Universitat d'Alacant
Universidad de Alicante
Departament d'Enginyeria Civil
Departamento de Ingeniería Civil

EXAMPLE OF TIME-DEPENDENT CONCRETE ANALYSIS IN SAP2000 v14

This is an example of a time-dependent concrete analysis using the non-linear staged construction feature of SAP2000. A 5 m high concrete column is built with a circular cross-section with a diameter of 0.35 m. The column is fixed at its base and free at the top. The concrete has a characteristic compressive strength f_{ck} = 25 MPa and is made with limestone aggregate and slowly hardening cement. The curing period is 2 days. At an age of 56 days a vertical load of 1000 kN is applied at the top of the column, which remains constant thereafter. The relative humidity of the atmosphere is 75%. The weight of the column will be neglected in the analysis. Obtain the vertical displacement at an age of 5000 days, for the following situations:

(a) Variation of the elastic modulus with time and concrete creep shall be taken into account.
(b) Concrete shrinkage shall also be considered.

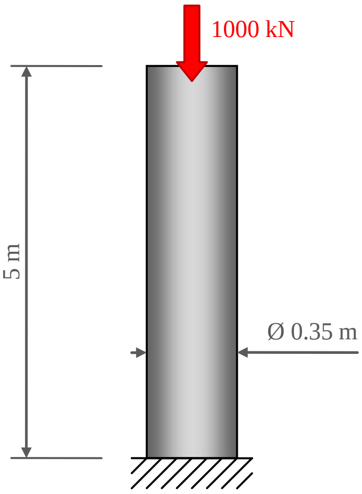

Fig. 1

In the solution by hand calculation the following results are obtained:

$$E_{ci} = 28808 \text{ MPa} \qquad E_{ci}(56\text{ d}) = 30457 \text{ MPa}$$

$$h = 175 \text{ mm} \qquad \varepsilon_{cs}(5000\text{ d}, 2\text{ d}) = -0.316‰$$

$$\varphi(5000\text{ d}, 56\text{ d}) = 1.78 \qquad t_{0,adjust} = 52.3 \text{ days}$$

$$\sigma_c(56\text{ d}) = 10.4 \text{ MPa}$$

(a) The total strain after 5000 days including creep is ε_{cs} (5000 d, 56 d) = 0.983‰. Therefore, the vertical displacement at the top would be Δz_c = 4.92 mm.
(b) The additional vertical displacement due to shrinkage would be Δz_s = 1.58 mm and the total vertical displacement at the top would be $\Delta z_c + \Delta z_s$ = 6.50 mm.

Using SAP2000 (link to YouTube tutorial) the following results are obtained:

(a) Δz_{top} = 4.87 mm by defining t_0 = 56 days (i.e., without adjustment). However, if the loading age is adjusted ($t_{0,adjust}$ = 52 days), then a value Δz_{top} = 4.93 mm is achieved.
(b) Δz_{top} = 6.50 mm (with $t_{0,adjust}$ = 52 days).

The formulation of ***Model Code 1990*** (CEB-FIP 90) for the time-dependent properties of concrete (elastic modulus, creep and shrinkage) is presented in the following sections.

MODULUS OF ELASTICITY ACCORDING TO CEB-FIP 90

The tangent modulus of elasticity at an age of 28 days can be estimated as follows:

$$E_{ci} = E_{c0}\left(\frac{f_{cm}}{f_{cm0}}\right)^{1/3}$$

where:

- E_{c0} = 21500 MPa
- $f_{cm} = f_{ck}$ + 8 MPa
- f_{cm0} = 10 MPa

Depending on the type of aggregate used in the mix, the tangent modulus of elasticity should be calculated multiplying $E_{ci}(t)$ with coefficient α_E, given in Table 1.

Table 1. Values of coefficient α_E

Aggregate type	α_E
Limestone	0.9
Dense limestone	1.2
Quartzitic	1
Sandstone	0.7
Basalt	1.2

The tangent modulus of elasticity at an age different from 28 days may be estimated as follows:

$$E_{ci}(t) = \beta_E(t)\,E_{ci} = \left[\beta_{cc}(t)\right]^{0.5} E_{ci}$$

where:

- t is the age of concrete (days), provided that seasonal variations produce mean temperatures between -20 °C and +40 °C.
- $\beta_{cc}(t) = \exp\left\{ s\left[1 - \left(\frac{28}{t}\right)^{0.5}\right]\right\}$
- s is a coefficient which depends on the type of cement, according to Table 2.

Table 2. Values of coefficient s

Rapid hardening high strength cement	Normal and rapid hardening cement	Slowly hardening cement
s = 0.2	s = 0.25	s = 0.38

CONCRETE SHRINKAGE ACCORDING TO CEB-FIP 90

The total shrinkage of concrete can be estimated as follows:

$$\varepsilon_{cs}(t,t_s)=\varepsilon_{cs0}\,\beta_s(t-t_s)$$

where:

- ε_{cs0} is the notional shrinkage coefficient
- $\beta_s(t-t_s)$ is the coefficient to describe the development of shrinkage with time
- t is the age of concrete (days)
- t_s is the age of concrete at the beginning of shrinkage

The notional shrinkage coefficient ε_{cs0} may be obtained from:

$$\varepsilon_{cs0}=\varepsilon_s(f_{cm})\,\beta_{RH}$$

with:

$$\varepsilon_s(f_{cm})=\left[160+10\,\beta_{sc}\left(9-\frac{f_{cm}}{f_{cm0}}\right)\right]\times 10^{-6}$$

and with:

$$\beta_{RH}=-1.55\left[1-\left(\frac{RH}{RH_0}\right)^3\right]$$

where:

- β_{sc} is a coefficient which depends on the type of cement, according to Table 3
- RH is the relative humidity of the ambient atmosphere (%)
- RH_0 = 100%

Table 3. Values of coefficient β_{sc}

Rapid hardening high strength cement	Normal and rapid hardening cement	Slowly hardening cement
β_{sc} = 8	β_{sc} = 5	β_{sc} = 4

The development of shrinkage with time is given by:

$$\beta_s(t-t_s)=\left[\frac{t-t_s}{350\,(h/h_0)^2+t-t_s}\right]^{0.5}$$

where:

- h is the notional size of the concrete member (mm), to be calculated as $h = \frac{2\,A_c}{u}$
- A_c is the cross-sectional area
- u is the perimeter of the cross-section in contact with the ambient atmosphere
- h_0 = 100 mm

CONCRETE CREEP ACCORDING TO CEB-FIP 90

If the service stresses of concrete in compression do not exceed 0.4 $f_{cm}(t_0)$, creep is assumed to be linearly related to stress. The stress dependent strain may be estimated as follows:

$$\varepsilon_{c\sigma}(t,t_0) = \sigma_c(t_0)\left[\frac{1}{E_{ci}(t_0)} + \frac{\varphi(t,t_0)}{E_{ci}}\right]$$

where:

- $\sigma_c(t_0)$ is the constant stress applied at time t_0
- $E_{ci}(t_0)$ is the tangent modulus of elasticity at the time of loading t_0
- E_{ci} is the tangent modulus of elasticity at the age of 28 days
- $\varphi(t,t_0)$ is the creep coefficient at an age t due to a constant stress applied at time t_0

The creep coefficient can be estimated as follows:

$$\varphi(t,t_0) = \varphi_0\ \beta_c(t-t_0)$$

where:

- φ_0 is the notional creep coefficient, which may be calculated from:

$$\varphi_0 = \left[1 + \frac{1 - {}^{RH}\!/_{RH_0}}{0.46\left({}^{h}\!/_{h_0}\right)^{1/3}}\right] \frac{5.3}{\left({}^{f_{cm}}\!/_{f_{cm0}}\right)^{0.5}} \frac{1}{0.1 + (t_0)^{0.2}}$$

- $\beta_c(t-t_0)$ describes the development of creep with time and is given by:

$$\beta_c(t-t_0) = \left(\frac{t-t_0}{\beta_H + t - t_0}\right)^{0.3}$$

$$\text{with } \beta_H = 150\left[1 + \left(1.2\,\frac{RH}{RH_0}\right)^{18}\right]\frac{h}{h_0} + 250 \ngtr 1500$$

The age at loading t_0 may have to be adjusted depending on the type of cement, as given by:

$$t_{0,adjust} = t_0\left[1 + \frac{9}{2 + (t_0)^{1.2}}\right]^{\alpha} \nless 0.5 \text{ days}$$

where the values of power α are given in Table 4.

Table 4. Values of power α

Rapid hardening high strength cement	Normal and rapid hardening cement	Slowly hardening cement
$\alpha = 1$	$\alpha = 0$	$\alpha = -1$

TIME-DEPENDENT PROPERTIES OF CONCRETE IN SAP2000 v14

The time-dependent properties for concrete in SAP2000 can be defined by the user according to the formulation of the ***Model Code 1990*** (CEB-FIP 90). Although the final draft of ***Model Code 2010*** (published in 2012) updated the formulation for creep and shrinkage, some recent versions of SAP2000, such as v17, also refer to CEB-FIP 90. Table 5 gives the values of the CEB-FIP 90 Parameters which must be chosen by the user in SAP2000 v14.

Table 5. CEB-FIP Parameters in SAP2000

Cement type coefficient	0.2	Rapid hardening high strength cement
	0.25	Normal and rapid hardening cement
	0.38	Slowly hardening cement
Relative humidity (%)	equations in CEB-FIP 90 are valid for $40\% \leq RH < 99\%$	
Notional size, h	$h = \frac{2\,A_c}{u}$	
Shrinkage coefficient, β_{sc}	8	Rapid hardening high strength cement
	5	Normal and rapid hardening cement
	4	Slowly hardening cement
Shrinkage start age (days)	t_s, does not need adjustment due to cement type	

In more recent versions of SAP2000 (e.g. v17) the notional size is not defined with the materials. Instead, it can be defined for each of the sections. This change is arguably more reasonable.

The formulation for creep in CEB-FIP 90 is associated to the tangent modulus of elasticity of concrete. Therefore, if time dependent analysis is performed with SAP2000, the tangent modulus of elasticity (E_{ci} , at an age of 28 days) should be defined in the material property data form. Otherwise, when only an elastic analysis of a concrete structure is carried out, the secant modulus of elasticity E_c may be used instead (E_c = 0.85 E_{ci}).