

# ANALYSIS OF THE ZONE CONNECTING CONSECUTIVE SECTORS IN GENERALIZED DISTILLATION COLUMNS BY USING THE PONCHON-SAVARIT METHOD

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**Abstract**— Ponchon-Savarit is a classical graphical method for the design of binary distillation columns. This method is still widely used, mainly with didactical purposes, though it is also valid for preliminary calculations. Nevertheless, no complete description has been found in books and situations such as different thermal feed conditions, multiple feeds, possibilities to extract by-products or to add or remove heat, are not always considered. In this work we provide, a systematic analysis of the Ponchon-Savarit method by developing generalized equations for the operating lines or difference points, as well as a consistent analysis of what may happen in the zone between two consecutive trays of the corresponding sectors separated by a lateral stream of feed, product, or a heat removal or addition. The graphical interpretation of all situations shown allows a clarifying view of the different possibilities in the rectifying column and completes the existing literature about this method.

**Keywords**— Distillation; Side Stream; Process Design; Heat Stages; Lateral Product.

## I. INTRODUCTION

Many references dealing with the design of binary distillation columns, describe the Ponchon-Savarit and the McCabe-Thiele methods. Equations for the operating lines (OL) and difference points (DP) are always developed for columns with single or multiple feed additions but product extractions and heat additions or withdrawals are not always considered.

Furthermore, when a feed stream is considered, whatever its physical condition, it is commonly assumed to be introduced to a single tray where it perfectly mixes with the vapor of the tray below and with the liquid of the tray above. The streams leaving this feeding stage are considered to be in equilibrium, as in any other theoretical stage. This separation between the liquid and vapor portions is considered to have a small influence in the calculations of the number of trays. Consequently, the equation that defines the change between two consecutive sectors in a column is not usually developed and the zone of such change of sector is not represented. Nevertheless, some authors have pointed out the necessity of incorporating the considerations of those partly vapor feeds to the graphical methods to develop generalized equations. Although it is not the general trend,

some references consider the zone between consecutive sectors for a two phase feed introduction to a distillation column (Ledanois and Olivera Fuentes, 1984; Wankat, 2012).

Additionally some references can be found in the literature dealing with the advantages of using the Ponchon-Savarit method and its geometrical concepts in particular cases, where molar latent heat depends on composition and heat of mixing is considerable, such as: multicomponent distillation, reactive distillation, quaternary liquid-liquid extraction, minimum reflux calculations and internal heat integrated distillation columns (HIDiC) (Reyes-Labarta *et al.*, 2012; Lee *et al.*, 2000; Marcilla *et al.*, 1997; 1999; Reyes *et al.*, 2000; Ho *et al.*, 2010, Wakabayashi and Hasebe, 2013).

In previous works (Reyes-Labarta *et al.*, 2014a), generalized equations for OL were systematically developed and analyzed for sectors and zones connecting consecutive sectors due to feeds, products or heat removals or additions on the McCabe-Thiele method, using a Generalized Feed Operating Line (GFOL) approach. In this paper, equations for OL and DP are systematically developed for the Ponchon-Savarit method with different side streams (feeds, products and/or heat removals or additions). This work widens the academic literature dealing with this subject. All the streams involved in the stages corresponding to the zones connecting consecutive sectors, as well as the corresponding DP, are unambiguously located in the enthalpy-composition diagrams.

## II. GENERALISED EQUATIONS

A generalised distillation column has a total condenser and several sectors, which are adiabatic zones without lateral feed or products streams. Two consecutive sectors  $k$  and  $k+1$ , are separated by a generalised feed ( $GF_k$ ) and we consider that only a single  $GF_k$  can be added or removed between two consecutive steps. The  $GF_k$  can be either a mass stream or an enthalpy stream. In the first case, we consider  $M_{GF_k}$  the mass flow rate,  $z_{GF_k}$  the molar fraction, and  $H_{GF_k}$  the specific enthalpy of such stream. In the case where there is not a net mass flow in or out the column, we consider that  $E_{GF_k}$  is the heat flow added or removed to the column by an intermediate heat exchanger.

Considering all the possibilities, as a summary the generalized in or out feed considered and their character-

istics are shown in Table 1. In the Ponchon-Savarit method variation of  $\lambda_{GFk}$  with composition is considered:  $\lambda_{GFk}=H_{Fk}-h_{Fk}$  where  $H=f(y)$  and  $h=f(x)$ .

On the other hand,  $V_{k+1,i+1}$  is the vapour flow of composition  $y_{k+1,i+1}$  that leaves tray  $i+1$  of section  $k+1$ , in equilibrium with  $L_{k+1,i+1}$ .  $L_{k+1,i}$  is the liquid flow of composition  $x_{k+1,i}$  that leaves tray  $i$  of section  $k+1$ ,  $D$  is the distillate flow of composition  $x_D$ ,  $R$  is the residue flow of composition  $x_R$ , and  $L_D$  is the liquid reflux to the column. A sector  $k$  has  $NTk$  trays and the streams have the subscript corresponding to the tray they leave. In each sector we use the subscript 0 for the stream entering its first and last trays.

**A. Operative line (OL) and Difference Point (DP) in a sector of a generalized column**

As in the McCabe-Thiele method, the equations corresponding to the OL of a general rectification column are deduced in order to establish a relationship between the composition of a vapour stream ascending from a stage and that of the liquid descending from the upper stage. A total mass balance between tray  $i$  of section  $k+1$  and the condenser is done:

$$V_{k+1,i+1} - L_{k+1,i} = D - \sum_{s=1}^k M_{GFs} = \Delta_{k+1} \tag{1}$$

where  $\Delta_{k+1}$  represents the difference between the vapour and liquid mass flows crossing between two consecutive trays in sector  $k+1$  being a fictitious net flow, so called *difference point* (DP) in the Ponchon-Savarit method, that is used for solving the calculation of the number of stages. For each adiabatic sector of a column these net mass and enthalpy flows are constant.

A mass balance over the more volatile component results in:

$$V_{k+1,i+1} \cdot y_{k+1,i+1} - L_{k+1,i} \cdot x_{k+1,i} = D \cdot x_D - \sum_{s=1}^k M_{GFs} \cdot z_{GFs} = \Delta_{k+1} z_{k+1} \tag{2}$$

The composition  $z_{k+1}$  of the fictitious stream  $\Delta_{k+1}$  is given by the following expression, which allows obtaining the abscise of  $\Delta_{k+1}$  in the enthalpy-composition diagram:

$$z_{k+1} = \left( D \cdot x_D - \sum_{s=1}^k M_{GFs} \cdot z_{GFs} \right) / \Delta_{k+1} \tag{3}$$

An enthalpy balance provides:

$$V_{k+1,i+1} \cdot H_{k+1,i+1} - L_{k+1,i} \cdot h_{k+1,i} = D \cdot h_D + Q_D - \sum_{s=1}^k M_{GFs} \cdot H_{GFs} - \sum_{s=1}^k E_{GFs} = \Delta_{k+1} M_{k+1} \tag{4}$$

where  $Q_D$  is the heat removed in the condenser. The net enthalpy flow in sector  $k+1$  is represented by  $\Delta_{k+1} M_{k+1}$ .

Obviously,  $M_{k+1}$  (ordinate of  $\Delta_{k+1}$  in the enthalpy-composition diagram) is given by:

$$M_{k+1} = \left( D \cdot h_D + Q_D - \sum_{s=1}^k M_{GFs} \cdot H_{GFs} - \sum_{s=1}^k E_{GFs} \right) / \Delta_{k+1} \tag{5}$$

This DP ( $z_{k+1}$ ,  $M_{k+1}$ ) is the common point of all the operative lines of sector  $k+1$ . Using Eq. (1) and (2):

$$L_{k+1,i} / V_{k+1,i+1} = (y_{k+1,i+1} - z_{k+1}) / (x_{k+1,i} - z_{k+1}) \tag{6}$$

And with Eq. (1) and (4):

$$L_{k+1,i} / V_{k+1,i+1} = (H_{k+1,i+1} - M_{k+1}) / (h_{k+1,i} - M_{k+1}) \tag{7}$$

Equation (7) corresponds to the *operative line* (OL) of sector  $k+1$  of a rectification column that allows the calculation of the subsequent stages belonging to that sector. In the specific enthalpy-composition diagram they are straight lines that pass through the points  $(x_{k+1,i}, h_{k+1,i})$  representing saturated liquid streams that leave stage  $i$  of sector  $k+1$ ,  $(y_{k+1,i+1}, H_{k+1,i+1})$  representing saturated vapour streams that leave next stage  $i+1$  of sector  $k+1$ ; and  $(z_{k+1}, M_{k+1})$  representing the fictitious stream  $\Delta_{k+1}$ , net mass flow of sector  $k+1$  of the column.

The above definitions have been deduced generally and they can be applied to any stage in any sector of the column. Location of  $\Delta_{k+1}$  in the enthalpy-concentration plot is defined by  $M_{k+1}$  and  $z_{k+1}$  (Eq. 3 and 5). In sector  $k+1$ , the calculation algorithm for the stages alternates the use of the OL with the equilibrium calculations. Nevertheless, consecutive sectors must be connected in the column and it must be specified where to use each one of the corresponding DP and how to calculate the vapour stream leaving the first stage of each new sector when that of the liquid leaving the last stage of the upper sector is known, in order to obtain the minimum number of stages for the specified separation.

**III. DIFFERENCE POINT (DP) OF A GENERALISED FEED  $GF_k$  FOR CONNECTING TWO CONSECUTIVE SECTORS IN A COLUMN**

The analysis of the change of sector in most references of the Ponchon-Savarit method is limited to the location of the point that we have named as FP (Fig. 1). Nevertheless, such analysis though approximate is unrealistic and does not allow clear understanding of the different operations and processes that may be involved whenever a generalised feed is introduced into a distillation column. The analysis we will present could be viewed as unnecessarily complex for applications and the results expected from the Ponchon-Savarit method. We believe however, that it has a fundamental didactical value and definitely helps the understanding of the processes that may be involved in any given generalized

Table 1. Compilation of the different cases presented for a generalized feed stream ( $GF_k$ ) and its characteristics.

Name and flow rates	Thermal condition	Molar fraction	$V_{GFk}$ [molar fraction]	$L_{GFk}$ [molar fraction]
Feed $M_{GFk} > 0$ $M_{GFk} \cdot H_{GFk}$	$q_{GFk} > 1$	$z_{GFk}$		
	$q_{GFk} = 1$	$x_{GFk}$	$(1 - q_{GFk}) \cdot M_{GFk}$	$q_{GFk} \cdot M_{GFk}$
	$0 < q_{GFk} < 1$	$z_{GFk}$	$[V_{GFk}]_{\text{equilibrium}}$	$[X_{GFk}]_{\text{equilibrium}}$
	$q_{GFk} = 0$	$y_{GFk}$		
	$q_{GFk} < 0$	$z_{GFk}$		
Product $M_{GFk} < 0$ $M_{GFk} \cdot H_{GFk}$	$q_{GFk} = 0$	$y_{GFk}$	$(1 - q_{GFk}) \cdot M_{GFk}$	0
			$[V_{GFk}]_{\text{equilibrium}}$	
Product $M_{GFk} < 0$ $M_{GFk} \cdot h_{GFk}$	$q_{GFk} = 1$	$x_{GFk}$	0	$q_{GFk} \cdot M_{GFk}$
				$[X_{GFk}]_{\text{equilibrium}}$
Heat add. $M_{GFk} = 0$ $E_{GFk} > 0$	--	$x_{GFk}$	$E_{GFk} / \lambda_{GFk} > 0$	$-E_{GFk} / \lambda_{GFk} < 0$
			$[Y_{GFk} = x_{GFk}]$	$[X_{GFk}]$
Heat rem. $M_{GFk} = 0$ $E_{GFk} < 0$	--	$y_{GFk}$	$E_{GFk} / \lambda_{GFk} < 0$	$-E_{GFk} / \lambda_{GFk} > 0$
			$[Y_{GFk}]$	$[X_{GFk} = y_{GFk}]$

feed operation, and the awareness of the type of approximation normally carried out. In other way, the method could be misunderstood or difficult to be explained.

The location of such an FP point is not sufficient as a reference for all the possible feed conditions and operations. Instead, the value must be calculated of the difference between the vapour stream ascending from the first stage of the sector below the feed and the liquid stream descending from the last stage of the sector above the feed. The zone connecting consecutive sectors (that we will name ZCCS hereinafter) must be considered. Liquid and vapour portions  $L_{GFk}$  and  $V_{GFk}$  generated by the  $GF_k$  introduction are equilibrium streams for mass feed (as shown, for example, in Fig. 1a where  $GF_k=MF_k$ ) or products, or streams with the same compositions for heat additions or extractions.

In the enthalpy-composition diagram (e.g. Fig. 1a), two sectors  $k$  and  $k+1$  (with their corresponding  $\Delta_k$  and  $\Delta_{k+1}$  DPs) are connected by the ZCCS which domain is defined by the ordinate and abscise ( $z_k^c$ ,  $M_k^c$ ) of its characteristic DP ( $\Delta_k^c$ ) and two compositions over the saturated liquid curve  $IP_k$  and  $IP_{k+1}$  that are determined by the side stream (i.e. the  $GF_k$ ) that separates both consecutive sectors.

The ZCCS, for any side stream, has a DP defined as:

$$\Delta_k^c = V_{k+1,1} - L_{k,Ntk} = V_{k,0} - V_{GFk} - L_{k,Ntk} = \Delta_k - V_{GFk} \quad (8)$$

$$\Delta_k^c = V_{k+1,1} - L_{k,Ntk} = V_{k+1,1} - L_{k+1,1} + L_{GFk} = L_{GFk} + \Delta_{k+1} \quad (9)$$

where  $L_{k,Ntk}$  corresponds the liquid flow that leaves tray Ntk (the last one) of section  $k$ .

According to Eq. 8 and 9,  $\Delta_k^c$  is the graphical intersection point of two straight lines:  $\frac{\Delta_k V_{GFk}}{\Delta_k}$  and  $\frac{\Delta_{k+1} L_{GFk}}{\Delta_{k+1}}$ .

The abscise and ordinate of  $\Delta_k^c$  are given by the corresponding enthalpy and mass balances:

$$M_k^c = (\Delta_k M_k - V_{GFk} H_{GFk}) / \Delta_k^c = (L_{GFk} H_{GFk} + \Delta_{k+1} M_{k+1}) / \Delta_k^c \quad (10)$$

$$z_k^c = (\Delta_k z_k - V_{GFk} y_{GFk}) / \Delta_k^c = (L_{GFk} x_{GFk} + \Delta_{k+1} z_{k+1}) / \Delta_k^c \quad (11)$$

The  $\Delta_k^c$  must be used only once in the step-by-step calculation, with the meaning that the feed has entered the column in just one stage, and the new liquid and vapour streams must be correspondingly updated, as in the case of the McCabe-Thiele construction, to obtain the minimum number of stages. The intersections of the two lines  $\frac{\Delta_k V_{GFk}}{\Delta_k}$  and  $\frac{\Delta_{k+1} L_{GFk}}{\Delta_{k+1}}$ , which have been used to define  $\Delta_k^c$  and to limit the ZCCS, with the saturated liquid curve also determine the characteristic points  $IP_k$  and  $IP_{k+1}$  of this ZCCS (Fig. 1).

For a *mass feed stream*  $V_{GFk}$  is the vapour generated from the generalized feed ( $GF_k$ ) in equilibrium with  $L_{GFk}$ . In the case of a *heat side stream*, the difference is that the vapour and liquid ( $V_{GFk}$ ,  $L_{GFk}$ ) have the same composition  $x_{GFk} = y_{GFk}$  and are not in equilibrium.

The intersection of the OL  $\frac{\Delta_k V_{GFk}}{\Delta_k}$  with the saturated liquid curve provides  $IP_k$ :

$$(y_{GFk} - z_{GFk}) / (x_{IPk} - z_{GFk}) = (H_{GFk} - M_k) / (h_{IPk} - M_k)$$

equivalent to :

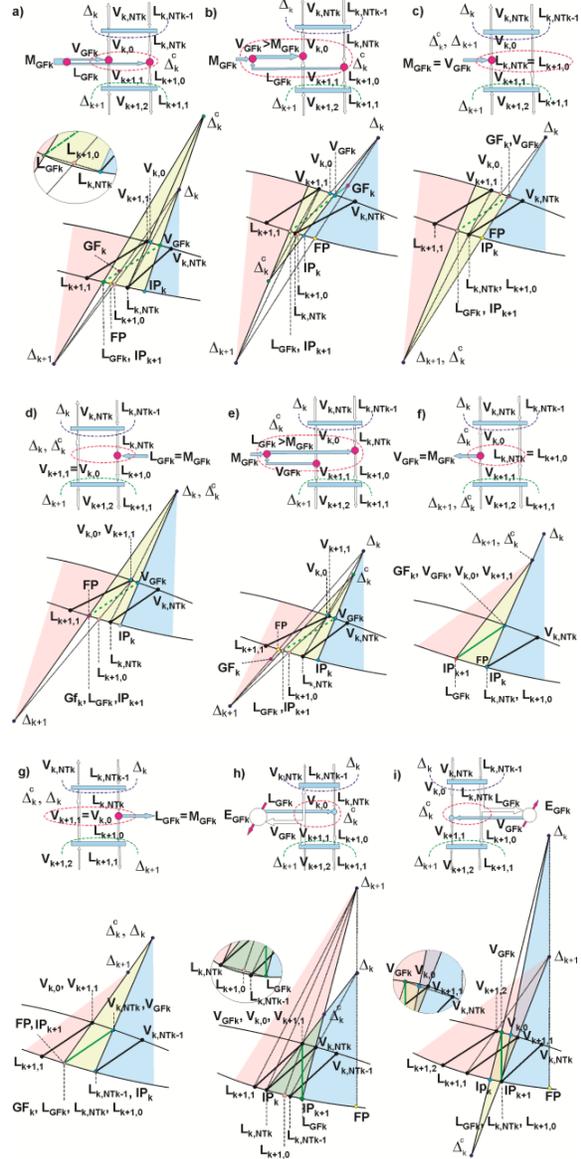


Figure 1. Ponchon-Savarit y-x diagrams for a  $GF_k$ : a)  $MF_k > 0$ ,  $0 < q_{GFk} < 1$ ; b)  $MF_k > 0$ ,  $q_{GFk} < 0$ ; c)  $MF_k > 0$ ,  $q_{GFk} = 0$ ; d)  $MF_k > 0$ ,  $q_{GFk} = 1$ ; e)  $MF_k > 0$ ,  $q_{GFk} > 1$ ; f)  $MF_k < 0$ ,  $q_{GFk} = 0$ ; g)  $MF_k < 0$ ,  $q_{GFk} = 1$ ; h)  $EF_k < 0$ ; i)  $EF_k > 0$ .

$$(y_{GFk} - z_{GFk}^c) / (x_{IPk} - z_{GFk}^c) = (H_{GFk} - M_k^c) / (h_{IPk} - M_k^c) \quad (12)$$

and the intersection of  $\frac{\Delta_k L_{GFk}}{\Delta_{k+1}}$  with the saturated liquid curve gives  $IP_{k+1}$ :

$$(z_{GFk}^c - z_{k+1}) / (x_{IP_{k+1}} - z_{k+1}) = (M_k^c - M_{k+1}) / (h_{IP_{k+1}} - M_{k+1}) \quad (13)$$

In the following sections we will analyse in detail the different possibilities of mass feeds, considering their thermal conditions or their liquid fractions, the removal of a liquid or vapour product as well as the heat addition to a liquid stream or heat removal from a vapour stream in a rectifying sector, i.e.: a sector above the main mass feed.  $\Delta_k^c$  will be represented for all cases of generalized feeds and the optimum position for the side stream, given by  $IP_k$  or  $IP_{k+1}$ , will be analysed.

#### IV. SYSTEMATIC ANALYSIS OF THE CHANGES OF SECTOR

A systematic analysis of the different possible situations, according to the previous generalised equations, is presented in Fig. 1. The first column shows the scheme of the ZCCS together with all the existing intermediate streams. For the case of generalized mass feed streams ( $GF_k$ ), the compositions of the streams developed in the rectification column do not generally match with any of the vapour ( $V_{GF_k}$ ) or liquid ( $L_{GF_k}$ ) portions generated from  $GF_k$  (particular cases when there exists any coincidence can be consulted in Reyes-Labarta *et al.*, 2014b). For other types of input streams, different from mass feeds, it must be considered that the stream to be removed (when the  $GF_k$  is a mass product) or to be heated or cooled (for the case of heat addition or removal) must actually exist in the column. Consequently, when solving a particular case of a design calculation, the specifications of the column must be adapted to the already existing streams.

The analysis of all cases presented in this section not only shows the relationships occurring among the streams involved at the ZCCS, but also between them and the rest of streams at the previous or subsequent stages.

##### A. Mass feed

This is the case most frequently considered in the literature. In order to connect two consecutive sectors  $\Delta_{k+1}$  and  $\Delta_k$  separated by a feed,  $\Delta_k^C$  is obtained. The abscise and ordinate of  $\Delta_k^C$  are given by the corresponding enthalpy and mass balances (Eq. 10 and 11).

First column of Fig. 1a-e shows all streams developed in the change of sector in the column for mass feed streams at different thermal conditions. According to Fig. 1a-e, the tie-line  $\frac{y_{GF_k}x_{GF_k}}{y_{GF_k}x_{GF_k}}$  represents the equilibrium separation taking place at the feed stream:  $x_{GF_k}$  is the composition of  $L_{GF_k}$  which is the liquid in equilibrium with a vapour  $V_{GF_k}$  of composition  $y_{GF_k}$ . The intersections of  $\frac{\Delta_k^C y_{GF_k}}{\Delta_k^C L_{GF_k}}$  and  $\frac{\Delta_k^C L_{GF_k}}{\Delta_k^C y_{GF_k}}$  with the bubble-point curve  $h=f(x)$  provide  $IP_k$  and  $IP_{k+1}$ , respectively, that are the characteristic points of the  $\Delta_k^C$ . It can be observed that the  $\Delta_k^C$  covers the amplitude of a tie-line in the equilibrium diagram. This  $\Delta_k^C$  must be used only once in the step-by-step calculation, with the meaning that the feed has entered the column and the new liquid and vapour streams must be correspondingly updated.

Depending on the feed thermal condition or the liquid fraction of the feed ( $q_{GF_k}$ ), five subcases may be found.

##### Liquid and vapour mixture in equilibrium ( $0 < q_{GF_k} < 1$ )

We will start with the case of a feed, that when entering the column generates a mixture of a liquid  $L_{GF_k}$  and vapour  $V_{GF_k}$  in equilibrium, where the liquid fraction generated ( $q_{GF_k}$ ) is comprised between 0 and 1. Figure 1a represents the situation of such a feed. They show the DPs of sectors  $k$  and  $k+1$  (i.e.:  $\Delta_k$  and  $\Delta_{k+1}$ ) and the  $\Delta_k^C$ . The operating line for the change of sector is used to locate of streams coherently: the vapour fraction  $V_{GF_k}$

joins the vapour coming from the stage below ( $V_{k,0} = V_{k+1,1} + V_{GF_k}$ ), which implies that  $V_{k,0}$  (of composition  $y_{k,0}$ ) is aligned between  $V_{k+1,1}$  (of composition  $y_{k+1,1}$ ) and  $V_{GF_k}$  (of composition  $y_{GF_k}$ ); whereas the feed liquid fraction  $L_{GF_k}$  joins the liquid coming from the plate immediately above ( $L_{k+1,0} = L_{k,NTk} + L_{GF_k}$ ) which implies that  $L_{k+1,0}$  ( $x_{k+1,0}$ ) is aligned between  $L_{k,NTk}$  ( $x_{k,NTk}$ ) and  $L_{GF_k}$  ( $x_{GF_k}$ ). In this case, neither of the intermediate streams  $V_{k,0}$  nor  $L_{k+1,0}$  (of compositions  $y_{k,0}$  and  $x_{k+1,0}$ ) are equilibrium streams, so they are located on the corresponding straight lines defined outside the  $H/y$  and  $h/x$  curves, as shown in magnification of diagram in Fig. 1a, unless such equilibrium enthalpy lines were straight lines. This analysis of the change of sector is somewhat different from what is presented in some text books that use the point FP as reference.

##### Superheated vapour ( $q_{GF_k} < 0$ ).

In the diagram of Fig. 1b,  $GF_k$  is located in the region over the dew point curve  $H=f(y)$  and consequently, the vapour  $y_{GF_k}$  composition is lower than the composition of the feed ( $z_{GF_k}$ ). Since  $V_{GF_k} > M_{GF_k}$ , and according to Eq. (10),  $\Delta_k^C$  is nearer the equilibrium line than  $\Delta_k$  and  $\Delta_{k+1}$ , which is unfavourable for the separation. Note that FP is now out of the interval defined by  $IP_k$  and  $IP_{k+1}$ . In this case,  $y_{k+1,1} < y_{GF_k} < y_{k,NTk}$ . The situation of the intermediate streams is represented analogously to the previous case of the mixture of liquid and vapour in equilibrium. Nevertheless, as stated before, for the superheated vapour feed it can be observed that  $V_{k,0}$  ( $y_{k,0}$ ) is aligned between  $V_{k+1,1}$  ( $y_{k+1,1}$ ) and  $V_{GF_k}$  ( $y_{GF_k}$ ) but  $L_{k+1,0}$  ( $x_{k+1,0}$ ) is aligned but not between  $L_{k,NTk}$  ( $x_{k,NTk}$ ) and  $L_{GF_k}$  ( $x_{GF_k}$ ) since  $L_{GF_k}$  is a negative stream.

##### Saturated vapour ( $q_{GF_k} = 0$ )

In this case  $M_{GF_k} = V_{GF_k}$  and the vapour composition of the feed  $z_{GF_k}$  is the same as  $y_{GF_k}$  (Fig. 1c). The diagrams of Fig. 1c shows that  $\Delta_k^C$  coincides with  $\Delta_{k+1}$ , according to Eq. (9), because  $L_{GF_k} = 0$ . This is the only thermal condition for the feed where  $IP_k$  coincides with the FP point so any of both approaches gives the same results. The liquid falling from the last tray of sector  $k$  has the same composition as the liquid arriving to the first tray of the next sector ( $x_{k,NTk} = x_{k+1,0}$ ). The location of the intermediate streams is represented analogously to the previous case. It can be observed that the relationships among the intermediate streams are fulfilled.

##### Saturated liquid ( $q_{GF_k} = 1$ )

In Fig. 1d  $M_{GF_k} = L_{GF_k}$  is a saturated liquid located in the  $h=f(y)$  curve, the  $\Delta_k^C$  matches up with  $\Delta_k$  because  $V_{GF_k} = 0$ . In this case the FP point does not coincide with  $IP_k$  but with  $IP_{k+1}$ . The composition of vapour streams  $V_{k,0}$  and  $V_{k+1,1}$  are the same:  $y_{k,0} = y_{k+1,1}$ .

##### Undercooled liquid ( $q_{GF_k} > 1$ )

In the diagram of Fig. 1e  $M_{GF_k}$  is an undercooled liquid, thus located under the  $h=f(y)$  curve. Since  $L_{GF_k} > M_{GF_k}$  and according to Eq. (10),  $\Delta_k^C$  is nearer the  $H/y$  and  $h/x$  curves than  $\Delta_k$  and  $\Delta_{k+1}$ , which is unfavourable for the separation at the specific stage of the change of sector. As expected,  $L_{k+1,0}$  ( $x_{k+1,0}$ ) is aligned between  $L_{k,NTk}$

( $x_{k,NTk}$ ) and  $L_{GFk}$  ( $x_{GFk}$ ) but the composition of  $V_{k,0}$  ( $y_{k,0}$ ) is lower than both of  $V_{k+1,1}$  ( $y_{k+1,1}$ ) and of  $V_{GFk}$  ( $y_{GFk}$ ).

### B. Mass product

Changes of sectors due to saturated vapour and saturated liquid products are analysed in this section (Fig. 1f,g). The extraction of a product in the rectifying section provokes  $\Delta_{k+1}$  to be nearer the  $H/y$  and  $h/x$  curves than  $\Delta_k$ , consequently more stages are needed, as compared to the case of a mass feed addition.

For mass feeds, it was considered that the stream to be added could coincide or differ from one of the streams in equilibrium with the feed ( $L_{GFk}$  or  $V_{GFk}$ ). In the case of mass products, the side stream specifications must coincide with one of the streams present in the column.

#### Saturated vapour ( $q_{GFk}=0$ )

In Fig. 1f,  $\Delta_k^C$  matches up with  $\Delta_{k+1}$  because  $q_{GFk} = 0$  and  $L_{GFk}=0$ . As in the case of a saturated vapour stream feed,  $IP_k$  coincides with the FP point.

Figure 1f shows that when a vapour product  $V_{GFk}$  is extracted from  $V_{k+1,1}$  of a column ( $y_{GFk} = y_{k+1,1}$ ), the liquid stream compositions of  $L_{k,NTk}$  ( $x_{k,NTk}$ ) and  $L_{k+1,0}$  ( $x_{k+1,0}$ ) are the same. Any other situation is nonsense since a non-existing stream can never be extracted.

#### Saturated liquid ( $q_{GFk}=1$ )

In Fig. 1g,  $\Delta_k^C$  coincides with  $\Delta_k$  because  $q_{GFk}=1$  and  $V_{GFk}=0$ . As in the case of a feed, for a saturated liquid feed stream  $IP_{k+1}$  and the FP point are coincident. Figure 1g shows that the relationships among compositions of intermediate streams are analogously fulfilled:  $x_{GFk} = x_{k,NTk}$  and  $y_{k,0} = y_{k+1,1}$  for the only possible case of the extraction of a liquid stream in the ZCCS where the vapour stream compositions remain invariable.

### C. Heat removal or addition in an enrichment sector

When a heat flow is removed from a vapour  $V_{GFk}$  of composition  $y_{GFk}$  causing the corresponding condensation, the flow entering the stage above decreases by  $V_{GFk} = EF_k/\lambda$ , and consequently the liquid flow entering the stage below increases by  $L_{GFk} = -EF_k/\lambda$  ( $EF_k < 0$  according to Table 1), both streams  $L_{GFk}$  and  $V_{GFk}$  having the same composition  $x_{GFk} = y_{GFk}$ . When heat is added to the liquid  $L_{GFk}$  of composition  $x_{GFk}$  causes a vaporization that implies a liquid flow decrease and the consequent vapour flow increase ( $EF_k > 0$  according to Table 1).

#### Heat removal from a vapour stream in an enrichment sector

Because of the liquid flow increase,  $\Delta_k^C$  is nearer the  $H/y$  and  $h/x$  curves than  $\Delta_k$  and  $\Delta_{k+1}$  is more separated than  $\Delta_k$ , as shown in Fig. 1h. The effect of the heat removal in the enrichment section of a column favours the separation. The coherent construction shown in the diagram allows the fulfilment of the relationship among the streams flowing at the ZCCS:  $L_{k+1,0}$  ( $x_{k+1,0}$ ) is aligned between  $L_{k,NTk}$  ( $x_{k,NTk}$ ) and  $L_{GFk}$  ( $x_{GFk}$ ), as shown in the corresponding magnification, and the composition of all vapour streams ( $V_{GFk}$ ,  $V_{k,0}$ ,  $V_{k+1,1}$ ) coincide:  $y_{GFk} = y_{k,0} = y_{k+1,1}$ .

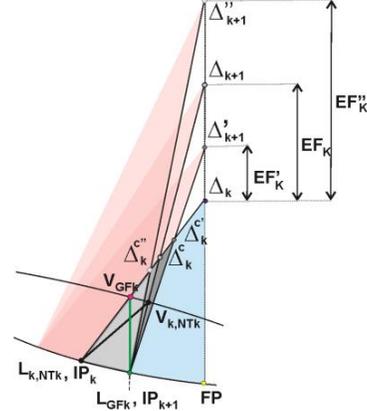


Figure 2. Effect of different  $E_{GFk}$ .

#### Heat addition to a liquid stream in an enrichment sector

All the features related to this case are shown in Fig. 1i. The global effect of the heat addition in the rectifying section of a column is unfavourable since the mixture of vapours at the ZCCS yields a vapour  $V_{k,0}$  richer in the less volatile component (the composition of  $V_{k+1,1}$  ( $y_{k+1,1}$ ) is greater than the composition of  $V_{k,0}$  ( $y_{k,0}$ )). The coherent construction shown in the diagram allows the fulfilment of the relationship among the streams:  $V_{k,0}$  ( $y_{k,0}$ ) is aligned between  $V_{k+1,1}$  ( $y_{k+1,1}$ ) and  $V_{GFk}$  ( $y_{GFk}$ ), as shown in the corresponding magnification, and the composition of all liquid streams ( $L_{GFk}$ ,  $L_{k+1,0}$ ,  $L_{k,NTk}$ ) coincide.

### D. $\Delta_k^C$ zones and use of $IP_k$ and $IP_{k+1}$ as references for the change of sector.

Heat removals or additions are different to mass streams where the liquid and vapour originated from the feed are in equilibrium, which defines a  $\Delta_k^C$  domain that always comprises a tie-line (Fig. 1a-g). However, in case of heat streams, the  $\Delta_k^C$  domain can be greater or smaller than the zone corresponding to a tie-line, depending on the  $E_{GFk}$  value. Figure 2 shows an example where, for the case of a heat extraction and a given tie-line separation, different  $E_{GFk}$  are considered and thus, different sizes of the  $\Delta_k^C$  zone are obtained: smaller as the extracted heat increases ( $E_{GFk}' > E_{GFk} > E_{GFk}$ ).

Note that for any side stream, sector domains are continuous over the saturated liquid curve. For mass extractions or additions sector zones are also continuous over the saturated vapour curve. In these cases  $IP_{k+1}$  is located at a smaller composition than  $IP_k$ . For heat streams, however, it can be noted that  $z_{k+1} = z_k$  according to Eqs. 4, 6, 9-11 and  $IP_{k+1}$  is located to the right of  $IP_k$  so both  $\Delta_k$  and  $\Delta_{k+1}$  zones are overlapped on the saturated vapour curve (Fig. 1h,i).

Note the difference between mass feed, on one hand, and mass product and heat streams, on the other, regarding the use of  $\Delta_k^C$  in calculations.  $\Delta_k^C$  is used in all cases once in the step by step construction but, contrary to a feed mass streams, where it can be used at any intermediate point of its domain, for mass extraction or for heat extractions or additions only the OLs of  $\Delta_k^C$  that coincide with one of the corresponding limits of the domain belonging to sector  $\Delta_k$  or  $\Delta_{k+1}$  can be used.

Though in this work the graphical method has been analysed, the generalized equations have been implemented analytically with Excel and *Visual Basic for Application* macros in a tool that allows the design of a binary column with ten possible generalized feed streams, which has been used to study different examples. From that study we can state that the result obtained (i.e. number of stages) by using the Ponchon-Savarit method with this *strict* analysis may be nearly the same, though depending on the equilibrium of the particular system analysed and other variables. However, it provides a clear analysis of what is happening at the column, thus facilitating the comprehension of the method.

### V. CONCLUSIONS

In this work a closer to reality analysis of what may happen when changing sector in the column, is carried out. We present generalised equations for the corresponding OL and DP for the different sectors in a column as well as those for the ZCCS due to feeds, products or heat removals or additions. All the streams involved in the ZCCS, as well as the OL, are located in the Ponchon-Savarit diagrams.

The presented procedure has a fundamental didactical value since it provides a clear analysis of what is happening at the column, and allows being aware of the type of approximation normally carried out, thus facilitating the comprehension of the method, that otherwise could be misunderstood or difficult to be explained.

### COMPLEMENTARY INFORMATION

A website of self-learning about the Ponchon-Savarit method for the design of distillation columns can be consulted at <http://iq.ua.es/Ponchon/>, as well as a resume of the Extension of the Ponchon and Savarit Method for Designing Ternary Rectification Columns (<http://hdl.handle.net/10045/14600>, [../14601](http://hdl.handle.net/10045/14601), [../10023](http://hdl.handle.net/10045/10023)).

### REFERENCES

Ho, T.J., C.T. Huang, L.S. Lee and C.T. Chen, "Extended Ponchon-Savarit method for graphically analyzing and designing internally heat-integrated distillation columns," *Ind. Eng. Chem. Res.*, **49**, 350-358 (2010).

Ledanois, J.M. and C. Olivera-Fuentes, "Modified Ponchon-Savarit and McCabe-Thiele methods for distillation of Two-Phase Feeds," *Ind. Eng. Chem. Process Des. Dev.*, **23**, 1-6 (1984).

Lee, J.W., S. Hauan, K.M. Lien and A.W. Westerberg, "A graphical method for designing reactive distillation columns: I. the Ponchon-Savarit method," *Proc. of the Royal Society A: Mathematical, Physical & Eng. Sciences*, **456**, 1953-1964 (2000).

Marcilla, A., Gómez, A. and Reyes, J., "New method for designing distillation columns of multicomponent mixtures", *Lat. Am. Appl. Res.* **27**, 51-60 (1997).

Marcilla, A., A. Gómez, J.A. Reyes and M.M. Olaya, "New method for quaternary systems liquid-liquid extraction tray to tray design," *Ind. Eng. Chem. Res.*, **38**, 3083-3095 (1999).

Reyes, J.A., A. Gómez and A. Marcilla, "Graphical concepts to orient the Minimum Reflux Ratio Calculation on Ternary Mixtures Distillation," *Ind. Eng. Chem. Res.*, **39**, 3912-3919 (2000).

Reyes-Labarta, J.A., J.A. Caballero and A. Marcilla, "A novel hybrid simulation-optimization approach for the optimal design of multicomponent distillation columns," *Computer Aided Chemical Engineering*, **30**, 1257-1261 (2012).

Reyes-Labarta, J.A., M.D. Serrano and A. Marcilla, "Analysis of the Connecting Zone between Consecutive Sections in Distillation Columns Covering Multiple Feeds, Products and Heat Transfer Stages," *Lat. Am. Appl. Res.*, **44**, 307-312 (2014a). (Sup.material: <http://hdl.handle.net/10045/53405>; and [../10045/23195](http://hdl.handle.net/10045/23195)).

Reyes-Labarta, J.A., M.D. Serrano and A. Marcilla, *Graphical Representation of the Changes of Sector for Particular Cases in the Ponchon Savarit Method*, Open Acad. Repository of the University of Alicante, <http://hdl.handle.net/10045/42101> (2014b).

Wakabayashi, T. and S. Hasebe, "Design of heat integrated distillation column by using H-xy and T-xy diagrams," *Comp. and Chem. Eng.*, **56**, 174-183 (2013).

Wankat, P.C., *Separation Process Engineering. Includes Mass Transfer Analysis*, (3<sup>rd</sup> ed.), Prentice Hall, Massachusetts (2012).

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