagogical and content knowledge. Furthermore, professional noticing can be considered a process by which teachers make sense of what occurs during instruction and make plans to respond to what students are doing. The studies by Wilkerson and colleagues and Pino-Fan and colleagues describe different aspects of teacher knowledge emphasizing the conceptual references used in the design of a data collection instrument. The articles in this issue thus address two relevant aspects in teacher education: teacher knowledge and the knowledge-based reasoning process, as well as the contexts in which the pre-service teachers' learning takes place.

Amador and Carter's study investigates how pre-service primary teachers notice when they engage in iterations of live teaching observation throughout the lesson study cycle. This article underlines the affordances and constraints of Lesson Study for encouraging the pre-service primary teachers' verbalization of noticing. In this way, Amador and Carter present an innovative notion to include noticing of students' mathematical thinking as a theoretical construct supporting the Lesson Study process. They underline that the support of a knowledgeable other is a critical factor in the Lesson Study process and they raise generative issues about teacher learning from new perspectives, for example: when and how new knowledge can be presented to pre-service teachers in the field experience, what role university tutors (facilitators) should play, and how supervision sessions should be organized.

Amador and Carter describe the lesson study cycle the following way: planning the lesson; teaching the lesson; observing all other members of the lesson study team, with emphasis on students' mathematical thinking, guided by a Lesson Observation Form; ending with a lesson study analysis. Each pair of pre-service teachers participates at each stage of this cycle on a weekly basis. The examination of the lesson study analysis meeting allowed identifying factors that determined how noticing was afforded (facilitator-initiated prompts and turn-taking following prompts) and factors that restrained the noticing (content shifts and knowledgeable other expertise).

The research on how to enhance noticing in the field experience provides insight on how pre-service teachers' noticing can be improved or limited. In particular, it sheds light on how the

conversational components in Lesson Study, as part of a field experience course, can become essential for pre-service teacher progress. Findings from this study underscore the relevance of the institutional context and the facilitator in developing pre-service teachers' noticing.

The other two articles of this issue focus on teacher knowledge and on how to describe it. The study by Wilkerson et al. addresses what mathematics teachers identify as mathematical models. The study by Pino-Fan et al. describes how to design an instrument to assess teacher's mathematics knowledge from a sound theoretical perspective. Both articles adopt a highly methodological perspective while supporting the design of instruments based on theoretical frameworks. Wilkerson et al. drew from a knowledge-in-pieces/conceptual ecology perspective to document the teacher's notions of modelling; and Pino-Fan et al. built their reflections from a specific model of mathematics teachers' knowledge (Didactic-Mathematical Knowledge, DMK) derived from the Onto-Semiotic-Approach.

The study by Wilkerson, Bautista, Tobin, Brizuela and Cao identifies what mathematics teacher attend to when describing the constitution of a mathematical model, and how their attention shifts as they engage in activities to construct a model and to sort a collection of representational artefacts considered as models. The focus of this study is on teachers' knowledge and approaches to what constitutes a mathematical model. Describing the middle mathematics teachers' perspective on mathematical models, emphasizing the links between mathematics and science teaching, helps to understand how modelling might be enacted in the classroom. Considering recent recommendations on curriculum development based on science, technology, engineering and mathematics (STEM) from an interdisciplinary and applied approach, this study draws our attention to the role of teacher knowledge. The issues explored here are key in this approach. They raise questions as to what model features and purposes teachers attend to when constructing a mathematical model, or when deciding whether something is a mathematical model, and how their attention to different features and purposes shifts as they engage in different tasks.
One relevant input from this study is the interview protocol used. The procedure mixes e-mail and face-to-face interviews. In the first step, teachers predict the behaviour of physical phenomena (what happens to the temperature of hot coffee that is left on a table); then they test their prediction by carrying out an experiment and take notes on what they noticed during the exploration to use them when they are interviewed face-to-face. In a second task, teachers sort out a collection of representational artefacts (graphs, diagrams, verbal descriptions, mathematical equations, etc.) that could be used to represent the models of coffee cooling. This data collection procedure structure allows Wilkerson and his colleagues to identify shifts in teachers' ideas about the model.
Findings from this study reveal the dynamic character of teachers' knowledge about modelling, and provide new references to understand how STEM-approach curricula might be developed. The theoretical framework for this research allowed authors to gain new insight into how teachers' knowledge about modelling should be interpreted in a more flexible way. In particular, they assume that "an individual teacher may have multiple, apparently distinct interpretations of modelling that draw from the same set of resources, but manifest differently across tasks". The authors used two constructs: *features – what models are and what they look like - and purposes – what models are for and how we use them*- to describe teachers' activation of conceptual resources. The use of these constructs allowed reporting teachers' perspectives on mathematical modelling, illustrating pattern shifts across the task of constructing a model and sorting a set of models.

The study by Pino-Fan, Godino and Font describes the design and implementation of an instrument to assess some aspects of the mathematics teacher knowledge needed to teach the concept of derivative. From an onto-semiotic approach on mathematical activity, the authors

describe how to analyze the mathematical dimension of the content knowledge and the specialized knowledge of the content knowledge (epistemic fact) as part of the pedagogical content knowledge, re-named here as *didactical dimension* of teacher knowledge. The epistemic facet of the Didactical dimension of knowledge (pedagogical content knowledge) includes representations of mathematical objects, different procedures to solve a task, linking mathematical objects, justifications and argumentation. This allows identifying the knowledge at play during the task resolution. The Onto-Semiotic approach is used to justify both the prior analysis of the tasks in the instrument and the pre-service teachers' answers. The theoretical approach allows designing an instrument to collect data that provides a large amount of information, rich in mathematical details of mathematical activity carried out by pre-service teachers and the mathematical objects involved in their mathematical practice. This wealth of collected information is exemplified by the detailed analysis of one of the tasks and shows the potential of the adopted approach.

The studies by Wilkerson and colleagues and Pino-Fan and colleagues have similar implications for teacher education and some relation with Amador and Carter's study. The rich description of teacher knowledge is necessary to reflect on adequate contexts to develop the skills of attending mathematical elements in tasks, interpreting students' mathematical thinking, and to learn how to support teaching decisions. Although the three studies focus on different types of teachers - pre-service primary teachers, middle school mathematics teachers enrolled in a professional Development program and pre-service secondary teachers -, the three studies provide a solid background to reflect on mathematics teacher education in a more informed way. Furthermore, the three articles in this issue increase our knowledge of three relevant areas of mathematics teacher education and also provide insights into conceptual and methodological approaches to research design. These contribute to strengthening the research field of mathematics teacher education.